



*INF4820: Algorithms for
Artificial Intelligence and
Natural Language Processing*

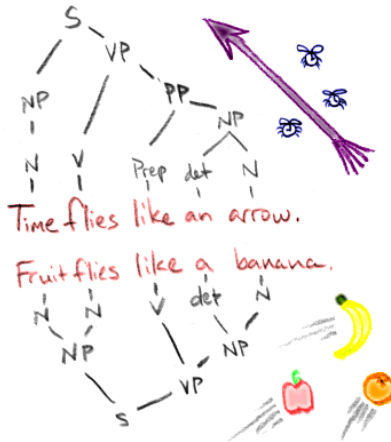
Chart Parsing

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Syntactic Structure and Ambiguity



(Courtesy of the *Speculative Grammarian*, –the journal of satirical linguistics.)

Context-Free Grammars



Formally, a CFG is a quadruple: $G = \langle C, \Sigma, P, S \rangle$

- ▶ C is the set of categories (aka *non-terminals*),
 - ▶ $\{S, NP, VP, V\}$
- ▶ Σ is the vocabulary (aka *terminals*),
 - ▶ $\{\text{Kim, snow, adores, in}\}$
- ▶ P is a set of category rewrite rules (aka *productions*)

$S \rightarrow NP VP$

$NP \rightarrow \text{Kim}$

$VP \rightarrow V NP$

$NP \rightarrow \text{snow}$

$V \rightarrow \text{adores}$

- ▶ $S \in C$ is the *start symbol*, a filter on complete results;
- ▶ for each rule $\alpha \rightarrow \beta_1, \beta_2, \dots, \beta_n \in P$: $\alpha \in C$ and $\beta_i \in C \cup \Sigma$

The Grammar of Spanish

$S \rightarrow NP VP \quad \{VP(NP)\}$

$VP \rightarrow V NP \quad \{V(NP)\}$

$VP \rightarrow VP PP \quad \{PP(VP)\}$

$PP \rightarrow P NP \quad \{P(NP)\}$

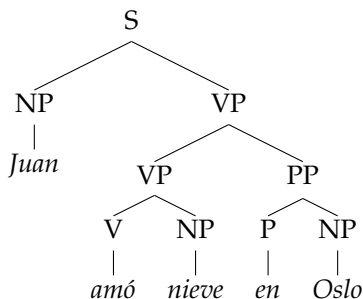
$NP \rightarrow \text{"nieve"} \quad \{\text{snow}\}$

$NP \rightarrow \text{"Juan"} \quad \{\text{John}\}$

$NP \rightarrow \text{"Oslo"} \quad \{\text{Oslo}\}$

$V \rightarrow \text{"amó"} \quad \{\lambda b \lambda a \text{ adore}(a, b)\}$

$P \rightarrow \text{"en"} \quad \{\lambda d \lambda c \text{ in}(c, d)\}$



Juan amó nieve en Oslo

Probabilistic Context-Free Grammars

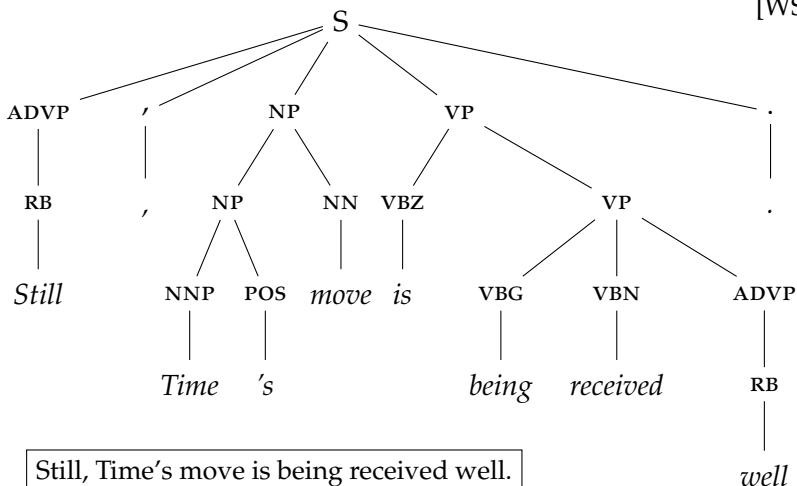


- ▶ We are interested, not just in which trees apply to a sentence, but also to which tree is **most likely**.
- ▶ Probabilistic context-free grammars (PCFGs) augment CFGs by adding probabilities to each production, e.g.
 - ▶ $S \rightarrow NP VP$ 0.6
 - ▶ $S \rightarrow NP VP PP$ 0.4
- ▶ These are conditional probabilities — the probability of the right hand side (RHS) given the left hand side (LHS)
 - ▶ $P(S \rightarrow NP VP) = P(NP VP|S)$
- ▶ We can learn these probabilities from a treebank, again using Maximum Likelihood Estimation.

