Wavelet Coding & JPEG 2000

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Contributions by

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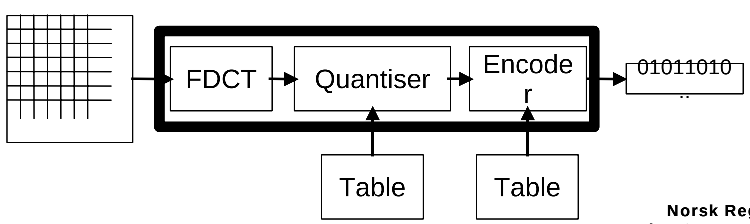
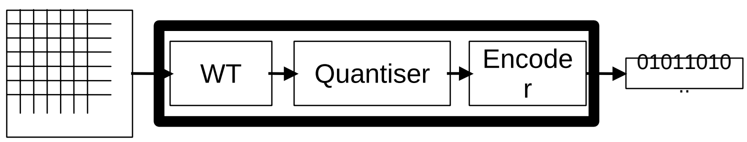
JPEG revisited

- JPEG …
  - Uses DCT on 8x8 blocks
  - Visible blocks for high compression rates
  - Is not good for compression rate of > 40:1
  - Low frequencies are not considered
  - provides competitive performance for the medium bit-rate range



Wavelet-Coding

- Good quality up to 60:1
- Linear degradation for higher compression rates
- No visible blocks
- JPEG 2000 Standard - ISO 15444
- JPEG-2000 mostly provides improved performance at for low and high bit-rates



Wavelets - Introduction

- Wavelets are used for
  - compression,
  - data modelling,
  - data analysis
  - signal processing
- Wavelets are suitable for representation of digital data in many applications.

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One-dimensional Case !

Wavelets

Model signals with:

- Fourier row development (whole area):

$$f(x)=a_0+\sum a_i \cos(u\,s(x))+b_i \sin(u\,s(x))$$

- DCT-based (8x8 blocks):

$$f(x)=a_0+\sum a_i \cos(u\,s(x))$$

- Wavelet-based (hierarchical):

$$f(x)=a_0+\sum a_i u_i(x)$$

- wavelets use piecewise constant base functions
- Average of wavelet function is 0

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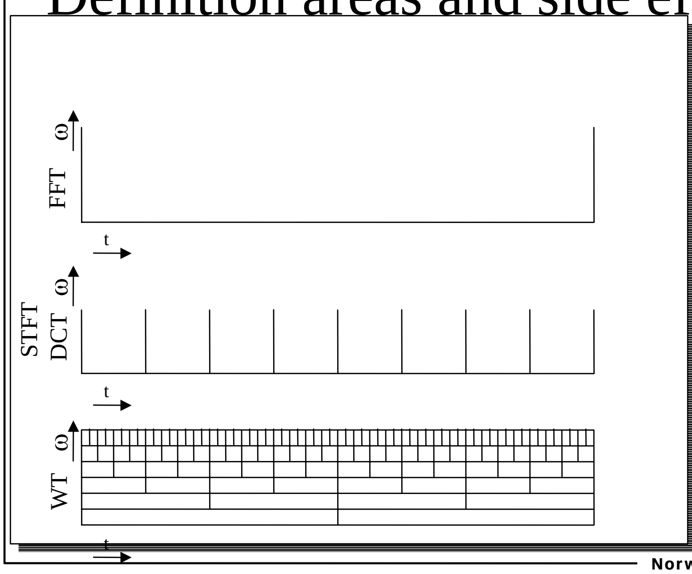
Definition areas and side effects



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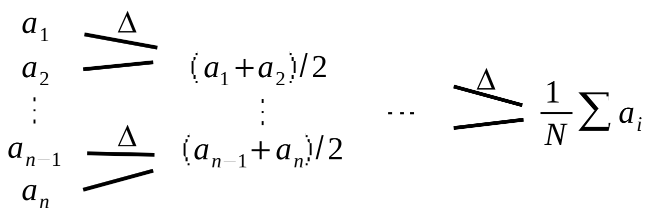
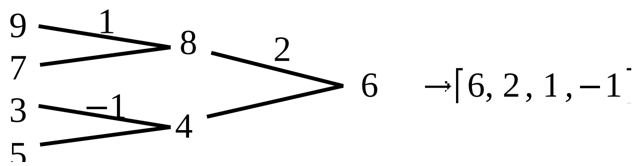
One-dimensional Case !

Wavelet Transform (1)





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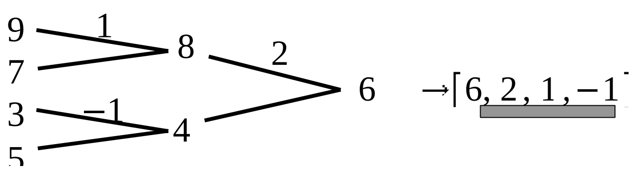
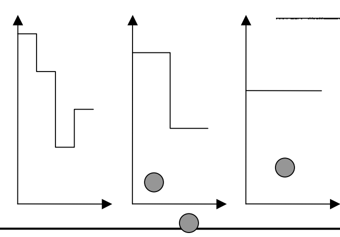
One-dimensional Case !

