

INF5490 RF MEMS

L3: Modelling, design og
analyse

Oversikt over forelesningen

- Metoder for å modellere RF MEMS
 - **1. Enkle matematiske modeller**
 - Eks. parallell plate kondensator
 - **2. Konvertering til elektriske ekvivalenter**
 - Eks. Mekanisk resonator
 - Samvirke mellom ulike energi-domener
 - **3. Analyse ved Finite Element Methods**
 - Eks. fra CoventorWare
 - Modellering av bulk mikromaskinert prosess

1. Enkle matematiske modeller

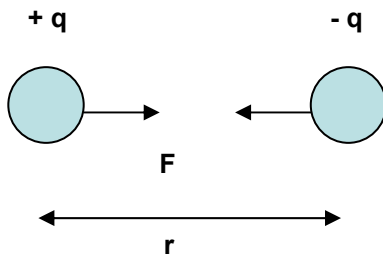
- Ligninger, formler beskriver fysiske fenomener
 - Approksimasjon
 - Eksplisitte løsninger for enkle problemer
 - Linearisering rundt et operasjonspunkt
 - Direkte simulering av ligningssett
 - Typisk differensialligninger
 - Løses ved iterasjoner
- + Gir konstruktøren design-innsikt
 - Hvordan endre funksjonalitet ved parametervariasjoner
 - Kan benyttes til innledende ”overslag”

Eks. på matematiske modeller

- Viktige ligninger for RF MEMS komponenter:
 - → Parallell plate kondensator
- **Elektrostatisk** aktivering av kondensator med bevegelig plate opphengt i fjær
- Beregning av ”**pull-in**”
 - Formler og figurer →

Elektrostatikk

Elektrisk kraft mellom ladninger: **Coulombs lov**



$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

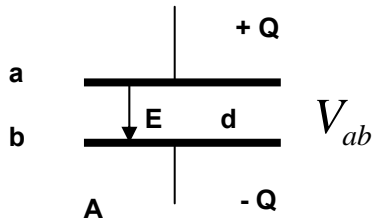
Elektrisk felt = kraft pr. enhetsladning $\bar{E} = \frac{\bar{F}}{q_0}$

Arbeid utført av en kraft = endring i potensial-energi $W_{a \rightarrow b} = \int_a^b \bar{F} \cdot d\bar{l} = U_a - U_b$

Potensial, V = potensial-energi pr. enhetsladning $V = \frac{U}{q_0}$

Spenning = potensial-differansen $V_a - V_b = \int_a^b \bar{E} \cdot d\bar{l}$

Kapasitans



Definisjon av kapasitans

$$C = \frac{Q}{V_{ab}}$$

Overflate ladningstetthet = σ

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A} \cdot \frac{1}{\epsilon_0}$$

Spenning

$$V_{ab} = E \cdot d = \frac{Q}{A\epsilon_0} \cdot d$$

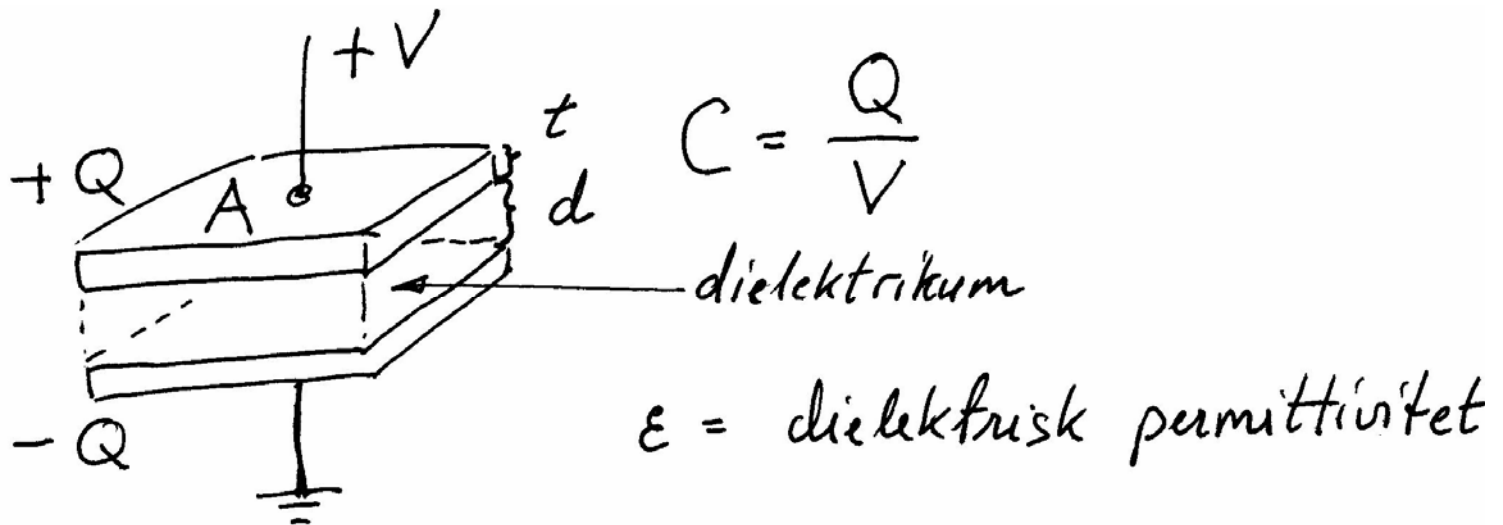
$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$

Energi lagret i en kondensator, C ,

som lades opp til en spenning V_0 ved strøm $i = \dot{Q} = C \frac{dV}{dt}$

$$U = \int v \cdot i \cdot dt = \int v \cdot C \frac{dv}{dt} \cdot dt = C \int_0^{V_0} v \cdot dv = \frac{1}{2} C V_0^2 = \frac{\epsilon_0 A}{2d} V_0^2$$

Parallell plate kondensator



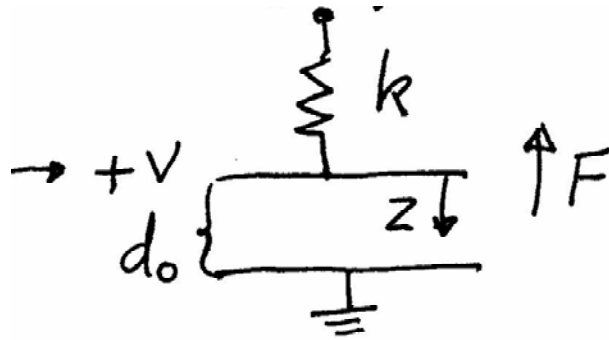
Tiltrekningskraft mellom platene

$$F = -\frac{\partial U}{\partial d} = -\frac{\partial}{\partial d} \left(\frac{\epsilon A}{2d} V^2 \right) = \frac{\epsilon A V^2}{2d^2}$$

Bevegelig kondensator-plate

- Forutsetninger for beregninger:
 - Anta luft mellom platene
 - Fjær holder øvre plate
 - Fjærkonstant: k
 - Spenning settes på
 - Elektrostatisk tiltrekning
 - Ved likevekt
 - **Kreftene oppover og nedover balanserer →**

Kreftene balanserer



$k =$ fjærkonstant

$$F_{\text{spring}} = k \cdot x$$

tøyning utfra
likevekt

$d_0 =$ gap ved 0V og null fjærutstrekning

$$d = d_0 - z$$

$$z = d_0 - d$$

Kraft på øvre plate ved V og d :

$$F_{\text{net}} = - \frac{\epsilon A V^2}{2 d^2} + k (d_0 - d) = 0 \text{ ved likevekt}$$

Stabilitet

- Hvordan kreftene utvikler seg når d minker
 - Anta en liten perturbasjon (endring) i gapet ved konstant spenning

$$\delta F_{net} = \left. \frac{\partial F_{net}}{\partial d} \right| \cdot \delta d$$

$$\delta F_{net} = \left(\frac{\epsilon A V^2}{d^3} - k \right) \delta d$$

Anta at gapet minker $\delta d < 0$

Hvis kraften oppover også minker, er systemet **USTABILT!**

$$\delta F_{net} < 0,$$

Stabilitet, forts.

Stabilitetsbetingelse:

$$\left. \frac{\partial F_{net}}{\partial d} \right|_V < 0$$

$$k > \frac{\epsilon A V^2}{d^3}$$

Pull-in når:

$$k = \frac{\epsilon A V_{PI}^2}{d_{PI}^3}$$

Pull-in

$$F_{net} = 0$$

$$\frac{\epsilon A V_{PI}^2}{2 d_{PI}^2} = k (d_0 - d_{PI})$$

↑
= $\frac{\epsilon A V_{PI}^2}{d_{PI}^3}$

Pull-in oppstår når:

$$d_{PI} = \frac{2}{3} d_0$$

$$V_{PI} = \sqrt{\frac{8 k d_0^3}{27 \epsilon A}}$$



To likevekts-punkter

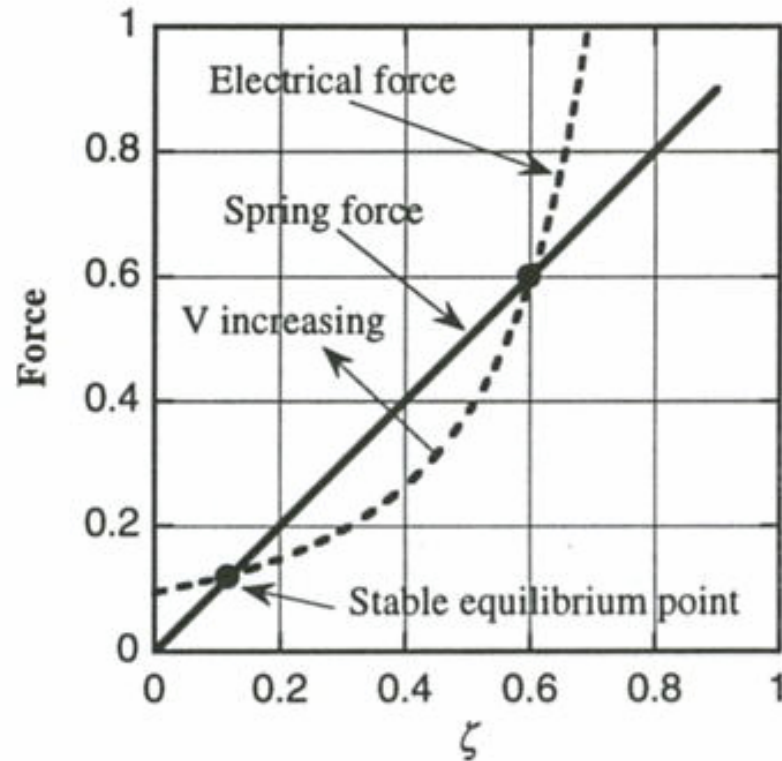


Figure 6.7. Electrical and spring forces for the voltage-controlled parallel-plate electrostatic actuator, plotted for $V/V_{PI} = 0.8$.

