Black-Box, Functional Testing

Introduction

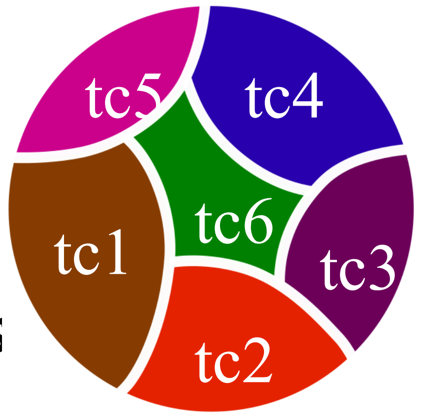
- Based on the definition of what a program's specification, as opposed to its structure
- Does the implementation correctly implement the functionality as per the given system specifications?
- The notion of coverage can also be applied to functional testing
- Rigorous specifications have another benefit, they help functional testing, e.g., categorize inputs, derive expected outputs
- In other words, they help test case generation and test oracles

Outline

- Equivalence Class Partitioning
- Boundary-Value Analysis
- Category-Partition
- Decision tables
- Cause-Effect Graphs
- Logic Functions

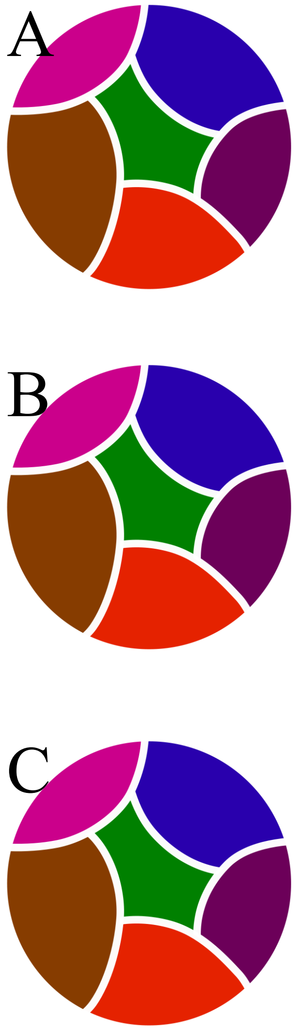
Equivalence Class Testing

- Motivation: we would like to have a sense of complete testing and we would hope to avoid test redundancy
- Equivalence classes: partitions of the input set in which input data have the same effect on the program (e.g., the result in the same output)
- Entire input set is covered: completeness
- Disjoint classes: to avoid redundancy
- Test cases: one element of each equivalence class
- But equivalence classes have to be chosen wisely …
- Guessing the likely system behavior is needed



A SUT's input set

Weak/Strong Equivalence Class Testing



- For an example SUT, suppose there are three input variables from three domains: A, B, C
- $A = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_m$ where $a_i \in A_i$
- $B = B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_n$ where $b_i \in B_i$
- $C = C_1 \cup C_2 \cup C_3 \cup \ldots \cup C_o$ where $c_i \in C_i$
- Weak Equivalence Class Testing: Choosing one variable value from each equivalence class (one a_i, b_i, and c_i) such that all classes are covered. # of test cases?
  - $\max(|A|, |B|, |C|)$
- Strong Equivalence Class Testing: Is based on the Cartesian product of the partition subsets ($A \times B \times C$), i.e., testing all interactions of all equivalence classes.

  # of test cases?
  - $|A| \times |B| \times |C|$
- Examples next…

Example of Weak Equivalence Class Testing (WECT)

- Number of WETCs needed=Max number of equivalence classes among {A, B, C}
- 4 WETCs are enough.

Test Case	A	B	C
WETC1	a1	b1	c1
WETC2	a2	b2	c2
WETC3	a3	b3	c1
WETC4	a1	b4	c2

Example of Strong Equivalence Class Testing (SECT)

- |A| = 3
- |B| = 4
- |C| = 2

- # of test cases=3x4x2=24

Test Case	A	B	C
SETC1	a1	b1	c1
SETC2	a1	b1	c2
SETC3	a1	b2	c1
SETC4	a1	b2	c2
SETC5	a1	b3	c1
SETC6	a1	b3	c2
SETC7	a1	b4	c1
SETC8	a1	b4	c2
SETC9	a2	b1	c1
SETC10	a2	b1	c2
SETC11	a2	b2	c1
SETC12	a2	b2	c2
SETC13	a2	b3	c1
SETC14	a2	b3	c2
SETC15	a2	b4	c1
SETC16	a2	b4	c2
SETC17	a3	b1	c1
SETC18	a3	b1	c2
SETC19	a3	b2	c1
SETC20	a3	b2	c2
SETC21	a3	b3	c1
SETC22	a3	b3	c2
SETC23	a3	b4	c1
SETC24	a3	b4	c2

`NextDate` Example

- `NextDate` is a function with three variables: `month, day, year`. It returns the date of the day after the input date. Limitation: 1812-2012
- Treatment Summary: if it is not the last day of the month, the next date function will simply increment the day value. At the end of a month, the next day is 1 and the month is incremented. At the end of the year, both the day and the month are reset to 1, and the year incremented. Finally, the problem of leap year makes determining the last day of a month interesting.

NextDate Equivalence Classes

- M1 = { month: month has 30 days}
- M2 = { month: month has 31 days}
- M3 = { month: month is February}

- D1 = {day: 1<= day <= 28}
- D2 = {day: day = 29}
- D3 = {day: day = 30}
- D4 = {day: day = 31}

- Y1 = {year: year = 1900}
- Y2 = {year: 1812 <= year <= 2012 AND (year != 1900) AND (year mod 4 = 0)}
- Y3 = {year: (1812 <= year <= 2012 AND year mod 4 != 0)}

NextDate
Weak Equivalence Class Testing (WECT) - Test Cases

- #WECT test cases=maximum partition size (D)=4

Test Case ID	Month	Day	Year	Output
WETC1	6	14	1900	6/15/1900
WETC2	7	29	1912	7/30/1912
WETC3	2	30	1913	Invalid Input date (not possible)
WETC4	6	31	1900	Invalid Input date

NextDate

Strong Equivalence Class Testing (SECT) - Test Cases

- #SECT test cases= partition size (D) x partition size (M) x partition size (Y) = 3x4x3=36 test cases

Test Case ID	Month	Day	Year	Expected Output
SE1	6	14	1900	6/15/1900
SE2	6	14	1912	6/15/1912
SE3	6	14	1913	6/15/1913
SE4	6	29	1900	6/30/1900
SE5	6	29	1912	6/30/1912
SE6	6	29	1913	6/30/1913
SE7	6	30	1900	7/1/1900
SE8	6	30	1912	7/1/1912
SE9	6	30	1913	7/1/1913
SE10	6	31	1900	ERROR
SE11	6	31	1912	ERROR
SE12	6	31	1913	ERROR
SE13	7	14	1900	7/15/1900
SE14	7	14	1912	7/15/1912
SE15	7	14	1913	7/15/1913
SE16	7	29	1900	7/30/1900
SE17	7	29	1912	7/30/1912
SE18	7	29	1913	7/30/1913
SE19	7	30	1900	7/31/1900
SE20	7	30	1912	7/31/1912
SE21	7	30	1913	7/31/1913
SE22	7	31	1900	8/1/1900
SE23	7	31	1912	8/1/1912
SE24	7	31	1913	8/1/1913
SE25	2	14	1900	2/15/1900
SE26	2	14	1912	2/15/1912
SE27	2	14	1913	2/15/1913
SE28	2	29	1900	ERROR
SE29	2	29	1912	3/1/1912
SE30	2	29	1913	ERROR
SE31	2	30	1900	ERROR
SE32	2	30	1912	ERROR
SE33	2	30	1913	ERROR
SE34	2	31	1900	ERROR
SE35	2	31	1912	ERROR
SE36	2	31	1913	ERROR

Discussion

- If error conditions are a high priority, we should extend strong equivalence class testing to include both valid (E) and invalid inputs (U)
  - E.g., year | year < 1812 or year > 2012
- ECT is appropriate when input data defined in terms of ranges and sets of discrete values
- SECT makes the assumption that the variables are independent – dependencies will generate "error" test cases
- Possibly too many of them …
- See the category-partition and decision table techniques next to address this issue

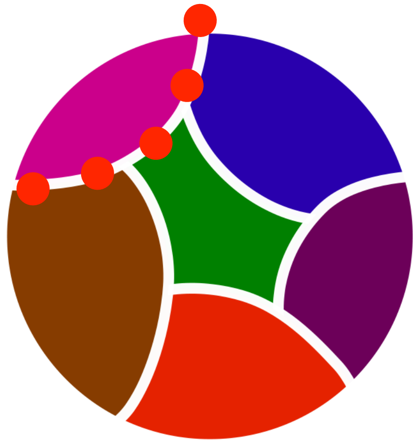
Boundary Value Testing

Motivations

- We have partitioned input domains into suitable classes, on the assumption that the behavior of the program is "similar"
- Some typical programming errors happen to be at the boundary between different classes
- This is what boundary value testing focuses on
- Simpler but complementary to previous techniques

Errors at the boundaries

- Experience indicates that programmers make mistakes in processing values at and near the boundaries of equivalence classes.
- For example, suppose that method M is required to compute a function f1 when $x \leq 0$ is true and function f2 otherwise. However, M has an error due to which it computes f1 for $x < 0$ and f2 otherwise.
- Obviously, this fault is revealed, though not necessarily, when M is tested against $x=0$ but not if the input test set is, for example, {-4, 7} derived using equivalence partitioning. In this example, the value $x=0$, lies at the boundary of the equivalence classes $x \leq 0$ and $x > 0$.



Boundary Value Analysis

- Boundary value analysis is a test selection technique that targets faults in applications at the boundaries of equivalence classes.
- While equivalence partitioning selects tests from within equivalence classes, boundary value analysis focuses on tests at and near the boundaries of equivalence classes.
- Certainly, tests derived using either of the two techniques may overlap.

