INF4820: Algorithms for Artificial Intelligence and Natural Language Processing

Probabilities and Language Models

Stephan Oepen & Milen Kouylekov

Language Technology Group (LTG)

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Recall: *N*-Gram Language Models

- Previous context can help predict the next thing in a sequence;
- Rather than use the whole previous context, the **Markov** assumption says that the whole history can be approximated by the last $n-1$ elements;
- An n-gram language model predicts the n-th word, conditioned on the $n-1$ previous words;
- Maximum Likelihood Estimation uses relative frequencies to approximate the conditional probabilities needed for an n-gram model;

Bigram MLE Example

"I want to go to the beach"

w_1	w_2	$C(w_1 w_2)$	$C(w_1)$	$P(w_2 \mid w_1)$
$\langle S \rangle$	I	1039	24243	0.0429
I	want	46	4131	0.0111
want	to	101	210	0.4810
to	go	128	9778	0.0131
go	to	59	383	0.1540
to	the	1192	9778	0.1219
the	beach	14	22244	0.0006

What's the probability of *Others want to go to the beach* ?

Problems with MLE of *N*-Grams

- Data sparseness: many perfectly acceptable *n*-grams will not be observed
- Zero counts will result in a estimated probability of 0

- Remedy—reassign some of the probability mass of frequent events to less frequent (or unseen) events.
- Known as **smoothing** or **discounting**
- The simplest approach is **Laplace** ('add-one') smoothing:

$$P_{\mathrm{L}}(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n) + 1}{C(w_{n-1}) + V}$$

Bigram MLE Example with Laplace Smoothing

"Others want to go to the beach"

w_1	w_2	$C(w_1 w_2)$	$C(w_1)$	$P(w_2 \mid w_1)$	$P_L(w_2 \mid w_1)$
$\langle S \rangle$	I	1039	24243	0.0429	0.01934
$\langle S \rangle$	Others	17	24243	0.0007	0.00033
I	want	46	4131	0.0111	0.00140
Others	want	0	4131	0	0.00003
want	to	101	210	0.4810	0.00343
to	go	128	9778	0.0131	0.00328
go	to	59	383	0.1540	0.00201
to	the	1192	9778	0.1219	0.03035
the	beach	14	22244	0.0006	0.00029

$$P_{\mathrm{L}}(w_n \mid w_{n-1}) = \frac{C(w_{n-1} w_n) + 1}{C(w_{n-1}) + 29534}$$

N-Gram Summary

- The likelihood of the next word depends on its context.
- We can calculate this using the chain rule:

$$P\left(w_1^N\right) = \prod_{i=1}^{N} P\left(w_i | w_1^{i-1}\right)$$

- In an n-gram model, we approximate this with a Markov chain:

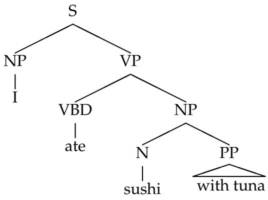
$$P\left(w_1^N\right) \approx \prod_{i=1}^{N} P\left(w_i | w_{i-n+1}^{i-1}\right)$$

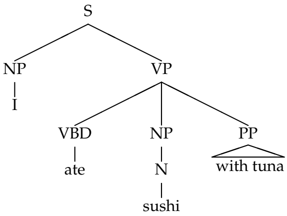
- We use Maximum Likelihood Estimation to estimate the conditional probabilities.
- Smoothing techniques are used to avoid zero probabilities.

