

INF5490 RF MEMS

L8: RF MEMS resonatorer II

Dagens forelesning

- Lateralt vibrerende resonator:
Kam-resonatoren
 - **Virkemåte**
 - Detaljert **modellering**
 - A) phasor-modellering
 - B) modellering ved konvertering mellom mekanisk og elektrisk energidomene

Lateral og vertikal bevegelse

- Lateral bevegelse i resonatoren
 - Parallelt med substratet
 - Foldet beam kam-struktur
- Vertikal bevegelse (neste forelesning)
 - Loddrett på substratet
 - Clamped-clamped beam (c-c beam)
 - Dobbeltinnspent bjelke
 - free-free beam (f-f beam)

Foldet beam kam-struktur

- Utviklet ved Univ. of California, Berkeley
 - Av de tidligste resonator design som ble produsert
 - Kam-strukturer brukt i mange MEMS systemer
 - For eksempel benyttet i **aksellerometer**
 - Kapasitansen mellom kammene påvirkes av aksellerasjonen

Kam-resonator

- Fast forankret kam + bevegelig, opphengt kam
- Opphengt ved foldede fjærer, kompakt utlegg
- Total-kapasitansen mellom kammene kan varieres
- Påtrykt spenning (+ eller -) genererer elektrostatisk kraft mellom venstre anker-kam og "shuttle"-kam. Plate trekkes til venstre i planet

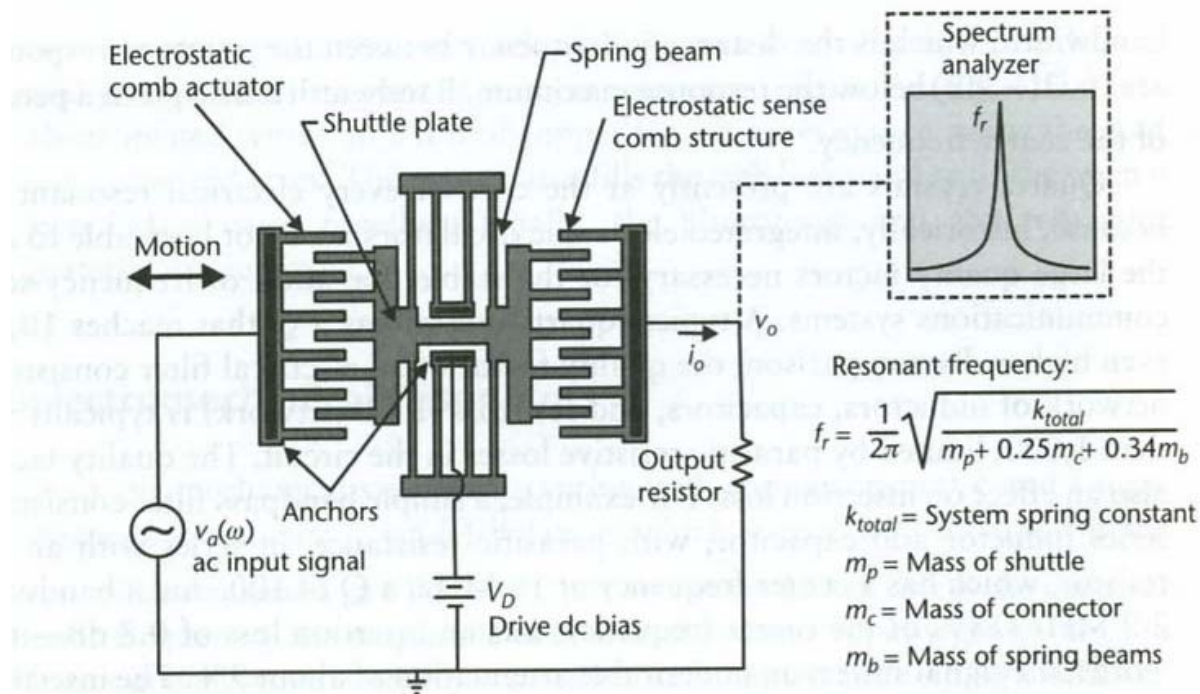
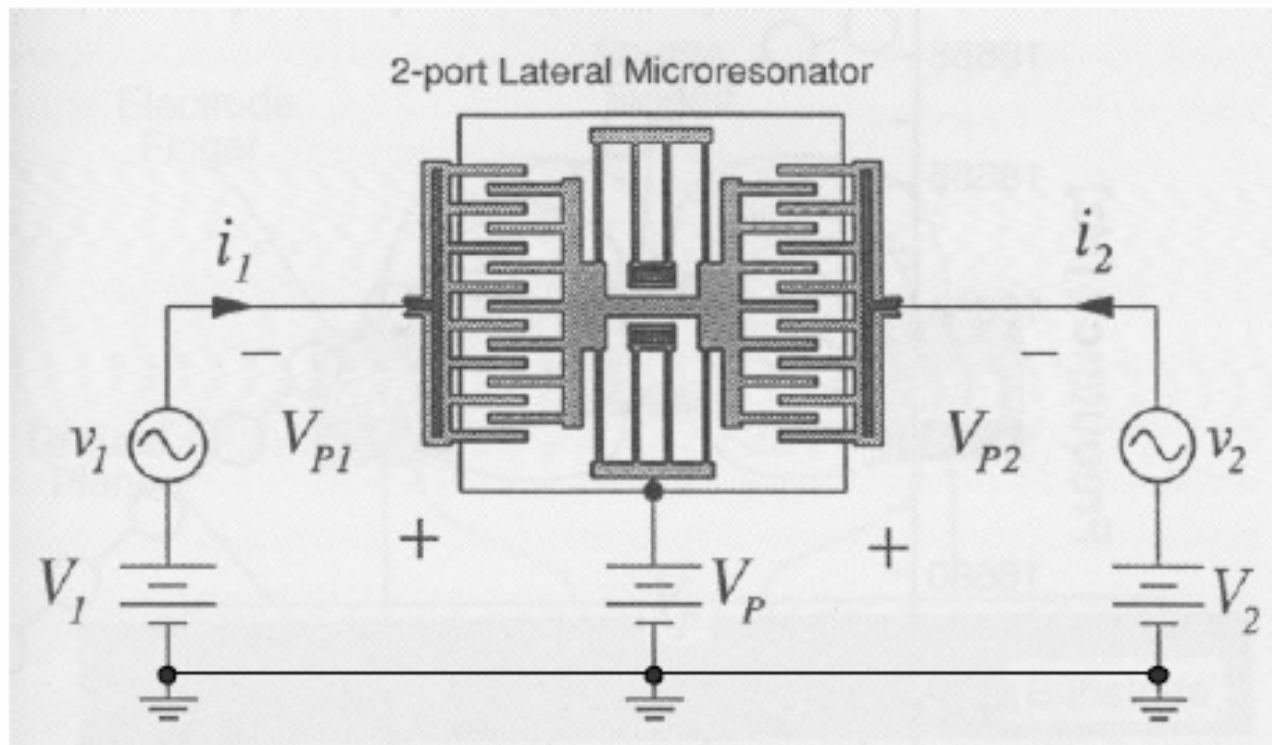


Figure 7.9 Illustration of a micromachined folded-beam comb-drive resonator. The left comb drive actuates the device at a variable frequency ω . The right capacitive-sense-comb structure measures the corresponding displacement by turning the varying capacitance into a current, which generates a voltage across the output resistor. There is a peak in displacement, current, and output voltage at the resonant frequency.