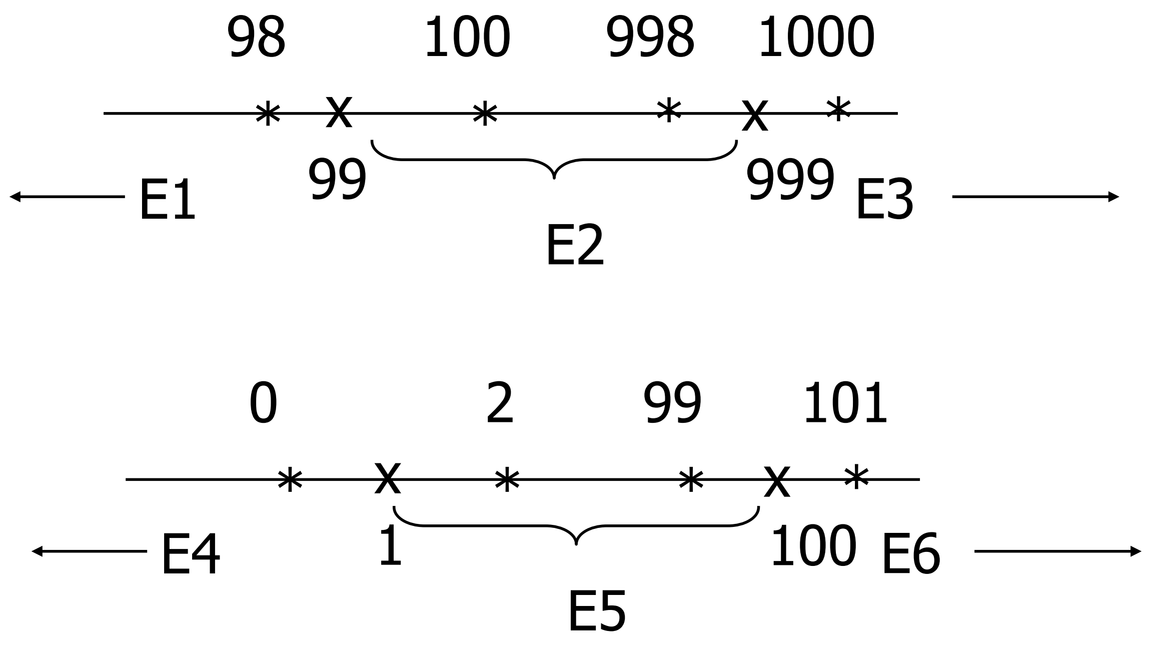
Example: 1. Create equivalence classes

Function findPrice() has two parameters: an item code must be in the range 99..999 and quantity in the range 1..100,

Equivalence classes for code:
- E1: Values less than 99.
- E2: Values in the range.
- E3: Values greater than 999.

Equivalence classes for qty:
- E4: Values less than 1.
- E5: Values in the range.
- E6: Values greater than 100.

Example: 2. Identify boundaries



Equivalence classes and boundaries for findPrice. Boundaries are indicated with an x. Points near the boundary are marked *.

Example: 3. Construct test set

Test selection based on the boundary value analysis technique requires that tests must include, for each variable, values at and around the boundary. Consider the following test set:

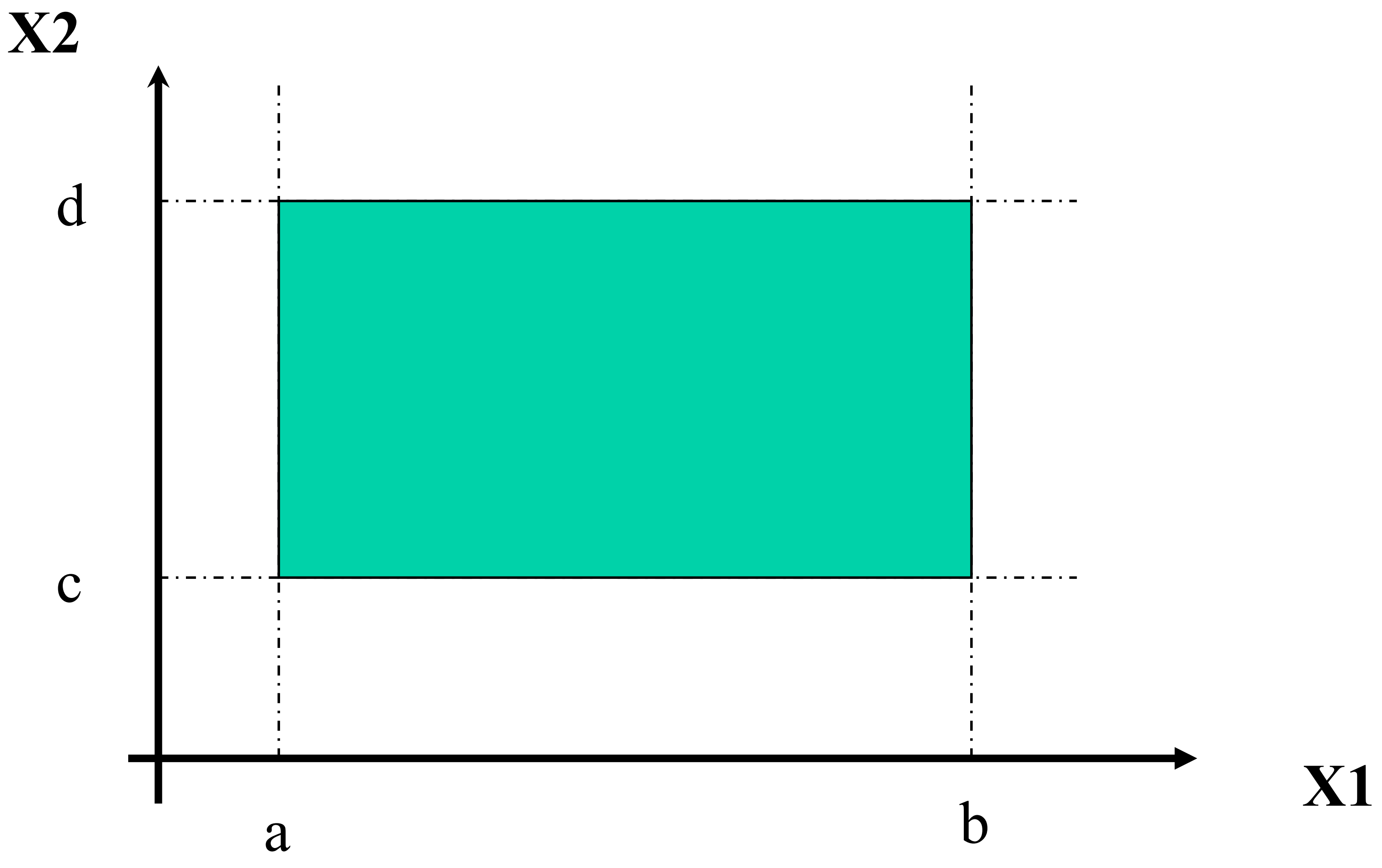
```
T={    t1: (code=98, qty=0),
       t2: (code=99, qty=1),
       t3: (code=100, qty=2),
       t4: (code=998, qty=99),
       t5: (code=999, qty=100),
       t6: (code=1000, qty=101)
}
```

Illegal values of code and qty included.

Principles

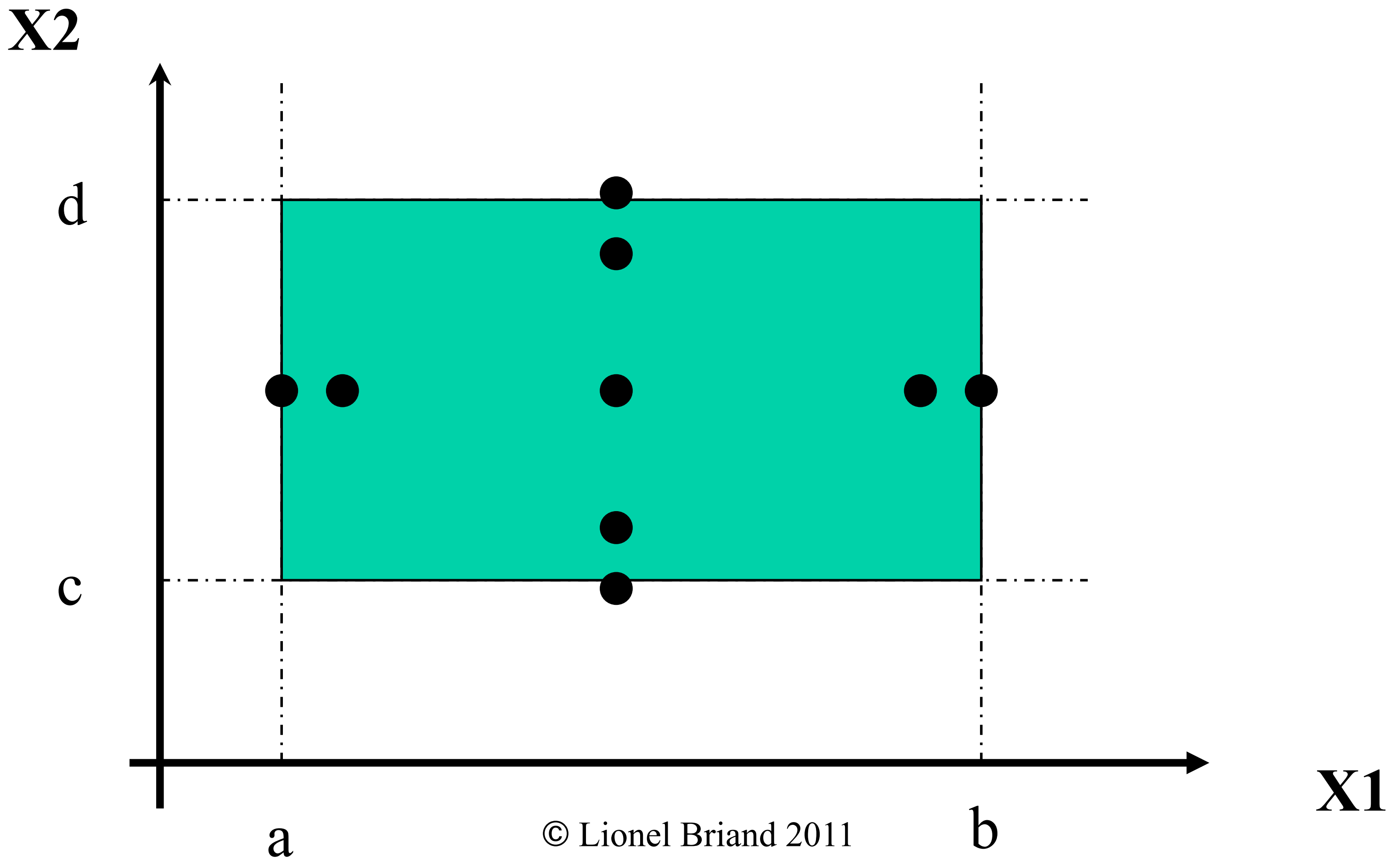
- Input variable values (within a class) at their minimum, just above the minimum, a nominal value, just below their maximum, and at their maximum.
- Convention: min, min+, nom, max-, max
- Hold the values of all but one variable at their nominal values, letting one variable assume its extreme value