$$\zeta = 1 - d/d_0$$

Senturia

Pull-in

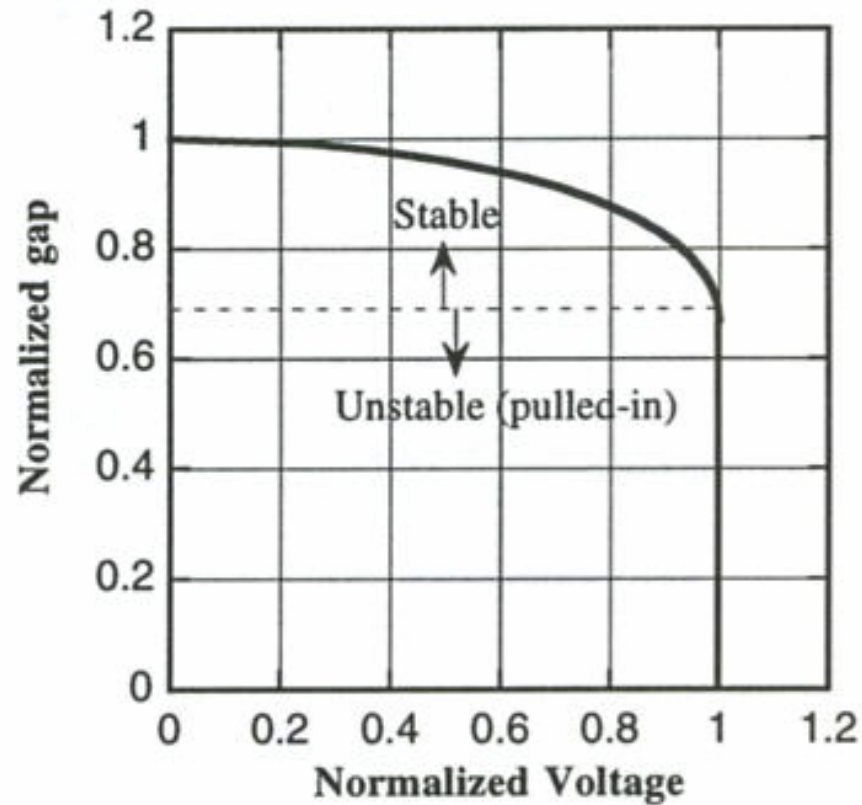


Figure 6.8. Normalized gap as a function of normalized voltage for the electrostatic actuator.

2. Konvertering til elektriske ekvivalenter

- Mekanisk oppførsel kan modelleres ved **elektriske kretselementer**
 - Mekanisk struktur → forenklinger → ekvivalent elektrisk krets
 - Mulighet for å binde sammen elektrisk og mekanisk domene
- Motivasjon for konvertering
 - Et rikt utvalg av analyse-verktøy finnes
 - Eks. SPICE
 - Modellering og sam-simulering av elektroniske og mekaniske systemer forenkles

Konvertering til elektriske ekvivalenter, forts.

- I det følgende gjennomgås:
 - Litt bakgrunn fra kretsteori
 - Konverteringsprinsipper
 - Ulike analogier eksisterer
 - Eksempel på en konvertering
 - Mekanisk resonator
 - Kobling og samvirke mellom ulike energidomener

Kretsteori

- Grunnleggende kretselementer: R, C, L
- Strøm og spennings-ligninger for grunnelementene (ved lave frekvenser)
 - Ohms lov, C og L-ligninger
 - $V = RI$, $I = C \, dV/dt$, $V = L \, dI/dt$
 - **Laplace** transformasjon
 - Fra differensial-ligninger til algebraiske (s-polynomer)
 - → Komplekse impedanser: R, $1/sC$, sL
- Kirchhoffs ligninger
 - Σ **strøm** inn i noder = 0, Σ **spenning** rundt løkker = 0

Effort - flow

- Elektriske kretser beskrives av et **variabelsett**:
 - Spenning V : **across** eller **effort**-variabel
 - Strøm I : **through** eller **flow**-variabel
- En effort ("innsats") -variabel driver en flow-variabel gjennom en impedans, Z
- Eks. på e,f-par i andre energi-domener:
 - Kraft og hastighet
 - Kraftmoment (torque) og angulær hastighet

Through- og across-variable

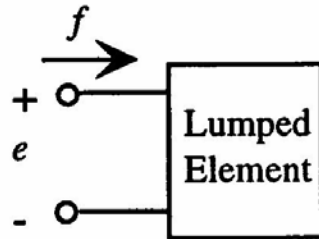


Figure 5.2. Sign conventions for one-port elements in the $e \rightarrow V$ convention.

Kretselement modelleres som 1-port med terminaler

Samme strøm ($f = \text{flow}$) inn og ut og gjennom (through) elementet

$f = \text{flow} = \text{through-variable}$

$e = \text{effort} = \text{across-variable}$

Positiv flow inn i terminal der samme terminal definerer **positiv effort**

Konjugerte power-variable: e,f

- Anta konverteringer mellom energidomener der energien bevarer!
- Egenskaper
 - $e * f = \text{effekt (power)}$
 - $e / f = \text{impedans}$
- Generalisert **displacement** representerer tilstanden (state), f. eks. posisjon eller ladning

$$f(t) = \dot{q}(t)$$

$$- e * q = \text{energi}$$

$$q(t) = \int_{t_0}^t f(t)dt + q(t_0)$$

Generalisert momentum

$$p(t) = \int_{t_0}^t e(t) dt + p(t_0)$$

– p * f = energi

Energi-domener, analogier

- Det eksisterer ulike energi-domener
 - Elektrisk, elastisk, termisk, for væsker etc.
- ***For hvert energidomene er det mulig å definere et sett **konjugerte power-variable** som kan brukes som basis for en diskret-komponent modellering (lumped modelling) med krets-elementer som er ekvivalente***
- Tabell 5.1 Senturia ->

Eks. på konjugerte power variable

Energy Domain	Effort	Flow	Momentum	Displacement
Mechanical translation	Force F	Velocity \dot{x}, v	Momentum p	Position x
Fixed-axis rotation	Torque τ	Angular velocity ω	Angular momentum J	Angle θ
Electric circuits	Voltage V, v	Current I, i	...	Charge Q
Magnetic circuits	Magnetomotive force MMF	Flux rate $\dot{\phi}$...	Flux ϕ
Incompressible fluid flow	Pressure P	Volumetric flow Q	Pressure momentum Γ	Volume V
Thermal	Temperature T	Entropy flow rate \dot{S}	...	Entropy S

Mekanisk energidomene

$$e = F \quad (\text{kraft})$$

$$f = v, \dot{x} \quad (\text{hastighet})$$

$$g = x \quad (\text{posisjon}) = \int \dot{x} dt$$

$$p = p \quad (\text{momentum}) = \int F dt$$

(kraft x tid)

$$e \cdot f \rightarrow F \cdot \dot{x} = \frac{F \Delta x}{\Delta t} = \frac{\text{arbeid}}{\text{tid}} = \text{effekt}$$

$$e \cdot g \rightarrow F \cdot x = \text{kraft} \times \text{vei} = \text{arbeid} = \text{energi}$$

$$p \cdot f \rightarrow p \cdot \dot{x} = m v \cdot v = m v^2 = \text{energi}$$

Elektrisk energidomene

$$e = V \quad (\text{spenning})$$

$$f = I \quad (\text{strøm})$$

$$q = \int I dt = Q \quad (\text{ladning})$$

$$p = n.a.$$

$$e \cdot f \rightarrow V \cdot I = \text{effekt}$$

$$e \cdot q \rightarrow V \cdot Q = V \int I dt = \text{energi}$$

$e \rightarrow V$ - konvensjonen

- **Senturia** og **Tilmans** holder seg til $e \rightarrow V$ – konvensjonen
- Eks. elektriske og mekaniske kretser
 - $e \rightarrow V$ (spenning) tilsvarende F (kraft)
 - $f \rightarrow I$ (strøm) tilsvarende v (hastighet)
 - $q \rightarrow Q$ (posisjon) tilsvarende x (posisjon)
 - $e * f =$ "power" som tilføres elementet

H. Tilmans, Equivalent circuit representation of electromagnetical transducers:
I. Lumped-parameter systems, J. Micromech. Microeng., Vol. 6, pp 157-176, 1996

Andre konvensjoner

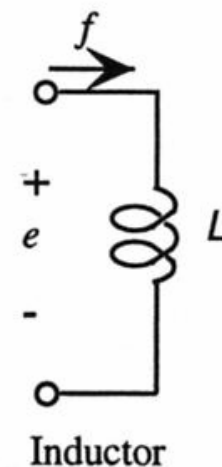
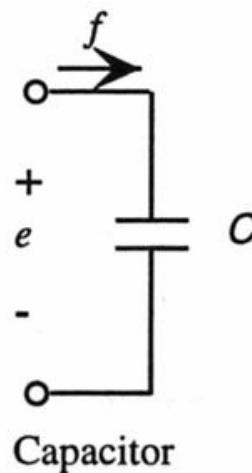
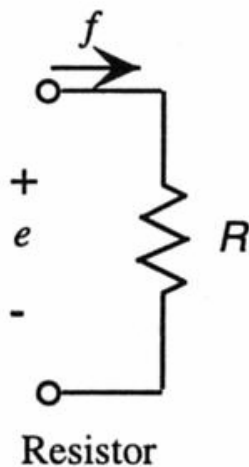
- Det finnes ulike konvensjoner for å definere **through-** eller **across-variable**

Table 5.2. Different conventions for assigning circuit variables.

