INF5390 – Kunstig intelligens

Logical Agents

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Outline

- Knowledge-based agents
- The Wumpus world
- Knowledge representation
- Logical reasoning
- Propositional logic
- Wumpus agent
- Summary

AIMA Chapter 7: Logical Agents

Knowledge-based agents

- *Knowledge-based* agents are able to:
  - Maintain a description of the environment
  - Deduce a course of action that will achieve goals
- Knowledge-based agents have:
  - A **knowledge base**
  - **Logical reasoning** abilities
- The performance of a knowledge-based agent is determined by its knowledge base

Knowledge bases

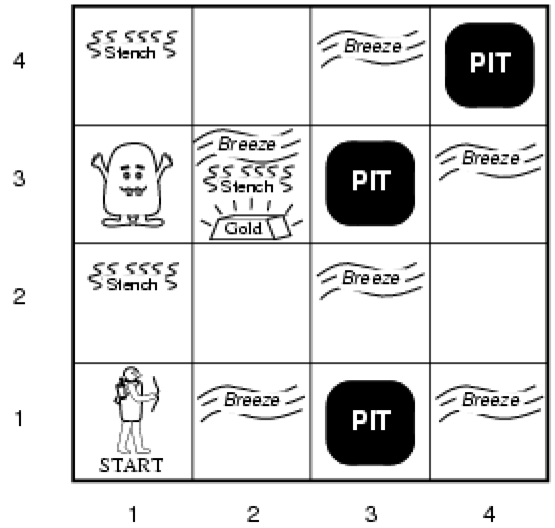
- A *knowledge base* is a set of representations of facts about the world, called *sentences*, expressed in a *knowledge representation language*
- Knowledge base (KB) interface
    - TELL(*KB*, *fact*) - Add a new fact
    - *fact* <= ASK(*KB*, *query*) - Retrieves a fact
    - RETRACT(*KB*, *fact*) - Removes a fact
- A knowledge-base agent can be built by TELLing it what it needs to know (*declarative* approach)
- The agent can be used to solve problems by ASKing questions

Generic knowledge-based agent

```
function KB-AGENT(percept) returns an action
persistent: KB, the agent's knowledge base
            t, a counter, initially 0, indicating time
TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
action <= ASK(KB, MAKE-ACTION-QUERY(t))
TELL(KB, MAKE-ACTION-SENTENCE(action, t))
t <= t + 1
return action
```

Example - The Wumpus world

- Wumpus
  - Stench
- Pits
  - Breeze
- Gold
  - Glitter
- Agent
  - Move
  - Shoot



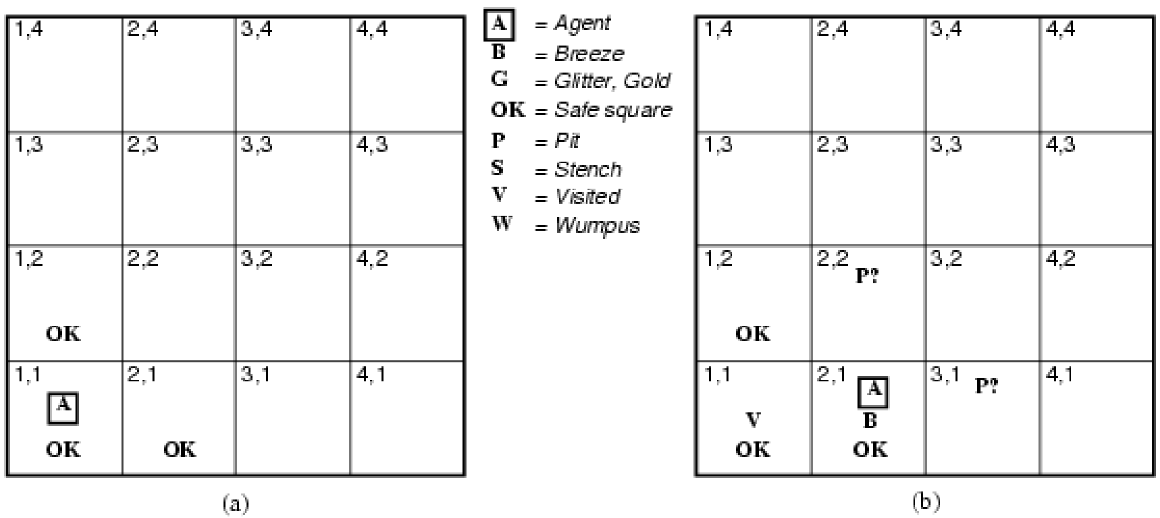
- Performance
  - +1000 for gold, -1000 fall in pit/eaten, -1 for action, -10 using arrow
- Environment
  - 4x4 grid, start in [1,1]
- Actuators
  - Move forward, turn left 90°, turn right 90°, grab gold, shoot (one) arrow
- Sensors
  - [Stench?, Breeze?, Glitter?, Wall?, Killed?]