Wavelet Transform (2)



Resolution	Averages	Detail Coefficients
4	[9 7 3 5]	
2	[8 4]	[1 −1]
1	[6]	[2]



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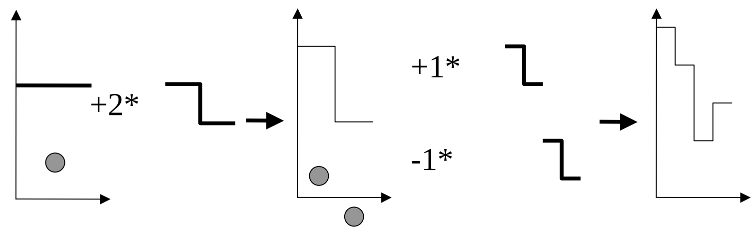
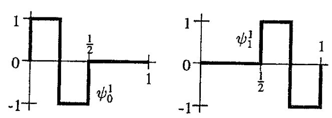
One-dimensional Case !

Wavelet Transform (3)

[6, 2, 1, -1]





Haar Wavelet

One-dimensional Case !

V^2-Basis and Haar Wavelet

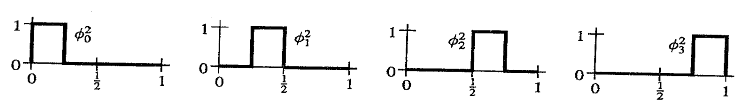
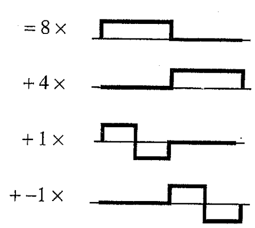


FIGURE 2.2 The box basis for V^2.

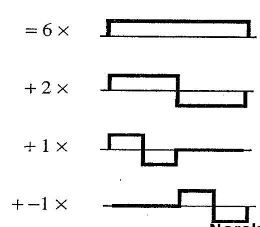
$$f(x)=c_0^2\Phi_0^2(x)+c_1^2\Phi_1^2(x)+c_2^2\Phi_2^2(x)+c_3^2\Phi_3^2(x)$$

$$[c_0^2, c_1^2, c_2^2, c_3^2] = [9,7,3,5]$$

$$\mathcal{I}(x) = c_0^1\,\phi_0^1(x) + c_1^1\,\phi_1^1(x) + d_0^1\,\psi_0^1(x) + d_1^1\,\psi_1^1(x)$$



$$\mathcal{I}(x) = c_0^0\,\phi_0^0(x) + d_0^0\,\psi_0^0(x) + d_0^1\,\psi_0^1(x) + d_1^1\,\psi_1^1(x)$$



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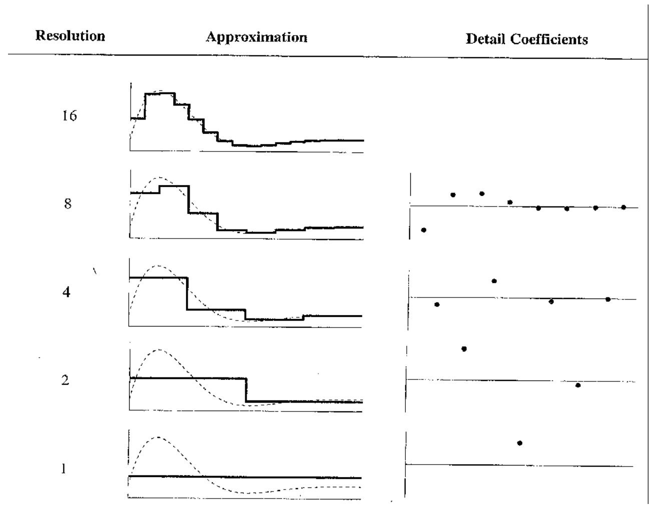
One-dimensional Case !

Another Example



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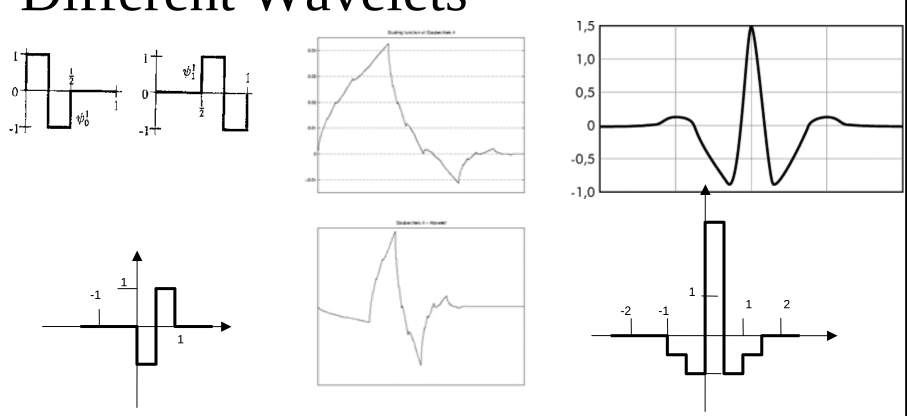
One-dimensional Case !

Different Wavelets



Haar Wavelet

Daubechies Wavelet
= 9/7 Wavelet

Lazy Wavelet
= 5/3 Wavelet

Average of wavelet function is 0

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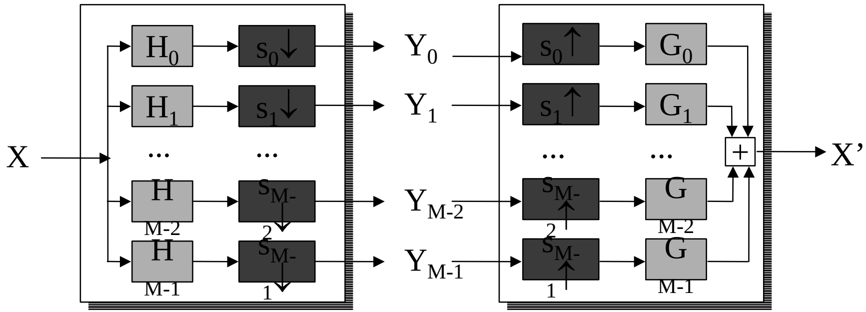
One-dimensional
Case !