Detaljert modellering

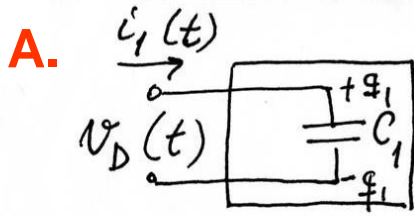
- Modellering av **lateral kam-struktur**
 - Phasor-modellering ala [UoC, Berkeley](#)
 - Supplert med utfyllende mellomregninger
 - Konvertering mellom energidomener
 - Stoff fra [UCLA](#)
- I forelesning L9 modelleres **c-c beam** med referanse til boka
 - T. Itoh et al: RF Technologies for Low Power Wireless Communications”, kap. 12: ”Transceiver Front-End Architectures Using Vibrating Micromechanical Signal Processors”, by Clark T.-C. Nguyen

The Lateral Resonator as a “Two-Port”



Beregningsprosedyre

- **A.** Betrakt kammen som en to-port og undersøk først inngangsporten
- **B.** Når kammen beveger seg, vil inngangskapasitansen ha et fast + et variabelt ledd
- **C.** Finn inngangsstrømmen som funksjon av forskyvningen, X , ved bevegelse
- **D.** Beregn inngangs-admittansen, Y ("motional admittance")
 - **D1.** Y uttrykkes ved X
 - **D2.** X er avhengig av den elektrostatiske kraften, F , samt m , b og k
 - **D3.** F er avhengig av påtrykt spenning, V
- **E.** Sett inn og finn et uttrykk for Y (dynamisk oppførsel)
- **F.** Sammenlign med Y for en L-C-R-gren og finn ekvivalente elementer
- **G.** Sett opp ekvivalentkretsen for inngangsporten
- **H.** Finn utgangsstrømmen for den gitte inngangen
- **I.** Beregn forholdet mellom utgangs- og inngangs-strøm ("forward current gain")
- **J.** Sett opp en to-port ekvivalent-krets
- **K.** Sett opp en komplett to-port-modell



$$\varphi_1 = C_1 \cdot v_D$$

$$\dot{\varphi}_1(t) = i_1(t) = C_1 \cdot \frac{dv_D}{dt} + v_D \cdot \frac{dC_1}{dt}$$

$$v_D(t) = V_1 + v_i(t) - V_P$$

$$= -V_{P1} + v_i(t) = -V_{P1} + v_i \cos \omega t$$

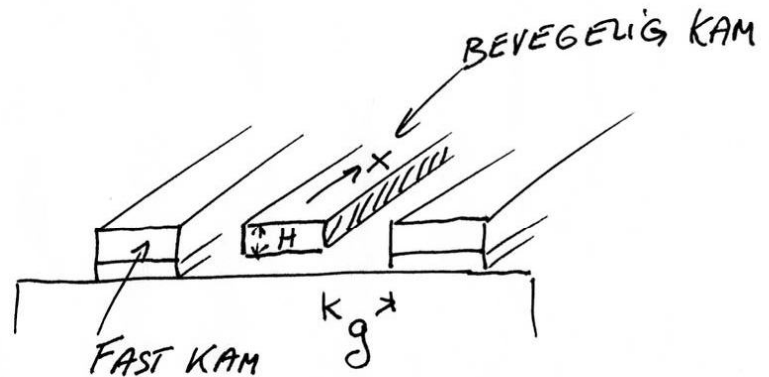
$$V_{P1} = V_P - V_1 \quad (\text{positiv när } V_P > V_1)$$

B.

$$C_1(t) = C_{01} + C_{m1}(t)$$

↑
DC-komponent

$$C_1 = \frac{\epsilon_0 A}{g} = \epsilon_0 \frac{x \cdot 2H}{g} \cdot n$$



$$C_1(t) = C_{01}(\text{fast}) + C_{m1}(\text{prop. med } x(t)) = C_{01} + \frac{\partial C_1}{\partial x} \cdot x(t)$$

(Linearisering, gennedt: $f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \dots$)

C. inngangsskemma:

$$i_1(t) = C_1 \frac{dv_D}{dt} + v_D \cdot \frac{dC_1}{dt}$$

$$= C_1 \frac{dv_1(t)}{dt} + (-V_{P1} + v_1(t)) \cdot \frac{\partial C_1}{\partial x} \cdot \frac{\partial x}{\partial t} = \left[C_{01} + \frac{\partial C_1}{\partial x} \cdot x(t) \right] \frac{dv_1(t)}{dt} + \dots$$

$$= C_{01} \frac{dv_1(t)}{dt} + \frac{\partial C_1}{\partial x} \cdot x(t) \frac{dv_1(t)}{dt} - V_{P1} \frac{\partial C_1}{\partial x} \cdot \frac{\partial x}{\partial t} + v_1(t) \cdot \frac{\partial C_1}{\partial x} \cdot \frac{\partial x}{\partial t}$$

$$= C_{01} \frac{dv_1(t)}{dt} + \frac{\partial C_1}{\partial x} \left(x \frac{\partial v_1}{\partial t} + v_1 \frac{\partial x}{\partial t} \right) - V_{P1} \frac{\partial C_1}{\partial x} \cdot \frac{\partial x}{\partial t}$$

$$\underbrace{\frac{\partial}{\partial t} (x \cdot v_1)}_{\sim \cos^2 \omega t = \frac{1}{2} (1 + \cos 2\omega t)} \quad \text{der} \quad \begin{array}{l} v_1 = v_0 \cos \omega t \\ x = x_0 \cos \omega t \end{array}$$

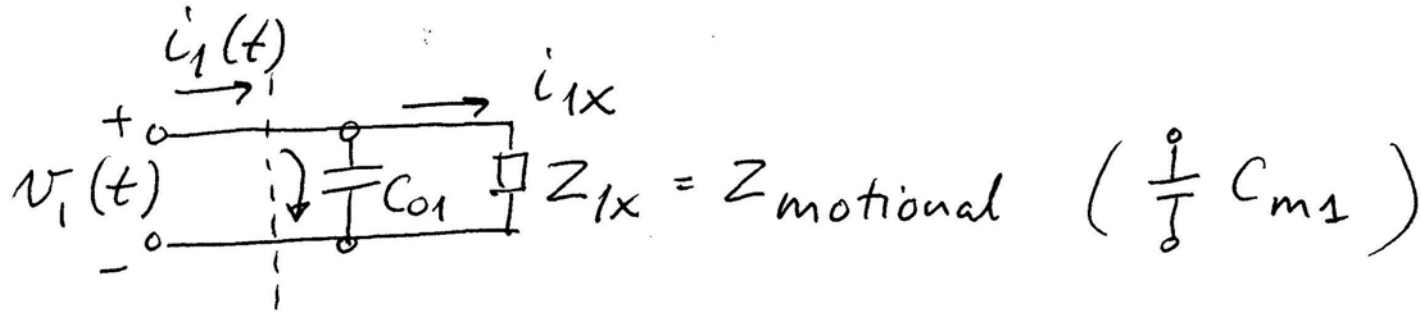
faller bort ved $\frac{\partial}{\partial t}$

double frekwensen
Lite ledd utenfor
 ω -resonansen

$$i_1(t) \approx C_{01} \cdot \frac{\partial v_1(t)}{\partial t} - V_{P1} \frac{\partial C_1}{\partial x} \cdot \frac{\partial x(t)}{\partial t}$$

Strom in i' DC-kapazitätsum

"Motional current"



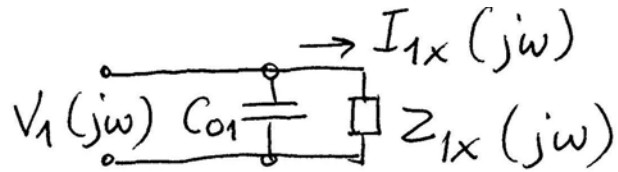
"Motional current"

$$i_{1x}(t) = -V_{P1} \frac{\partial C_1}{\partial x} \cdot \frac{\partial x(t)}{\partial t} \quad \left(= -V_{P1} \cdot \frac{\partial C_1}{\partial t} \right)$$

Phasor-form av "motional current"

$$I_{1x}(j\omega) = -V_{P1} \cdot \frac{\partial C_1}{\partial x} \cdot j\omega \cdot X(j\omega)$$

D.



input "motional admittance"

D1.

$$Y_{1x}(j\omega) = \frac{I_{1x}(j\omega)}{V_1(j\omega)} = -V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot \frac{x(j\omega)}{V_1(j\omega)}$$

← forskyvning (displacement)
 ← spenning

D2/3.

$$Y_{1x}(j\omega) = -V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot \frac{x(j\omega)}{F_d(j\omega)} \cdot \frac{F_d(j\omega)}{V_1(j\omega)}$$

→
 $F_d(j\omega)$ er avhengig av m, b og k

↑
 Spenning V_1 forårsaker en elektrostatiske kraft F_d

D3.