Today

Determining

- which string is most likely: ✓
  - *How to recognise speech* vs. *How to wreck a nice beach*
- which tag sequence is most likely for *flies like flowers*:
  - **NNS VB NNS** vs. **VBZ P NNS**
- which syntactic analysis is most likely:





Parts of Speech

- Known by a variety of names: part-of-speech, POS, lexical categories, word classes, morphological classes, . . .
- 'Traditionally' defined semantically (e.g. "nouns are naming words"), but more accurately by their distributional properties.

http://chronicle.com/blogs/linguafranca/2012/06/20/being-a-noun/

- Open-classes
  - New words created/updated/deleted all the time
- Closed-classes
  - Smaller classes, relatively static membership
  - Usually function words

Open Class Words

- Nouns: dog, Oslo, scissors, snow, people, truth, cups
  - proper or common; countable or uncountable; plural or singular; masculine, feminine or neuter; . . .
- Verbs: fly, rained, having, ate, seen
  - transitive, intransitive, ditransitive; past, present, passive; stative or dynamic; plural or singular; . . .
- Adjectives: good, smaller, unique, fastest, best, unhappy
  - comparative or superlative; predicative or attributive; intersective or non-intersective; definite or indefinite; . . .
- Adverbs: again, somewhat, slowly, yesterday, aloud
  - intersective; scopal; discourse; degree; temporal; directional; comparative or superlative; . . .

Closed Class Words

- Prepositions: *on*, *under*, *from*, *at*, *near*, *over*, ...
- Determiners: *a*, *an*, *the*, *that*, ...
- Pronouns: *she*, *who*, *I*, *others*, ...
- Conjunctions: *and*, *but*, *or*, *when*, ...
- Auxiliary verbs: *can*, *may*, *should*, *must*, ...
- Interjections, particles, numerals, negatives, politeness markers, greetings, existential there ...

(Examples from Jurafsky & Martin, 2008)

POS Tagging

The (automatic) assignment of POS tags to word sequences

- non-trivial where words are ambiguous: *fly* (v) vs. *fly* (n)
- choice of the correct tag is *context-dependent*
- useful in pre-processing for parsing, etc; but also directly for text-to-speech synthesis: **con**tent (n) vs. con**tent** (adj)
- difficulty and usefulness can depend on the *tagset*
  - English
    - Penn Treebank (PTB)—45 tags: NNS, NN, NNP, JJ, JJR, JJS

    `http://bulba.sdsu.edu/jeanette/thesis/PennTags.html`
  - Norwegian
    - Oslo-Bergen Tagset—multi-part: ⟨subst appell fem be ent⟩

    `http://tekstlab.uio.no/obt-ny/english/tags.html`

Labelled Sequences

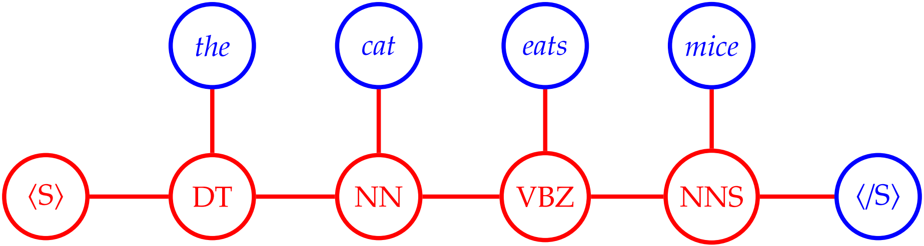
- We are interested in the probability of sequences like:

flies	like	the	wind		flies	like	the	wind
NNS	VB	DT	NN	**or**	VBZ	P	DT	NN

- In normal text, we see the words, but not the tags.
- Consider the POS tags to be underlying skeleton of the sentence, unseen but influencing the sentence shape.
- A structure like this, consisting of a **hidden** state sequence, and a related **observation** sequence can be modelled as a *Hidden Markov Model*.