Input Domain of Function F



Boundary Analysis Test Cases

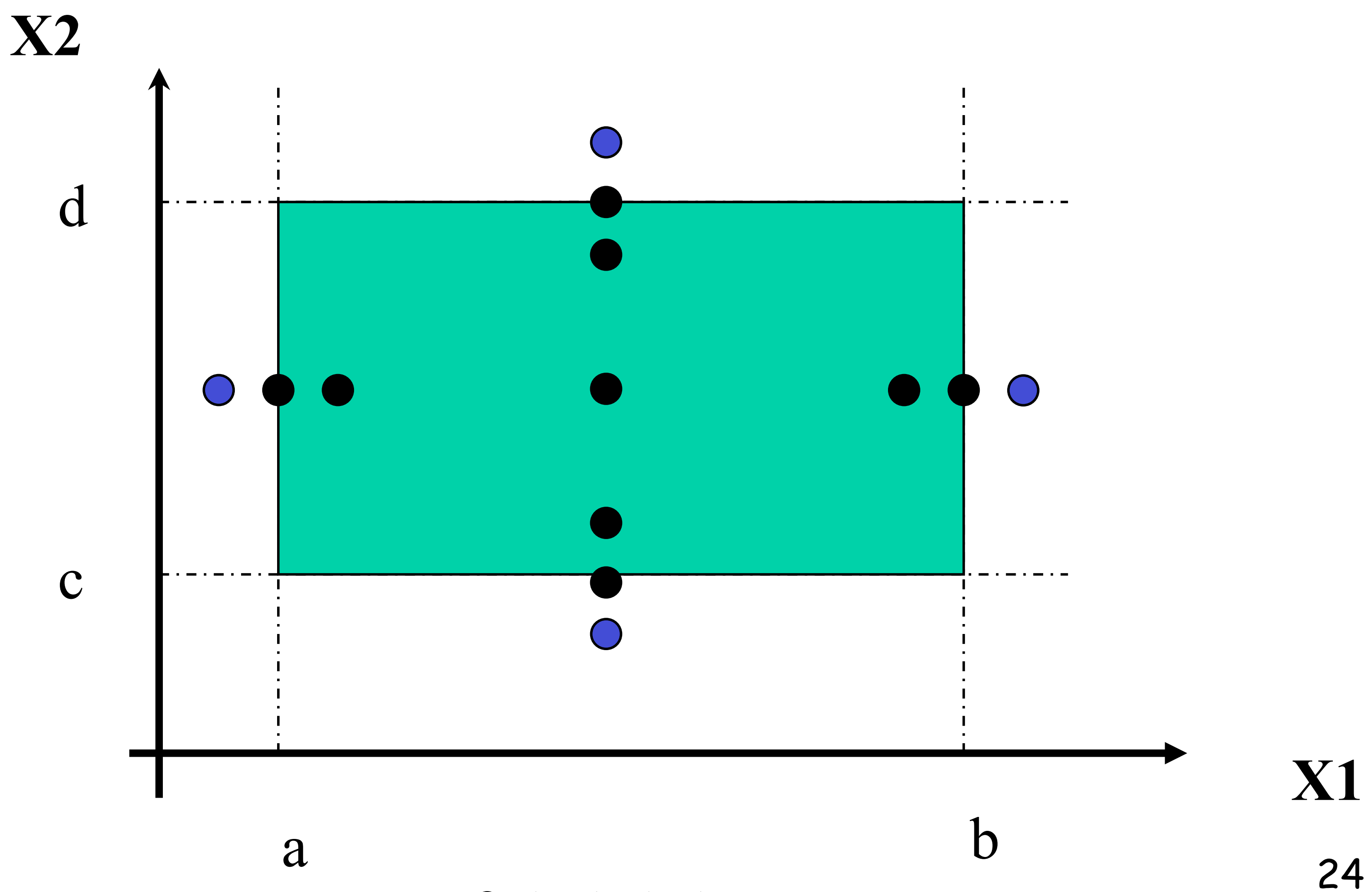
- Test set ={<$x1_{nom}$, $x2_{min}$>, <$x1_{nom}$, $x2_{min+}$>, <$x1_{nom}$, $x2_{nom}$>, <$x1_{nom}$, $x2_{max-}$>, <$x1_{nom}$, $x2_{max}$>, <$x1_{min}$, $x2_{nom}$>, <$x1_{min+}$, $x2_{nom}$>, <$x1_{max-}$, $x2_{nom}$>, <$x1_{max}$, $x2_{nom}$>}



General Case and Limitations

- A function with n variables will require 4n + 1 test cases
- Works well with variables that represent bounded physical quantities
- No consideration of the nature of the function and the meaning of variables
- Rudimentary technique that is amenable to robustness testing