- Agent goal: Find gold and not get killed!
- Play at: http://mostplays.com/play/Wumpus_World_1585

Classification of the Wumpus world

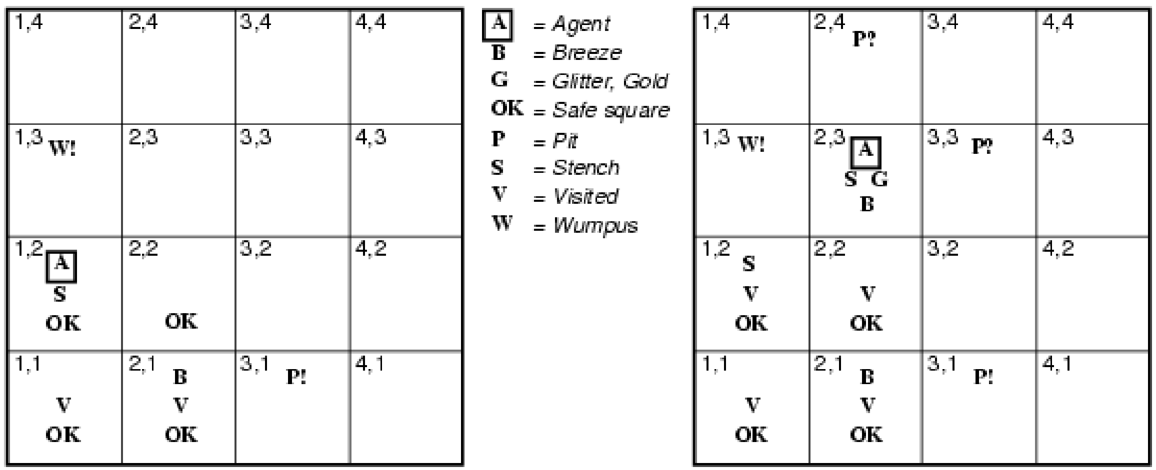
- Partially observable
  - Some aspects not directly observable, e.g. position of Wumpus
- Single-agent
  - Self
- Deterministic
  - Next state given by current state and action
- Sequential
  - Reward may require many steps
- Static
  - Wumpus does not move
- Discrete
  - Everything discretized
- Known
  - Effect of actions known

Exploring the Wumpus world



- Initial percept: [None, None, None, None, None]
- Deduction: [1,2] and [2,1] are OK
- Action: Move right
- Second percept: [None, Breeze, None, None, None]
- Deduction: Pit in [2,2] or [3,1]
- Action sequence: Turn back and go to [1,2]

Exploring the Wumpus world (cont.)



(a) (b)

- Fourth percept: [Stench, None, None, None, None]
- Deduction: Wumpus must be in [1,3], pit in [3,1]
- Action: Move right, etc.