Hidden Markov Models

The generative story:



$$P(S,O) = P(\text{DT}|\langle S\rangle)\, P(\text{the}|\text{DT})\, P(\text{NN}|\text{DT})\, P(\text{cat}|\text{NN})\, P(\text{VBZ}|\text{NN})\, P(\text{eats}|\text{VBZ})\, P(\text{NNS}|\text{VBZ})\, P(\text{mice}|\text{NNS})\, P(\langle /S\rangle|\text{NNS})$$

Hidden Markov Models

For a bi-gram HMM, with O_1^N:

$$P(S, O) = \prod_{i=1}^{N+1} \mathbf{P(s_i|s_{i-1})P(o_i|s_i)} \quad \text{where} \quad s_0 = \langle S \rangle,\ s_{N+1} = \langle /S \rangle$$

- The **transition probabilities** model the probabilities of moving from state to state.
- The **emission probabilities** model the probability that a state *emits* a particular observation.

Using HMMs

The HMM models the process of generating the labelled sequence. We can use this model for a number of tasks:

- $P(S, O)$ given S and O
- $P(O)$ given O
- S that maximises $P(S|O)$ given O
- $P(s_x|O)$ given O
- We can also learn the model parameters, given a set of observations.

Estimation

As so often in NLP, we learn an HMM from labelled data:

Transition probabilities

Based on a training corpus of previously tagged text, with tags as our state, the MLE can be computed from the counts of observed tags:

$$P(t_i|t_{t-1}) = \frac{C(t_{i-1}, t_i)}{C(t_{i-1})}$$

Emission probabilities

Computed from relative frequencies in the same way, with the words as observations:

$$P(w_i|t_j) = \frac{C(t_i, w_j)}{C(t_i)}$$

Implementation Issues

$$\begin{aligned} P(S,O) &= P(s_1|\langle S\rangle)P(o_1|s_1)P(s_2|s_1)P(o_2|s_2)P(s_3|s_2)P(o_3|s_3)\ldots \\ &= 0.0429 \times 0.0031 \times 0.0044 \times 0.0001 \times 0.0072 \times \ldots \end{aligned}$$

- Multiplying many small probabilities → underflow
- Solution: work in log(arithmic) space:
  - $\log(AB) = \log(A) + \log(B)$
  - hence $P(A)P(B) = \exp(\log(A) + \log(B))$
  - $\log(P(S,O)) = -1.368 + -2.509 + -2.357 + -4 + -2.143 + \ldots$

The issues related to MLE / smoothing that we discussed for n-gram models also applies here …

Ice Cream and Global Warming

Missing records of weather in Baltimore for Summer 2007

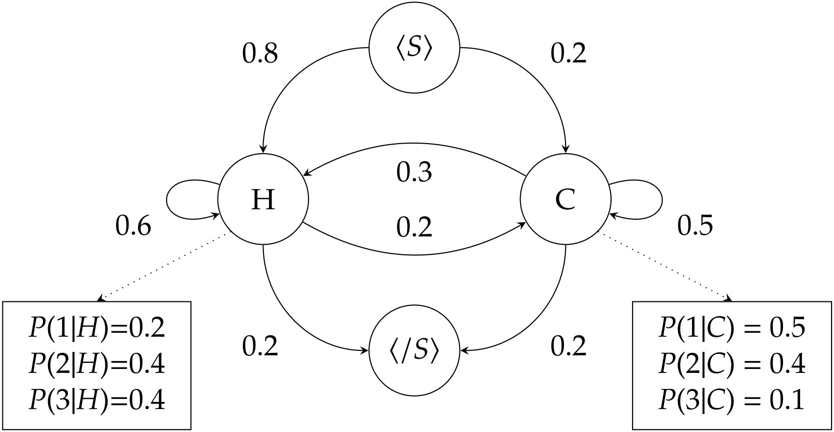
- Jason likes to eat ice cream.
- He records his daily ice cream consumption in his diary.
- The number of ice creams he ate was influenced, but not entirely determined by the weather.
- Today's weather is partially predictable from yesterday's.

A Hidden Markov Model!

with:

- Hidden states: $\{H, C\}$ (plus pseudo-states $\langle S \rangle$ and $\langle /S \rangle$)
- Observations: $\{1, 2, 3\}$

Ice Cream and Global Warming



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- $P(s_x|O)$ given O
- We can also learn the model parameters, given a set of observations.