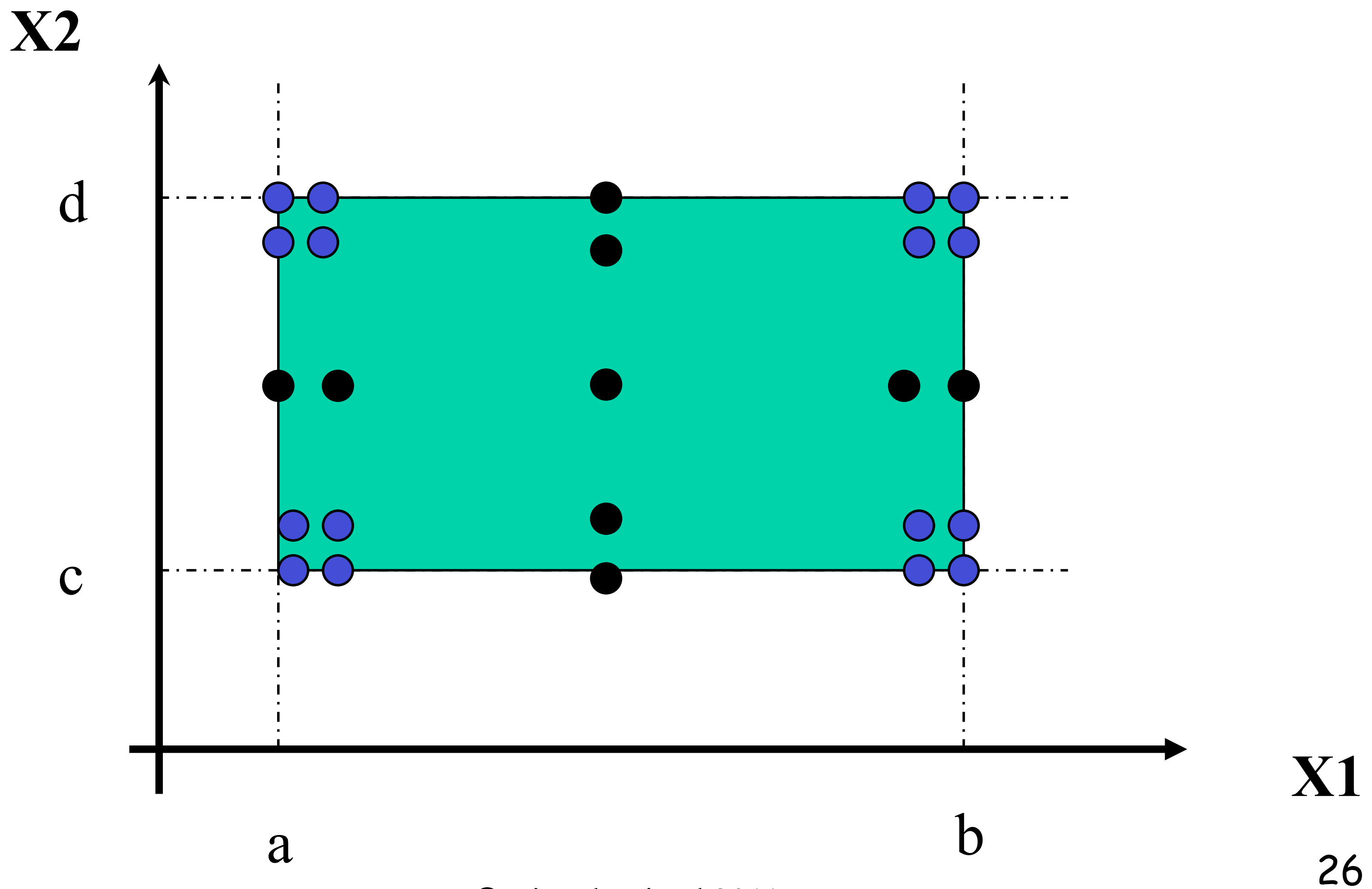
Robustness Testing



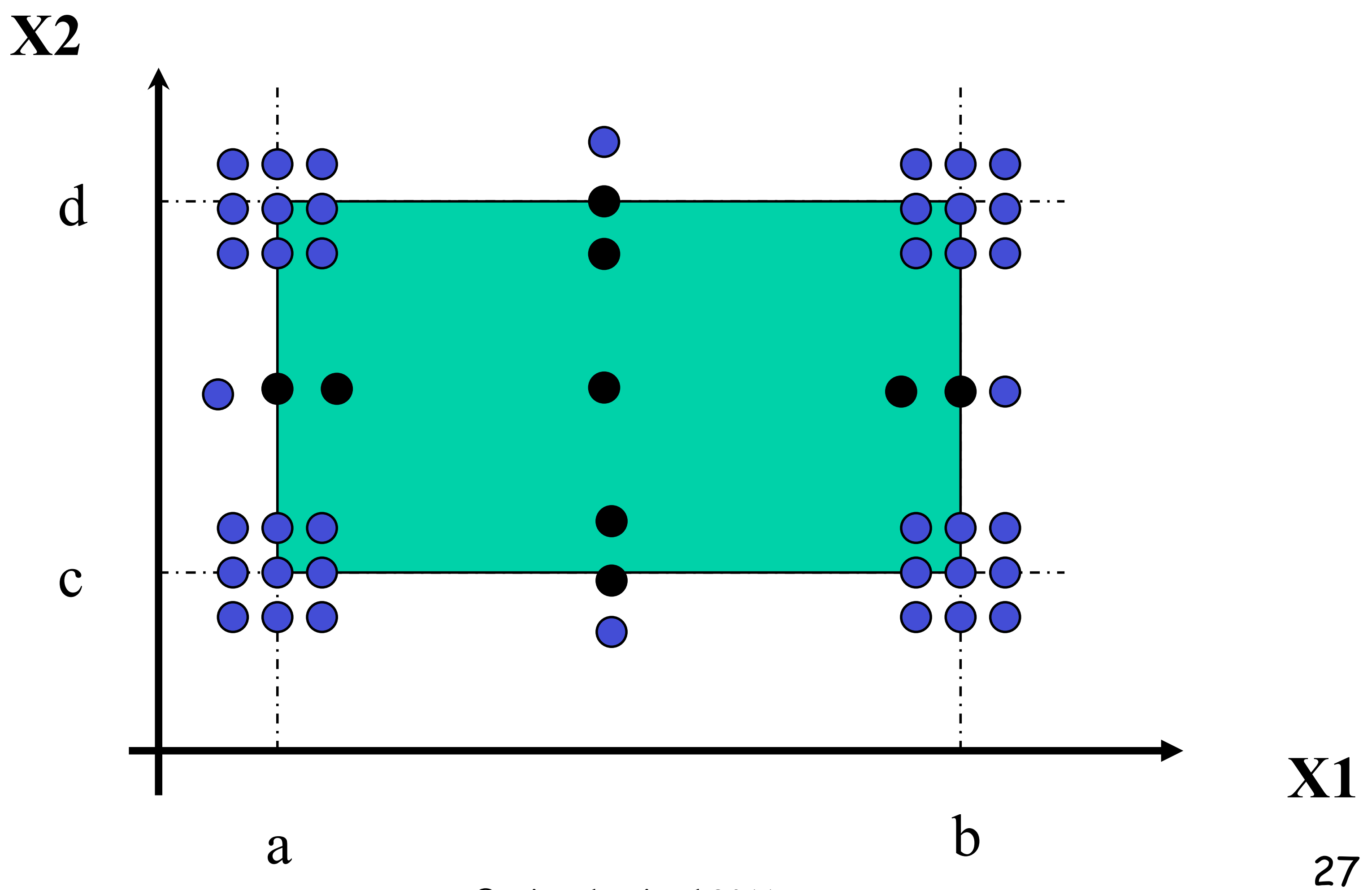
Worst Case Testing (WCT)

- Boundary value analysis makes the common assumption that failures, most of the time, originate from one fault related to an extreme value
- What happens when more than one variable has an extreme value?
- Idea comes from electronics in circuit analysis
- Cartesian product of {min, min+, nom, max-, max}
- Clearly more thorough than boundary value analysis, but much more effort: 5^n test cases
- Good strategy when physical variables have numerous interactions, and where failure is costly
- Even further: Robust Worst Case Testing

WCT for 2 variables



Robust WCT for 2 variables



Category-Partition Testing

Steps

- Extends and combine ECT, boundary value analysis.
- The system is divided into individual "functions" (use cases) that can be independently tested
- The method identifies the *parameters* of each "function" and, for each parameter, identifies distinct *categories*
- Besides parameters, *environment characteristics,* under which the function operates (characteristics of system state), can also be considered, e.g., versions of libraries.
- *Categories* are major properties or characteristics
- The *categories* are further subdivided into *choices* in the same way as equivalence partitioning is applied (value subdomains)

Small Example

- Function: Sorting an array
- Characteristics:
  - Length of array (Len)
  - Type of elements
  - Max value
  - Min value
  - Position of max value (Max pos)
  - Position of min value
- Choices for Max pos: {1, 2..Len-1, Len}

Steps (II)

- The *constraints* operating between choices are then identified, I.e., how the occurrence of one choice can affect the existence of another
  - E.g., in the array sorting example, if Len = 0, then the rest does not matter
- *Test frames* are generated which consist of the allowable combinations of choices in the categories (test specifications)
- Test frames are then converted into *test data*

Constraints

- *Properties, Selectors* associated with choices

Category A	
ChoiceA1	[property X, Y, Z]
ChoiceA2	
Category B	
ChoiceB1	
ChoiceB2	[if X and Z]

Special annotation: [Error], [Single]

Trivial Example

- Specification: The program prompts the user for a positive integer in the range 1 to 20 and then for a string of characters of that length. The program then prompts for a character and returns the position in the string at which the character was first found or a message indicating that the character was not present in the string. The user has the option to search for more characters.

Parameters and Categories

- Three parameters: integer x (length), the string a, and the character c
- For x the categories are "in-range" (1-20) or "out-of-range"
- Categories for a: minimal, maximal, intermediate length
- Categories for c: character appears at the beginning, middle, end of string, or does not occur in the string

Choices

- Integer x, out-of-range: 0, 21
- Integer x, in-range: 1, 2-19, 20
- String a: 1, 2-19, 20
- Character c: first, middle, last, does not occur
- Combined boundary analysis, robustness and equivalence class partitioning

Formal Test Specifications

x:

1)	0	[error]
2)	1	[property stringok, length1]
3)	2-19	[property stringok, midlength]
4)	20	[property stringok, length20]
5)	21	[error]

a:

1)	Length 1	[if stringok and length1]
2)	Length 2-19	[if stringok and midlength]
3)	Length 20	[if stringok and length20]

c:

1)	At first position in string	[if stringok]
2)	At last position in string	[if stringok and not length1]
3)	In middle of string	[if stringok and not length1]
4)	Not in string	[if stringok]

Test Frames and Cases

x 1	x = 0
x 2a1c1	x = 1, a = 'A', c = 'A'
x 2a1c4	x = 1, a = 'A', c = 'B'
x 3a2c1	x = 7, a = 'ABCDEFG', c = 'A'
x 3a2c2	x = 7, a = 'ABCDEFG', c = 'G'
x 3a2c3	x = 7, a = 'ABCDEFG', c = 'D'
x 3a2c4	x = 7, a = 'ABCDEFG', c = 'X'
x 4a3c1	x = 20, a = 'ABCDEFGHIJKLMNOPQRST', c = 'A'
x 4a3c2	x = 20, a = 'ABCDEFGHIJKLMNOPQRST', c = 'T'
x 4a3c3	x = 20, a = 'ABCDEFGHIJKLMNOPQRST', c = 'J'
x 4a3c4	x = 20, a = 'ABCDEFGHIJKLMNOPQRST', c = 'X'
x 5	x = 21

Criteria Using Choices

- *All Combinations (AC):* This is what was shown in the previous example, what is typically done when using category-partition. One value for every choice of every parameter must be used with one value of every (possible) choice of every other category.
- *Each choice (EC):* This is a weaker criterion. One value from each choice for each category must be used at least in one test case.
- *Base Choice (BC):* This criterion is a compromise. A base choice is chosen for each category, and a first base test is formed by using the base choice for each category. Subsequent tests are chosen by holding all but one base choice constant (I.e., we select a non-base choice for one category) and forming choice combinations by covering all non-base choices of the selected category. This procedure is repeated for each category.
- The base choice can be the simplest, smallest, first in some ordering, or most likely from an end-user point of view, e.g., in the previous example, character c occurs in the middle of the string, length x is within 2-19.

