EXERCISES FOR INF3320 AND INF4320

BÉZIER CURVES AND SPLINE CURVES

C. DYKEN AND M.S. FLOATER

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1. Show that if $t_0 < t_1 < t_2$, the Vandermonde matrix

$$V = \begin{pmatrix} 1 & t_0 & t_0^2 \\ 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \end{pmatrix}$$

is non-singular by showing that its determinant can be written as

$$(t_1 - t_0)(t_2 - t_0)(t_2 - t_1).$$

2. Recall the cubic Lagrange basis functions

$$L_i(t) = \prod_{\substack{j=0 \\ j \neq i}}^{3} \frac{t - t_j}{t_i - t_j}, \qquad i = 0, 1, 2, 3.$$

What is the value of L_i at the points $t = t_0, t_1, t_2, t_3$?

3. What are the values and first derivatives of the Hermite polynomials H_0, H_1, H_2, H_3, defined in the lecture, at the end points $t = 0$ and $t = 1$?

4. Let $\mathbf{p}$ and $\mathbf{q}$ be two Bezier curves of degree d,

$$\mathbf{p}(t) = \sum_{i=0}^{d} \mathbf{p}_i B_{i,d}(t), \qquad \mathbf{q}(t) = \sum_{i=0}^{d} \mathbf{q}_i \tilde{B}_{i,d}(t),$$

where

$$B_{i,d}(t) = \binom{d}{i} \lambda^i (1 - \lambda)^{d-i}, \qquad \tilde{B}_{i,d}(t) = \binom{d}{i} \tilde{\lambda}^i (1 - \tilde{\lambda})^{d-i},$$

and

$$\lambda = \frac{t - a}{b - a}, \qquad \tilde{\lambda} = \frac{t - b}{c - b},$$

and $a < b < c$ and define a spline curve $\mathbf{r} : [a, c] \to \mathbb{R}^n$ by

$$\mathbf{r}(t) = \begin{cases} \mathbf{p}(t) & a \leq t \leq b, \\ \mathbf{q}(t) & b < t \leq c. \end{cases}$$

What are the conditions on the control points for G^1 and G^2 continuity?