Convention	Across Variable	Through Variable	Product	Principal Use
$e \rightarrow V$	e	f	power	electric circuit elements
$f \rightarrow V$	f	e	power	mechanical circuit elements
Thermal	T	\dot{Q}	Watt-Kelvin	thermal circuits
HDL	q	e	energy	HDL circuit representation of mechanical elements

Generaliserte krets-elementer

- **En-port** krets-elementer
 - R dissiperende element
 - C, L energi-lagrings elementer
 - Elementene kan ha en **generell funksjon!**
 - Kan brukes i **forskjellige energi-domener**



Generalisert kapasitans

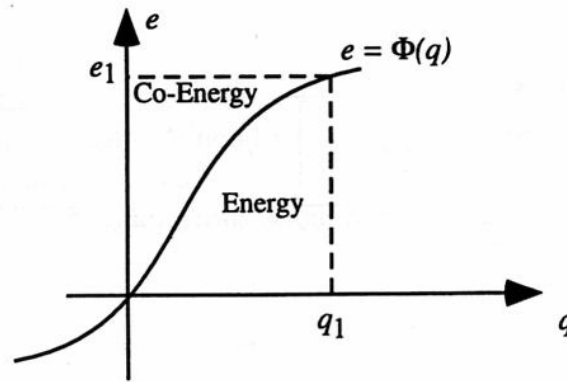


Figure 5.5. Illustrating energy and co-energy for a generalized capacitor.

Sammenlign med et forenklet tilfelle:
- en **lineær** kondensator

$$Q = V \cdot C$$

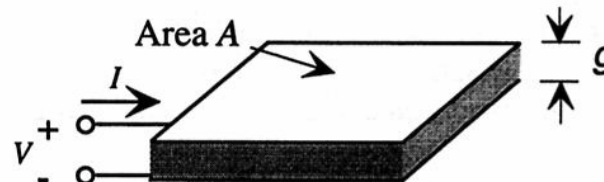
$$V = \frac{1}{C} \cdot Q$$

\Downarrow

$$e = \frac{1}{C} \cdot q$$

$$C = \frac{\epsilon A}{g}$$

definisjon av C



Generalisert kapasitans, forts.

Kapasitans assosieres med lagret **potensiell energi**

$$\mathcal{W}(q_1) = \int_0^{q_1} e \, dq = \int_0^{q_1} \Phi(q) \, dq \quad (5.10)$$

Co-energy:

$$\mathcal{W}^*(e) = eq - \mathcal{W}(q) \quad (5.11)$$

$$\mathcal{W}^*(e_1) = \int_0^{e_1} q \, de = \int_0^{e_1} \Phi^{-1}(e) \, de \quad (5.12)$$

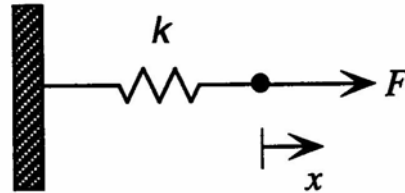
Energi lagret i parallell-plate kondensator

Energy:
$$W(Q) = \int_0^Q e \cdot dq = \int_0^Q \frac{q}{C} \cdot dq = \frac{Q^2}{2C}$$

Co-energy:
$$W^*(V) = \int_0^V q \cdot de = \int_0^V C \cdot v \cdot dv = \frac{CV^2}{2}$$

$$W^*(V) = W(Q) \quad \text{for lineær kapasitans}$$

Mekanisk fjær



Hook's lov: $F = k \cdot x$

Lagret energi: $W(x_1) = \int_0^{x_1} F(x)dx = \frac{1}{2}kx_1^2$ (5.18)

Sammenlign med kondensator $W(Q) = \frac{1}{2} \cdot \frac{1}{C} \cdot Q^2$

Q displacement
 x_1 displacement

→ 1/C tilsvarer k

”Compliance”

- ”Compliance” = ”ettergivenhet”

$$C_{spring} = \frac{1}{k}$$

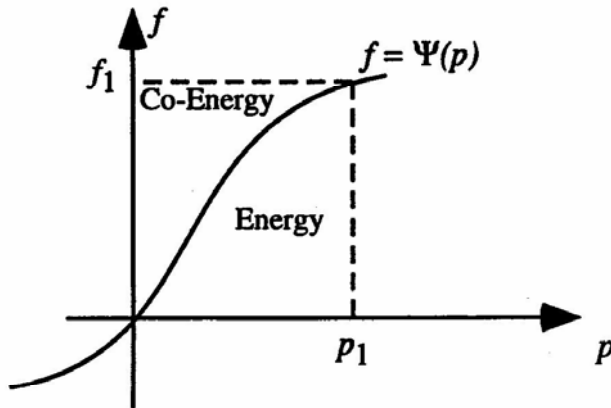
- Stiv fjær → liten kondensator
- Myk fjær → stor kondensator

Generalisert induktans

Energi er også definert som:

$$\int e dt = \text{flow} \times \text{momentum}$$

\uparrow \uparrow \uparrow
 v v $m \cdot v$



Energi = lagret kinetisk energi

$$W(p_1) = \int_0^{p_1} f(p) dp$$

Eks.: Elektrisk spole

Co-energy: $W^*(f) = \int_0^{f_1} p(f) df$



$$V = L \frac{dI}{dt}$$

$$p = \int e dt = \int V dt = \int L \frac{dI}{dt} dt = \int L dI$$

$$p(f) = p(I) = LI$$

$$W^*(f_1) = W^*(I_1) = \int_0^{I_1} L \cdot I \cdot dI = \frac{1}{2} L I_1^2$$

Analogien mellom masse (mekanisk "inertance") og induktans L

Et mekanisk system har **lineært momentum**: $p = mv$

Flow: $\phi = v = \frac{p}{m}$

$$W(p_1) = \int_0^{p_1} f(p) dp = \int_0^{p_1} \frac{p}{m} dp = \frac{p_1^2}{2m}$$

Co-energy:

$$W^*(v_1) = \int_0^{v_1} p(v) dv = \int_0^{v_1} (mv) dv = \frac{1}{2} m v_1^2$$

Analogi mellom m og L

$$W^*(f_1) = W^*(I_1) = \int_0^{I_1} L \cdot I \cdot dI = \frac{1}{2} L I_1^2$$

Sammenlign med: $W^*(v_1) = \frac{1}{2} m v_1^2$

$$I_1 = \text{flow}$$

$$v_1 = \text{---}$$

L tilsvarer m

m = L "inertance"

Mekanisk "inertance" (treghet) = masse m
har analogi til induktans L

Sammenkobling av elementer

- $e \rightarrow V$ har to grunnleggende prinsipper
 - Elementer som deler en felles *flow* og derved en felles variasjon i *displacement* er koblet i **serie**
 - Elementer som deler en felles *effort* er koblet i **parallell**

Eks. på sammenkobling:

”Direkte omforming”

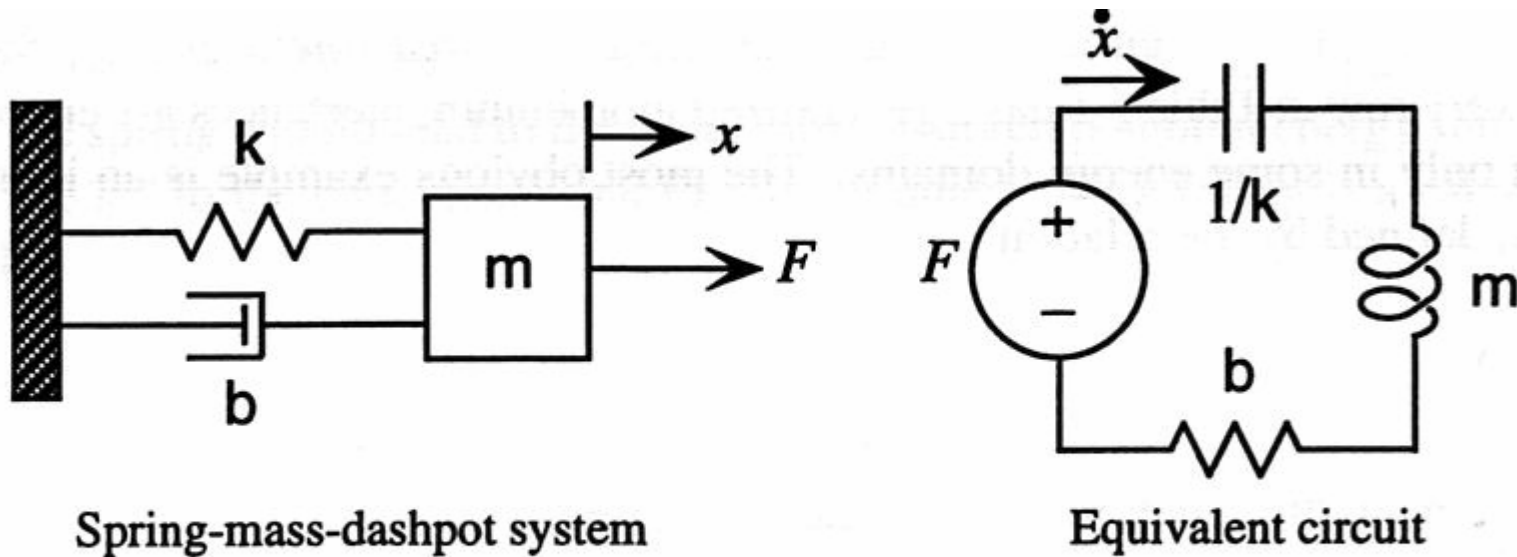
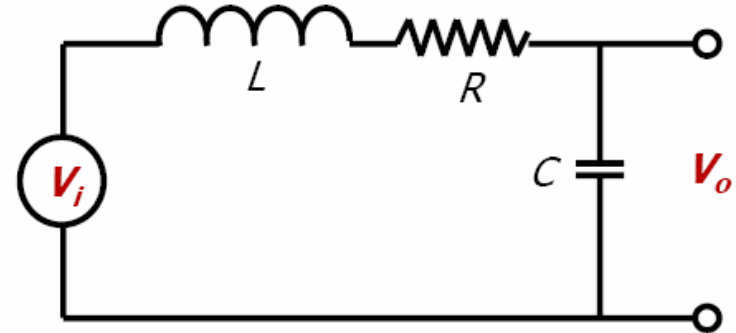
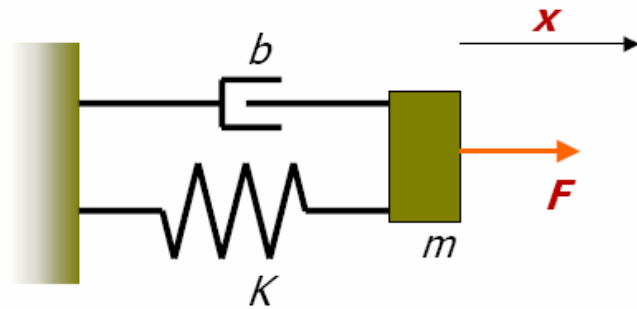


Figure 5.9. Translating mechanical to electrical representations.

