Assignment 1 for INF4360, 2011
Bezier curves

Michael S. Floater

To be completed by Tuesday 27 September. Solutions can be handed to me in the lecture or sent electronically to `michaelf@ifi.uio.no`.

1. Show that the Bernstein polynomial $B_i^n(x)$ has only one maximum in $[0, 1]$, namely at $x = i/n$.

2. The Bernstein approximation to a function $f : [0, 1] \to \mathbb{R}$ of order n is the polynomial $g : [0, 1] \to \mathbb{R}$ defined by

$$g(x) = \sum_{i=0}^{n} f(i/n) B_i^n(x).$$

   Show that if f is a polynomial of degree $m \leq n$ then g has degree m.

3. Show that

$$\Delta^i \mathbf{c}_0 = \sum_{k=0}^{i} (-1)^{i-k} \binom{i}{k} \mathbf{c}_k.$$

4. Show that

$$n(n-1)\cdots(n-k)x^{k+1} = \sum_{i=0}^{n} i(i-1)\cdots(i-k) B_i^n(x).$$