

Assignment 2 for INF4360, 2012

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To be completed by Tuesday 23 October. Solutions can be handed to me in the lecture or sent electronically to `michaelf@ifi.uio.no`.

1. Let $\mathbf{p}(t) = \sum_{i=0}^n \mathbf{c}_i B_i^n(t)$, $\mathbf{c}_i \in \mathbb{R}^d$, be a Bezier curve in \mathbb{R}^d . Show that its length,

$$L = \int_0^1 \|\mathbf{p}'(t)\| dt,$$

is less than or equal to the length of its control polygon.

2. Consider the two tensor-product Bezier surfaces,

$$\mathbf{p}(s, t) = \sum_{i=0}^m \sum_{j=0}^n \mathbf{c}_{ij} B_i^m(s) B_j^n(t), \quad (s, t) \in [0, 1] \times [0, 1],$$

$$\mathbf{q}(s, t) = \sum_{i=0}^m \sum_{j=0}^n \mathbf{d}_{ij} B_i^m(s-1) B_j^n(t), \quad (s, t) \in [1, 2] \times [0, 1],$$

with control points $\mathbf{c}_{ij}, \mathbf{d}_{ij} \in \mathbb{R}^3$. What are the conditions on the control points that ensure that \mathbf{p} and \mathbf{q} join with C^1 continuity on the common edge $s = 1$, $0 \leq t \leq 1$?

3. B-splines: find the coefficients $c_i \in \mathbb{R}$ such that

$$t^2 = \sum_{i=1}^n c_i N_i^3(t), \quad t_4 \leq t \leq t_{n+1},$$

over some knot vector t_1, t_2, \dots, t_{n+4} ,

4. (a) Write a computer program which, given a, b , with $a < b$, and $\mathbf{c}_0, \dots, \mathbf{c}_n \in \mathbb{R}^2$, plots the Bezier curve $\mathbf{p}(t) = \sum_{i=0}^n \mathbf{c}_i B_i(u)$, where $u = (t - a)/(b - a)$.

(b) Use the program to plot the composite curve $\mathbf{s} : [0, 2] \rightarrow \mathbb{R}^2$, with pieces

$$\mathbf{p}(t) = \sum_{i=0}^2 \mathbf{c}_i B_i^2(t), \quad 0 \leq t < 1,$$

$$\mathbf{q}(t) = \sum_{i=0}^2 \mathbf{d}_i B_i^2(t - 1), \quad 1 \leq t < 2,$$

where $\mathbf{c}_0 = (-1, 1)$, $\mathbf{c}_1 = (-1, 0)$, $\mathbf{c}_2 = (0, 0)$, and $\mathbf{d}_0 = (0, 0)$, $\mathbf{d}_1 = (1, 0)$, $\mathbf{d}_2 = (2, 1)$. What is the order of continuity of \mathbf{s} at $(0, 0)$?

(c) The *curvature* of a parametric curve \mathbf{r} in \mathbb{R}^2 is

$$\kappa(\mathbf{r}(t)) = \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{\|\mathbf{r}'(t)\|^3},$$

where $(a_1, a_2) \times (b_1, b_2) := a_1 b_2 - a_2 b_1$. What are the curvatures of \mathbf{p} and \mathbf{q} at $(0, 0)$?