Signal-Analysis / -Synthesis



$$\sum_i \frac{1}{s_i} = 1$$

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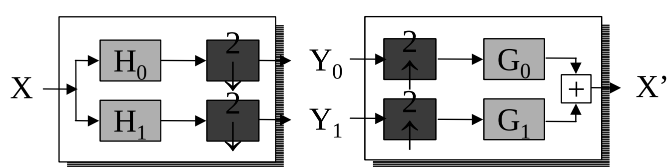
One-dimensional
Case !

Analysis/Synthesis



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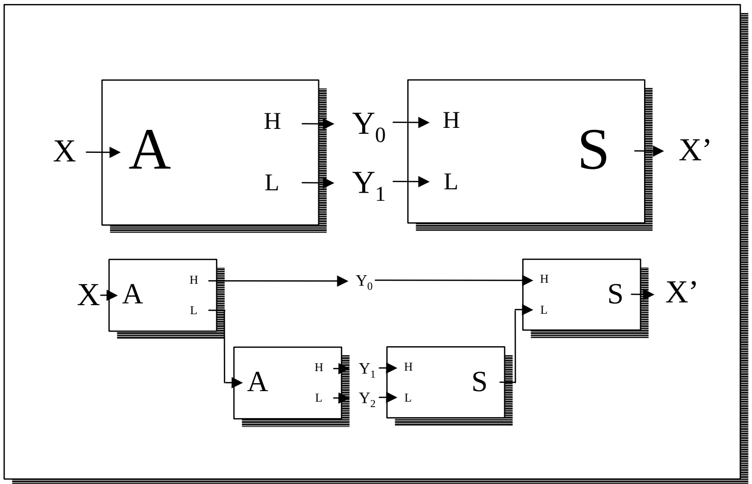
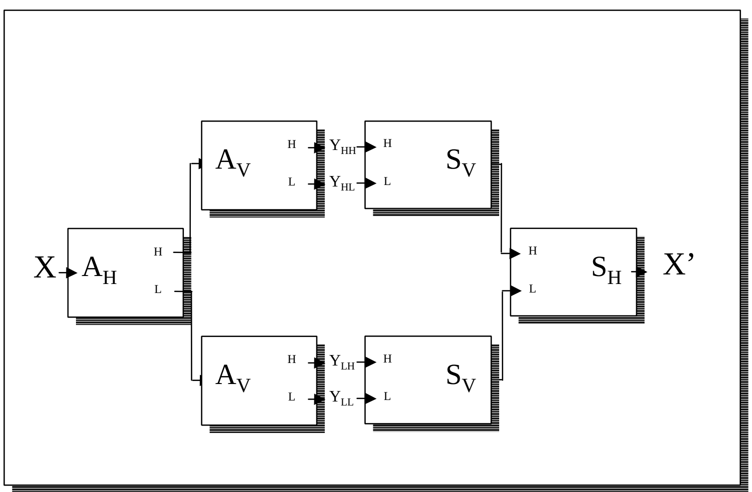
One-dimensional Case !

Analysis/Synthesis



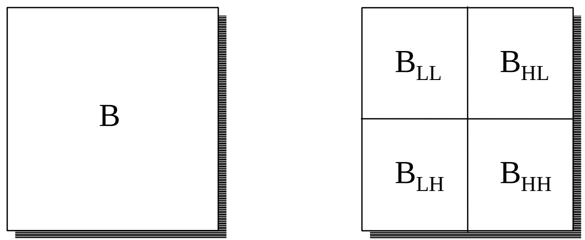
Two-dimensional Case !

Transformation of Images



Two-dimensional Case !

Transformation of Images



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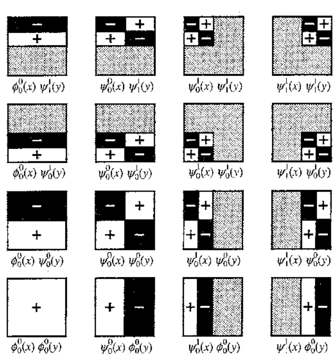
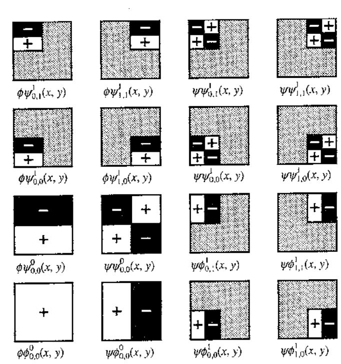
Two-dimensional Case !

Construction of Haar Basis



Standard construction of Haar basis



Non-Standard construction of Haar basis

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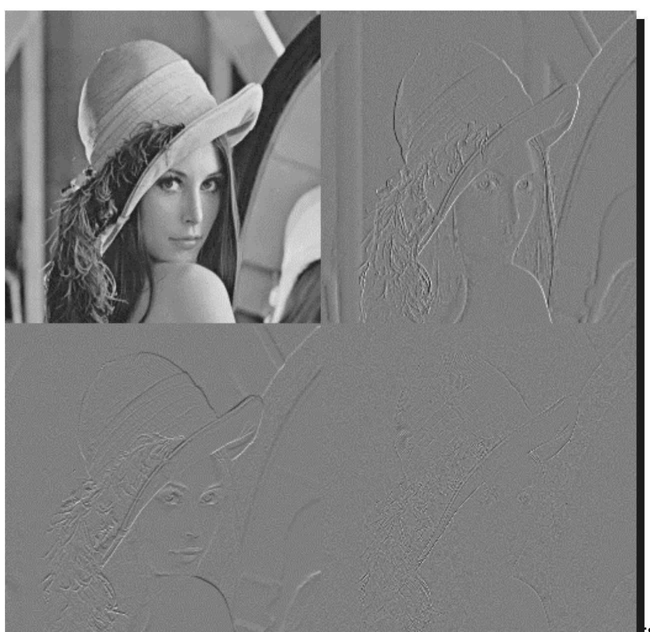