Conclusions

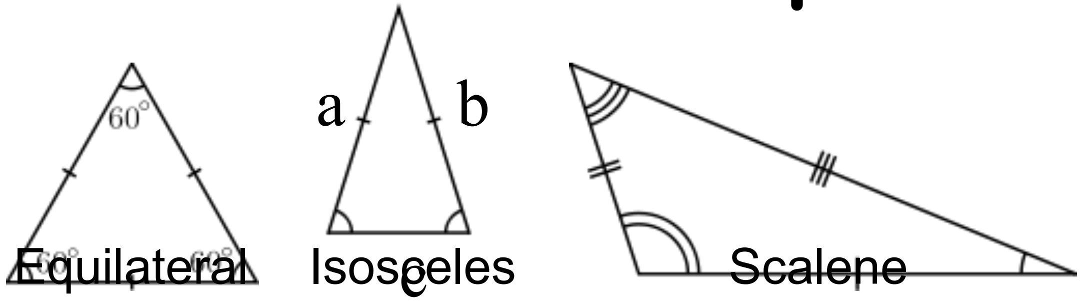
- Identifying parameters and environments conditions, and categories, is heavily relying on the experience of the tester
- Makes testing decisions explicit (e.g., constraints), open for review
- Combine boundary analysis, robustness testing, and equivalence class partitioning
- Once the first step is completed, the technique is straightforward and can be automated
- Techniques for test case reduction makes it useful for practical testing

Decision Tables

Motivations

- Help express *test requirements* in a directly usable form
- Easy to understand and support the systematic derivation of tests
- Support automated or manual generation of test cases
- A particular response or response subset is to be selected by evaluating many related conditions
- Ideal for describing situations in which a number of combinations of actions are taken under varying sets of conditions, e.g., control systems

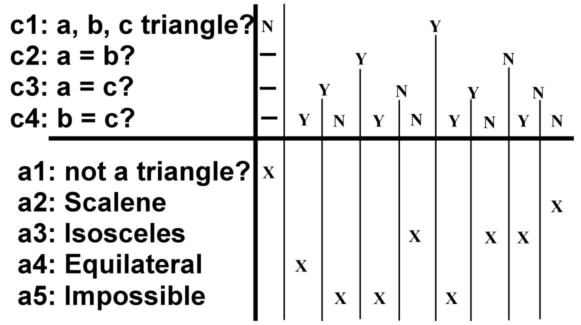
Table Example



c1: a, b, c triangle?	N	Y	Y	Y	Y	Y	Y	Y	Y
c2: a = b?	–	Y	Y	Y	Y	N	N	N	N
c3: a = c?	–	Y	Y	N	N	Y	Y	N	N
c4: b = c?	–	Y	N	Y	N	Y	N	Y	N
a1: not a triangle?	X								
a2: Scalene									X
a3: Isosceles					X		X	X	
a4: Equilateral		X							
a5: Impossible			X	X		X			

[simula . research laboratory]

Structure



c1: a, b, c triangle?	N	Y							
c2: a = b?	–	Y				N			
c3: a = c?	–	Y		N		Y		N	
c4: b = c?	–	Y	N	Y	N	Y	N	Y	N
a1: not a triangle?	X								
a2: Scalene									X
a3: Isosceles					X		X	X	
a4: Equilateral		X							
a5: Impossible			X	X		X			

- Condition section lists *conditions* and combinations thereof
- Condition expressed relationship among *decision variables*
- Action section list *responses* to be produced when corresponding combinations of conditions are true
- Limitations: Resultant actions are determined by the *current* values of decision variables!
- Actions are *independent* of input order and the order in which conditions are evaluated
- Actions may appear more than once but each combination of conditions is unique

Table Structure

Condition	c1	True			False		
	c2	True		False	True		False
	c3	T	F	–	T	F	–
Action	a1	X	X		X		
	a2	X				X	
	a3		X		X	X	
	a4			X			X

Rule

Truth Table

conditions											
c1: a < b + c?	F	T	T	T	T	T	T	T	T	T	T
c2: b < a + c?	-	F	T	T	T	T	T	T	T	T	T
c3: c < a + b?	-	-	F	T	T	T	T	T	T	T	T
c4: a = b?	-	-	-	T	T	T	T	F	F	F	F
c5: a = c?	-	-	-	T	T	F	F	T	T	F	F
c6: b = c?	-	-	-	T	F	T	F	T	F	T	F
a1: Not a triangle	X	X	X								
a2: Scalene											X
a3: Isosceles							X		X	X	
a4: Equilateral				X							
a5: Impossible					X	X		X			

Test Cases

Case ID	a	b	c	Expected Output
TC1	4	1	2	Not a triangle
TC 2	1	4	2	Not a triangle
TC 3	1	2	4	Not a triangle
TC 4	5	5	5	Equilateral
TC 5	?	?	?	Impossible
TC 6	?	?	?	Impossible
TC 7	2	2	3	Isosceles
TC 8	?	?	?	Impossible
TC 9	2	3	2	Isosceles
TC 10	3	2	2	Isosceles
TC 11	3	4	5	Scalene

Ideal Usage Conditions

- One of several distinct responses is to be selected according to distinct cases of input variables
- These cases can be modeled by mutually exclusive Boolean expressions on the input variables
- The response to be produced does not depend on the order in which input variables are set or evaluated (e.g., events are received)
- The response does not depend on prior inputs or outputs

Scale

- For n conditions, there may be at most 2^n *variants* (unique combinations of conditions and actions)
- But, fortunately, there are usually much fewer *explicit* variants …
- "Don't care" values in decision tables help reduce the number of variants
- "Don't care" can correspond to several cases:
  - The inputs are necessary but have no effect
  - The inputs may be omitted
  - Mutually exclusive cases (type-safe exclusions)

Special Cases

- "can't happen" : reflect some assumption that some inputs are mutually exclusive, or that they cannot be produced in the environment, e.g., an insurance policy be issued to a person older than 100.
- A chronic source of bugs, e.g., Ariane 5
- "can't happen" do occur because of programming errors and unexpected change effects
- "don't know" condition reflect an incomplete model, e.g., due to incomplete documentation
- Most of the time, they are specification bugs

Cause-Effect Graphs

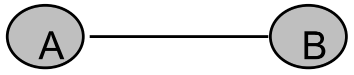
Definition

- Graphical technique that helps derive decision tables
- Aim at supporting interaction with domain experts and the reverse engineering of specifications, for the purpose of testing.
- Identify causes (conditions on inputs, stimuli) and effects (outputs, changes in system state)
- Causes have to be stated in such a way to be either true or false (Boolean expression)
- Specifies explicitly (environmental, external) constraints on causes and effects
- Help select more "significant" subset of input-output combinations and build smaller decision tables

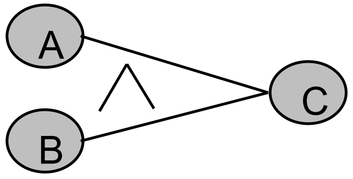
Structure of Cause-Effect Graphs

- A node is drawn for each cause and effect
- Nodes placed on opposite side of a sheet
- A line from a cause to an effect indicates that the cause is a necessary condition for the effect
- If a single effect has two or more causes, the logical relationship of the causes is annotated by symbols for logical *and* (^) and logical *or* (v) placed between lines
- A cause whose negation is necessary is shown by a logical *not* (~)
- A single cause may be necessary for many effects; a single effect may have many necessary causes.
- Intermediate nodes may be used to simplify the graph and its construction

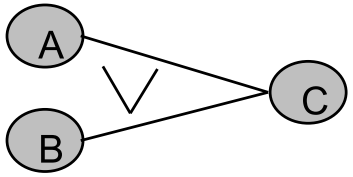
Notation



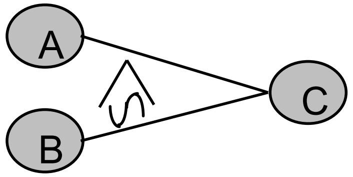
If A then B



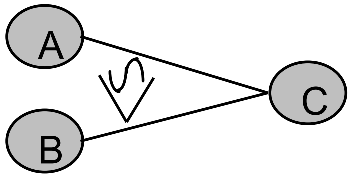
AND: If (A and B) then C



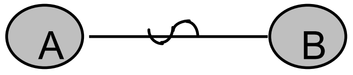
OR: If (A or B) then C



NAND: If not (A and B) then C

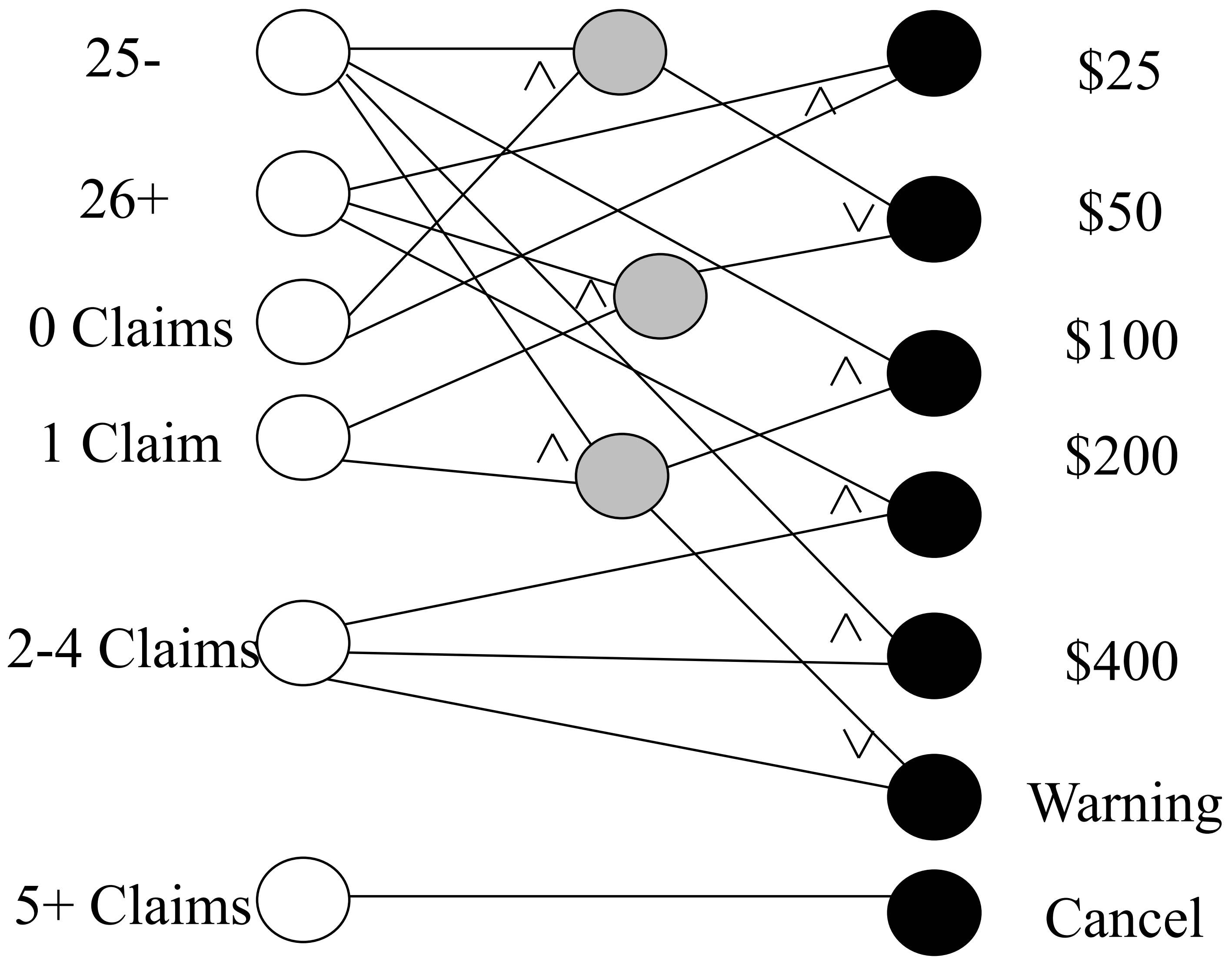


NOR: If (neither A nor B) then C



NOT: If (not A) then B

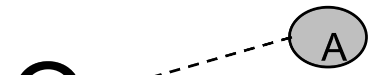
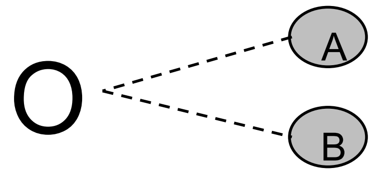
Insurance Renewal Example