Estimating PCFGs (1/3)



[WSJ 2350]



Estimating PCFGs (2/3)



(S	RB → Still	1
(ADVP (RB "Still"))	AVP → RB	2
(, ",")	, → ,	1
(NP	NNP → Time	1
(NP (NNP "Time") (POS "'s"))	POS → 's	1
(NN "move"))	NP → NNP POS	1
(VP	NN → move	1
(VBZ "is")	NP → NP NN	1
(VP	VBZ → is	1
(VBG "being")	VBG → being	1
(VP	VCN → received	1
(VBN "received")	RB → well	1
(ADVP (RB "well"))))	VP → VBN ADVP	1
(\ . ".")	VP → VBG VP	1
	\. → .	1
	S → ADVP , NP VP \.	1
	START → S	1

Estimating PCFGs (3/3)



Once we have counts of all the rules, we turn them into probabilities.

$S \rightarrow ADVP \mid, \mid NP VP \setminus.$	50	$S \rightarrow NP VP \setminus.$	400
$S \rightarrow NP VP PP \setminus.$	350	$S \rightarrow VP !$	100
$S \rightarrow NP VP S \setminus.$	200	$S \rightarrow NP VP$	50

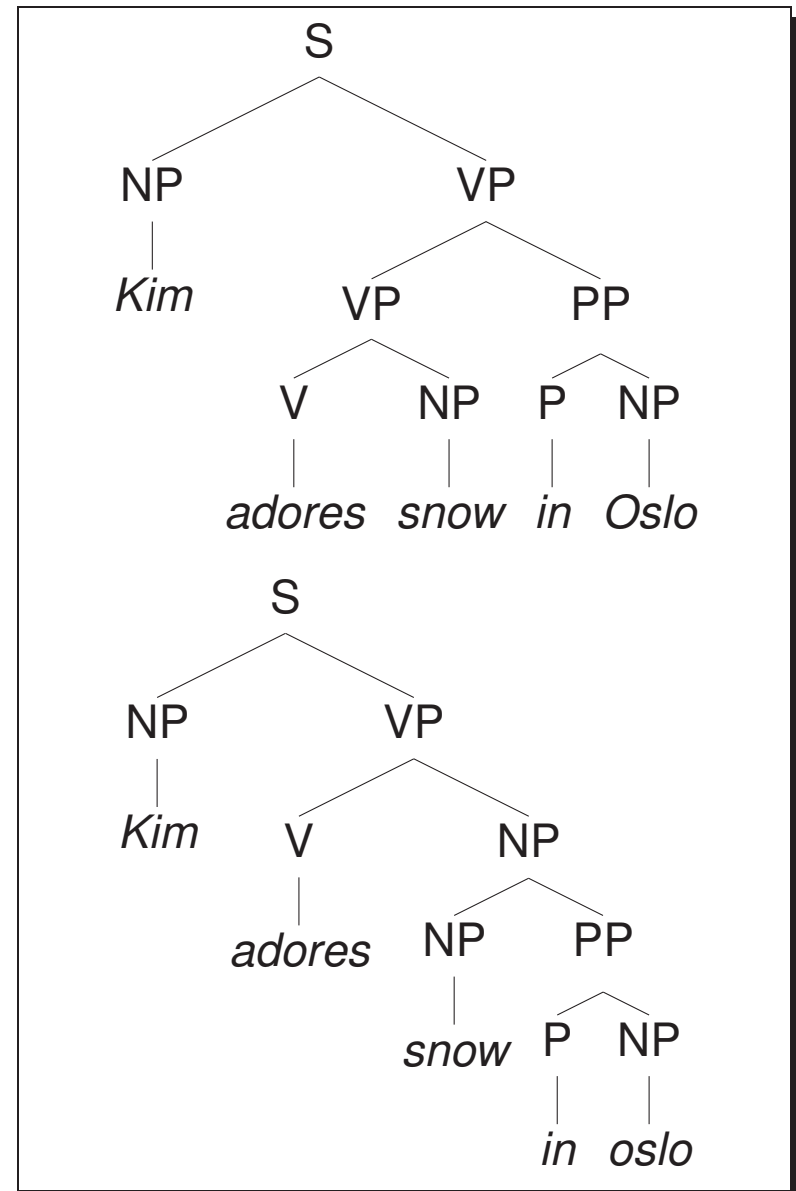
$$\begin{aligned}P(S \rightarrow ADVP \mid, \mid NP VP \setminus.) &\approx \frac{C(S \rightarrow ADVP \mid, \mid NP VP \setminus.)}{C(S)} \\ &= \frac{50}{1150} \\ &= 0.0435\end{aligned}$$