Forholdet mellem kraft og spenning findes av:

$$\text{Pot. energi } U = \frac{1}{2} C_1 v_D^2(t)$$

↑ uafhængig av x

$$F = \frac{\partial U}{\partial x} = \frac{1}{2} v_D^2(t) \cdot \frac{\partial C_1}{\partial x} \quad \leftarrow \text{ikke-lineært forhold}$$

Linearisering rundt DC-punktet:

$$F = F_0 + f \cdot \cos \omega t \quad v_D = -V_{P1} + v_i \cos \omega t$$

Indsættning:

$$\begin{aligned} F_0 + f \cos \omega t &= \frac{1}{2} (-V_{P1} + v_i \cos \omega t)^2 \cdot \frac{\partial C_1}{\partial x} \\ &= \frac{1}{2} [V_{P1}^2 - 2 V_{P1} v_i \cos \omega t + v_i^2 \cos^2 \omega t] \cdot \frac{\partial C_1}{\partial x} \end{aligned}$$

cos 2ωt-ledd

Se på AC-leddene:

$$f \cos \omega t = -V_{P1} \cdot v_i \cos \omega t \cdot \frac{\partial C_1}{\partial x}$$

$$f_{d,w} = -V_{P1} \cdot \frac{\partial C_1}{\partial x} \cdot v_i(t) \quad \Leftarrow \text{LINEÆRT FORTHOLD!}$$

i phasor-form

$$F_d(j\omega) = -V_{P1} \frac{\partial C_1}{\partial x} \cdot V_1(j\omega)$$

$$\Rightarrow \frac{F_d(j\omega)}{V_1(j\omega)} = -V_{P1} \cdot \frac{\partial C_1}{\partial x} \quad \text{ders. D3. er funnet.}$$

D2.

Forhold mellom forskyvning og kraft :

$$\frac{x(s)}{F_d(s)} = \frac{1}{m s^2 + b s + k} = \frac{1/m}{s^2 + \frac{b}{m} s + \frac{k}{m}} = \frac{1}{k} \cdot \frac{k/m}{s^2 + \frac{b}{m} s + \frac{k}{m}}$$

$$\text{Sette } \omega_0^2 = k/m \quad \text{og} \quad \frac{b}{m} = \frac{\omega_0}{Q} \quad \text{der } Q = \frac{\sqrt{k/m}}{b/m} = \frac{\sqrt{km}}{b}$$

$$\frac{x(s)}{F_d(s)} = \frac{1}{k} \cdot \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \xrightarrow{s=j\omega} \frac{1}{k} \cdot \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + j \frac{\omega_0 \cdot \omega}{Q}}$$

$$\frac{x(j\omega)}{F_d(j\omega)} = \frac{1}{k} \frac{1}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] + j \frac{\omega}{Q \omega_0}}$$

E. $Y_{1x}(j\omega) = -V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot \frac{x(j\omega)}{F_d(j\omega)} \cdot \frac{F_d(j\omega)}{V_1(j\omega)}$ input admittance

$$= -V_{P1} \cdot \frac{\partial C_1}{\partial x} \cdot j\omega \cdot \frac{1/k}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] + j \frac{\omega}{Q\omega_0}} \cdot \left(-V_{P1} \frac{\partial C_1}{\partial x}\right)$$

hence: $\boxed{\eta = V_{P1} \frac{\partial C_1}{\partial x}}$

$$Y_{1x}(j\omega) = \eta^2 j\omega \cdot \frac{1/k}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] + j \frac{\omega}{Q\omega_0}}$$

oder $I_{1x}(j\omega) = [\quad] \cdot V_1(j\omega)$

Dette gir

$$Y(j\omega) = \frac{j\omega c}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] + j \frac{\omega}{Q\omega_0}}$$

Sammenlign med:

$$Y_{1x}(j\omega) = \eta^2 \frac{j\omega \cdot 1/k}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] + j \frac{\omega}{Q\omega_0}}$$

Dette gir:

$$C_{x1} = \eta^2 / k$$

$$\omega_0^2 = \frac{k}{m} = \frac{1}{Lc} \Rightarrow L_{x1} = \frac{1}{c} \cdot \frac{m}{k} = \frac{k}{\eta^2} \cdot \frac{m}{k} = \frac{m}{\eta^2}$$

$$RC = \frac{1}{Q \cdot \omega_0} = \frac{1}{Q \sqrt{\frac{k}{m}}} \Rightarrow R_{x1} = \frac{1}{c} \cdot \frac{1}{Q \sqrt{\frac{k}{m}}} = \frac{k}{\eta^2} \cdot \frac{\sqrt{m}}{Q \sqrt{k}} = \frac{\sqrt{km}}{Q \cdot \eta^2}$$

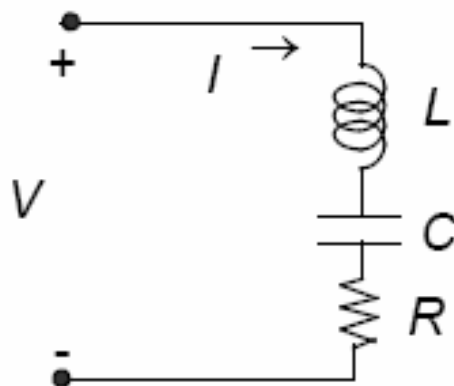
η = elektromekanisk koblingskoeffisient

Ved resonans opphever impedansene for L og C hverandre

$$I_{x1} = V_1 / R_{x1}$$

F. Series L - C - R Admittance

The current through an L - C - R branch is:

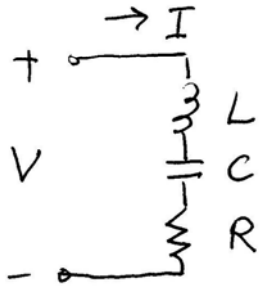


$$\frac{I(j\omega)}{V(j\omega)} = \frac{j\omega C}{1 - (\omega / \omega_o)^2 + j(\omega RC)}$$

$$\omega_o^{-2} = LC$$

Match terms in motional admittance \rightarrow find equivalent elements

F. Strøm gjennom L-C-R - gren



$$V = I \left(sL + \frac{1}{sC} + R \right)$$

$$\frac{I(s)}{V(s)} = \frac{sC}{s^2LC + sRC + 1}$$

$$Y(j\omega) = \frac{I(j\omega)}{V(j\omega)} = \frac{j\omega C}{-\omega^2 LC + j\omega RC + 1}$$

innføres $\omega_0^2 = \frac{1}{LC}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Y(j\omega) = \frac{j\omega C}{\left[1 - \left(\frac{\omega}{\omega_0} \right)^2 \right] + j\omega RC}$$

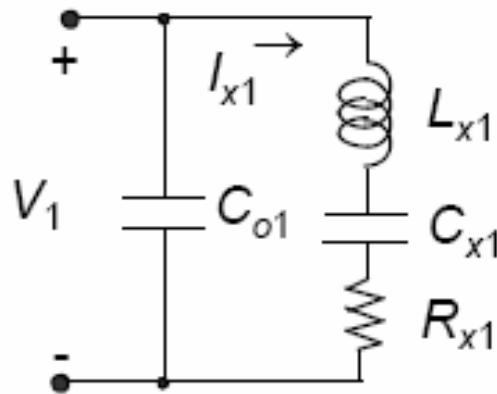
$$= \frac{\omega}{Q\omega_0} \quad \left(RC = \frac{1}{Q\omega_0} \right)$$

$$\text{dvs } Q = \frac{1}{\omega_0 RC} = \frac{\sqrt{LC}}{RC} = \sqrt{\frac{L}{C}} \cdot \frac{1}{R}$$