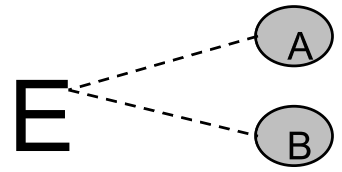
Another Table Example Insurance Renewal

	Condition Section		Action Section		
Variant	**Claims**	**Age**	**Premium Increase $**	**Send Warning**	**Cancel**
1	0	25-	50	No	No
2	0	26+	25	No	No
3	1	25-	100	Yes	No
4	1	26+	50	No	No
5	2 to 4	25-	400	Yes	No
6	2 to 4	26+	200	Yes	No
7	5+	Any	0	No	Yes

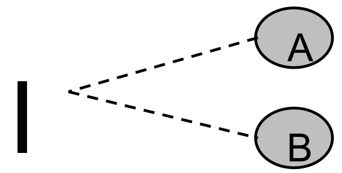
Additional Constraints

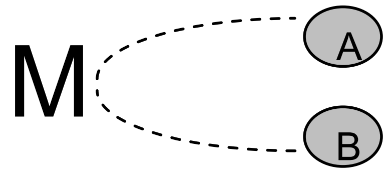
EXACTLY ONE of A and B must be true



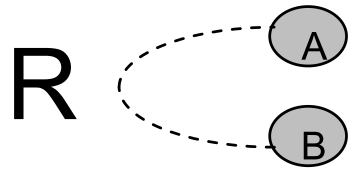
AT MOST ONE of A and B may be true



AT LEAST ONE of A and B must be true



A MASKS B, I.e., A => NOT B



A REQUIRES B, I.e., A => B

Another Example

- Input: The syntax of the function is LEVEL(A,B) where A is the height in meters of the water behind the dam and B is the number of centimeters of rain in the last 24-hour period.
- Processing: The function calculates whether the water level is (1) within a normal range, (2) too high, (3) too low
- Outputs: one of the following messages
  - LEVEL = SAFE (for normal and low)
  - LEVEL = HIGH
  - INVALID PARAMETERS

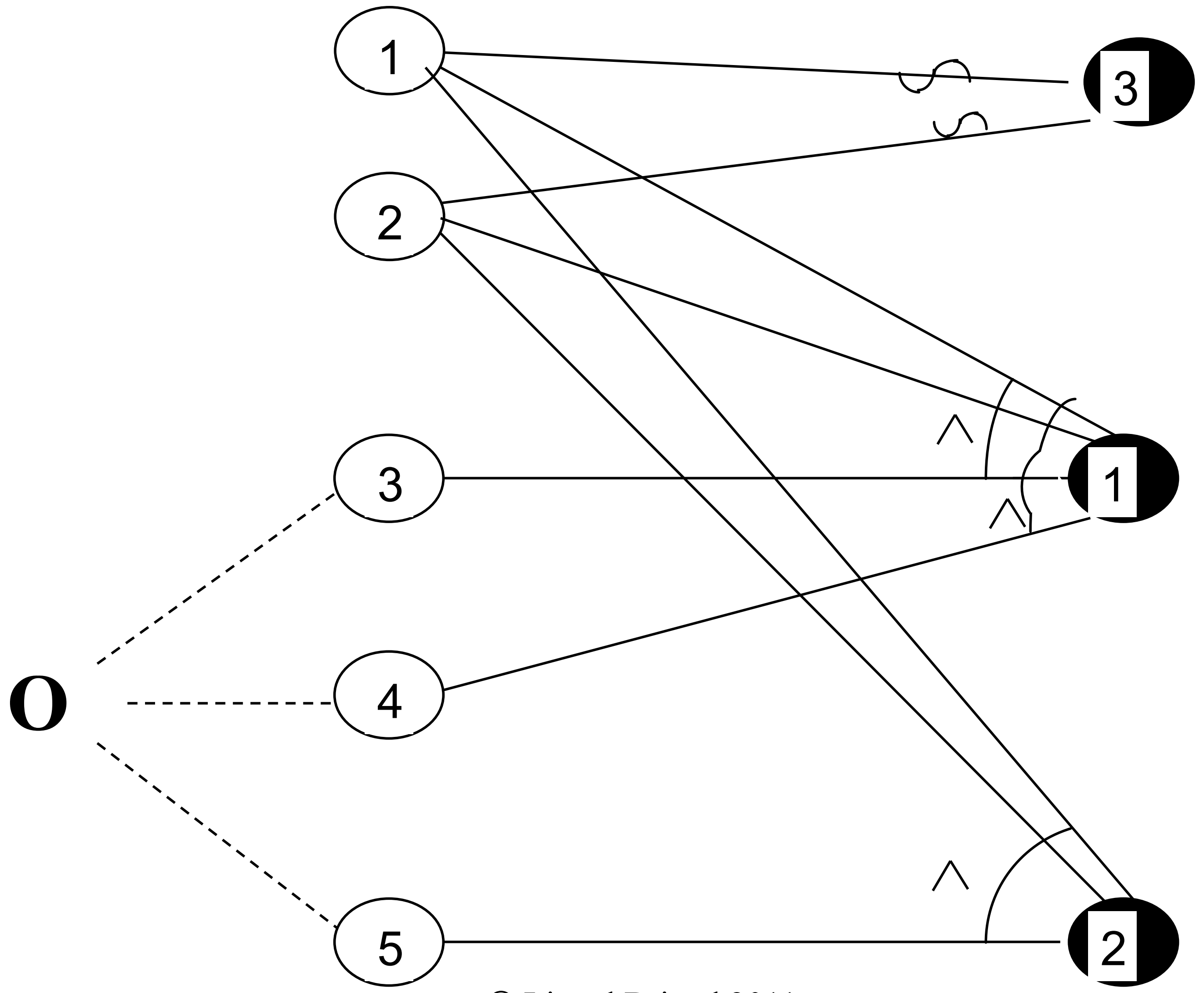
Identifying Causes

1. The value of A is within acceptable range
2. The value of B is within acceptable range
3. The parameter A and B are real numbers such that the water level is calculated to be LOW
4. The parameter A and B are real numbers such that the water level is calculated to be NORMAL
5. The parameter A and B are real numbers such that the water level is calculated to be HIGH

Identifying Effects

1. "LEVEL = SAFE" is displayed on screen
2. "LEVEL = HIGH" is displayed on screen
3. "INVALID PARAMETERS" is printed out

Cause-Effect Graph for LEVEL



Deriving a Decision Table

- A row for each cause or effect
- The columns correspond to test cases (variants)
- Examine each effect and listing all combinations (conjunctions) of causes (subject to constraints) that can lead to that effect
- Create a column for each possible combination of causes
- For each combination, determine the state of other effects
- Two separate lines flow into effect E3, each corresponding to a test case, four lines flow into E1 but correspond to only two combinations

LEVEL Decision Table

Decision Table for Cause-and-effect Graph

	Test 1	Test 2	Test 3	Test 4	Test 5
Cause 1	T	T	T	F	T
Cause 2	T	T	T	-	F
Cause 3	T	F	F	-	-
Cause 4	F	T	F	-	-
Cause 5	F	F	T	-	-
Effect 1	P	P	A	A	A
Effect 2	A	A	P	A	A
Effect 3	A	A	A	P	P

Process

- The specification is divided into workable pieces
- The causes and effects are identified from the specification
- Causes are linked to effects
- The graph is annotated with constraints describing impossible combinations of causes and/or effects
- The graph is used to generate a limited-entry decision table
- The columns of the table are converted into test cases

Discussion

- Aids in selecting , in a systematic way, a high yield of test cases
- The cause-Effect graph can be used to identify all *possible* combinations of causes and checking whether the effect corresponds to the specification
- It provides a test oracle and specifies constraints on outputs (effects), helping detecting wrong system states and output/action combinations
- If the graph is too large, for each admissible combination of effects, find some combinations of causes that cause that combination of effects by tracing back through the graph, e.g., we merged LOW and NORMAL effects in LEVEL(A,B)
- Because of additional constraints on graph, can be more restrictive than straight decision tables
- A beneficial side effect is that it points out incompleteness and ambiguities in the specifications

Testing Logic Functions or Predicates

Definitions

- A *predicate* (or logic function) is an expression that evaluates to a boolean value
- Predicates may contain boolean variables, non-boolean variables that are compared with the comparator operators {>, <, =, …}, and function calls (return Boolean value)
- The internal predicate structure is created by *logical operators* {not, and, or, …}
- A *clause* is a predicate that does not contain any of the logical operators, e.g., (a<b), boolean variable
- Predicates may be written in different, logically equivalent ways (Boolean algebra)