Mechanical / Electrical Systems



Input : external force F

Output : displacement x

$$m\ddot{x}(t) + b\dot{x}(t) + Kx(t) = F$$

m mass, b damping, K stiffness

Transfer function :

$$H(s) = \frac{x}{F} = \frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{K}{m}}$$

Input : voltage V_i

Output : voltage V_o

$$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = V_i$$

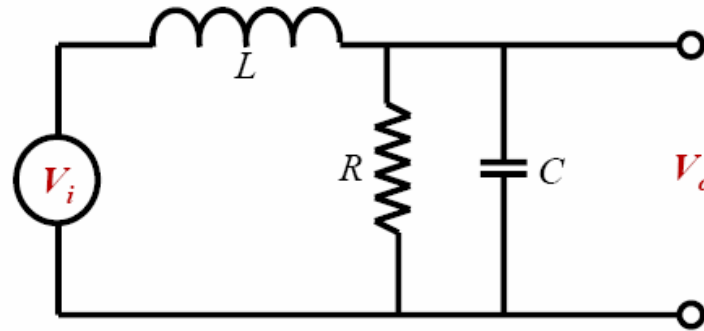
L induct., R resist., C capacit.

Transfer function :

$$H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Mechanical / Electrical Systems

Alternative circuit:



Input : voltage V_i

Output : voltage V_o

$$L\ddot{q}(t) + \frac{L}{RC}\dot{q}(t) + \frac{1}{C}q(t) = V_i$$

L inductance, R resistance, C capacitance

Transfer function :

$$H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Resonators

- Analogy between mechanical and electrical system:
 - Mass m - inductivity L
 - Spring K - capacitance C
 - Damping b - resistance R (depending where R is placed in circuit)
- Solution to 2nd order differential equation:

$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

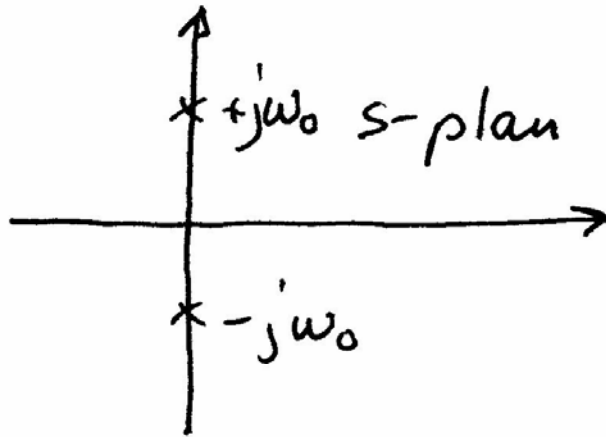
$$\omega_0 = 2\pi f_0 \text{ natural frequency}$$

$$\omega_0 = \sqrt{\frac{K}{m}} \text{ mechanical system, } \omega_0 = \sqrt{\frac{1}{LC}} \text{ electrical system}$$

$$Q \text{ quality factor}$$

Udempet system

$$H(s) = \frac{\omega_0^2}{s^2 + \omega_0^2} = \frac{\omega_0^2}{(s + j\omega_0)(s - j\omega_0)}$$



$$|H(j\omega_0)| = \infty$$

$$H(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

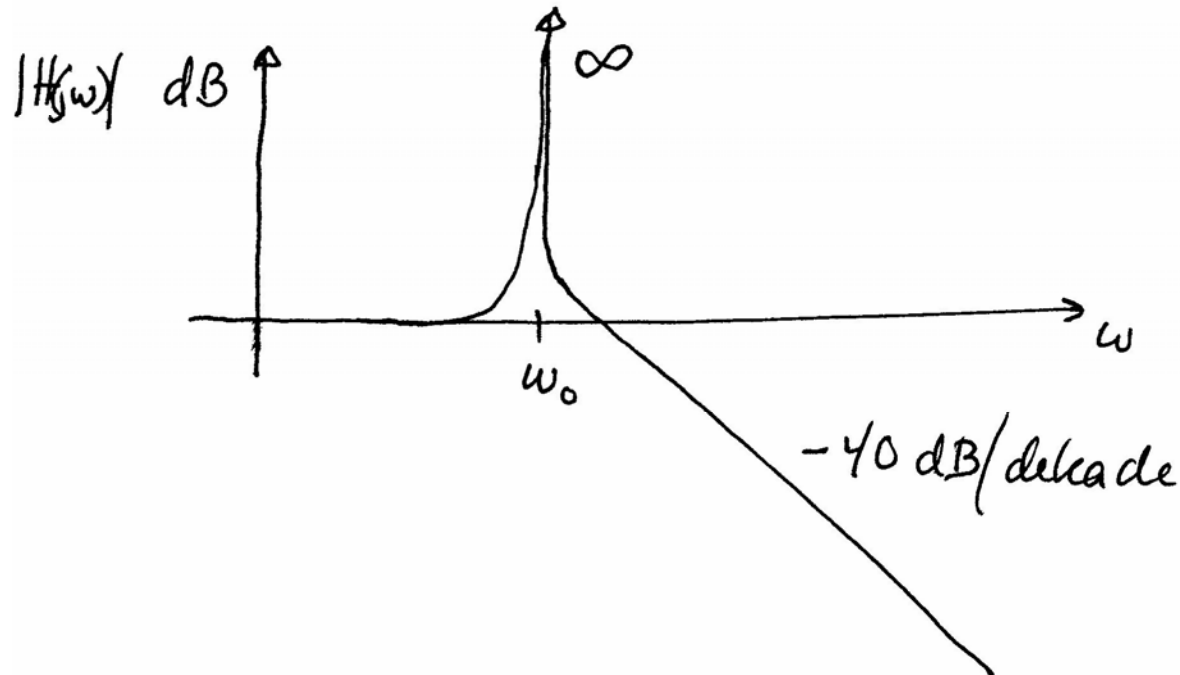
$$\omega_0 = \sqrt{\frac{1}{LC}}, \omega_0 = \sqrt{\frac{k}{m}}$$

Udempet system, forts.

$$H(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

$$|H(j\omega)| = 1 \quad \text{når} \quad \omega \ll \omega_0 \quad 0 \text{ dB}$$

$$|H(j\omega)| = -\left(\frac{\omega_0}{\omega}\right)^2 \quad \text{når} \quad \omega \gg \omega_0 \quad -40 \text{ dB/dekade}$$

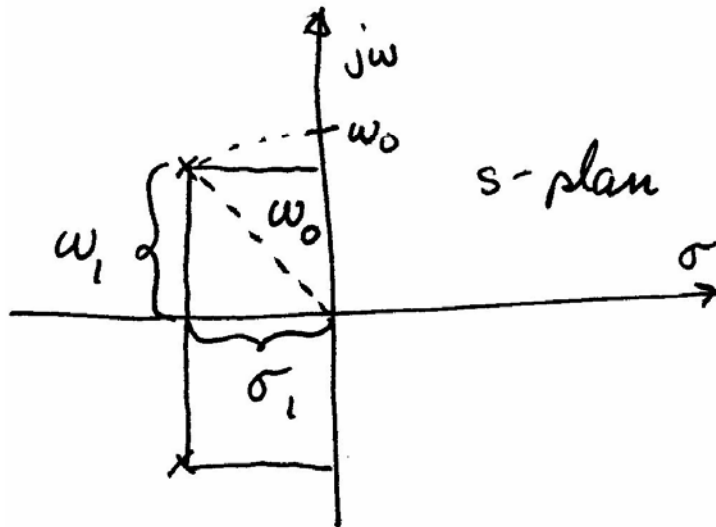


Ved demping

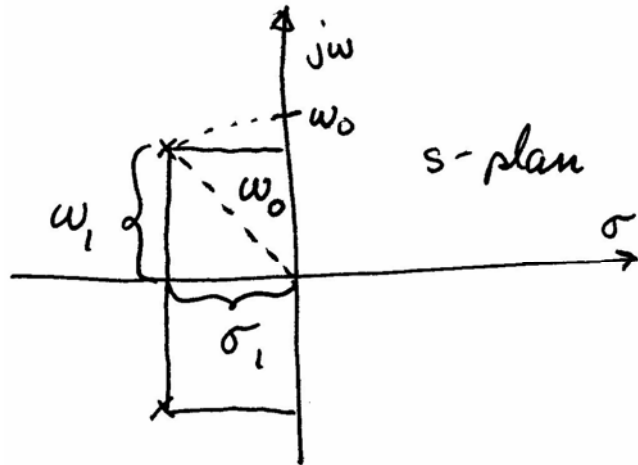
$$s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0$$

$$s = -\frac{\omega_0}{2Q} \pm j \omega_0 \sqrt{1 - \frac{1}{4Q^2}}$$

$$= -\sigma_1 \pm j \omega_1$$



Dempet system, forts.



$$\frac{\omega_0}{Q} = \frac{b}{m} = \frac{1}{\tau}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\sigma_1 = \frac{1}{2\tau} = \frac{b}{2m}$$

$$\omega_1 = \omega_0 \sqrt{1 - \frac{1}{4\omega_0^2 \tau^2}} = \omega_0 \sqrt{1 - \frac{b^2}{4km}}$$

$$\omega_1^2 + \sigma_1^2 = \omega_0^2$$

Mechanical Resonator

- Frequency and phase shift under damping:

$$x(t) = Ae^{-t/2\tau} \cos(\omega_1 t + \varphi)$$

$$\tau = m/b \text{ damping time}$$

$$\omega_1 = \omega_0 \sqrt{1 - \frac{1}{4\omega_0^2 \tau^2}} = \omega_0 \sqrt{1 - \frac{b^2}{4Km}}$$

φ phase shift

- Energy dissipation:

$$E(t) = E_0 e^{-t/\tau}$$

Hva betyr "damping time"?