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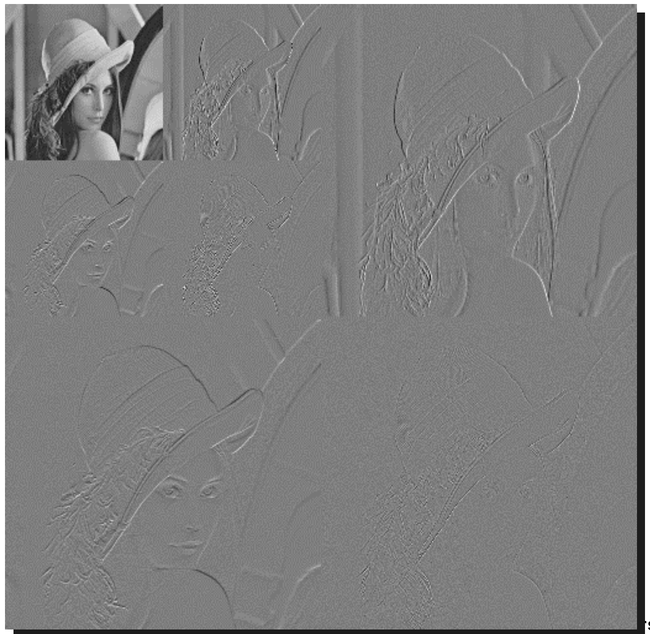

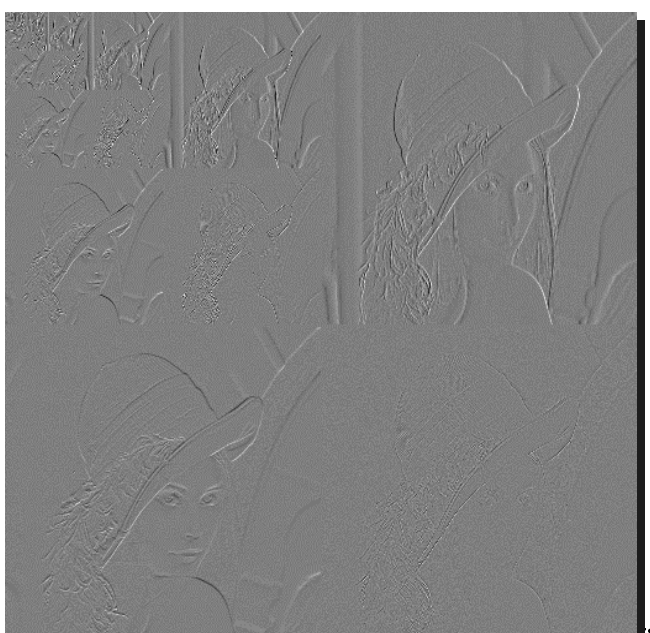

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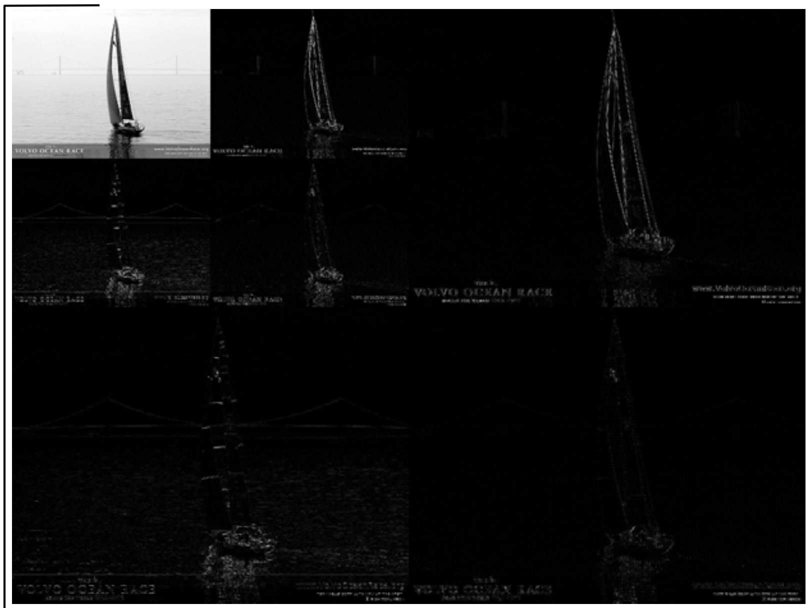

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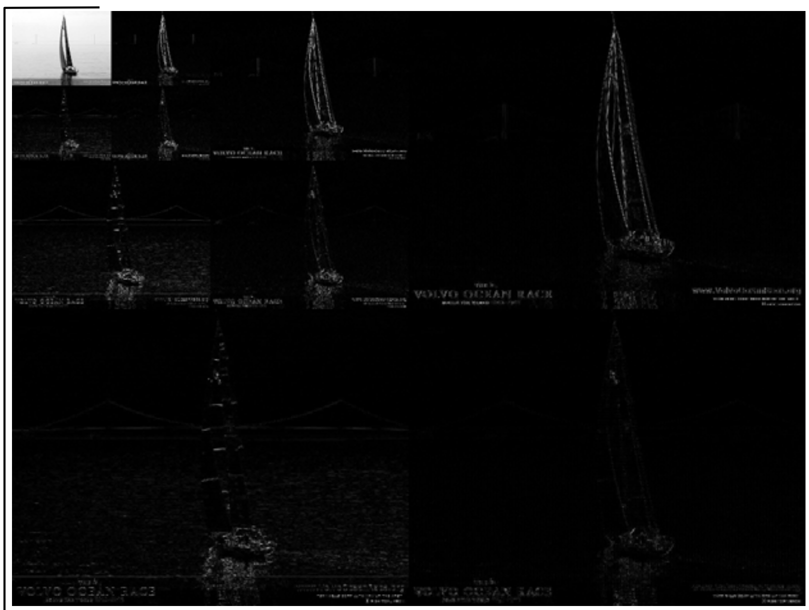