G. Equivalent Circuit for Input Port

A series L-C-R circuit results in the identical expression \rightarrow
find equivalent values L_{x1} , C_{x1} , and R_{x1}

$$L_{x1} = \frac{m}{\eta^2} \quad C_{x1} = \frac{\eta^2}{k} \quad R_{x1} = \frac{\sqrt{km}}{Q\eta^2} \quad \eta = V_{p1} \frac{\partial C_1}{\partial x} = \text{electromechanical coupling coefficient}$$



At resonance, the impedances of the inductance and the capacitance *cancel out* \rightarrow

$$I_{x1} = \frac{V_1}{R_{x1}}$$

H. Beregning av utgangsstrømmen

Impf. inngangsstrøm pga. bevegelsen: $i_{1x}(t) = -V_{P1} \cdot \frac{\partial C_1}{\partial t}$

Denne bevegelsen gjør at utgangskapasitansen C_2 også endrer seg

Utgangsstrøm pga. bevegelsen ($v_2 = 0V$, kortsluttet)

$$i_2(t) = -V_{P2} \frac{\partial C_2}{\partial t} = -V_{P2} \frac{\partial C_2}{\partial x} \cdot \frac{\partial x}{\partial t}$$

i phasor-form:

$$I_2(j\omega) = -V_{P2} \frac{\partial C_2}{\partial x} \cdot j\omega \cdot X(j\omega)$$

$$\uparrow X(j\omega) = \frac{1/k}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] + j \frac{\omega}{Q\omega_0}} \cdot F_d(j\omega)$$

$$F_d(j\omega) = -V_{P1} \cdot \frac{\partial C_1}{\partial x} \cdot V_1(j\omega)$$

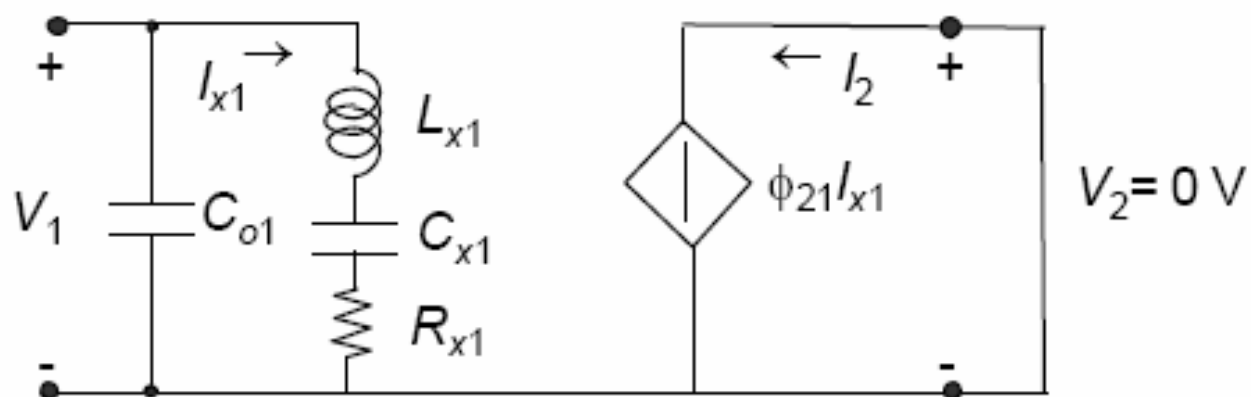
$$\Rightarrow I_2(j\omega) = \frac{V_{P1} \cdot V_{P2} \cdot \frac{\partial C_1}{\partial x} \cdot \frac{\partial C_2}{\partial x}}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] + j \frac{\omega}{Q\omega_0}} \cdot j\omega \cdot \frac{1}{k} \cdot V_1(j\omega)$$

I. "Forward current gain"

$$\phi_{21} = \frac{I_2(j\omega)}{I_{x1}(j\omega)} = \frac{-V_{P2} \frac{\partial C_2}{\partial x} \cdot j\omega \cdot X(j\omega)}{-V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot X(j\omega)} = \frac{V_{P2}}{V_{P1}} \cdot \frac{\frac{\partial C_2}{\partial x}}{\frac{\partial C_1}{\partial x}}$$

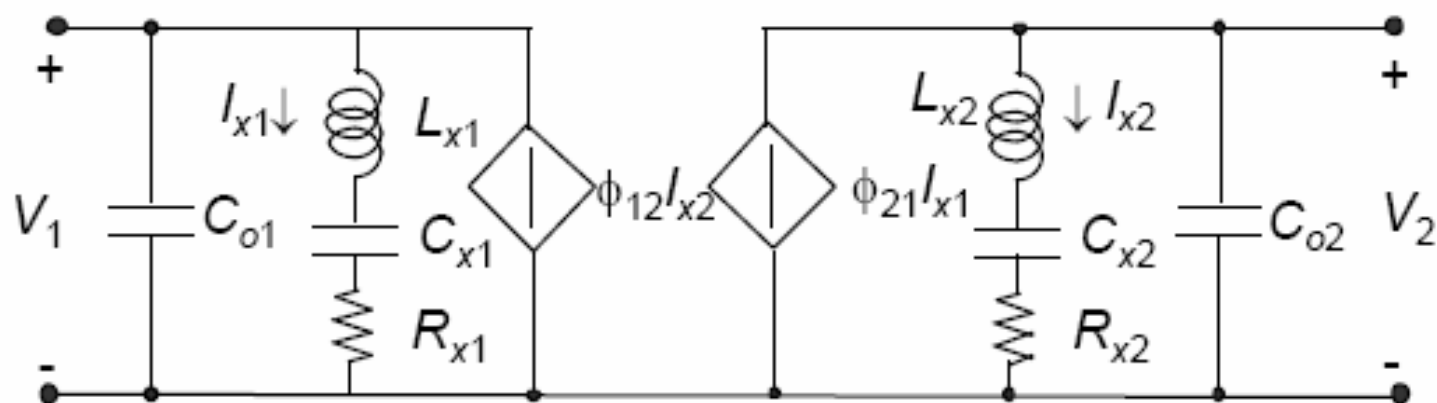
$$I_2(j\omega) = \phi_{21} \cdot I_{x1}(j\omega), \quad V_2 = 0$$

J. Two-Port Equivalent Circuit ($v_2 = 0$)



K.