τ = damping time

$$e^{-t/2\tau} \Big|_{t=\tau} = e^{-1/2} = \frac{1}{\sqrt{e}}$$

Effekten

$$|x(t)|^2 \Big|_{t=\tau} = \frac{1}{e}$$

$$x(t) = A e^{-t/2\tau} \cos(\omega_1 t + \varphi)$$

$$x(0) = A \cdot \cos \varphi \quad \text{initialbetingelser}$$

Q-faktor og dempetid

Generell ligning

$$s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0$$

$$\Rightarrow s^2 + \frac{1}{\tau} s + \omega_0^2 = 0$$

$$Q = \omega_0 \tau$$

$$\tau = \frac{m}{b} \quad \text{mekanisk}$$

$$\tau = \frac{L}{R} \quad \text{elektrisk}$$

$$Q_{\text{mek}} = \frac{\omega_0 m}{b}$$

$$Q_{\text{el}} = \frac{\omega_0 L}{R}$$

Quality Factor

- How fast does energy dissipate?
- What is the maximum amplitude for a given frequency?

Definition: Quality factor (Q factor)

Ratio of stored energy and lost energy: $Q = 2\pi \frac{E}{|\Delta E|} = 2\pi \frac{\tau}{T} = \omega_0 \tau$

Mechanical system: $Q = \omega_0 \frac{m}{b} = \frac{\sqrt{Km}}{b}$

Similar for electric systems: (a) $Q = \omega_0 \frac{L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$

(b) $Q = \omega_0 RC = R \sqrt{\frac{C}{L}}$

Q-faktor, forts.

Impedans i seriegren $R + j\omega L$
 $\underbrace{\hspace{1.5cm}}_{re} \quad \underbrace{\hspace{1.5cm}}_{imag}$

$$Q_d = \frac{Z_{imag}}{Z_{re}} = \frac{\omega_0 L}{R}$$

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost}} = \omega_0 \tau = 2\pi f_0 \cdot \tau$$
$$= 2\pi \frac{\tau}{T_0}$$

$T_0 = \text{perioden}$

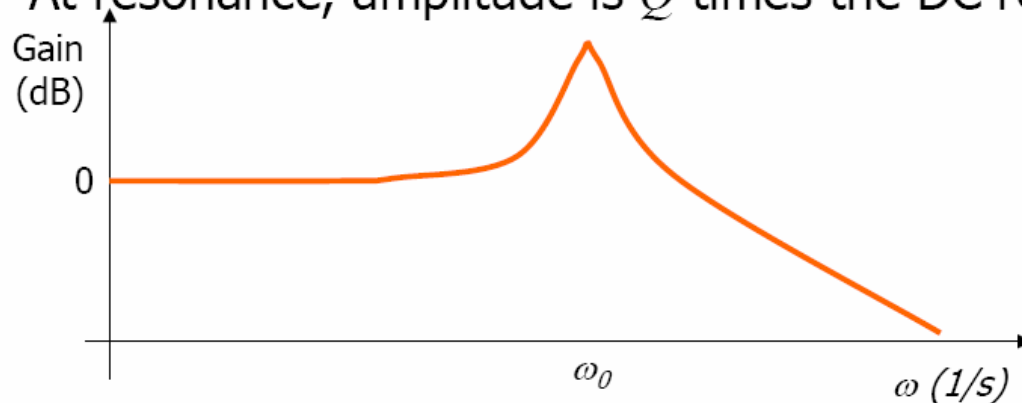
Quality Factor

- How fast does energy dissipate?

$$\tau = \frac{Q}{\omega_0} \quad \tau = \frac{m}{b} \text{ (mechanical)}$$

- What is the maximum amplitude for a given frequency?

At resonance, amplitude is Q times the DC response



Amplituden ved resonans

$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H(j\omega) = \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + j \frac{\omega \omega_0}{Q}}$$

$$|H(j\omega_0)| = \left| \frac{\omega_0^2}{0 + j \frac{\omega_0^2}{Q}} \right| = Q$$

Interaksjon mellom energidomener

- **Sammenkobling** av forskjellige energidomener hvor det ikke er energitap
 - 1. Hver av energidomenene omformes til sin elektriske ekvivalent
 - 2. Transformator og gyrator benyttes til sammenkobling
 - 3. Transformator kan "fjernes"
 - komponentverdiene må regnes om til nye verdier
 - vindingstall ("turn ratio") er en sentral parameter

Interaksjon mellom energidomener

- Lineære 2-port elementer er
 - Transformator og gyrator

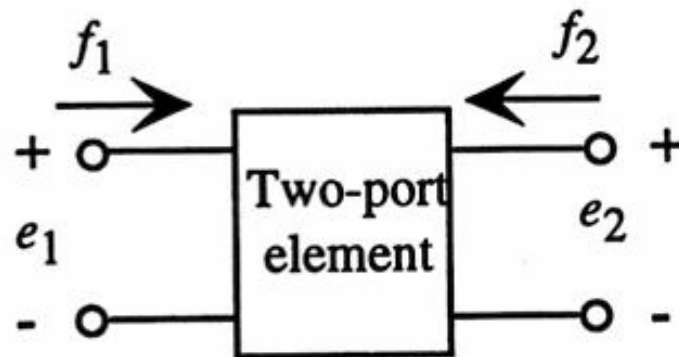


Figure 5.11. General two-port element.

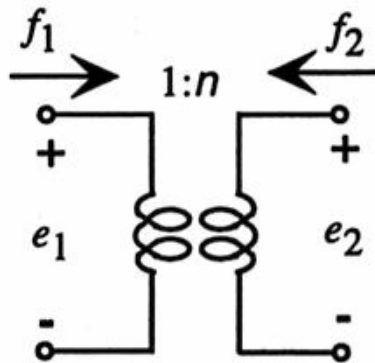
$$e_1 f_1 + e_2 f_2 = 0$$

(5.41)

Transformer

TRANSFORMER:

$$\begin{pmatrix} e_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \begin{pmatrix} e_1 \\ f_1 \end{pmatrix} \quad (5.42)$$



Transformer

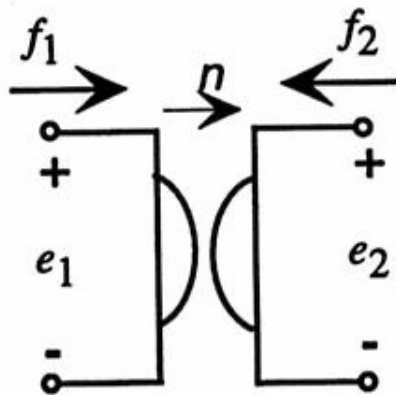
$$e_2 = n \cdot e_1$$
$$f_2 = -\frac{1}{n} f_1$$

n = "turns ratio"

Gyrator

GYRATOR:

$$\begin{pmatrix} e_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} 0 & n \\ -\frac{1}{n} & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ f_1 \end{pmatrix} \quad (5.43)$$

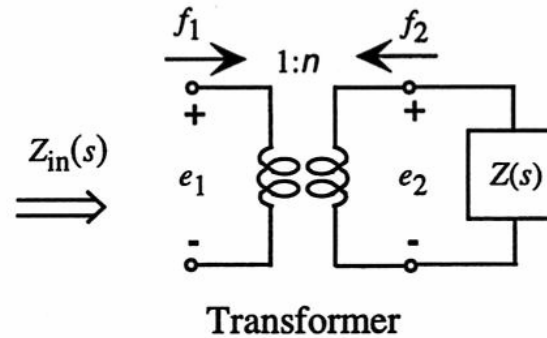


Gyrator

$$e_2 = n \cdot f_1$$
$$f_2 = -\frac{1}{n} e_1$$

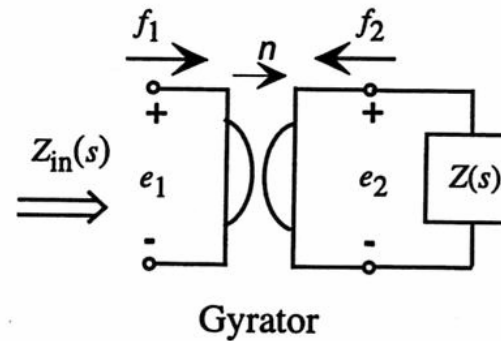
Impedanser transformereres

$$Z_{in}(s) = \frac{Z(s)}{n^2}$$



n^2 = koblingskoeff mellom energidomenene

$$Z_{in}(s) = \frac{n^2}{Z(s)}$$



F. eks

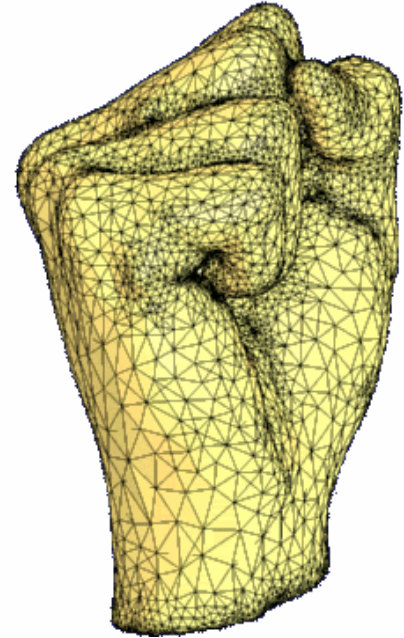
$$\frac{1}{sC} \rightarrow sL$$



3. Analyse ved Finite Element Methods

- Typiske trekk
 - Oppdeling i små-elementer: "meshing"
 - Løse matematiske ligninger for interaksjonen mellom elementene
 - Mange iterasjoner utføres før stabil løsning oppnås
- + Mer realistiske resultater
 - Matematiske modeller er approksimasjoner
 - Utilstrekkelige ved komplekse sammenhenger og strukturer
 - Jmfr. bøyning av bjelke: ladningsfordeling \leftrightarrow kraft
- Bruk av FEM-simuleringer
 - CoventorWare
 - Eksempler fra modellering av bulk prosess \rightarrow