Parsing with CFGs: Moving to a Procedural View

$S \rightarrow NP VP$
 $VP \rightarrow V \mid V NP \mid VP PP$
 $NP \rightarrow NP PP$
 $PP \rightarrow P NP$
 $NP \rightarrow Kim \mid snow \mid Oslo$
 $V \rightarrow adores$
 $P \rightarrow in$

All Complete Derivations

- are rooted in the start symbol S ;
- label internal nodes with categories $\in C$, leafs with words $\in \Sigma$;
- instantiate a grammar rule $\in P$ at each local subtree of depth one.



Recursive Descend: A Naïve Parsing Algorithm

Control Structure

- top-down: given a parsing goal α , use all grammar rules that rewrite α ;
- successively instantiate (extend) the right-hand sides of each rule;
- for each β_i in the RHS of each rule, recursively attempt to parse β_i ;
- termination: when α is a prefix of the input string, parsing succeeds.

(Intermediate) Results

- Each result records a (partial) tree and remaining input to be parsed;
- complete results consume the full input string and are rooted in S ;
- whenever a RHS is fully instantiated, a new tree is built and returned;
- all results at each level are combined and successively accumulated.



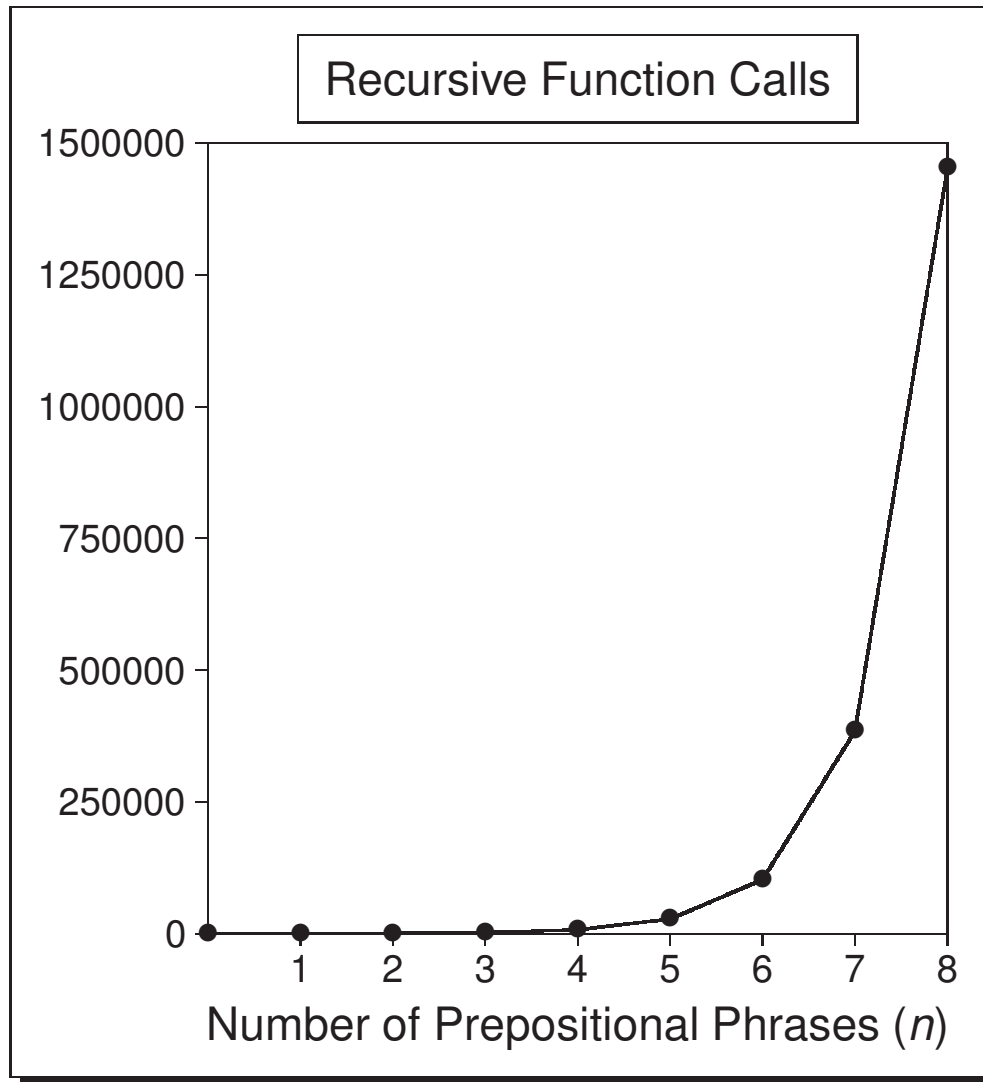
The Recursive Descent Parser

```
(defun parse (input goal)
  (if (equal (first input) goal)
      (let ((edge (make-edge :category (first input))))
        (list (make-parse :edge edge :input (rest input))))
      (loop
        for rule in (rules-deriving goal)
        append (extend-parse (rule-lhs rule) nil (rule-rhs rule) input))))
```

```
(defun extend-parse (goal analyzed unanalyzed input)
  (if (null unanalyzed)
      (let ((tree (cons goal analyzed)))
        (list (make-parse :tree tree :input input)))
      (loop
        for parse in (parse input (first unanalyzed))
        append (extend-parse
                  goal (append analyzed (list (parse-tree parse)))
                  (rest unanalyzed)
                  (parse-input parse)))))
```



