

Kurs INF5080

# Compression

Ifi, UiO

Norsk Regnesentral

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10110100



## This part of the course ...

- ... is held at Ifi, UiO ...  
(Wolfgang Leister)
- ... and at contains material from  
University College Karlsruhe  
(Peter Oel, Clemens Knoerzer)



## Content

- Information Theory
- Data Compression



## Preview

- Image Formats
  - JPEG / JIFF
  - Wavelet Compression
  - Fractal Compression
- Video Formats
  - MJPEG
  - MPEG



## Multimedia data types

- Images
- Graphics
- Video / Image sequences
- Audio
- 3D-Data
- Text / Documents
- others



## Multimedia data types

- Images
- Graphics
- Video / Image sequences
- Audio
- 3D-Data
- Text / Documents
- others



## Lossy Coding

- Applicable only for data types like:
  - Images
  - Films (image sequences)
  - Audio
- Use physiological capabilities and limitations of the senses to design compression methods



## Capabilities of the senses

- Eye
  - Eye recognises frequencies
  - Brightness is better recognised than colours.
  - Movement and flicker is recognised very strongly!
- Ear
  - Densely situated frequencies cover each other.
  - Frequencies characteristic of the ear



## Information Theory

- Symbols:  $A = a_0, a_1, \dots, a_{N-1}$
- Coding:  $C = c_0, c_1, \dots, c_{N-1}$ 
  - trivial Coding (constant Code Length)
    - $c_i = b_{i_0}, b_{i_1}, \dots, b_{i_{M-1}}$  for  $i \in [0, N-1]$
    - $b_{ij} \in \{0, 1\}$  (binary)
    - $M = \lceil \log_2 N \rceil$  (Code length, number of bits)
- Distribution:  $P = p_0, p_1, \dots, p_{N-1}$ 
  - probability for symbols

## Information Theory

Information content: (Entropy)

$$H_0 = -\sum_{i=0}^{N-1} p_i \log p_i \quad (\text{Basis} = 2 \Rightarrow \text{bit/codeword})$$

Example: Uniform distribution

$$p_i = \frac{1}{N}$$

$$H_0 = -\sum_{i=0}^{N-1} \frac{1}{N} \log \frac{1}{N} = -\log \frac{1}{N} = \log N$$

# Information theory

- Coding:  $C = c_0, c_1, \dots, c_{N-1}$

–  $c_i = b_{i0}, b_{i1}, \dots, b_{i_{l_i-1}}$

- $i \in [0, N-1]$
- $b_{ij} \in \{0, 1\}$
- $l_i = \text{length of code word } c_i$

- average code length:

$$L = \sum_{i=0}^{N-1} p_i l_i \geq H_0$$



# Information Theory

- When is  $L = H_0$  ? i.e.: average code Length = Entropy

$$l_i = -\log_2 p_i$$

$$p_i = \frac{1}{2^{l_i}} = \frac{1}{2^k}$$

uniform distribution

- What if  $p_i \neq \frac{1}{2^k}$  for all  $i$  ?

not uniform distribution



## Information Theory

- What if  $p_i \neq \frac{1}{2^k}$  ?

non uniform  
distribution

- Huffman Coding
- Group events into one code word:  
 $(a_1, \dots, a_n) \rightarrow c_i$
- Every  $(a_i)$ -combination must be available:  
 $N^n$  code words
- Arithmetic coding often better suitable

Intermezzo ...

## Why Data compression?

- An image says more than a thousand words ...
- An image needs more space than a thousand words ...
- Data are contain redundancy ...
- Humans love redundancy ...

# Compression techniques

lossless

run length encoding

with loss



# Compression techniques

lossless

run length encoding

with loss

## run length encoding:

```
00001110000000011000010111111000000000  
(4x0)(3x1)(8x0)(2x1)(4x0)(1x1)(1x0)(6x1)(10x0)  
4,3,8,2,4,1,1,6,10
```

## Examples:

- PCX
- Fax
- JPEG



# Compression techniques

lossless

run length encoding  
optimal Codes

with loss



# Compression techniques

lossless

run length encoding  
optimal Codes

Code words have different lengths

Length of code word dependent on probability:

(high probability → short Codewort)

(low probability → long Codewort)

Global and fixed code word table



Code word table is part of coding

## Compression techniques

### Example: Huffman-Coding:

**1.Step:** Events to be coded are sorted by probabilities (rising order).

**2.Step:** The events with least probabilities are removed from list, united to one event, and added sorted into list with sum of both probabilities.