Finite Element Methods



- Features
 - + good precision
 - + coupled electrostatic/ mech interaction
 - + can cope with irregular topologies
 - - insight into parameters influence is lost
 - - only small parts are practical
- Critical issues
 - proper system selection, building the 3D model
 - partitioning (meshing) , simulation parameters

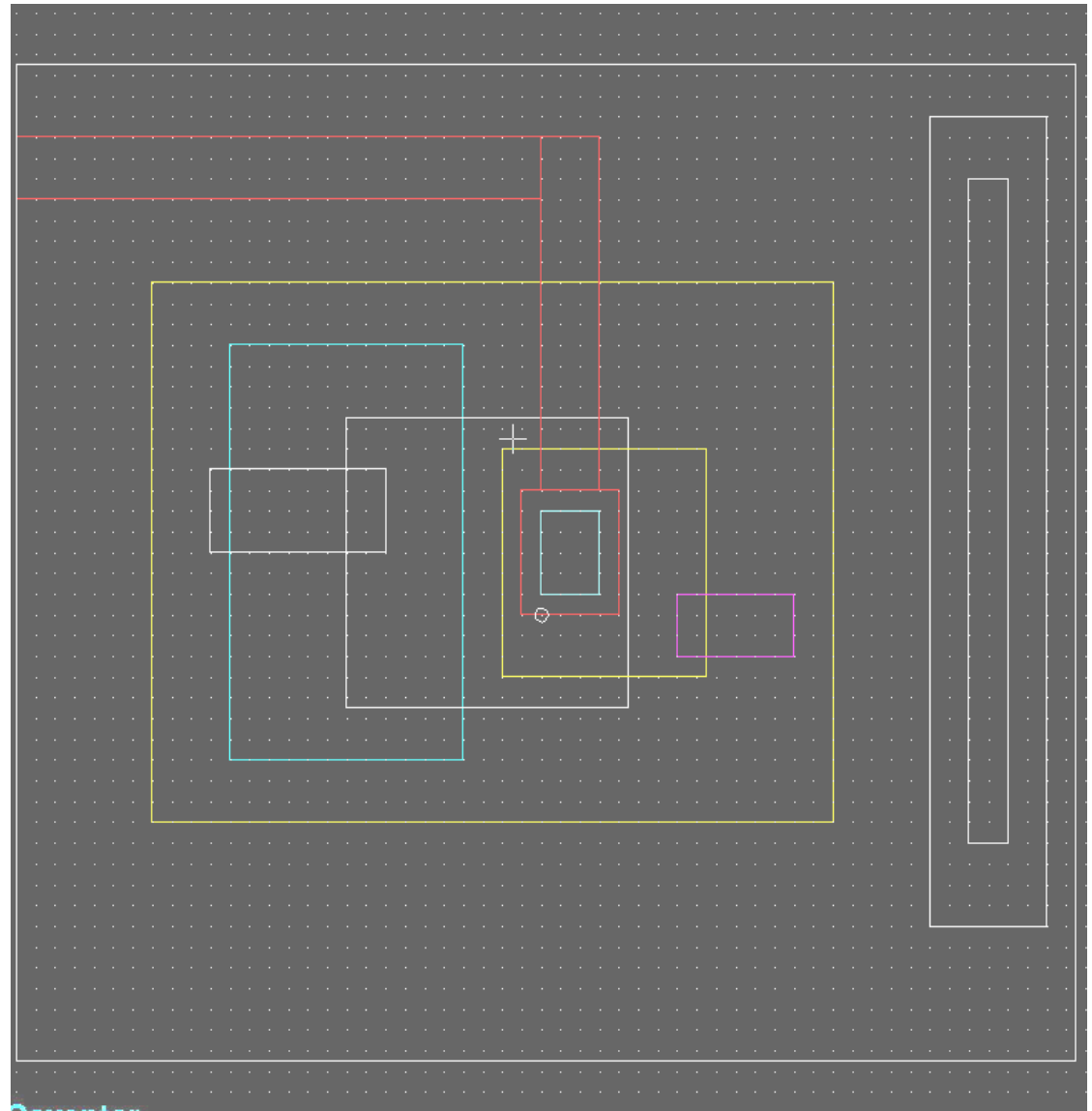
3D model building: process specification

Step	Action	Type	Layer Name	Material	Thic...	Color	Mask Name/ Polarity	Depth	Offset	Sidewall Angle	Comment
0	Base		Substrate	SILICON	10.0	blue	GND				
1	Etch	Back, Substr...				cyan	BETCH	- 10.0	0.0	0.0	
2	Deposit	Stacked	Layer1	SILICON	0.01	blue					
3	Deposit	Stacked	Layer2	SILICON	8.0	blue					
4	Etch	Front, Last L...				yellow	NOWEL	- 8.0	0.0	0.0	
5	Deposit	Planar	Layer3	SILICON	0.0	yellow					
6	Etch	Front, Partial				white	BUCON	- 4.0	0.0	0.0	
7	Etch	Front, Partial				pink	BURES	- 1.0	0.0	0.0	
8	Deposit	Planar	Layer4	SILICON	0.0	white					
9	Etch	Front, Partial				pink	BURES	- 1.0	0.0	0.0	
10	Deposit	Planar	Layer5	SILICON	0.0	pink					
11	Deposit	Stacked	Layer6	SILICON	3.0	green					
12	Etch	Front, Last L...				oran...	SUCON	- 3.0	0.0	0.0	
13	Etch	Front, Partial				mag...	SURES	- 1.0	0.0	0.0	
14	Deposit	Planar	Layer7	SILICON	0.0	oran...					
15	Etch	Front, Partial				mag...	SURES	- 1.0	0.0	0.0	
16	Deposit	Planar	Layer8	SILICON	0.0	mag...					
17	Etch	Front, By Depth				lemo...	NOSUR	- 1.0	0.0	0.0	
18	Deposit	Planar	Layer9	SILICON	0.0	gray					
19	Deposit	Stacked	Layer10	THERM_OXIDE	2.0	tan					
20	Etch	Front, Last L...				dlodg...	COHOL	- 2.0	0.0	0.0	
21	Etch	Front, Last L...				light...	NOBOA	- 2.0	0.0	0.0	
22	Deposit	Conformal	Layer11	ALUMINUM	1.0	red					
23	Etch	Front, Last L...				red	MCOND	+ 1.0	0.0	0.0	

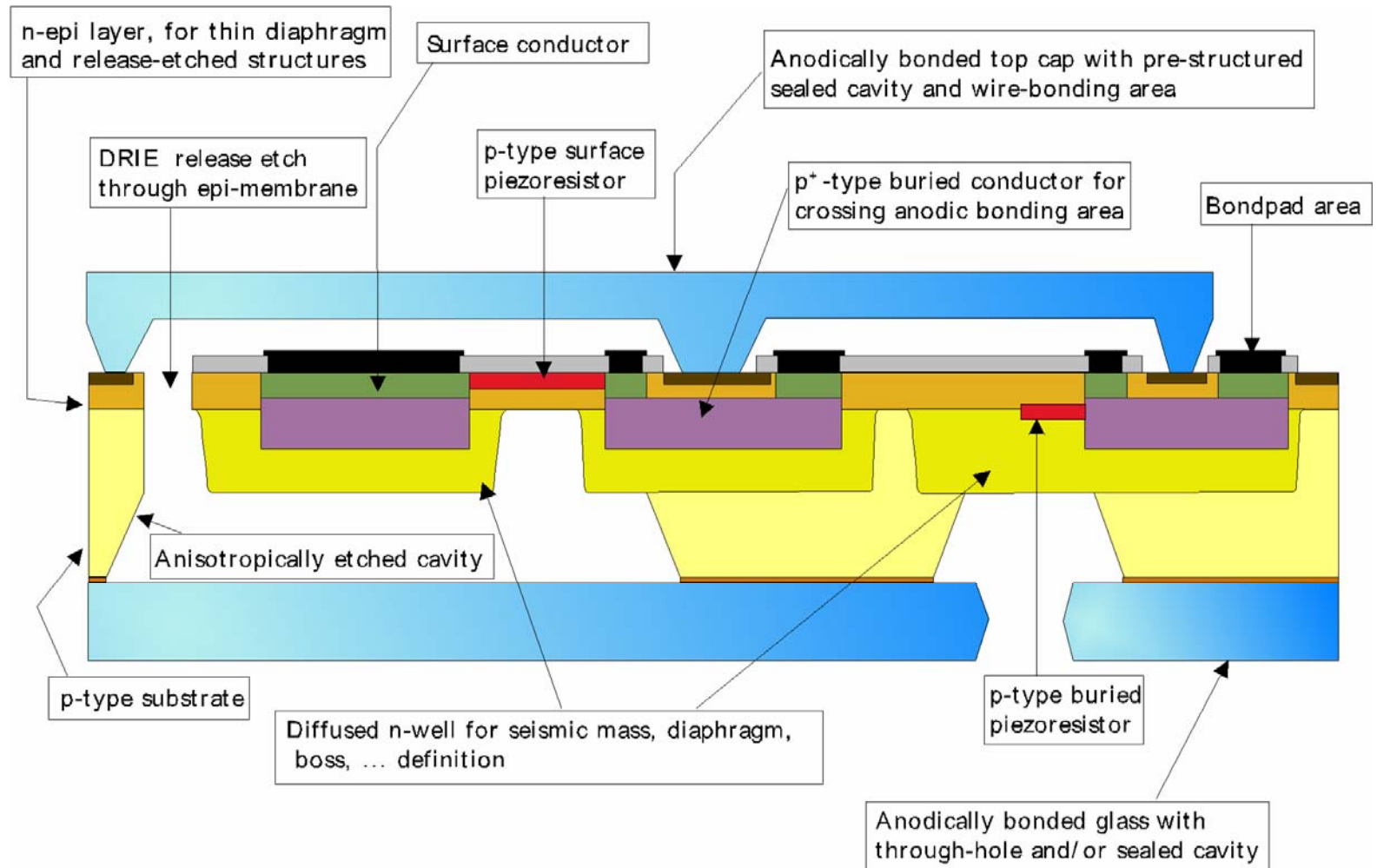
- Specify a **process file** which matches an actual foundry process
 - simplifications
 - realistic: essential process features included
- --> **pseudo layers**

