EXERCISES FOR INF3320 AND INF4320

BEZIER CURVES AND SURFACES

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1. Let $\mathbf{p}_0 = (-1, 1)$, $\mathbf{p}_1 = (1, 1)$, $\mathbf{p}_2 = (1, 0)$ be the control points of a quadratic Bezier curve $\mathbf{p}$.

   (a) Evaluate $\mathbf{p}$ at $t = \frac{1}{4}$ using the de Casteljau algorithm.

   (b) Evaluate $\mathbf{p}$ at $t = \frac{1}{4}$ using recursion on the basis functions.

   Solution: *(a)*

   - *Level 0:* $\mathbf{p}_i^0 = \mathbf{p}_i$, $i = 0, 1, 2$.
   - *Level 1:* $\mathbf{p}_0^1 = (3/4)\mathbf{p}_0^0 + (1/4)\mathbf{p}_1^0 = (-1/2, 1)$, $\mathbf{p}_1^1 = (3/4)\mathbf{p}_1^0 + (1/4)\mathbf{p}_2^0 = (1, 3/4)$.
   - *Level 2:* $\mathbf{p}(1/4) = \mathbf{p}_0^2 = (3/4)\mathbf{p}_0^1 + (1/4)\mathbf{p}_1^1 = (-1/8, 15/16)$.

   (b)

   - *Level 0:* $B_{0,0} = 1$.
   - *Level 1:* $B_{0,1} = (3/4)B_{0,0} = 3/4$, $B_{1,1} = (1/4)B_{0,0} = 1/4$.
   - *Level 2:* $B_{0,2} = (3/4)B_{0,1} = 9/16$, $B_{1,2} = (1/4)B_{0,1} + (3/4)B_{1,1} = 3/8$, $B_{2,2} = (1/4)B_{1,1} = 1/16$.

   $$\mathbf{p}(1/4) = (9/16)(-1, 1) + (3/8)(1, 1) + (1/16)(1, 0) = (-1/8, 15/16).$$

2. Express a quadratic Bezier curve $\mathbf{p}(t) = \sum_{i=0}^{2} \mathbf{p}_i B_{0,2}(t)$ in monomial form, i.e., in the form $\mathbf{p}(t) = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{a}_2 t^2$.

   Solution:

   $$\begin{aligned} \mathbf{p}(t) &= \mathbf{p}_0 B_{0,2}(t) + \mathbf{p}_1 B_{1,2}(t) + \mathbf{p}_2 B_{2,2}(t) \\ &= \mathbf{p}_0 (1-t)^2 + \mathbf{p}_1 2t(1-t) + \mathbf{p}_2 t^2 \\ &= \mathbf{p}_0 (1 - 2t + t^2) + \mathbf{p}_1 (2t - 2t^2) + \mathbf{p}_2 t^2 \\ &= \mathbf{p}_0 + 2(\mathbf{p}_1 - \mathbf{p}_0)t + (\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}_2)t^2 \end{aligned}$$

3. Express a quadratic polynomial $\mathbf{p}(t) = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{a}_2 t^2$ in Bezier form, i.e., in the form $\mathbf{p}(t) = \sum_{i=0}^{2} \mathbf{p}_i B_{0,2}(t)$.

   Solution:

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{a}_2 t^2 \\ &= \mathbf{a}_0(B_{0,2}(t) + B_{1,2}(t) + B_{2,2}(t)) + \mathbf{a}_1(B_{1,2}(t)/2 + B_{2,2}(t)) + \mathbf{a}_2 B_{2,2}(t) \\ &= \mathbf{a}_0 B_{0,2}(t) + (\mathbf{a}_0 + \mathbf{a}_1/2)B_{1,2}(t) + (\mathbf{a}_0 + \mathbf{a}_1 + \mathbf{a}_2)B_{2,2}(t).\end{aligned}$$

4. Show that the Bernstein polynomial $B_{i,d}$ attains its unique maximum at $t = i/d$.

   Solution: *Since*

$$B'_{i,d}(t) = \binom{d}{i}(it^{i-1}(1-t)^{d-i} - t^i(d-i)(1-t)^{d-i-1}) = \binom{d}{i} t^{i-1}(1-t)^{d-i-1}(i - td),$$

   $B_{i,d}$ *is monotonically increasing in* $(0, i/d)$ *and monotonically decreasing in* $(i/d, 1)$.

5. Start from `ex7-6_bezier.cpp.template` and implement the function `deCasteljauEval` which applies the de Casteljau algorithm to the Bezier curve defined by `src_points` at the parameter value `t` (the degree of the curve is implicitly given by how many points there are).