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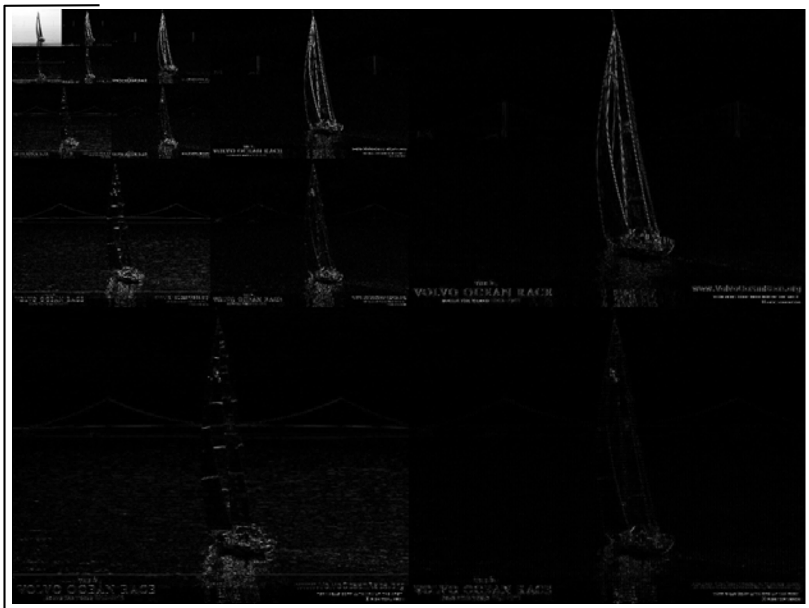

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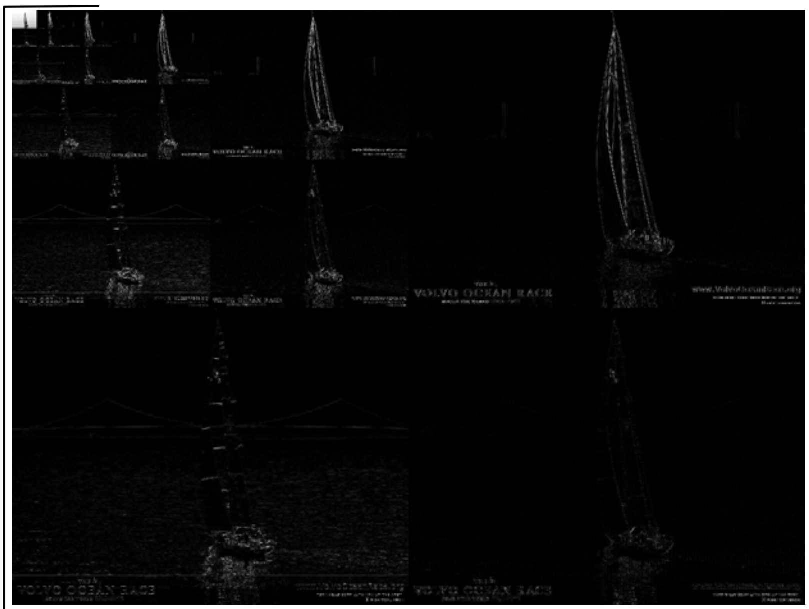

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JPEG 2000

- ISO/IEC 15444 consists of 10 parts:
  - Part 1: Core coding system (JP2, code stream)
  - Part 2: Extensions (JPX, others)
  - Part 3: Motion JPEG 2000
  - Part 4: Conformance Testing
  - Part 5: Reference software
  - Part 6: Compound image file format (JP2, JPX)
  - Part 7-10: JPSEC, JPIP, JP3D

JPEG 2000

- Improved R-D performance at low and medium bitrates
- Covers low bit-rate lossy coding up to lossless coding
- Progressive Transmission Capability (resolution & SNR)
- Region of interest (ROI) representation and editing
- Error Resilience

- Approach: Wavelet Transformation + EBCOT + Adaptive Arithmetic Coding
- Embedded Block Coding with Optimized Truncation

Source: Thomas Wiegand

Part 1: Core Coding system

- overview & 11 annexes:
  - Annex A - Code stream Syntax
  - Annex B - Data ordering
  - Annex C - Arithmetic entropy coding (MQ coder)
  - Annex D - Coefficient bit modeling
  - Annex E - Quantization
  - Annex F - Discrete wavelet transform of tile components
  - Annex G - DC level shifting and component transforms
  - Annex H - Coding of images with Regions of Interest
  - Annex I - JP2 file format syntax
  - Annex J - Examples and guidelines
  - Annex K - Bibliography