3D model building: layout

Make accompanying
layout

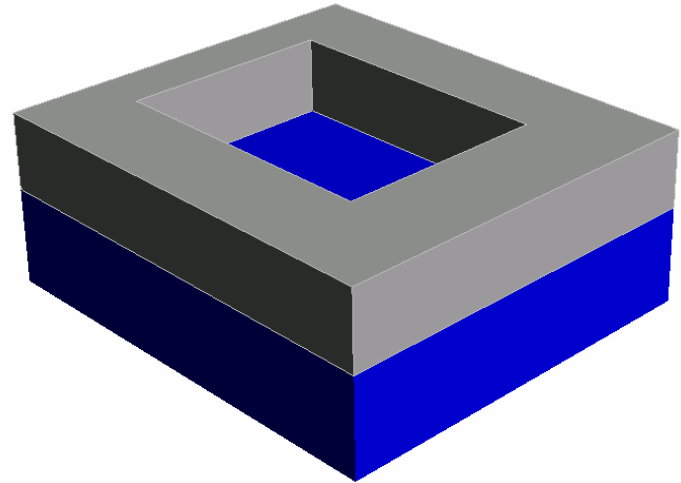


MultiMEMS, typical features



How to model the MultiMEMS bulk process in CoventorWare?

- Problem:
 - the process is not based on “stacking layers”
- Create a pseudo process!
 - simplified, but matching
 - transfer to a procedure of **stacking layers**
 - some layers with zero spacing
 - slicing the bulk material into sub-layers **in contact**
 - make etchings and re-fillings

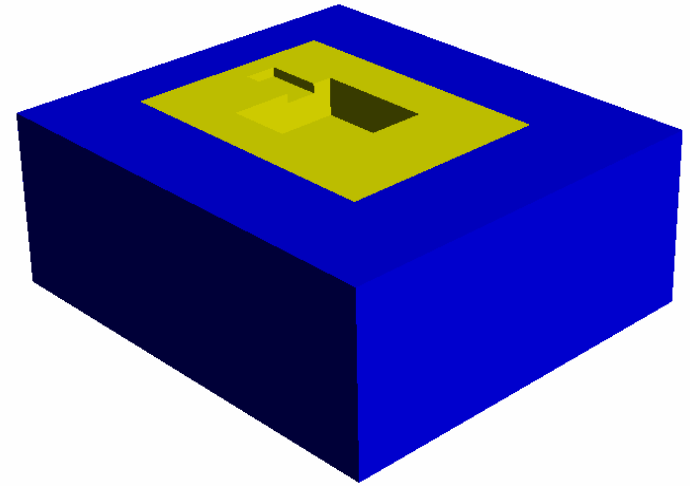


ProcessEditor: M:\Design_Files\testproject1\Devices\nlayers_c.proc

File Edit View Help

Step	Action	Type	Layer Name	Material	Thic...	Color	Mask Name/ Polarity	Depth	Offset	Sidewall Angle	Comment
0	Base		Substrate	SILICON	10.0	blue	GND				
1	Etch	Back, Substr...				cyan	BETCH	- 10.0	0.0	0.0	
2	Deposit	Stacked	Layer1	SILICON	0.01	blue					
3	Deposit	Stacked	Layer2	SILICON	8.0	blue					
4	Etch	Front, Last L...				yellow	NOWEL	- 8.0	0.0	0.0	

Two slices of the base material stacked. **N-well** opening



ProcessEditor: M:\Design_Files\testproject1\Devices\layers_c.proc

File Edit View Help

↑ ↓ [Color icons] [ENABLE/DISABLE] [X] [?]

Step	Action	Type	Layer Name	Material	Thic...	Color	Mask Name/ Polarity	Depth	Offset	Sidewall Angle	Comment
0	Base		Substrate	SILICON	10.0	blue	GND				
1	Etch	Back, Substr...				cyan	BETCH -	10.0	0.0	0.0	
2	Deposit	Stacked	Layer1	SILICON	0.01	blue					
3	Deposit	Stacked	Layer2	SILICON	8.0	blue					
4	Etch	Front, Last L...				yellow	NOVMEL -	8.0	0.0	0.0	
5	Deposit	Planar	Layer3	SILICON	0.0	yellow					
6	Etch	Front, Partial				white	BUCON -	4.0	0.0	0.0	
7	Etch	Front, Partial				pink	BURES -	1.0	0.0	0.0	

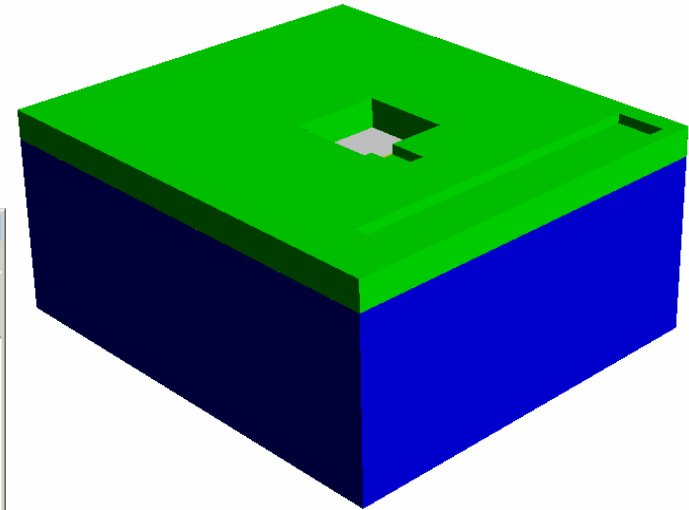
N-well in-filling. Etching holes for **buried conductor** implant and **buried resistor** implant

ProcessEditor: M:\Design_Files\testproject1\Devices\layers_c.proc

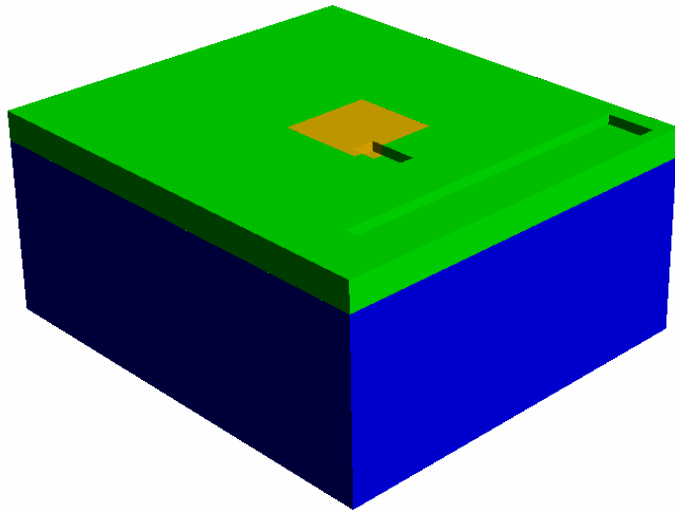
File Edit View Help

↑ ↓ [Color Swatches] [Enable/Disable] [Close] [Help]

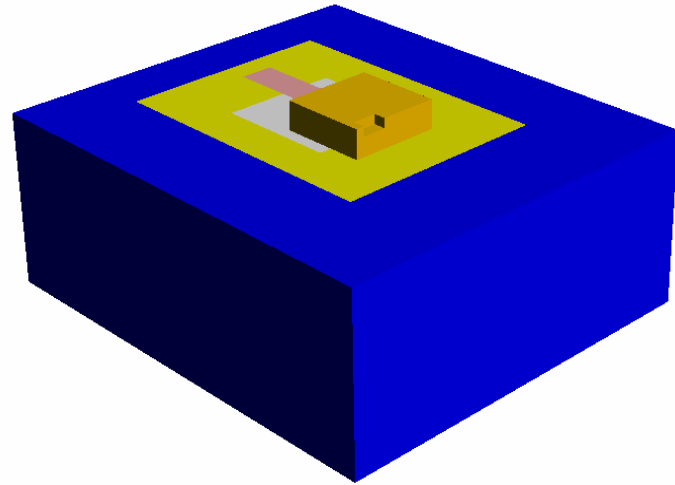
Step	Action	Type	Layer Name	Material	Thic...	Color	Mask Name/ Polarity	Depth	Offset	Sidewall Angle	Comment
0	Base		Substrate	SILICON	10.0	blue	GND				
1	Etch	Back, Substr...				cyan	BETCH -	10.0	0.0	0.0	
2	Deposit	Stacked	Layer1	SILICON	0.01	blue					
3	Deposit	Stacked	Layer2	SILICON	8.0	blue					
4	Etch	Front, Last L...				yellow	NOWEL -	8.0	0.0	0.0	
5	Deposit	Planar	Layer3	SILICON	0.0	yellow					
6	Etch	Front, Partial				white	BUCON -	4.0	0.0	0.0	
7	Etch	Front, Partial				pink	BURES -	1.0	0.0	0.0	
8	Deposit	Planar	Layer4	SILICON	0.0	white					
9	Etch	Front, Partial				pink	BURES -	1.0	0.0	0.0	
10	Deposit	Planar	Layer5	SILICON	0.0	pink					
11	Deposit	Stacked	Layer6	SILICON	3.0	green					
12	Etch	Front, Last L...				oran...	SUCON -	3.0	0.0	0.0	
13	Etch	Front, Partial				mag...	SURES -	1.0	0.0	0.0	
14	Deposit	Planar	Layer7	SILICON	0.0	oran...					
15	Etch	Front, Partial				mag...	SURES -	1.0	0.0	0.0	
16	Deposit	Planar	Layer8	SILICON	0.0	mag...					
17	Etch	Front, By Depth				lemo...	NOSUR -	1.0	0.0	0.0	