Quantifying the Complexity of the Parsing Task



Kim adores snow (in Oslo)ⁿ

<i>n</i>	trees	calls
0	1	46
1	2	170
2	5	593
3	14	2,093
4	42	7,539
5	132	27,627
6	429	102,570
7	1430	384,566
8	4862	1,452,776
⋮	⋮	⋮



Top-Down vs. Bottom-Up Parsing

Top-Down (Goal-Oriented)

- Left recursion (e.g. a rule like ‘ $VP \rightarrow VP PP$ ’) causes infinite recursion;
 - search is uninformed by the (observable) input: can hypothesize many unmotivated sub-trees, assuming terminals (words) that are not present;
- assume bottom-up as basic search strategy for remainder of the course.

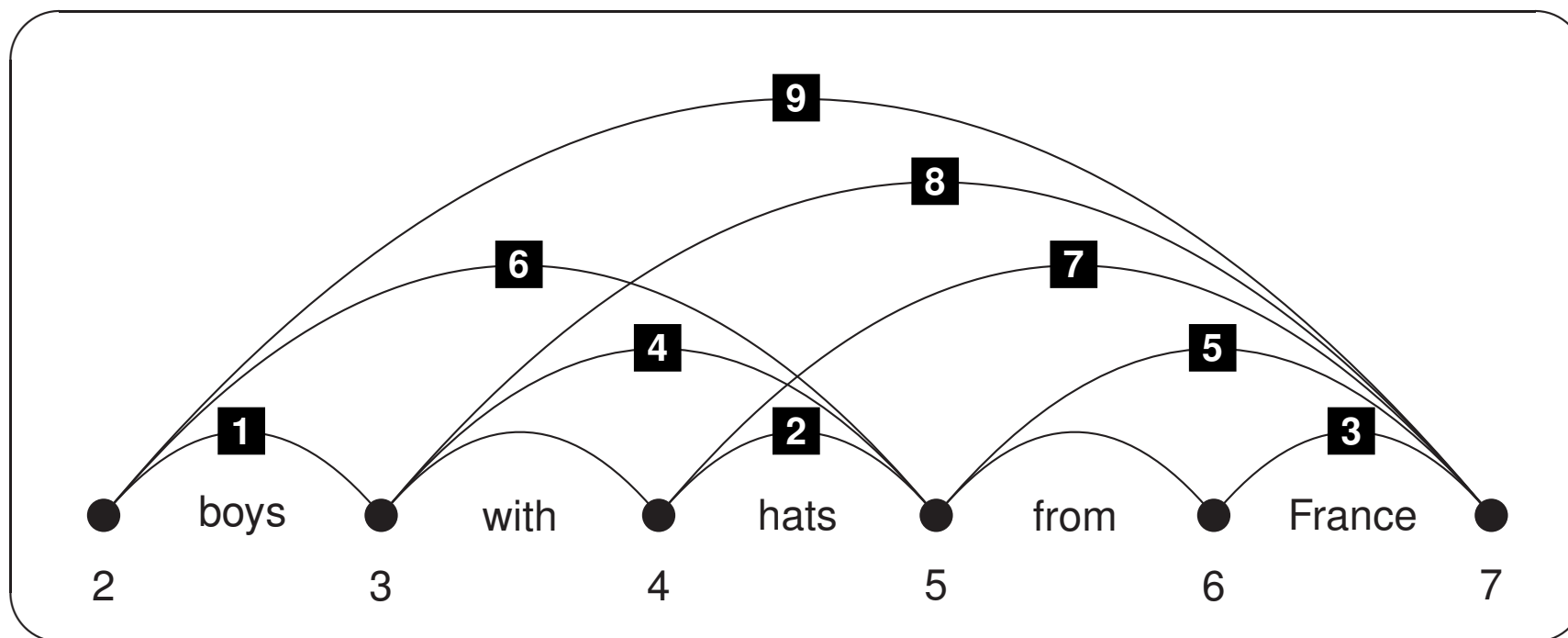
Bottom-Up (Data-Oriented)