Definitions II

- A logic function (predicate) maps from *n* boolean input variables (clauses) to 1 boolean output variable
- To make expressions easier to read we will use adjacency for the *and* operator, + for the *or* operator, and a ~ for the negation operator.
- Example: Enable or disable the ignition of a boiler based on four input variables
  - NormalPressure (A): pressure within safe operating limit?
  - CallForHeat (B): ambient temperature below set point?
  - DamperShut (C): exhaust duct is closed?
  - ManualMode (D): manual operation selected?
- Logic Function: Z = A(B~C+D) ~> Truth table

Boiler Truth Table I

Input Vector Number	Normal Pressure	CallFor Heat	Damper Shut	Manual Mode	Ignition
	A	B	C	D	Z
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0

Boiler Truth Table II

Input Vector Number	Normal Pressure	CallFor Heat	Damper Shut	Manual Mode	Ignition
	A	B	C	D	Z
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	1

Elements of Boolean Expressions

- *Boolean space:* The n-dimensional space formed by the input variables
- *Product term or conjunctive clause*: String of clauses related by the *and* operator
- *Sum-of-products or disjunctive normal form (DNF)*: Product terms related by the *or* operator
- *Implicant*: Each term of a sum-of-products expression – sufficient condition to fulfill for *True* output of that expression
- *Prime implicants*: An implicant such that no subset (proper subterm) is also an implicant
- *Logic minimization*: Deriving compact (irredundant) but equivalent boolean expressions, using boolean algebra

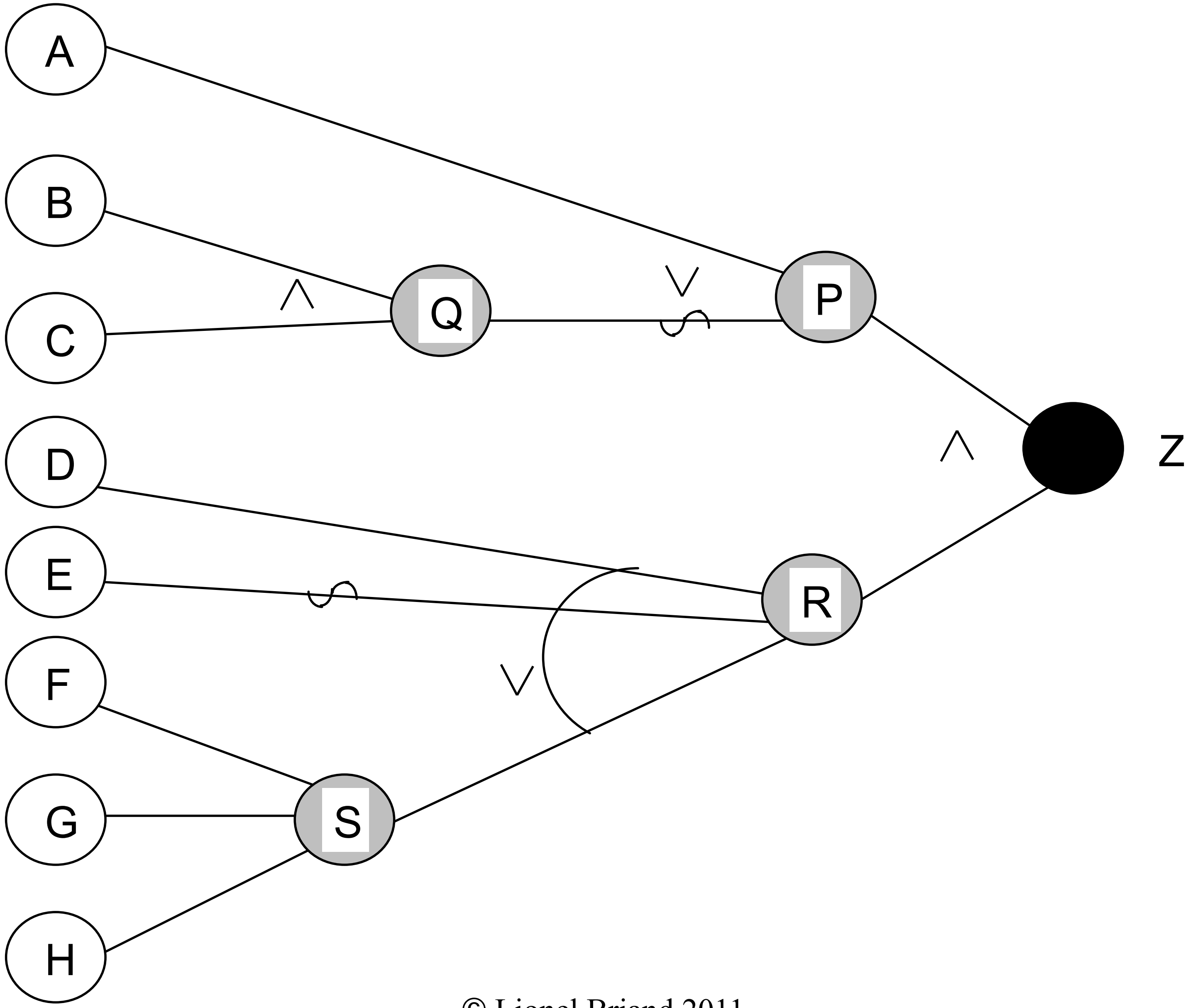
Boiler Example

- Logic function: Z=A(B~C+D)
- Sum-of-Product Form (DNF): Z=A (B~C+D) = AB~C+AD
- Implicants: AB~C, AD
- Prime implicant: AB~C = TTFx = {TTFT, TTFF}, AD=TxxT={TFFT, TFTT, TTFT, TTTT} => both terms are prime implicants

From Graph to Logic Function

- Once a cause-effect graph is reviewed and considered correct, we want to derive a logic function for the purpose of deriving test requirements (in the form of a decision table)
- One function (predicate, truth table) exists for each effect (output variable)
- If several effects are present, then the resulting decision table is a composite of several truth tables that happen to share decision/input variables and actions/effects
- Easier to derive a function for each effect separately
- Derive a Boolean function from the graph in a systematic way

Example



Generate a Logic Function

- Generate an initial function
  - Start from effect node
  - Backtrack through the graph
  - Substitute higher level clauses with lower level clauses and boolean expressions, until you reach cause nodes
- Transform into minimal, DNF form
  - Use boolean algebra laws to reduce boolean expressions
  - Re-express in sum-of-products form (disjunctive normal form)
  - There exist algorithms to do that automatically (Schneeweiss, Boolean Functions with Engineering Applications and Computer Programs, 1989)

Example

- Z = PR (effect)
- P = A + ~Q (intermediate)
- Q = BC (intermediate)
- R = D + ~E + S (intermediate)
- S = F + G + H (intermediate)
- Z = (A + ~(BC)) (D + ~E + (F+G+H)) (substitution)
- Z=(A+~B+~C)(D+~E+F+G+H) (De Morgan's law)
- Z = AD + A~E + AF + AG + AH + ~BD + ~B~E + ~BF + ~BG + ~BH + ~CD + ~C~E + ~CF + ~CG + ~CH (Distributive law is used to obtain sum-of-products)

Laws of Boolean Algebra

- Associative
  - (A+B)+C = A+(B+C), (AB)C = A(BC)
- Distributive
  - A+(BC) = (A+B)(A+C), A(B+C) = AB+AC
- De Morgan's laws
  - ~(A+B)=~A~B, ~(AB)=~A+~B
- Absorption
  - A + AB = A
  - A(A+B) = A
  - A+(~AB) = A+B, A(~A+B) = AB
  - AB+A~B = A
  - (A+B)(A+~B) = A
  - AB+AC+B~C = AC+B~C

Fault Model for Logic-based Testing

- Expression Negation Fault (ENF): The logic function is implemented as its negation
- Clause Negation Fault (CNF): A clause in a particular term is replaced by its negation
- Term Omission Fault (TOF): A particular term in the logic function is omitted.
- Operator Reference Fault (ORF): A binary operator *or* in the logic function is implemented as *and* or vice-versa
- Clause Omission Fault (COF): A clause in a particular term of the logic function is omitted
- Clause Insertion Fault (CIF): A clause not appearing in a particular term of a logic function is inserted in that term
- Clause Reference Fault (CRF): A clause in a particular term of a logic function is replaced by another clause not appearing in the term

Basic Test Criteria

- The goal is to test an implementation and make sure it is consistent with its specification, as modeled by the predicate (or graph)
- There exist a number of test coverage criteria that do not assume a disjunctive normal form for predicates:
  - Predicate coverage
  - Clause coverage
  - Combinatorial coverage
  - (in)active clause coverage
- Notation: P is set of predicates, C is set of clauses in P, Cp is the set of clauses in predicate p

Predicate Coverage

- Predicate coverage: *For each p ∈ P, we have two test requirements: p evaluates to true, and p evaluates to false.*
- For A(B~C+D) two test that satisfy Predicate Coverage are (1) (A=true, B=false, C=true, D=true), (2) (A=false, B=false, C=true, D=true),
- Problem: Individual clauses are not exercised

Clause Coverage

- Clause coverage: *For each $c \in C$, we have two test requirements: c evaluates to true, and c evaluates to false.*
- For (A+B)C, two tests that satisfy Clause Coverage: (1) (A=true, B=true, C=false), (2) (A=false, B=false, C=true),
- Note: Clause coverage does not subsume predicate coverage or vice-versa.

Example

- Z = A+B
- t1 = (A = true; B = true) => Z
- t2 = (A = true; B = false) => Z
- t3 = (A = false; B = true) => Z
- t4 = (A = false; B = false) => ~Z
- If we choose the pair of test cases T1 = {t1; t2}, it satisfies neither Clause Coverage (because A is never false) nor Predicate Coverage (because Z is never false).
- Test set T2 = {t2; t3} satisfies Clause Coverage, but not Predicate Coverage (because Z is never false).
- Test set T3 = {t2; t4} satisfies Predicate Coverage, but not Clause Coverage (because B is never true).
- Test set T4 = {t1; t4} is the only pair that satisfies both Clause Coverage and Predicate Coverage.