**3.Step:** Repeat Step 2 until only one element is contained in list.

**4.Step:** Code words are generated by marking the edges in the binary tree by 0 and 1. Read the code word from top to bottom.

## Compression techniques

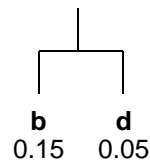
### Example Huffman-Coding:

<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>
0.2	0.15	0.4	0.05	0.2
<b>c</b>	<b>a</b>	<b>e</b>	<b>b</b>	<b>d</b>
0.4	0.2	0.2	0.15	0.05

# Compression techniques

Example: Huffman-Coding:

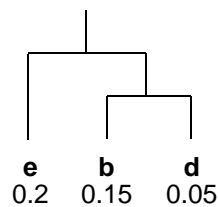
<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>
0.2	0.15	0.4	0.05	0.2
<b>c</b>	<b>a</b>	<b>e</b>	<b>b</b>	<b>d</b>
0.4	0.2	0.2	0.15	0.05
<b>c</b>	<b>a</b>	<b>e</b>	<b>bd</b>	
0.4	0.2	0.2	0.2	



# Compression techniques

Example: Huffman-Coding:

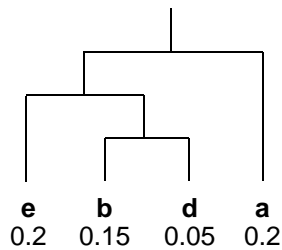
<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>
0.2	0.15	0.4	0.05	0.2
<b>c</b>	<b>a</b>	<b>e</b>	<b>b</b>	<b>d</b>
0.4	0.2	0.2	0.15	0.05
<b>c</b>	<b>a</b>	<b>e</b>	<b>bd</b>	
0.4	0.2	0.2	0.2	
<b>c</b>	<b>bde</b>	<b>a</b>		
0.4	0.4	0.2		



# Compression techniques

Example: Huffman-Coding:

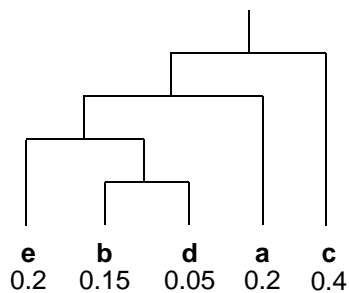
<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>
0.2	0.15	0.4	0.05	0.2
<b>c</b>	<b>a</b>	<b>e</b>	<b>b</b>	<b>d</b>
0.4	0.2	0.2	0.15	0.05
<b>c</b>	<b>a</b>	<b>e</b>	<b>bd</b>	
0.4	0.2	0.2	0.2	
<b>c</b>	<b>bde</b>	<b>a</b>		
0.4	0.4	0.2		
<b>bdea</b>	<b>c</b>			
0.6	0.4			



# Compression techniques

Example: Huffman-Coding:

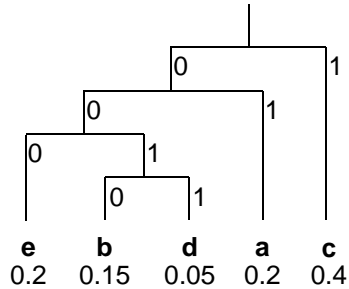
<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>
0.2	0.15	0.4	0.05	0.2
<b>c</b>	<b>a</b>	<b>e</b>	<b>b</b>	<b>d</b>
0.4	0.2	0.2	0.15	0.05
<b>c</b>	<b>a</b>	<b>e</b>	<b>bd</b>	
0.4	0.2	0.2	0.2	
<b>c</b>	<b>bde</b>	<b>a</b>		
0.4	0.4	0.2		
<b>bdea</b>	<b>c</b>			
0.6	0.4			
<b>bdeac</b>				
1.0				