- unary (left-recursive) rules (e.g. ‘ $NP \rightarrow NP$ ’) would still be problematic;
- lack of parsing goal: compute all possible derivations for, say, the input *adores snow*; however, it is ultimately rejected since it is not sentential;
- availability of partial analyses desirable for, at least, some applications.



A Key Insight: Local Ambiguity

- For many substrings, more than one way of deriving the same category;
- NPs: **1** | **2** | **3** | **6** | **7** | **9**; PPs: **4** | **5** | **8**; **9** \equiv **1** + **8** | **6** + **5**;
- *parse forest* — a single item represents multiple trees [Billot & Lang, 89].



The CKY (Cocke, Kasami, & Younger) Algorithm

```

for ( $0 \leq i < |input|$ ) do
   $chart_{[i,i+1]} \leftarrow \{\alpha \mid \alpha \rightarrow input_i \in P\};$ 
for ( $1 \leq l < |input|$ ) do
  for ( $0 \leq i < |input| - l$ ) do
    for ( $1 \leq j \leq l$ ) do
      if ( $\alpha \rightarrow \beta_1 \beta_2 \in P \wedge \beta_1 \in chart_{[i,i+j]} \wedge \beta_2 \in chart_{[i+j,i+l+1]}$ ) then
         $chart_{[i,i+l+1]} \leftarrow chart_{[i,i+l+1]} \cup \{\alpha\};$ 

```

$[0,2] \leftarrow [0,1] + [1,2]$

...

$[0,5] \leftarrow [0,1] + [1,5]$

$[0,5] \leftarrow [0,2] + [2,5]$

$[0,5] \leftarrow [0,3] + [3,5]$

$[0,5] \leftarrow [0,4] + [4,5]$

	1	2	3	4	5
0	NP		S		S
1		V	VP		VP
2			NP		NP
3				P	PP
4					NP



Limitations of the CKY Algorithm

Built-In Assumptions

- *Chomsky Normal Form* grammars: $\alpha \rightarrow \beta_1\beta_2$ or $\alpha \rightarrow \gamma$ ($\beta_i \in C$, $\gamma \in \Sigma$);
- breadth-first (aka exhaustive): always compute all values for each cell;
- rigid control structure: bottom-up, left-to-right (one diagonal at a time).

Generalized Chart Parsing

- Liberate order of computation: no assumptions about earlier results;
- *active edges* encode partial rule instantiations, 'waiting' for additional (adjacent and passive) constituents to complete: $[1, 2, VP \rightarrow V \bullet NP]$;
- parser can fill in chart cells in *any* order and guarantee completeness.



Chart Parsing — Specialized Dynamic Programming

Basic Notions

- Use *chart* to record partial analyses, indexing them by string positions;
- count inter-word vertices; CKY: chart row is *start*, column *end* vertex;
- treat multiple ways of deriving the same category for some substring as *equivalent*; pursue only once when combining with other constituents.

Key Benefits

- Dynamic programming (memoization): avoid recomputation of results;
- efficient indexing of constituents: no search by start or end positions;
- compute *parse forest* with exponential ‘extension’ in *polynomial* time.