- What does an intelligent agent need to know and how can it reason to succeed in the Wumpus world?

Requirements for knowledge representation

- Knowledge representation languages should be:
  - Expressive and concise
  - Unambiguous and independent of context
  - Able to express incomplete knowledge
  - Effective inference of new knowledge
- Existing languages not suited:
  - Programming languages - *precise* descriptions and recipes for machines
  - Natural languages - *flexible* communication between humans
- In AI, *logic* is used for knowledge representation

Knowledge representation languages

- Syntax
  - How are legal sentences in the language composed
- Semantics
  - What do the sentences *mean*
  - What is the *truth* of every sentence with respect to each *possible world* (also called a *model*)
- Entailment
  - The fact that sentences *logically follow* from other sentences
- Inference
  - How to derive new sentences that are entailed by known ones

Logical entailment

- Logical *entailment* between two sentences

$$\alpha \models \beta$$

means that β *follows logically* from α: in every *model* (possible world) in which α is true, β is also true

- We can also say that an entire KB (all sentences in the knowledge base) entails a sentence

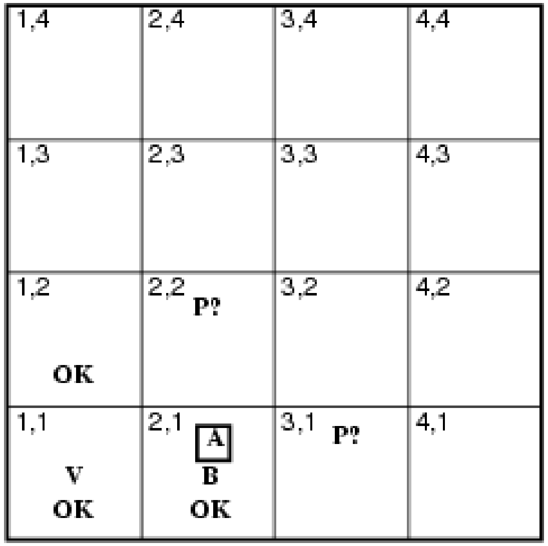
$$KB \models \beta$$

β *follows logically* from KB: in every *model* (possible world) in which KB is true, β is also true