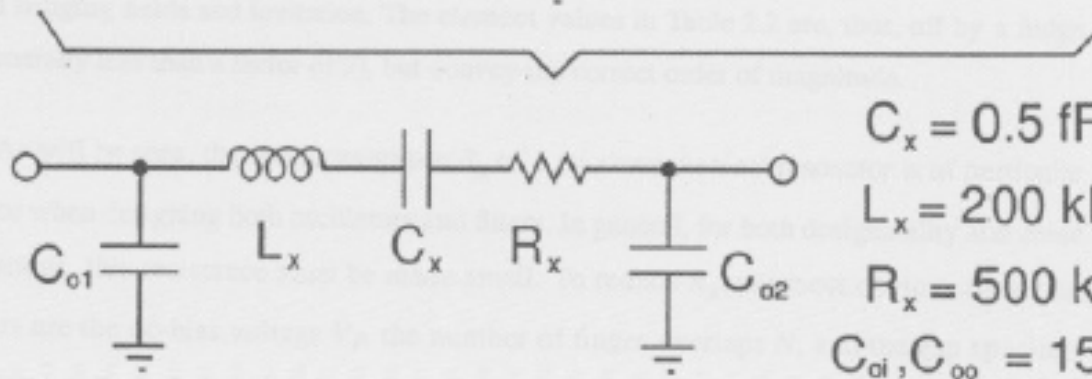
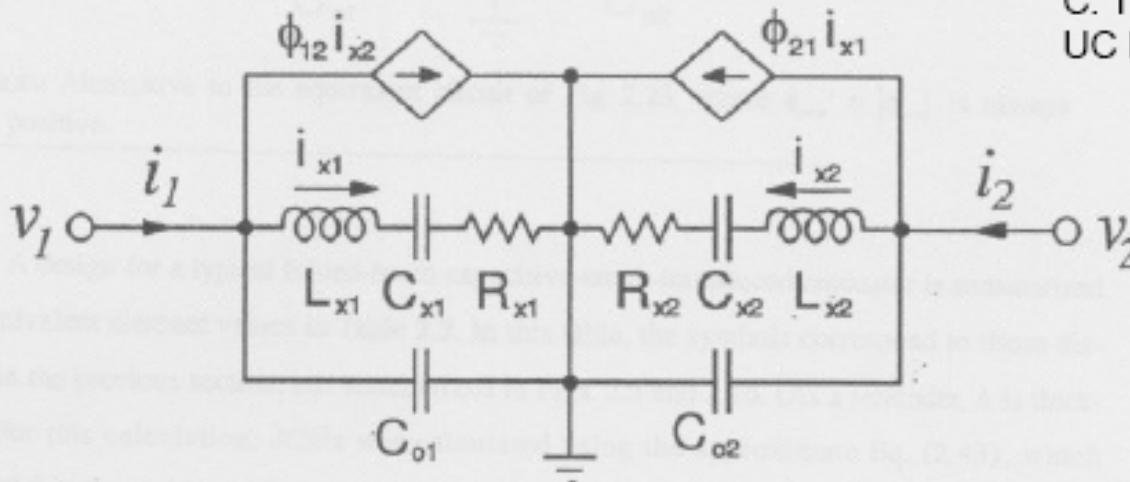
Complete Two-Port Model



Symmetry implies that modeling can be done from port 2, with port 1 shorted \rightarrow superimpose the two models

Equivalent Circuit for Symmetrical Resonator ($\phi_{21} = \phi_{12} = 1$)

C. T.-C. Nguyen, Ph.D.,
UC Berkeley, 1994



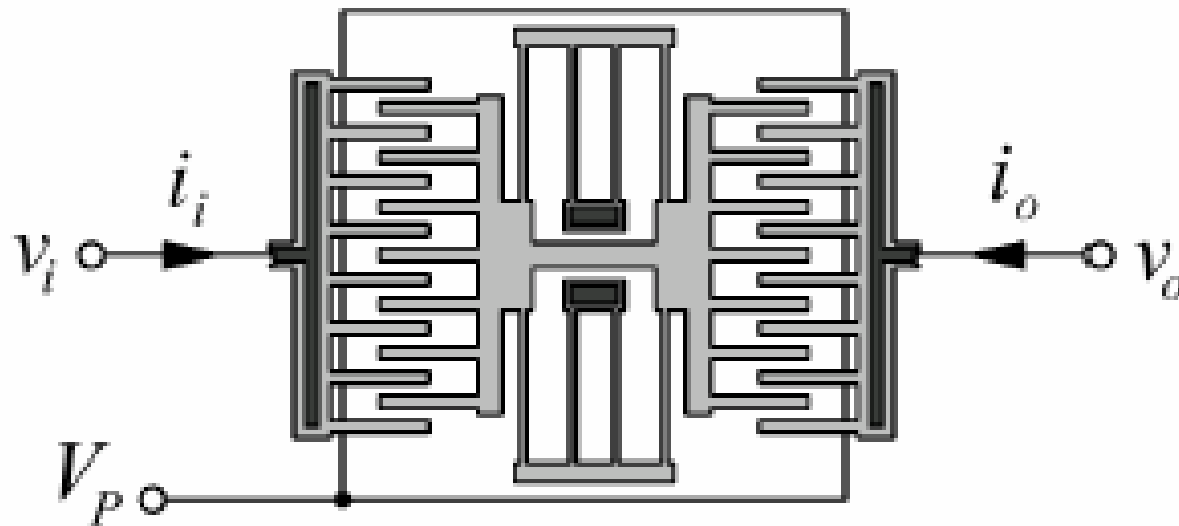
$C_x = 0.5 \text{ fF}$
 $L_x = 200 \text{ nH}$
 $R_x = 500 \text{ k}\Omega$
 $C_{o1}, C_{o2} = 15 \text{ fF}$

Alternativ modellering

- Benytte **konvertering** mellom mekanisk og elektrisk energidomene
 - Slides fra UCLA
- Utfylles ved notater →

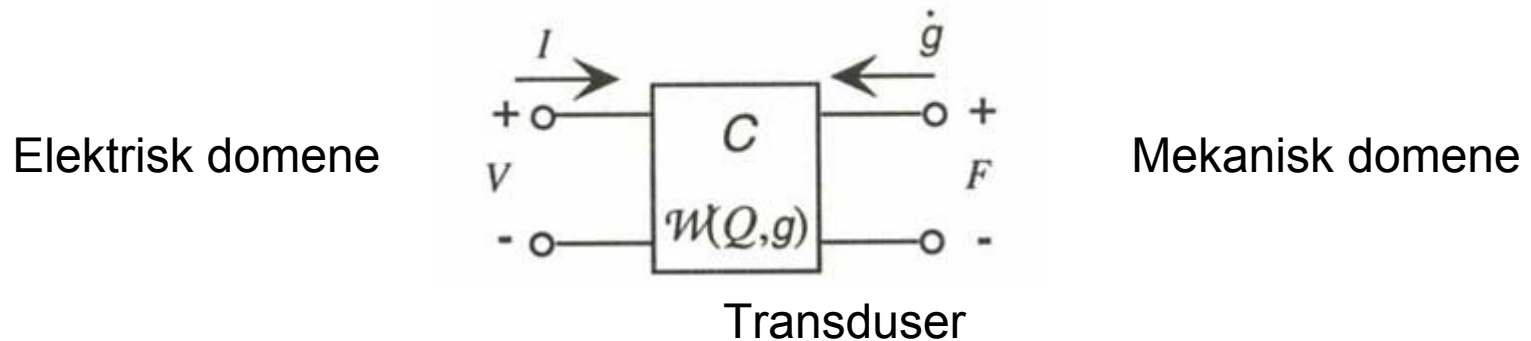
Two-Port Micromechanical Resonator Using Comb-Drive Actuator

2-port Lateral Microresonator



Konverteringer mellom energidomener

- Både vertikale og laterale resonator-strukturer kan beskrives ved **generalisert ikke-lineær kapasitans, C**



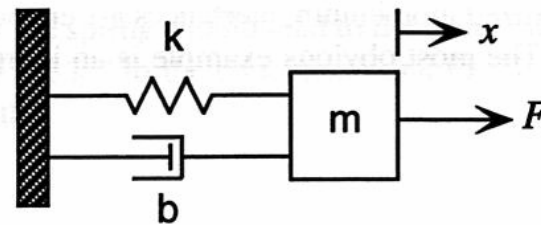
"Lumped elements" i det mekaniske domene transformeres til ekvivalente kretselementer i det elektriske domene

Eks. på konjugerte power variable

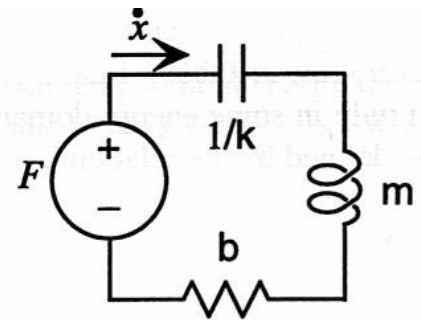
Energy Domain	Effort	Flow	Momentum	Displacement
Mechanical translation	Force F	Velocity \dot{x}, v	Momentum p	Position x
Fixed-axis rotation	Torque τ	Angular velocity ω	Angular momentum J	Angle θ
Electric circuits	Voltage V, v	Current I, i	...	Charge Q
Magnetic circuits	Magnetomotive force MMF	Flux rate $\dot{\phi}$...	Flux ϕ
Incompressible fluid flow	Pressure P	Volumetric flow Q	Pressure momentum Γ	Volume V
Thermal	Temperature T	Entropy flow rate \dot{S}	...	Entropy S

Prosedyre

- C er en generalisert ikke-lineær 2-port
- I det mekaniske domene modelleres de mekaniske egenskapene som "lumped" (diskrete) elektriske komponenter
 - $C = 1/k$
 - $L = m$
 - $R = b$
- Power-variablene
 - Effort = kraft
 - Flow = hastighet



Spring-mass-dashpot system



Equivalent circuit

Prosedyre, forts.