# Compression techniques

## Example: Huffman-Coding:

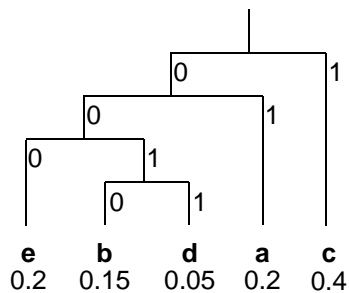
<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>
0.2	0.15	0.4	0.05	0.2
<b>c</b>	<b>a</b>	<b>e</b>	<b>b</b>	<b>d</b>
0.4	0.2	0.2	0.15	0.05
<b>c</b>	<b>a</b>	<b>e</b>	<b>bd</b>	
0.4	0.2	0.2	0.2	
<b>c</b>	<b>bde</b>	<b>a</b>		
0.4	0.4	0.2		
<b>bdea</b>	<b>c</b>			
0.6	0.4			
<b>bdeac</b>				
1.0				



# Compression techniques

## Example: Huffman-Coding:

<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>
0			5	0.2
<b>a</b>		01		
0	<b>b</b>	0010	5	0.05
<b>c</b>		1	1	
0	<b>d</b>	0011	2	
<b>e</b>		000		
0	<b>bdea</b>			
0.6	0.4			
<b>bdeac</b>				
1.0				



# Compression techniques

lossless

run length encoding  
optimal Codes  
adaptive Codes

with loss



# Compression techniques

lossless

run length encoding  
optimal Codes  
adaptive Codes

with loss

## Method by Ziv and Lempel

Code table is generated while coding

No need to transfer code table



# Compression techniques

lossless

run length encoding

Example: **a**b c a b a b a c

1	a
2	b
3	c
4	ab
5	
6	
7	
8	
...	

w: a | b  
↓  
Output: 1

with k

# Compression techniques

lossless

run length encoding

Example: a**b**c a b a b a c

1	a
2	b
3	c
4	ab
5	bc
6	
7	
8	
...	

w: b | c  
↓  
Output: 12

with k

# Compression techniques

lossless

run length encoding

with k

Example: a b c a b a b a c

1	a
2	b
3	c
4	ab
5	bc
6	ca
7	
8	
...	

w: c | a

Output: 123

# Compression techniques

lossless

run length encoding

with k

Example: a b c a b a b a c

1	a
2	b
3	c
4	ab
5	bc
6	ca
7	aba
8	
...	

w: a b | a

Output: 1234



# Compression techniques

lossless

run length encoding

Example: a b c a b a b a c

1	a
2	b
3	c
4	ab
5	bc
6	ca
7	aba
8	abac
...	

w: a b a c

Output: 12347

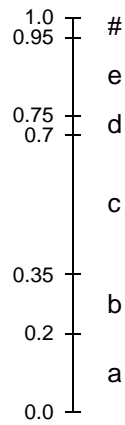
with k

## Arithmetic Coding

- Symbols are represented by probability intervals
- Symbol chains are represented by a concatenation of their probability intervals



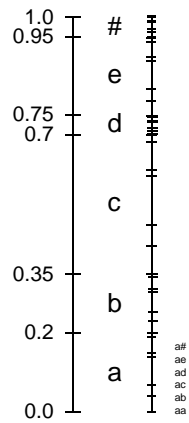
# Arithmetic Coding



A	a	b	c	d	e	#
P	0.2	0.15	0.35	0.05	0.2	0.05

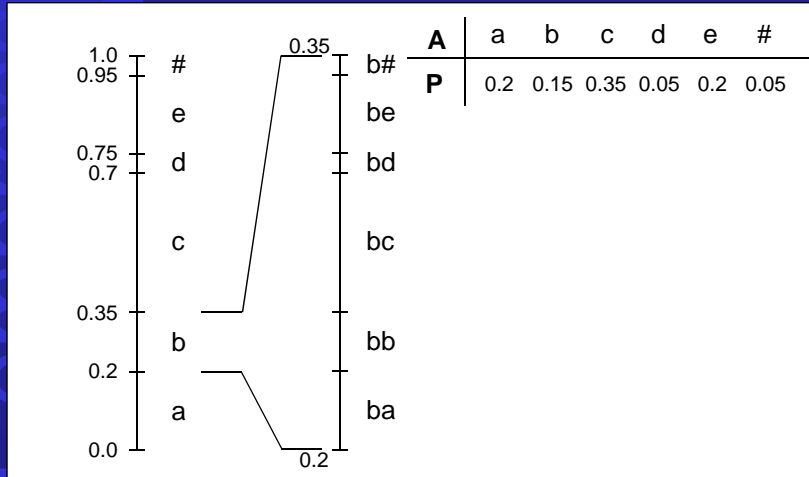
$\# = [0.95, 1.0)$  0.96.. 0.111111  
 $e = [0.75, 0.95)$  0.75 0.11  
 $d = [0.7, 0.75)$  0.71.. 0.10111  
 $c = [0.35, 0.7)$  0.5 0.1  
 $b = [0.2, 0.35)$  0.25 0.01  
 $a = [0.0, 0.2)$  0.125 0.001