JPEG 2000 Part 1

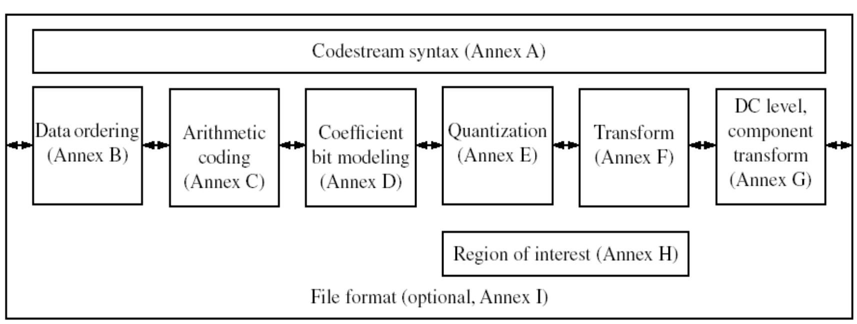

Figure 6-1 — Specification block diagram

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Encoding in JPEG 2000

- DC level component transform (G)
- image decomposition (tiles, sub-bands) (B)
- DFWT (F.3)
- Quantization (E)
- Coefficient bit modeling (D)
- Arithmetic coding (C.1, C.2) (MQ coder)
- Code stream Syntax (A)
- Encoding to JP2 file format (I)

Image Decomposition

- components
- tiles
- sub-bands
- precincts
- code blocks
- layer
- packet

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DC level component transform

- components
- tiles
- sub-bands
- precincts
- code blocks
- layer
- packet

- unsigned → signed values
- RCT: reversible component transform (used with 5-3 reversible WT)
- ICT: irreversible component transform (used with 9-7 irreversible WT)
- When first components are RGB: ICT is approximation to YC_bC_r transform.

Image Decomposition

- components
- tiles
- sub-bands
- precincts
- code blocks
- layer
- packet

Component

- Two-dimensional array of samples (1D)
- Color images consists of several components

Reference Grid

- components
- tiles
- sub-bands
- precincts
- code blocks
- layer
- packet

- Component has reference grid
- (0,0) upper left

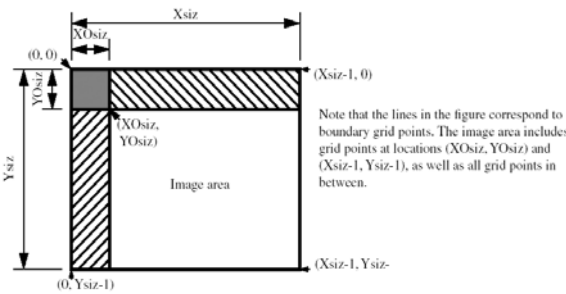

Figure B-1 — Reference grid diagram

Reference Grid

- components
- tiles
- sub-bands
- precincts
- code blocks
- layer
- packet

- Rectangular array of tiles
- Row by row

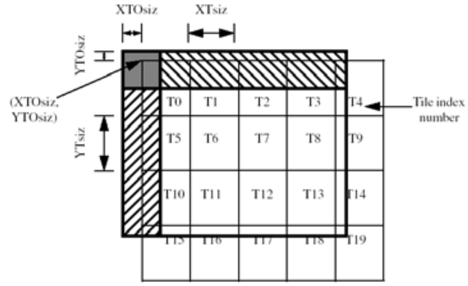

Figure B-2 — Tiling of the reference grid diagram

Sub-Bands

N_LLL, N_LHL, N_LLH, N_LHH, (N_L-1)HL, (N_L-1)LH, (N_L-1)HH, ... , 1HL, 1LH, 1HH

As illustrated in Figure F-19, all the sub-bands in the case where N_L=2 can be represented in the following way:

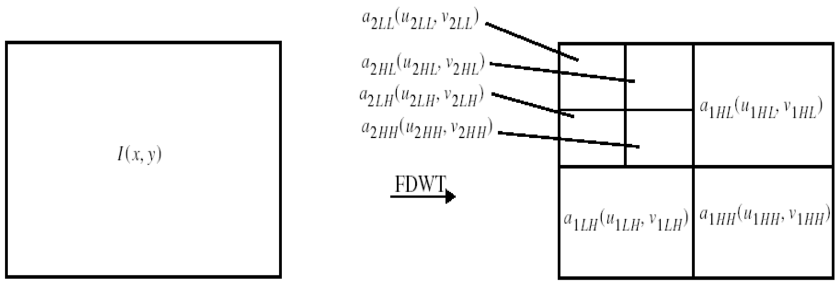

Figure F-19 — The FDWT (N_L=2)

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Precinct Partition

- components
- tiles
- sub-bands
- precincts
- code blocks
- layer
- packet

- Tiles are divided into precincts of size

$$(2^{PP_x}, 2^{PP_y})$$

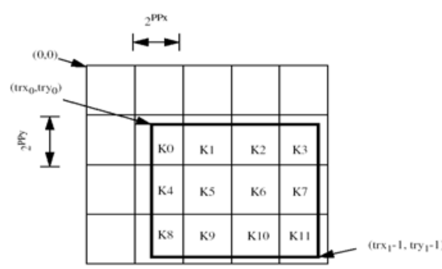

Figure B-6 — Precinct partition

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Code Blocks

- components
- tiles
- sub-bands
- precincts
- code blocks
- layer
- packet

- Each layer divided into code blocks
- Code blocks equal size for entire tile
- Coefficient bit modeling