- Ønsker å transformere alle komponenter til ekvivalente kretselementer i det elektriske domenet
 - Effort = spenning, V
 - Flow = strøm, I
- A. Må **linearisere** 2-port kapasitansen
 - -> Transformator
- B. Transformasjon av impedansene i det mekaniske domenet til det elektriske domenet
 - -> **Elektromekanisk koblingskoeffisient benyttes!**

Interaksjon mellom energidomener

- Lineære 2-port elementer
 - Bruk av transformator og gyrator

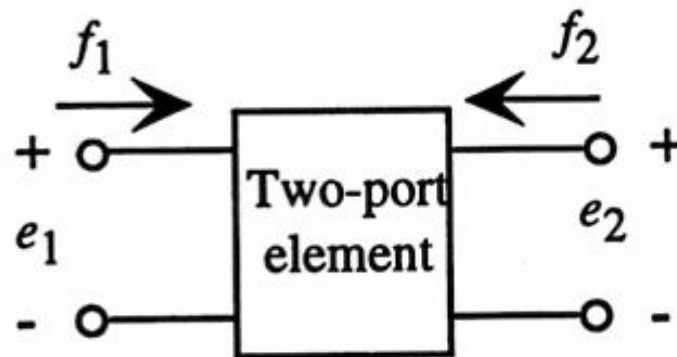


Figure 5.11. General two-port element.

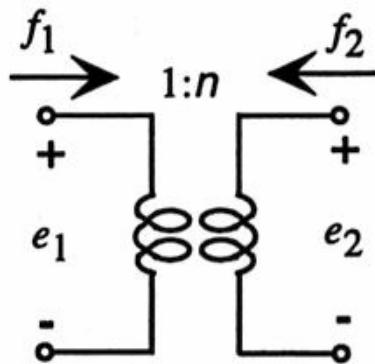
$$e_1 f_1 + e_2 f_2 = 0$$

(5.41)

Transformer

TRANSFORMER:

$$\begin{pmatrix} e_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \begin{pmatrix} e_1 \\ f_1 \end{pmatrix} \quad (5.42)$$



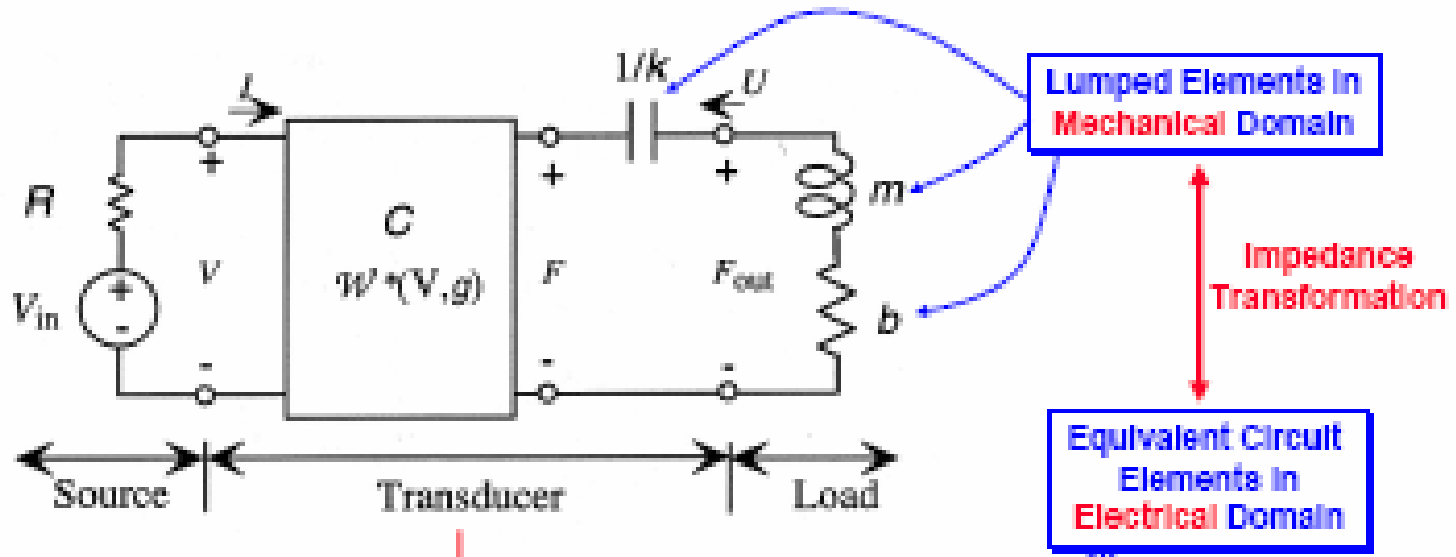
Transformer

$$e_2 = n \cdot e_1$$

$$f_2 = -\frac{1}{n} f_1$$

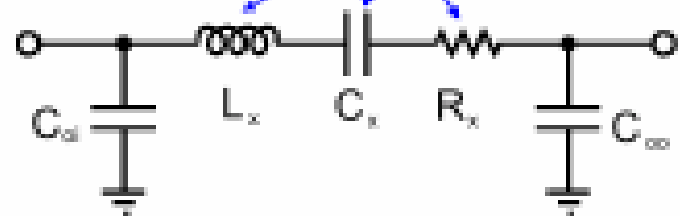
n = "turns ratio"

Lumped Element Model (Senturia's Book)



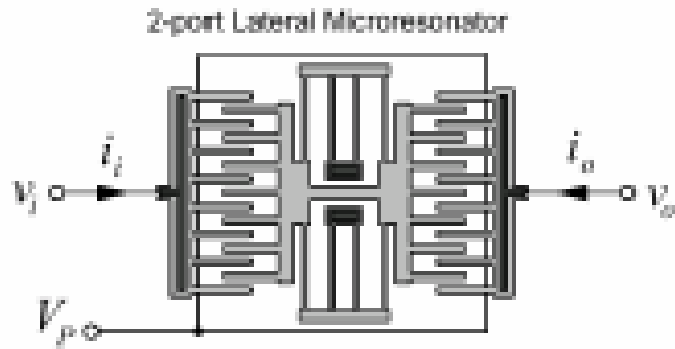
1. Linearize 2-port capacitor
2. Impedance transformation

(See HW problem for details)