Part-of-Speech Tagging

We want to find the tag sequence, given a word sequence. With tags as our states and words as our observations, we know:

$$P(S, O) = \prod_{i=1}^{N+1} P(s_i | s_{i-1}) P(o_i | s_i)$$

We want: $P(S|O) = \dfrac{P(S, O)}{P(O)}$

Actually, we want the state sequence that maximises $P(S|O)$:

$$S_{\text{best}} = \arg\max_{S} \frac{P(S, O)}{P(O)}$$

Since $P(O)$ always is the same, we can drop the denominator.

Decoding

Task

What is the most likely state sequence S, given an observation sequence O and an HMM.

HMM

$P(H\|\langle S\rangle) = 0.8$	$P(C\|\langle S\rangle) = 0.2$
$P(H\|H) = 0.6$	$P(C\|H) = 0.2$
$P(H\|C) = 0.3$	$P(C\|C) = 0.5$
$P(\langle /S\rangle\|H) = 0.2$	$P(\langle /S\rangle\|C) = 0.2$
$P(1\|H) = 0.2$	$P(1\|C) = 0.5$
$P(2\|H) = 0.4$	$P(2\|C) = 0.4$
$P(3\|H) = 0.4$	$P(3\|C) = 0.1$

if $O = 3\ 1\ 3$

$\langle S\rangle$	H	H	H	$\langle /S\rangle$	0.0018432
$\langle S\rangle$	H	H	C	$\langle /S\rangle$	0.0001536
$\langle S\rangle$	H	C	H	$\langle /S\rangle$	0.0007680
$\langle S\rangle$	H	C	C	$\langle /S\rangle$	0.0003200
$\langle S\rangle$	C	H	H	$\langle /S\rangle$	0.0000576
$\langle S\rangle$	C	H	C	$\langle /S\rangle$	0.0000048
$\langle S\rangle$	C	C	H	$\langle /S\rangle$	0.0001200
$\langle S\rangle$	C	C	C	$\langle /S\rangle$	0.0000500

Dynamic Programming

For (only) two states and a (short) observation sequence of length three, comparing all possible sequences is workable, but …

- for N observations and L states, there are L^N sequences
- we do the same calculations over and over again

Enter **dynamic programming**:

- records sub-problem solutions for further re-use
- useful when a complex problem can be described recursively
- examples: Dijkstra's shortest path, minimum edit distance, longest common subsequence, **Viterbi algorithm**

Viterbi Algorithm

Recall our problem:

$$\text{maximise } P(s_1 \dots s_n | o_1 \dots o_n) = P(s_1|s_0)P(o_1|s_1)P(s_2|s_1)P(o_2|s_2)\dots$$

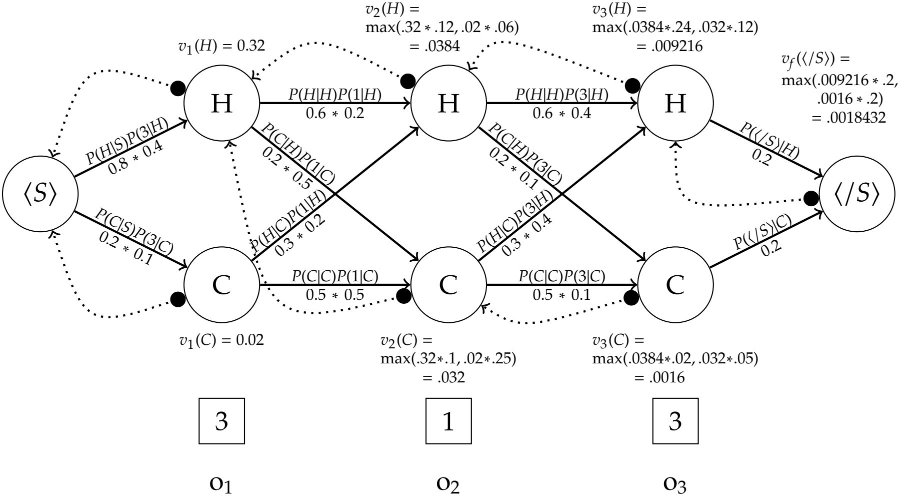
Our recursive sub-problem:

$$v_i(x) = \max_{k=1}^{L} [v_{i-1}(k) \cdot P(x|k) \cdot P(o_i|x)]$$

The variable $v_i(x)$ represents the maximum probability that the i-th state is x, given that we have seen O_1^i.

At each step, we record backpointers showing which previous state led to the maximum probability.

An Example of the Viterbi Algorithmn



⟨ H H H ⟩

Pseudocode for the Viterbi Algorithm

Input: *observations* of length N, *state set* of size L
Output: *best-path*
create a path probability matrix $viterbi[N, L+2]$
create a path backpointer matrix $backpointer[N, L+2]$
for each *state s from 1 to L* **do**
    $viterbi[1, s] \leftarrow trans(\langle S \rangle, s) \times emit(o_1, s)$
    $backpointer[1, s] \leftarrow 0$
end
for each *time step i from 2 to N* **do**
    for each *state s from 1 to L* **do**
        $viterbi[i, s] \leftarrow \max_{s'=1}^{L} viterbi[i-1, s'] \times trans(s', s) \times emit(o_i, s)$
        $backpointer[i, s] \leftarrow \arg\max_{s'=1}^{L} viterbi[i-1, s'] \times trans(s', s)$
    end
end
$viterbi[N, L+1] \leftarrow \max_{s=1}^{L} viterbi[s, N] \times trans(s, \langle /S \rangle)$
$backpointer[N, L+1] \leftarrow \arg\max_{s=1}^{L} viterbi[N, s] \times trans(s, \langle /S \rangle)$
return the path by following backpointers from $backpointer[N, L+1]$

Diversion: Complexity and O(N)

Big-O notation describes the complexity of an algorithm.

- it describes the worst-case *order of growth* in terms of the size of the input
- only the largest order term is represented
- constant factors are ignored
- determined by looking at loops in the code

Pseudocode for the Viterbi Algorithm

Input: *observations* of length N, *state set* of length L
Output: *best-path*
create a path probability matrix $viterbi[N, L+2]$
create a path backpointer matrix $backpointer[N, L+2]$
for each *state s from 1 to L* **do** L
    $viterbi[1, s] \leftarrow trans(\langle S \rangle, s) \times emit(o_1, s)$
    $backpointer[1, s] \leftarrow 0$
end
for each *time step i from 2 to N* **do** N
    for each *state s from 1 to L* **do** L
        $viterbi[i, s] \leftarrow \max_{s'=1}^{L} viterbi[i-1, s'] \times trans(s', s) \times emit(o_i, s)$ L
        $backpointer[i, s] \leftarrow \arg\max_{s'=1}^{L} viterbi[i-1, s'] \times trans(s', s)$
    end
end
$viterbi[N, L+1] \leftarrow \max_{s=1}^{L} viterbi[s, N] \times trans(s, \langle /S \rangle)$
$backpointer[N, L+1] \leftarrow \arg\max_{s=1}^{L} viterbi[N, s] \times trans(s, \langle /S \rangle)$
return the path by following backpointers from $backpointer[N, L+1]$ N

$$O(L^2 N)$$

Using HMMs

The HMM models the process of generating the labelled sequence. We can use this model for a number of tasks:

- $P(S, O)$ given S and O
- **$P(O)$ given O**
- S that maximises $P(S|O)$ given O
- $P(s_x|O)$ given O
- We can also learn the model parameters, given a set of observations.

Computing Likelihoods

Task

Given an observation sequence O, determine the likelihood $P(O)$, according to the HMM.