- Model checking: Can *check* entailment by reviewing all possible models

Model checking – Wumpus example

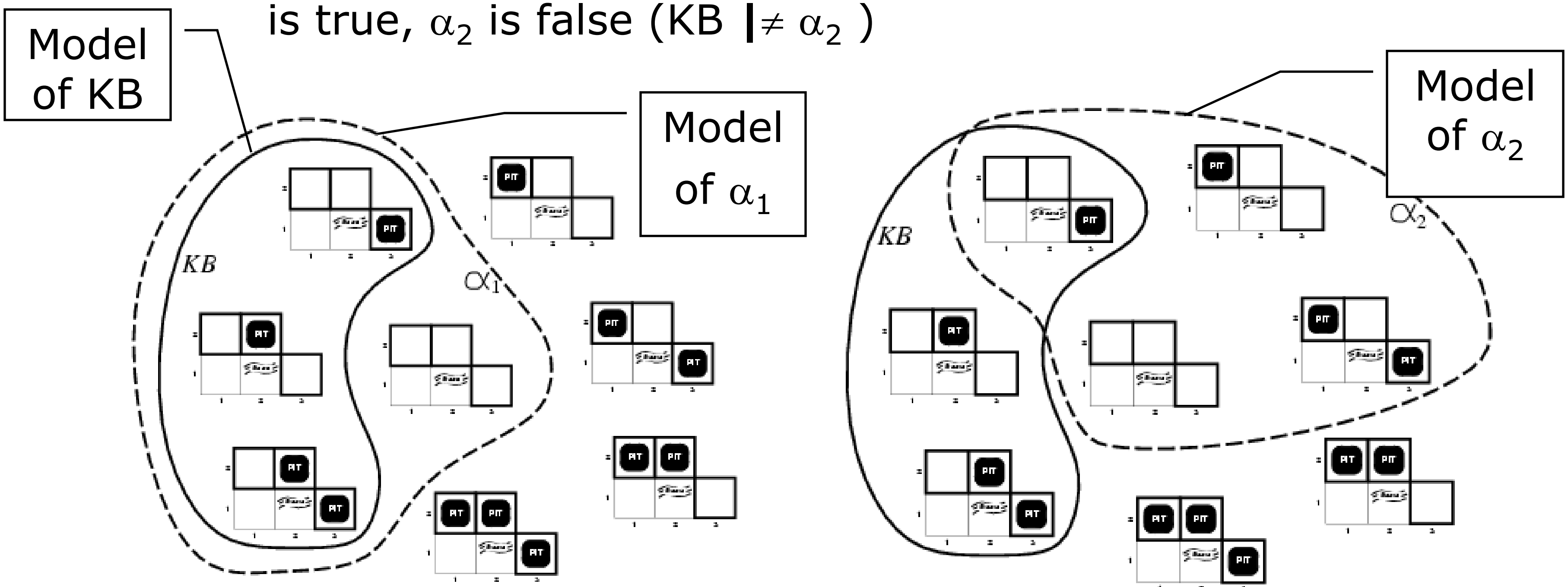
- Agent is in [2,1] and has detected a breeze
- Agent wants to know: Pits in [1,2], [2,2] and [3,1]?
- Each square may have a pit or not, i.e. there are $2^3 = 8$ models
- KB is false in any model that contradicts what the agent knows
- Only three models in which the KB is true (next slide)



(b)

Model checking example (cont.)

- Check two conclusions
  - √ α_1 = No pit in [1,2] – *True* since every model where KB is true, α_1 is also true (KB **|=** α_1)
  - √ α_2 = No pit in [2,2] – *False* since for some models where KB is true, α_2 is false (KB **|**≠ α_2)



Logical inference

- Logical *inference* between two sentences

$$\alpha \vdash \beta$$

  means that β *can be derived* from α by following an *inference algorithm* (can also say KB $\vdash \beta$)
- Model checking is an example of an inference algorithm

Inference and entailment

- "Entailment is like the needle being in the haystack; inference is like finding it"
- *Sound* inference: The inference algorithm only derives entailed sentences
  - √ Required property
  - √ Model checking is sound
- *Complete* inference: The inference algorithm can derive any entailed sentence
  - √ Desirable property
  - √ Not always possible

Propositional and first-order logic

- Propositional logic
  - Symbols represent true or false facts
  - More complex sentences can be constructed with Boolean connectives (and, or, ..)
- First-order logic
  - Symbols represent objects and predicates on objects
  - More complex sentences can be constructed with connectives and quantifiers (for-all, there-is, ..)
- In AI, both propositional and and first-order logic are heavily used

Propositional logic - syntax

$$
\begin{aligned}
Sentence &\rightarrow AtomicSentence \mid ComplexSentence \\
AtomicSentence &\rightarrow \text{True} \mid \text{False} \\
&\mid P \mid Q \mid R \\
ComplexSentence &\rightarrow (Sentence) \\
&\mid Sentence\ Connective\ Sentence \\
&\mid \neg Sentence \\
Connective &\rightarrow \wedge \mid \vee \mid \Leftrightarrow \mid \Rightarrow
\end{aligned}
$$

Logical connectives

- Λ (and)
  - √ Conjunction $P \Lambda Q$
- V (or)
  - √ Disjunction $P \vee Q$
- $\neg$ (not)
  - √ Negation $\neg P$
- <=> (equivalent)
  - √ Equivalence $(P \Lambda Q) <=> (Q \Lambda P)$
- => (implies)
  - √ Implication $(P \Lambda Q) => R$