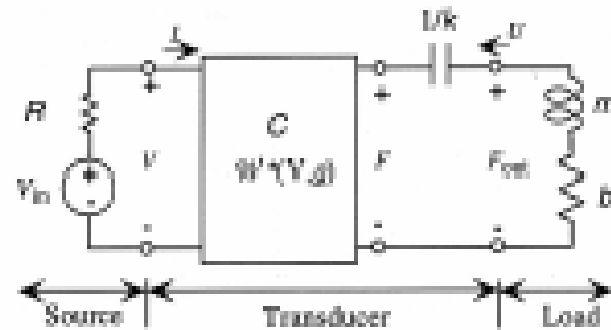


Linearized Transducers

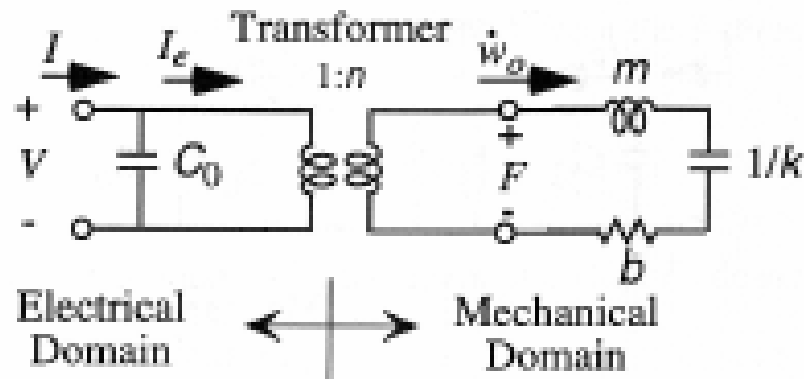
Physical Circuit



Equivalent Circuit (Nonlinear)



Linearized Equivalent Circuit



Prosedyre

- Se på relasjonen mellom "efforts" og "flows" i de 2 domeneene
- **Efforts:** beregningsprosedyre
 - 1. Start med potensiell energi
 - 2. Beregn kraft
 - 3. Perturbasjoner rundt DC-likevekt
 - 4. Finn forholdet mellom AC-leddene

Forhold mellom "efforts"

$$F = \frac{\partial W^*}{\partial x} = \frac{1}{2} V^2 \frac{\partial C}{\partial x}$$

$$F = F_{\text{mek}} + f \cdot \sin(\omega t)$$

$$V = V_{\text{mek}} + v \cdot \sin(\omega t)$$

$$F_{\text{mek}} + f \cdot \sin(\omega t) = \frac{1}{2} (V_{\text{mek}} + v \cdot \sin(\omega t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2} \left((V_{\text{mek}})^2 + 2 \cdot V_{\text{mek}} \cdot v \cdot \sin(\omega t) \right) \frac{\partial C}{\partial x}$$

$$f = V_{\text{mek}} \cdot \frac{\partial C}{\partial x} \cdot v \quad \leftarrow \text{AC term}$$

effort (mek domene) = konst. * effort (elektrisk domene)

Linearization – Small Signal Analysis

Relations between “Efforts”

$$F = \frac{\partial \mathcal{F}^*}{\partial x} = \frac{1}{2} V^2 \frac{\partial C}{\partial x}$$

$$F = F_{dc} + f \cdot \sin(\omega t)$$

$$V = V_{dc} + v \cdot \sin(\omega t)$$

$$F_{dc} + f \cdot \sin(\omega t) = \frac{1}{2} (V_{dc} + v \cdot \sin(\omega t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2} \left((V_{dc})^2 + 2 \cdot V_{dc} \cdot v \cdot \sin(\omega t) \right) \frac{\partial C}{\partial x}$$

$$f = V_{dc} \cdot \frac{\partial C}{\partial x} \cdot v \quad \leftarrow \text{AC terms}$$

Relations between “Flows”

$$Q = V \cdot C$$

$$I = V \cdot \frac{\partial C}{\partial t} = V \cdot \frac{\partial C}{\partial X} \cdot \frac{\partial X}{\partial t} = V \cdot \frac{\partial C}{\partial X} \cdot \dot{X}$$

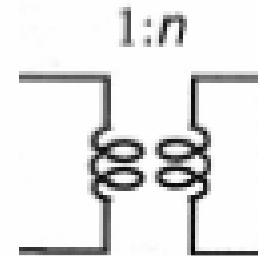
$$I = I_{dc} + i \cdot \sin(\omega t)$$

$$X = X_{dc} - x \cdot \sin(\omega t) \quad \text{Negative sign due to definition of flow direction}$$

$$i = -V_{dc} \frac{\partial C}{\partial X} \dot{x}$$

Linearized capacitive transducer is a Transformer

$$\begin{pmatrix} f \\ \dot{x} \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \begin{pmatrix} v \\ i \end{pmatrix}$$



Turn Ratio: $n = V_{dc} \frac{\partial C}{\partial X}$

flow (elektrisk domene) = - konst. * flow (mekanisk domene)

Strømretning, mek domene

- Flow i det mek domene er definert **positiv inn** mot 2-port C
- Velger at strømmen går **ut av** 2-port C
 - **Strøm går inn** i det elektriske domene
 - → forårsaker tiltrekningskraft på kammen
 - → fjæra strekkes
 - → potensiell energi bygges opp
 - → analogt med at 1/k-kondensatoren lades

 - Økning i strøm → ladning på kondensatoren øker → tiltrekningskraften øker → **"displacement" (x) minker**

En samsvarende relasjon både mellom "efforts" og "flows"

$$f = V_{dc} \cdot \frac{\partial c}{\partial x} \cdot v = n \cdot v \quad \text{der} \quad n = V_{dc} \cdot \frac{\partial c}{\partial x}$$

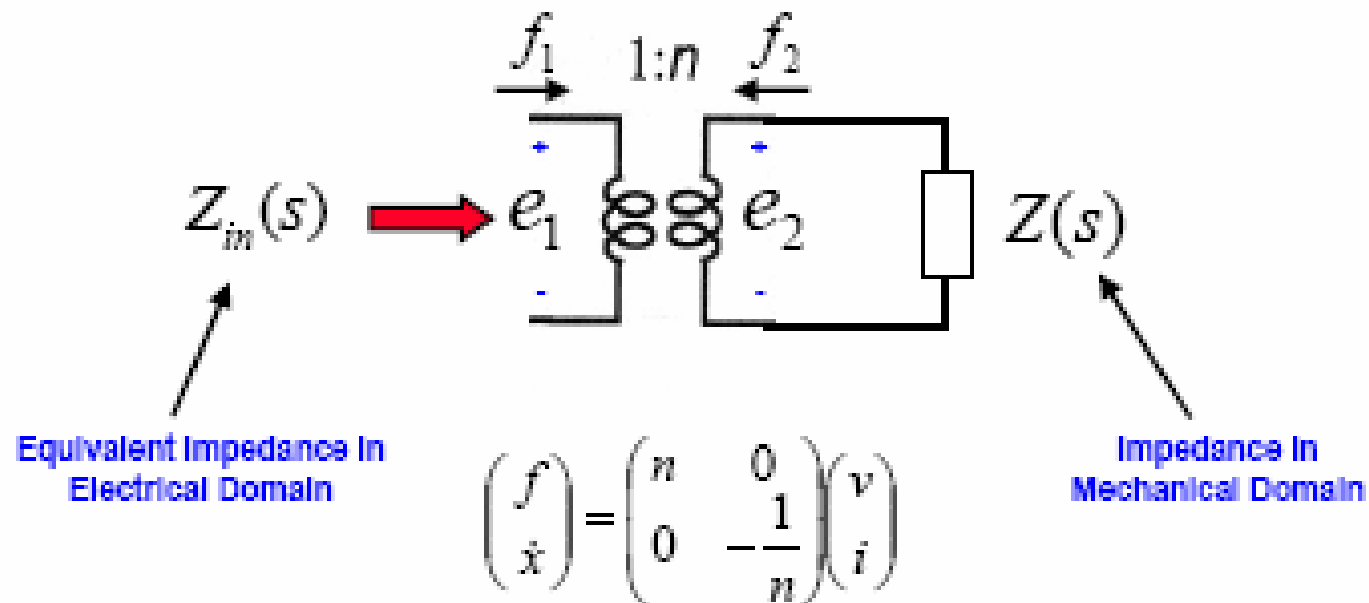
$$\dot{i} = -V_{dc} \cdot \frac{\partial c}{\partial x} \cdot \dot{x} = -n \cdot \dot{x} \quad \Rightarrow \quad \dot{x} = -\frac{1}{n} \cdot \dot{i}$$