Add **epi-layer**. Etch holes for **surface conductor** and **surface resistor**, -fill in.
Etch hole for n+ implant. (Implants are invisible)

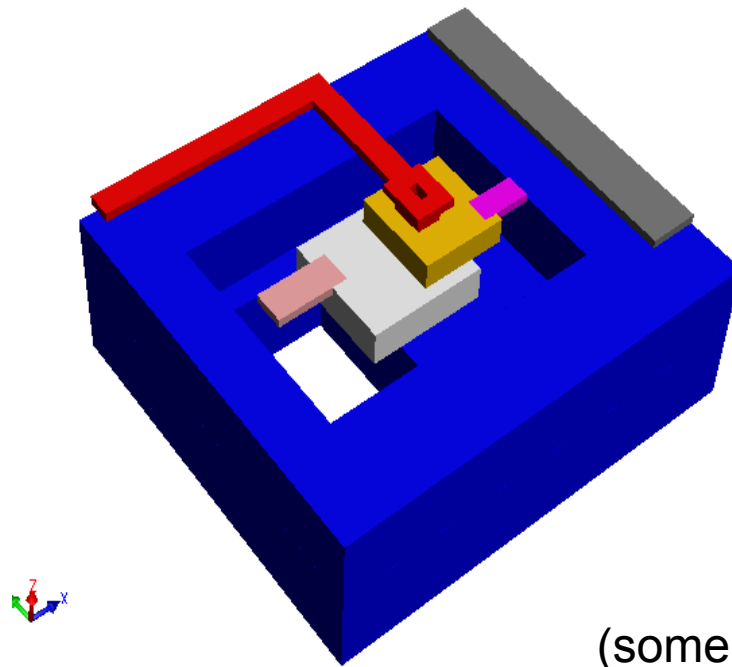


Surface conductor is made visible

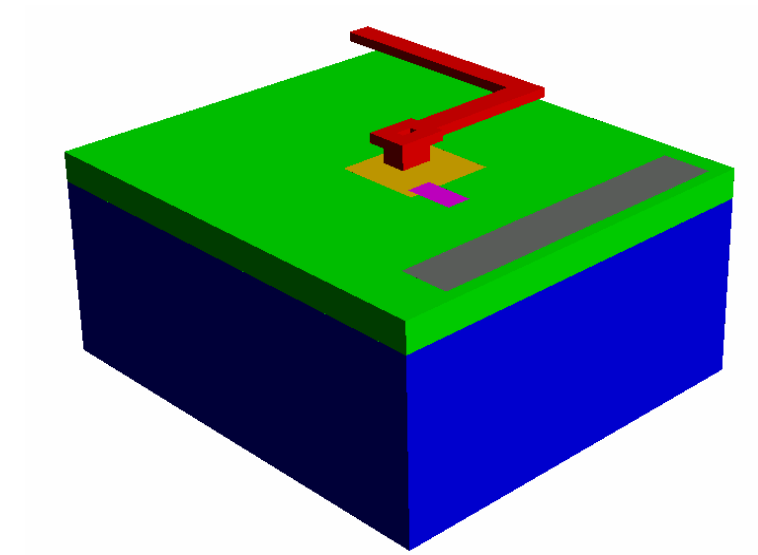
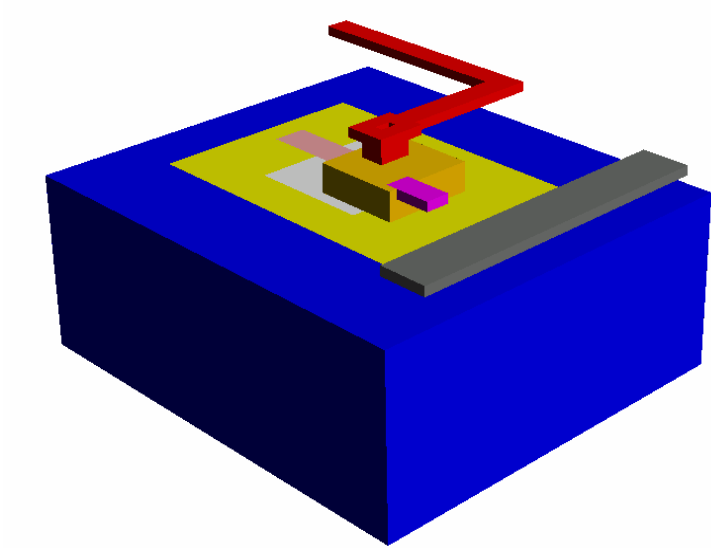


Epi-layer is invisible

3D model building: expansion



(some layers invisible)

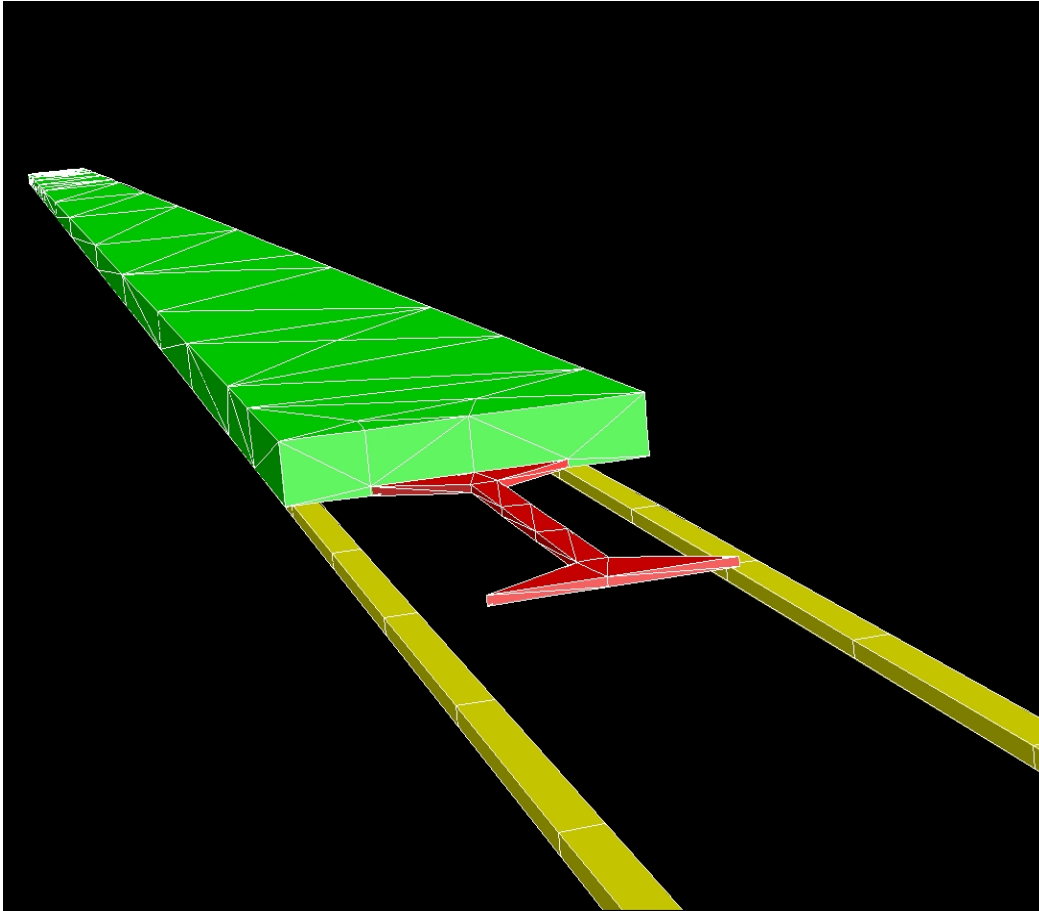


Complete structure with some layers made invisible

3D modelling procedure

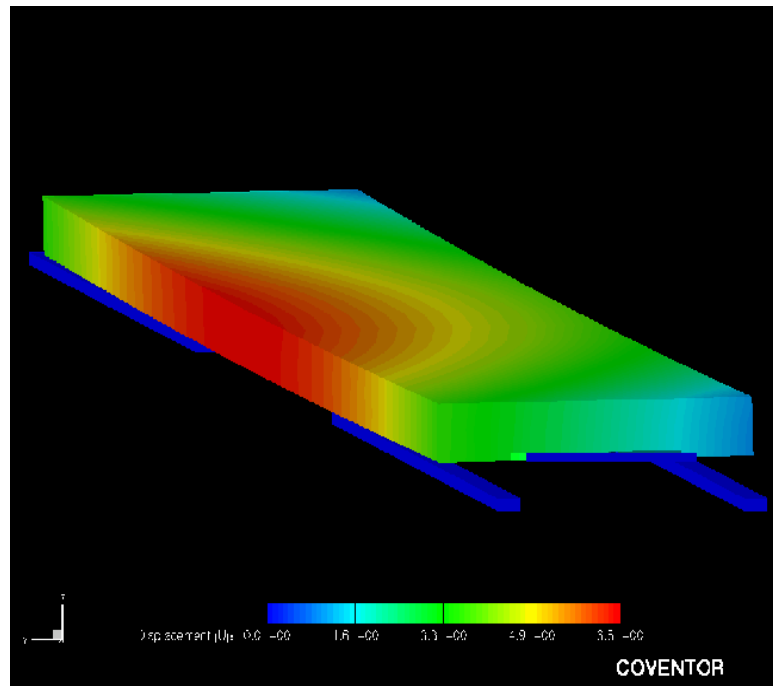
- To introduce one diffusion:
 - etch base material
 - fill in implanted material
 - **“deposit planar” with thickness = 0**
- To introduce multiple overlapping diffusions:
 - etch base material with all diffusion masks (the deepest first)
 - fill in the deepest implanted material
 - re-etch the remaining diffusion openings
 - fill in the next deepest implant etc.

Meshed model



- Mirror meshed by tetrahedrons
 - 23 μm , 3 μm
- Electrodes meshed by Manhattan bricks
 - 5 μm
- Rather coarse dim due to pull-in analysis

Mirror deflection, snapshot



Simulation: pull-in