Propositional logic - semantics

- Semantics defines the rules for determining the truth of a sentence with respect to a certain model
- A model in propositional logic fixes the truth (true or false) of every propositional symbol
- The truth of a complex sentence is given by the truth value of its parts and the connectives used

Truth table for logical connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P => Q$	$P \Leftrightarrow Q$
False	*False*	*True*	*False*	*False*	*True*	*True*
False	*True*	*True*	*False*	*True*	*True*	*False*
True	*False*	*False*	*False*	*True*	*False*	*False*
True	*True*	*False*	*True*	*True*	*True*	*True*

A simple KB for the Wumpus world

- Propositional symbols
  - $P_{i,j}$ is true if there is a pit in [i,j]
  - $B_{i,j}$ is true if there is a breeze in [i,j]
- The KB contains the following sentences (and more)
  - R1: $\neg P_{1,1}$ – No pit in [1,1]
  - R2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ – Breezy iff a pit in next cell
  - R3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$ – Same, etc.
  - R4: $\neg B_{1,1}$ – No breeze in [1,1]
  - R5: $B_{2,1}$ – Breeze in [2,1]

Propositional inference by checking

- Construct truth table with one row for each combination of truth values of the propositional symbols in the sentence
- Calculate truth value of the KB sentences for each row; if all are true, the row is a model of the KB
- All other sentences that are true in the same rows as the KB is true, are entailed by the KB

Wumpus - Model checking by truth table

P[1,2]	P[2,2]	P[3,1]	KB	α_1 =Not P[1,2]	α_2 =Not P[2,2]
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	**T**	T	F
F	T	F	**T**	T	F
F	F	T	**T**	T	T
F	F	F	F	T	T

Complexity of propositional inference

- The checking algorithm is sound and complete, but
  - Time complexity is $O(2^n)$
  - Space complexity is $O(n)$
- All known inference algorithms for propositional logic has worst-case complexity that is exponential in the size of inputs

Equivalence, validity and satisfiability

- Two sentences are *equivalent* if they are true in the same models
- A sentence is *valid* (necessarily true, tautological) if it is true in all possible models
- A sentence is *satisfiable* if it true in some model
- A sentence that is not satisfiable is *unsatisfiable* (contradictory)

Inference by applying rules

- An *inference rule* is a standard pattern of inference that can be applied to drive chains of conclusions leading to goal
- A sequence of applications of inference rules is called a *proof*
- Searching for proofs is similar (or in some cases identical) to problem-solving by search
- Using rules for inference is an alternative to inference by model checking

Inference rules $\frac{\alpha}{\beta}$ for propositional logic

- Modus ponens $$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$
- And-Elimination $$\frac{\alpha_1 \wedge \alpha_2 \wedge \mathrm{K} \wedge \alpha_2}{\alpha_1}$$
- Unit Resolution $$\frac{\alpha \vee \beta, \quad \neg\beta}{\alpha}$$
- Etc.

Ex.: Inference in the Wumpus world

- Want to show that there is no pit in [1,2] given R1-R5, i.e. $\neg P_{1,2}$
  - R6: $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$ – Biconditional elimination in R2 (def. of equivalence)
  - R7: $((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$ – And-Elimination in R6
  - R8: $(\neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1}))$ – Equivalence of contrapositives in R7
  - R9: $\neg(P_{1,2} \vee P_{2,1})$ – Modus ponens with R8 and R4
  - R10: $\neg P_{1,2} \wedge \neg P_{2,1}$– Morgan's law. Neither [1,2] nor [2,1] contains a pit
- The inference path could be found by *search* as an alternative to model checking

The resolution rule

- Takes two sentences with *complementary* parts and produces and new sentence with the part removed
- Example

$$\frac{P1,1 \lor P3,1 \qquad \neg P1,1 \lor \neg P2,2}{P3,1 \lor \neg P2,2}$$

- It can be shown that resolution is sound and complete
- Any complete search algorithm, using only resolution as inference rule, can derive any conclusion entailed by any knowledge base in propositional logic