Combinatorial Coverage

- Combinatorial coverage: *For each $p \in P$, we have test requirements for clauses in Cp to evaluate each possible combination of truth values*
- Subsumes predicate coverage
- There are $2^{|Cp|}$ possible assignments of truth values
- Problem: Impractical for predicates with more than a few clauses

Masking Effects

- When we introduce tests at the clause level, we want to have an effect on the predicate
- Logical expressions (clauses) can mask each others
- In the predicate AB, if B = *false*, B can be said to mask A, because no matter what value A has, AB will still be *false*.
- We need to consider circumstances under which a clause affects the value of a predicate, to detect possible implementation failures

Determination

- Determination: *Given a clause c_i in predicate p, called the major clause, we say that c_i* determines *p if the remaining minor clauses $c_j \in p$, j <> i have values so that changing the truth value of c_i changes the truth value of p.*
- We would like to test each clause under circumstances where it determines the predicate
- Test set T4 in previous slide satisfied both predicate and clause coverage but does not test neither A nor B effectively.

Active Clause Coverage

- **Active Clause Coverage (ACC):** *For each p∈P and each major clause $c_i \in Cp$, choose minor clauses c_j, j <> i so that c_i determines p. We have two test requirements for each c_i: c_i evaluates to true and c_i evaluates to false.*
- For example, for Z=A+B, we end up with a total of four test requirements, two for clause A and two for clause B.
- For clause A, A determines Z if and only if B is false. So we have the two test requirements {(A = *true; B =false*)*;* (A = *false; B = false*)}.
- For clause B, B determines Z if and only if A is false. So we have the two test requirements {(A = *false; B = true*)*;* (A = *false; B = false*)}, the latter in common with A.
- ACC almost identical to MCDC in code coverage
- The most important questions are whether (1) ACC should subsume PC, (2) the minor clauses c_j need to have the same values when the major clause c_i is true as when c_i is false.

Correlated ACC (CACC)

- For each $p \in P$ and each major clause $c_i \in Cp$, choose minor clauses c_j, $j <> i$ so that c_i determines p. There are two test requirements for each c_i: c_i evaluates to true and c_i evaluates to false. The values chosen for the minor clauses c_j must cause p to be true for one value of the major clause c_i and false for the other, that is, it is required that $p(c_i = true) < > p(c_i = false)$.
- CACC is subsumed by combinatorial clause coverage and subsumes clause/predicate coverage

Restricted ACC (RACC)

- For each $p \in P$ and each major clause $c_i \in Cp$, choose minor clauses c_j, $j <> i$ so that c_i determines p. There are two test requirements for each c_i : c_i evaluates to true and c_i evaluates to false. The values chosen for the minor clauses c_j must be the same when c_i is true as when c_i is false, that is, it is required that $c_j(c_i = true) = c_j(c_i = false)$ for all c_j .
- RACC makes it easier than CACC to determine the cause of the problem, if one is detected: major clause
- But is it common in specification to have constraints between clauses, making RACC impossible to achieve.
- This corresponds to MCDC for code coverage

Example

- Z=A(B+C)
- It would be possible to satisfy Correlated Active Clause Coverage with respect to clause *A* with the two test requirements:

  {(A = true; B = true; C = false);

  (A = false; B =false; C = true)}
- But it does not satisfy RACC:

  {(A = true; B = true; C = false);

  (A = false; B = true; C = false)}
- This case is easy …

Inactive Clause Coverage

- The Active Clause Coverage Criteria focus on making sure the major clauses do affect their predicates. A complementary criterion to Active Clause Coverage ensures that changing a major clause that should *not* affect the predicate does not, in fact, affect the predicate.
- **Inactive Clause Coverage (ICC):** *For each $p \in P$ and each major clause $c_i \in Cp$, choose minor clauses c_j , j <> i so that c_i does not determine p. There are four test requirements for c_i under these circumstances: (1) c_i evaluates to true with p true, (2) c_i evaluates to false with p true, (3) c_i evaluates to true with p false, and (4) c_i evaluates to false with p false.*
- ICC is subsumed by combinatorial clause coverage and subsumes clause/predicate coverage

Disjunctive Normal Form Coverage Criteria

- Here criteria assume the predicates have been re-expressed in a disjunctive normal form (DNF).
- What is interesting with DNF are the criteria that go with it.
- Criteria:
  - Implicant coverage
  - Prime implicant coverage
  - Variable negation strategy

Implicant Coverage (IC)

- IC: *Given DNF representations of a predicate p and its negation ~p, for each implicant, a test requirement is that the implicant evaluates to true.*
- This tests different situations in which an action should (not) be taken (e.g., a boiler turned on)
- p: AB+B~C
- ~p (one representation): ~B+~AC
- Four implicants: {AB, B~C, ~B, ~AC}
- Many test sets can satisfy this criterion, e.g., for ABC, respectively, we can use {TTF, FFT}
- IC subsumes predicate coverage, but not necessarily Active Clause Criteria.

Problems with IC

- A problem with IC is that tests might be chosen so that a single test satisfies multiple implicants. (see previous example)
- Although this lets testers minimize the size of test suites, it is a bad thing from the perspective of testing the unique contributions that each implicant might bring to a predicate.
- Thus we introduce a method to force a kind of “independence" of the implicants.

Prime Implicants

- The first step is to obtain a DNF form where each implicant can be satisfied without satisfying any other implicant.
- Fortunately, standard approaches already exist that can be used. A *proper subterm* of an implicant is the implicant with one or more clauses omitted.
- A *prime implicant* is an implicant such that no proper subterm of the implicant is also an implicant.
- Example: ABC+AB~C+B~C
- ABC is not a prime implicant because a proper subterm (AB) is also an implicant

Prime Implicant Coverage (PIC)

- Let us assume our DNF predicate only contains prime implicants
- An implicant is redundant if it can be omitted without changing the value (truth table) of the predicate.
- In AB+AC+B~C, AB is redundant
- PIC: *Given nonredundant, prime-implicant DNF representations of a predicate p and its negation ~p, for each implicant, a test requirement is that the implicant evaluates to true, while all other implicants evaluate to false.*

PIC Example & Discussion

- p: AB+B~C
- ~p: ~B+~AC
- Both are nonredundant, prime implicant representations
- The following test set satisfies PIC: {TTT, FTF, FFF, FTT}
- PIC is a powerful coverage criteria: none of the clause coverage criteria subsume PIC
- Though up to 2^{n-1} prime implicants, many predicates generate a modest number of tests for PIC
- It is an open question whether PIC subsumes any of the clause coverage criteria.

Variable Negation Strategy

- Goes even further than PIC
- *Unique true points*: variants that makes one and only one product term true
  - E.g., (TTFF) for the first product term in the boiler example (AB~C), AD is false
- *Near false points*: variants for each product term where one clause is negated such that the overall logic function evaluates to false
  - E.g., (TTTF) for AB~C where ~C is negated
- Such variants constitute Test Candidate Sets (TCS)
- Generate TCS for each product term in logic function
- The test suite is formed by selecting the smallest suite that covers all TCSs

Boiler Example

- Makes AB~C true but not AD: one unique true point is (TTFF), or (1100) in binary form, or {12} in decimal form
- Makes AD true but not AB~C: set of unique true points {9, 11, 15}
- Near false points for AB~C: {14}, {8}, {4,5} for negating ~C, B, and A, respectively
- Near false points for AD: {1, 3, 5, 7}, {8, 10, 14} for negating A and D, respectively
- Generate the variant set matrix and select test suite by covering all candidate sets {*} above
- Because one variant may belong to more than one candidate set, the number of tests required can be less than the cumulative number from candidate test sets

Variant Set Matrix

Var	1	2	3	4	5	6	7	TCS
0								
1							x	
2								
3							x	
4				x				
5				x			x	S
6								
7							x	
8			x			x		S
9					x			
10						x		
11					x			S
12	x							S
13								
14		x				x		S
15					x			

Discussion

- If one product term implementation does not evaluate to true when it should - implying that at least one clause in that product term does not evaluate to *true* when expected - test cases from the TCS (*unique true points*) corresponding to the term will be able to detect it, without masking effect from other clauses or terms
- If one product term implementation does not evaluate to false when it should, that is the negation of (at least) a clause has not the effect expected on the logic function (false), test cases from the TCS (*near false points*) corresponding to the negated clause will be able to detect it, without masking effect from other clauses or terms
- In a study by Weyuker et al, roughly *6 percent* of the All-Variant test suite (2^n) is needed to meet the *variable negation criteria*

VN Strategy versus Faults

- Expression Negation Fault (ENF): Any point in the Boolean space
- Clause Negation Fault (CNF): Any unique true point or near false point for the faulty term and clause negated
- Term Omission Fault (TOF): Any unique true point for the faulty term
- Operator Reference Fault (ORF):
  - *or* implemented as *and*: Any unique true point of one of the two terms
  - *and* implemented as *or*: any near false point of one of the two terms
- Clause Omission Fault (COF): Any near false point for the faulty term and clause omitted
- Clause Insertion Fault (CIF): *All* near false points and unique true points for the faulty term
- Clause Reference Fault (CRF): *All* near false points and unique true points for the faulty term

TCASII Study

- Weyuker and Goradia, May 1994, IEEE Transactions on Software Engineering
- TCASII, aircraft collision avoidance system
- 20 predicates/logic functions formed the specifications (in modified statechart notation)
- On average 10 distinct clauses per expression
- Five mutation operators, defined for boolean expressions, were used to seed faults in the specifications
- Random selection of test cases (same size) leads to an average mutation score of 42.7%
- The variable negation strategy is therefore doing much better with an average of 97.9%

Summary of BB Testing

- All techniques see a program as a mathematical function that maps inputs onto its outputs
- By order of sophistication: (1) boundary value analysis, (2) equivalence class testing, (3) Category-partition (4) Cause-effect graphs
- (1) Mechanical, (2) devise equivalence classes, (3) partitions, categories, and logical dependencies (4) logical dependencies between causes themselves, and causes and effects
- Less test cases with (3) or (4)
- Trade-off between test identification and test execution effort