# Arithmetic Coding



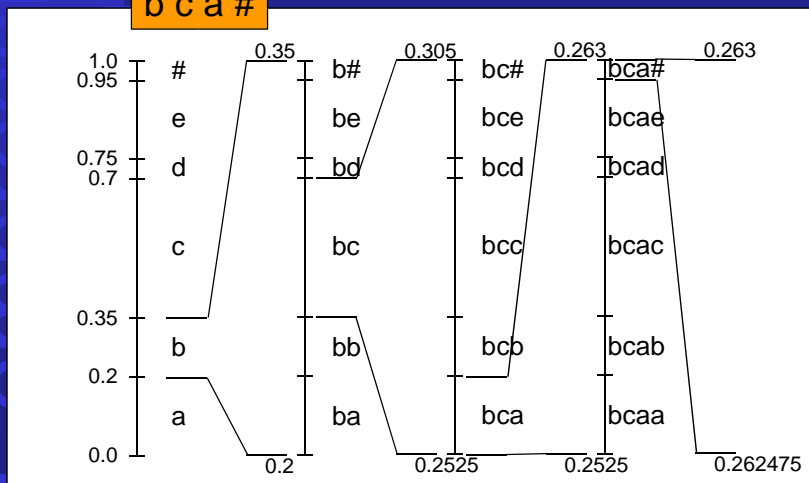
A	a	b	c	d	e	#
P	0.2	0.15	0.35	0.05	0.2	0.05

# Arithmetic Coding

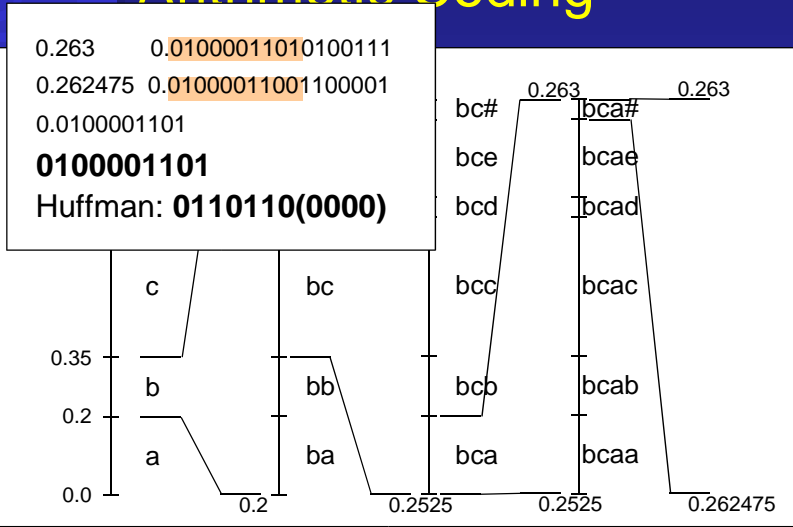


# Arithmetic Coding

**b c a #**



# Arithmetic Coding



# Compression techniques

lossless

run length encoding

optimal Codes

with I

## Quantising:

- Analogue    →    Digital
- 8 Bit        →    4 Bit
- RGB         →    Colour palette

## Clustering

# Compression techniques

lossless

run length encoding  
optimal Codes  
adaptive Codes

with loss

Quantising  
Clustering



# Compression techniques

lossless

run length encoding  
optimal Codes  
adaptive Codes

with loss

Quantising  
Clustering  
Descriptive

Transformation



## Image Data formats

lossless	with loss
PBM+	
GIF	-----> ?
PNG	
(JPEG)	JPEG
Wavelet Compr.	Wavelet Compr.



## The End of Lecture