Searching - forward and backward chaining

- Forward chaining
  - Start with sentences in KB and generate consequences
  - Uses inference rules in forward direction
  - Also called *data-driven* procedure
- Backward chaining
  - Start with goal to be proven and look for premises
  - Uses inference rules in backward direction
  - Also called *goal-directed* procedure
- Specialized search algorithms for propositional logic can be very efficient

Requirements for a Wumpus agent

- Able to deduce as far as possible state of the world based on percept history
- Must have complete logical model of effect of its actions
- Able to keep track of world efficiently without going back in all percept history at each step
- Use logical inference to construct plans that are guaranteed to achieve its goals

Keeping track of state of the world

- Knowledge base contains general knowledge of how the world works plus specific percept history
  - All percept need to be indexed by time *t* to capture changes in percepts: $\neg Stench^3$, $Stench^4$, …
- The term *fluent* is used for any state variable that changes over time
- *Effect axioms* are used to describe outcome of an action at the next time step
- *Frame problem*: Effect axioms fail to state what does *not* change as a result of an action
- Solved by writing *successor-state axioms* for fluents, e.g.
  - $\text{HaveArrow}^{t+1} \Leftrightarrow (\text{HaveArrow}^{t} \wedge \neg \text{Shoot}^{t})$
- Propositional agent requires *very large number* of rules

A hybrid Wumpus agent

function HYBRID-WUMPUS-AGENT(*percept*) **returns** an *action*

inputs: *percept*, a list, [*stench, breeze, glitter, bump, scream*]

persistent: *KB*, knowledge base, initially containing Wumpus “physics”

t, a counter, initially 0, indicating time; *plan*, an action sequence, initially empty

TELL(KB, new percept and temporal “physics” sentences for time *t)*

if *glitter* ***then*** *//* grab the gold, plan safe retreat and climb out

    plan <= [*Grab*]+PLAN-ROUTE(*current*, [1,1], *safe*)+[*Climb*]

if *plan* is empty **then** *//* plan safe route to safe unvisited square

    plan <= PLAN-ROUTE(*current*, *unvisited* Π *safe*, *safe*)

if *plan* is empty **and** $HaveArrow^t$ **then** *//* plan move to shoot at Wumpus

    plan <= PLAN-SHOT(*current*, *possible-wumpus*, *safe*)

if *plan* is empty **then** // go to a square that is not provably unsafe

    plan <= PLAN-ROUTE(*current*, *unvisited* Π *not-unsafe*, *safe*)

if *plan* is empty **then** // mission impossible, plan retreat and climb out

    plan <= PLAN-ROUTE(*current*, [1,1], *safe*)+[*Climb*]

action <= POP(*plan*)

TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*))

t <= *t* + 1

return *action*

function PLAN-ROUTE(*current, goals, allowed*)
    returns an action sequence using A* search

Summary

- Intelligent agents need *knowledge* stored in a *knowledge base* to reach good decisions
- A *knowledge-based agent* applies an *inference* mechanism to the KB to reach conclusions
- Knowledge is expressed in *sentences* in a *knowledge representation language* (with *syntax* and *semantics*)
- A sentence logically *entails* another sentence if the latter is true in all models where the first is true
- *Inference* is the process of deriving new sentences from old ones
- Inference should be *sound* (only true conclusions) and *complete* (all true conclusions)

Summary (cont.)

- *Propositional logic* consists of propositional symbols and logical connectives
- *Model-checking* can be used to check propositional entailment
- *Inference rules* can be used to find *proofs,* and *resolution* is a complete and sound rule
- *Fluents* can be used to express values of properties that change over time
- Propositional agents can be efficient for some domains, but do not scale to complex problems
- *Hybrid agents* combine propositional KB and reasoning with problems solving by search