Generalized Chart Parsing

- The parse *chart* is a two-dimensional matrix of *edges* (aka chart items);
- an edge is a (possibly partial) rule instantiation over a substring of input;
- the chart indexes edges by start and end string position (aka vertices);
- dot in rule RHS indicates degree of completion: $\alpha \rightarrow \beta_1 \dots \beta_{i-1} \bullet \beta_i \dots \beta_n$;
- *active* edges (aka *incomplete* items) — partial RHS: $[1, 2, VP \rightarrow V \bullet NP]$;
- *passive* edges (aka *complete* items) — full RHS: $[1, 3, VP \rightarrow V NP \bullet]$;

The Fundamental Rule

$$[i, j, \alpha \rightarrow \beta_1 \dots \beta_{i-1} \bullet \beta_i \dots \beta_n] + [j, k, \beta_i \rightarrow \gamma^+ \bullet] \\ \mapsto [i, k, \alpha \rightarrow \beta_1 \dots \beta_i \bullet \beta_{i+1} \dots \beta_n]$$



An Example of a (Near-)Complete Chart

	1	2	3	4	5
0	$NP \rightarrow NP \bullet PP$ $S \rightarrow NP \bullet VP$ $NP \rightarrow kim \bullet$				$S \rightarrow NP VP \bullet$
1		$VP \rightarrow V \bullet NP$ $V \rightarrow adores \bullet$	$VP \rightarrow VP \bullet PP$ $VP \rightarrow V NP \bullet$		$VP \rightarrow VP \bullet PP$ $VP \rightarrow VP PP \bullet$ $VP \rightarrow V PP \bullet$
2			$NP \rightarrow NP \bullet PP$ $NP \rightarrow snow \bullet$		$NP \rightarrow NP \bullet PP$ $NP \rightarrow NP PP \bullet$
3				$PP \rightarrow P \bullet NP$ $P \rightarrow in \bullet$	$PP \rightarrow P NP \bullet$
4					$NP \rightarrow NP \bullet PP$ $NP \rightarrow oslo \bullet$

0 *Kim* 1 *adores* 2 *snow* 3 *in* 4 *Oslo* 5



(Even) More Active Edges

	0	1	2	3
0	$S \rightarrow \bullet NP VP$ $NP \rightarrow \bullet NP PP$ $NP \rightarrow \bullet kim$	$S \rightarrow NP \bullet VP$ $NP \rightarrow NP \bullet PP$ $NP \rightarrow kim \bullet$		$S \rightarrow NP VP \bullet$
1		$VP \rightarrow \bullet VP PP$ $VP \rightarrow \bullet V NP$ $V \rightarrow \bullet adores$	$VP \rightarrow V \bullet NP$ $V \rightarrow adores \bullet$	$VP \rightarrow VP \bullet PP$ $VP \rightarrow V NP \bullet$
2			$NP \rightarrow \bullet NP PP$ $NP \rightarrow \bullet snow$	$NP \rightarrow NP \bullet PP$ $NP \rightarrow snow \bullet$
3				

- Include all grammar rules as *epsilon* edges in each $chart_{[i,i]}$ cell.
- after initialization, apply *fundamental rule* until fixpoint is reached.



Combinatorics: Keeping Track of Remaining Work

The Abstract Goal

- Any chart parsing algorithm needs to check all pairs of adjacent edges.

A Naïve Strategy

- Keep iterating through the complete chart, combining all possible pairs, until no additional edges can be derived (i.e. the fixpoint is reached);
- frequent attempts to combine pairs multiple times: deriving ‘duplicates’.

An Agenda-Driven Strategy

- Combine each pair exactly once, viz. when both elements are available;
- maintain *agenda* of new edges, yet to be checked against chart edges;
- new edges go into agenda first, add to chart upon retrieval from agenda.



In Conclusion—What Happened this Week

Syntactic Structure

- Languages (formal or natural) exhibit complex, hierarchical structures;
- grammars encode rules of the language: dominance and sequencing;
- context-free grammar ‘generates’ a language: strings and derivations;
- ambiguity in natural language grows exponentially: a search problem;
- bounding (or ‘packing’) of local ambiguity mandatory for tractability;
- chart parsing uses dynamic programming: free order of computation.

Coming up Next

- Viterbi adaptation to parse forest; PTB parsing; parser evaluation; quiz.