Compute the **sum over all possible state sequences**:

$$P(O) = \sum_{S} P(O, S)$$

For example, the ice cream sequence 3 1 3:

$$\begin{aligned} P(3\ 1\ 3) = \quad & P(3\ 1\ 3, \text{cold cold cold}) + \\ & P(3\ 1\ 3, \text{cold cold hot}) + \\ & P(3\ 1\ 3, \text{hot hot cold}) + \ldots \quad \Rightarrow O(L^N N) \end{aligned}$$

The Forward Algorithm

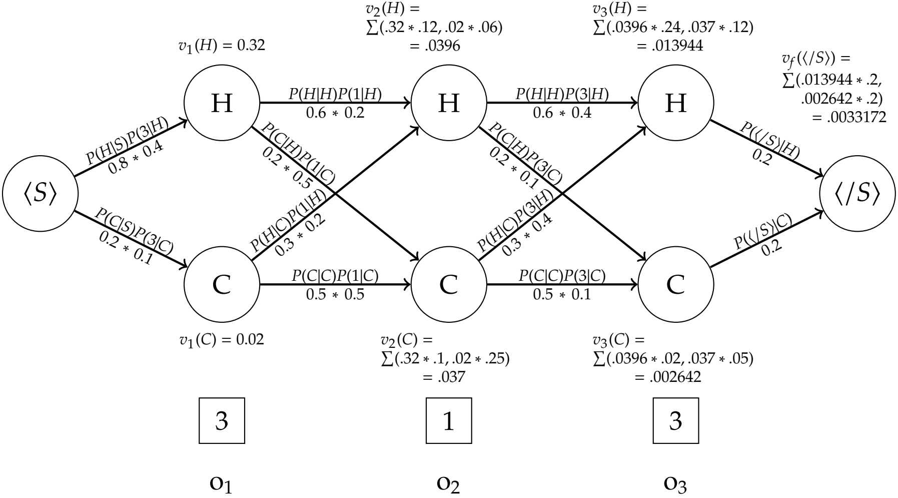
Again, we use **dynamic programming**—storing and reusing the results of partial computations in a **trellis** α.

Each cell in the trellis stores the probability of being in state s_x after seeing the first i observations:

$$\begin{aligned}\alpha_i(x) &= P(o_1 \ldots o_i, s_i = x) \\ &= \sum_{k=1}^{L} \alpha_{i-1}(k) \cdot P(x|k) \cdot P(o_i|x)\end{aligned}$$

Note $\sum$, instead of the max in Viterbi.

An Example of the Forward Algorithmn



$$P(3\,1\,3) = 0.0033172$$

Pseudocode for the Forward Algorithm

Input: *observations* of length N, *state set* of length L
Output: *forward-probability*
create a probability matrix $forward[N, L+2]$
for each *state s from 1 to L* **do**
  $forward[1, s] \leftarrow trans(\langle S \rangle, s) \times emit(o_1, s)$
end
for each *time step i from 2 to N* **do**
  for each *state s from 1 to L* **do**
    $forward[i, s] \leftarrow$
    $\sum_{s'=1}^{L} forward[i-1, s] \times trans(s', s) \times emit(o_t, s)$
  end
end
$forward[N, L+1] \leftarrow \sum_{s=1}^{L} forward[N, s] \times trans(s, \langle /S \rangle)$
return $forward[N, L+1]$

Tagger Evaluation

To evaluate a part-of-speech tagger (or any classification system) we:

- train on a labelled training set
- test on a *separate* test set

For a POS tagger, the standard evaluation metric is tag accuracy:

$$Acc = \frac{\text{number of correct tags}}{\text{number of words}}$$

The other metric sometimes used is *error rate*:

$$\textit{error rate} = 1 - Acc$$

Summary

Understand

- Why does dynamic programming save time, and what type of problems can it be used for?
- What is the complexity of the Viterbi algorithm?

Coming Up

- Context-free grammars
- Most likely trees