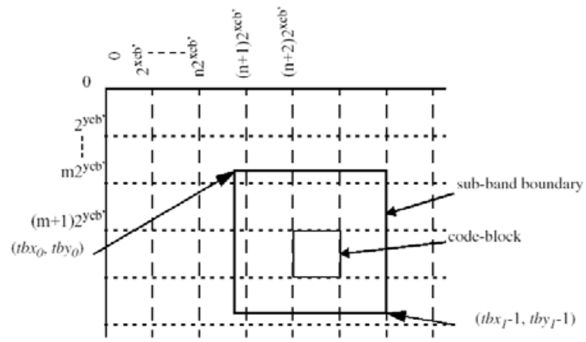

Figure B-7 — Code-blocks in sub-band *b*

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Layers

- components
- tiles
- sub-bands
- precincts
- code blocks
- layer
- packet

- One or more layers in code stream
- Each layer consists of consecutive bit-plane passes from each code block in the tile
- Each layer successively and monotonically improves image quality

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Packet

- components
- tiles
- sub-bands
- precincts
- code blocks
- layer
- packet

- 8 bit packet boundaries
- Contributions in raster order
- For resolution r=0:
  - LL band only
- For resolution r>0:
  - HL, LH, HH bands only
- Progression order

FDWT
Fast Discrete Wavelet Tra

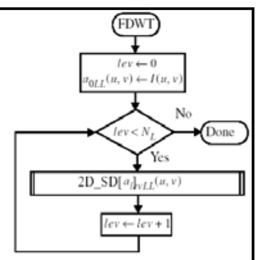

Figure F-20 — The FDWT Procedure

N_LLL, N_LHL, N_LLH, N_LHH, (N_L-1)HL, (N_L-1)LH, (N_L-1)HH, ... , 1HL, 1LH, 1HH

As illustrated in Figure F-19, all the sub-bands in the case where N_L=2 can be represented in the following way:

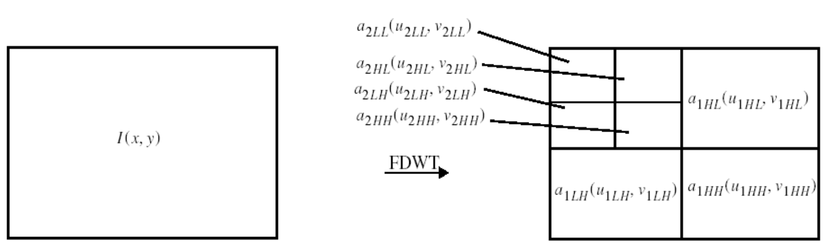

Figure F-19 — The FDWT (N_L=2)

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Wavelet transforms in JPEG2000

5/3 wavelet transform

$$X(2n) = Y_{ext}(2n) - \left\lfloor \frac{Y_{ext}(2n-1) + Y_{ext}(2n+1) + 2}{4} \right\rfloor \quad \text{for } \left\lfloor \frac{i_0}{2} \right\rfloor \le n < \left\lfloor \frac{i_1}{2} \right\rfloor + 1,$$

$$X(2n+1) = Y_{ext}(2n+1) + \left\lfloor \frac{X(2n) + X(2n+2)}{2} \right\rfloor \quad \text{for } \left\lfloor \frac{i_0}{2} \right\rfloor \le n < \left\lfloor \frac{i_1}{2} \right\rfloor.$$

9/7 wavelet transform

$$\begin{cases} X(2n) = KY_{ext}(2n) & [STEP1] \\ X(2n+1) = (1/K)Y_{ext}(2n+1) & [STEP2] \\ X(2n) = X(2n) - \delta(X(2n-1) + X(2n+1)) & [STEP3] \\ X(2n+1) = X(2n+1) - \gamma(X(2n) + X(2n+2)) & [STEP4] \\ X(2n) = X(2n) - \beta(X(2n-1) + X(2n+1)) & [STEP5] \\ X(2n+1) = X(2n+1) - \alpha(X(2n) + X(2n+2)) & [STEP6] \end{cases}$$

Extension of signal

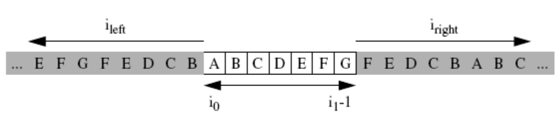
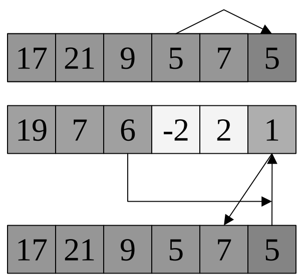


Figure F-15 — Periodic symmetric extension of signal

Extension for Haar Wavelet



- Extend with mirrored pixel
- Perform wavelet transform
- The 1 is redundant

Reconstruction

- Perform IWT
- The 1 is reconstructed from 6 and 5

Decomposition into sub-bands

Figure F-21 describes the input and output parameters of the 2D_SD procedure.

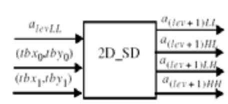


Figure F-21 — Inputs and outputs of the 2D_SD procedure

Figure F-22 illustrates the sub-band decomposition performed by the 2D_SD procedure.

$$Y(2n+1) = X_{ext}(2n+1) - \left\lfloor \frac{X_{ext}(2n) + X_{ext}(2n+2)}{2} \right\rfloor.$$

output signal Y are computed from the even values of
values of n such that $i_0 \le 2n < i_1$

$$Y(2n) = X_{ext}(2n) + \left\lfloor \frac{Y(2n-1) + Y(2n+1) + 2}{4} \right\rfloor$$

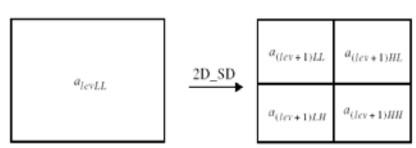


Figure F-22 — One-level decomposition into four sub-bands (2D_SD procedure)

IDWT
Inverse Discrete Wavelet Transform

The following ordering of sub-bands is used:

N_LLL, N_LHL, N_LLH, N_LHH, (N_L-1)HL, (N_L-1)LH, (N_L-1)HH, ... , 1HL, 1LH, 1HH

As illustrated in Figure F-2, all the sub-bands in the case where N_L=2 can be represented in the following way:

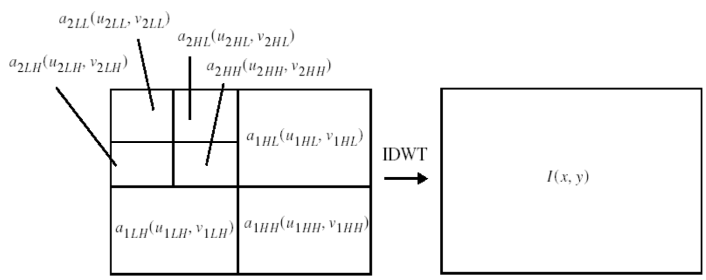


Figure F-2 — The IDWT (N_L=2)

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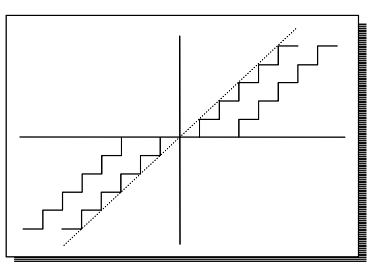
Quantization

- Divide by factor and use floor-function
- Irreversible case: Scalar Quantization with dead zone
- separate approach for every sub-band possible



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Arithmetic entropy coding

- Compatible with ISO/IEC 14492
- MQ-Coder

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Codestream syntax

- Markers: 0xFF + Opcode (0x01-0xFE)
- Marker segments: marker + parameter
- Marker segments include a length-field
- Headers: Collections of markers and marker segments
- Codestream: SOC – SIZ - ... - EOC
- Part: SOT - ... - SOD ... bitstream

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Construction of codestream

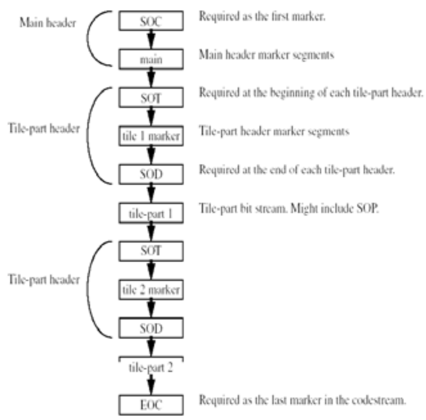

Figure A-2 — Construction of the codestream

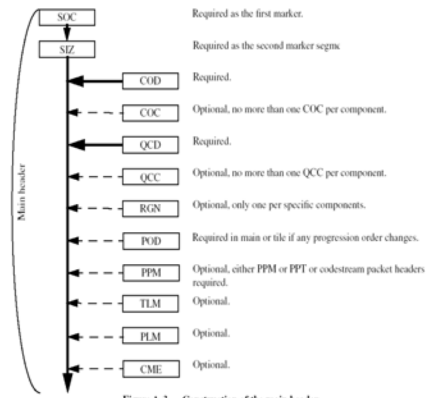

Figure A-3 — Construction of the main header

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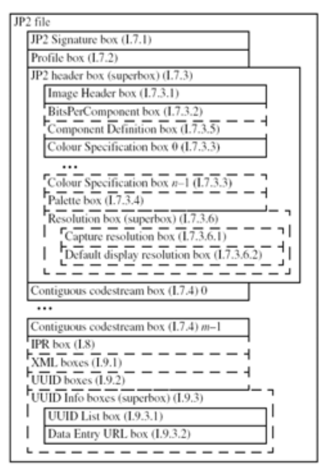
Coding into JP2 File Format

- Sequence of boxes
- Simple boxes (containing payload)
- Superboxes (containing other boxes)
- optional/mandatory boxes



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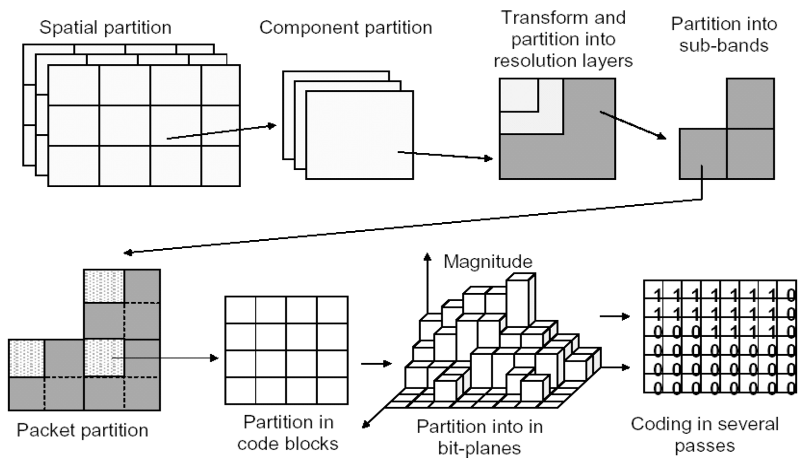
JPEG 2000 Process



Source: Thomas Wiegand

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JPEG vs JPEG2000 (Original)

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JPEG 2000
130 x compression

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JPEG
130 x compression

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Literature - Wavelets

- Daubechies: **Ten Lectures on Wavelets**, Society for Industrial and Applied Mathematics
- Fournier: **Wavelets an their Applications in Computer Graphics**, SIGGRAPH `94 Course Notes
- Stollnitz, DeRose, Salesin: **Wavelets for computer graphics: theory and applications**, Morgan Kaufman, 1996

End of Part

Thank you for your attention!