- **effort** (mekanisk domene) = n * **effort** (elektrisk domene)
- **flow** (mekanisk domene) = $-1/n$ * **flow** (elektrisk domene)
- En linearisert kapasitiv transduser er en **transformator**

Elektromekanisk koblingskoeffisient

- Har funnet
 - $f = [V_{dc} * dC/dx] * v$
 - $dx/dt = - [1/ (V_{dc} * dC/dx)] * i$
 - $n = V_{dc} * dC/dx$ (**elektromekanisk koblingskoeffisient**)
- Generelt er det et mer komplisert, ikke-lineært forhold mellom power-variablene i de to energi-domenene
 - Storskala: $F = \frac{1}{2} * V^2 * dC/dx$: **ulineært!**

Impedance Transformation



$$Z_{in}(s) = \frac{1}{n^2} Z(s)$$

Transformasjon av impedanser

Impedanstransformasjon

$$Z_{\text{elektrisk}} = \frac{\text{effort}}{\text{flow}} = \frac{v}{i}$$

Transformator - transformasjon : $\phi = n \cdot v$
 $\dot{x} = -\frac{1}{n} \cdot i$

$$Z_{\text{el}} = \frac{v}{i} = \frac{\frac{1}{n} \cdot \phi}{-n \cdot \dot{x}} = -\frac{1}{n^2} \cdot Z_{\text{mek}} \quad (\text{strøm inn i mek. domene})$$

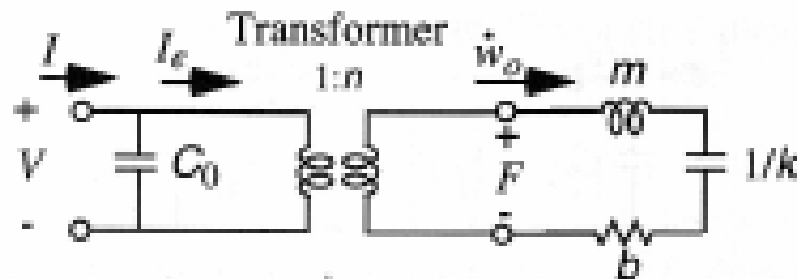
$$Z_{\text{el}} = \frac{1}{n^2} \cdot Z_{\text{mek}} \quad (\text{strøm ut i mek. domene})$$

Spole : $sL_{\text{el}} = \frac{1}{n^2} \cdot sL_{\text{mek}} = \frac{s \cdot m}{n^2} \Rightarrow L_{\text{el}} = \frac{m}{n^2}$

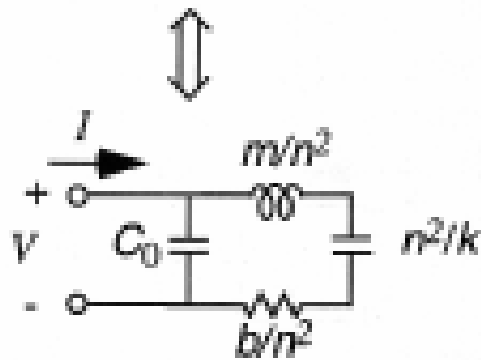
Motstand : $R_{\text{el}} = \frac{1}{n^2} \cdot R_{\text{mek}} = \frac{b}{n^2}$

Kondensator : $\frac{1}{sC_{\text{el}}} = \frac{1}{n^2} \cdot \frac{1}{sC_{\text{mek}}} = \frac{1}{n^2} \cdot \frac{k}{s} \Rightarrow C_{\text{el}} = \frac{n^2}{k}$

Small Signal Equivalent Circuit of Microresonators



Electrical Domain ↔ Mechanical Domain

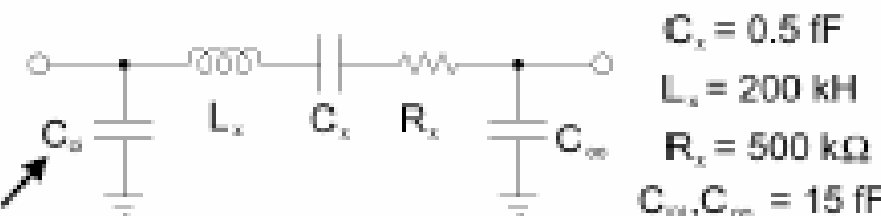
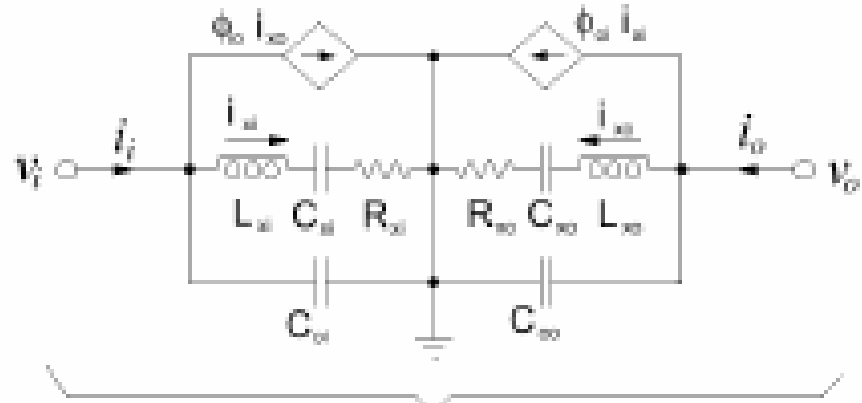


Equivalent Electrical Circuit

Unit of n^2/k is Farad

$$n = V_{dc} \frac{\partial C}{\partial x}$$

Equivalent Circuit of 2-Port Resonator (in Electrical Domain)



$C_s = 0.5 \text{ fF}$
 $L_s = 200 \text{ nH}$
 $R_s = 500 \text{ k}\Omega$
 $C_{01}, C_{02} = 15 \text{ fF}$

Fixed electrical
Capacitance
Between fixed comb
And ground plane

$$C_{sM} = \frac{\eta_n^2}{k} \quad R_{sM} = \frac{\sqrt{kM}}{Q\eta_n^2} \quad \eta_n = V_{Fe} \frac{\partial C_s}{\partial x}$$

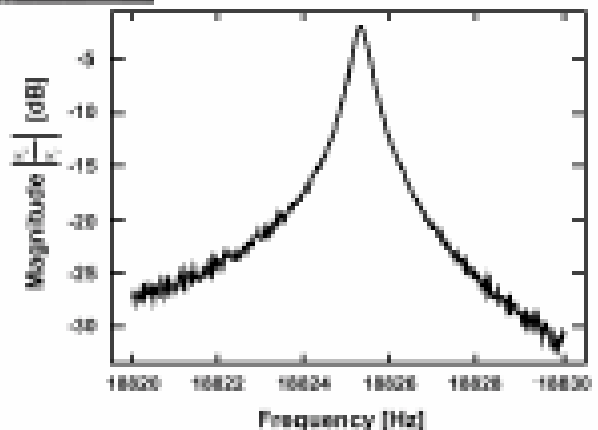
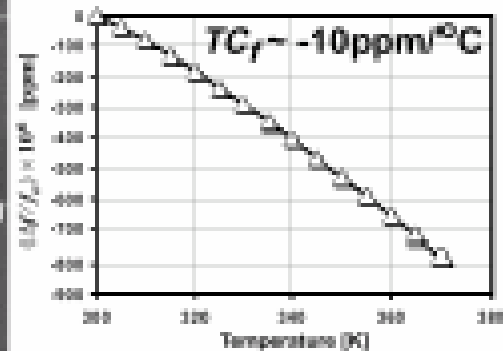
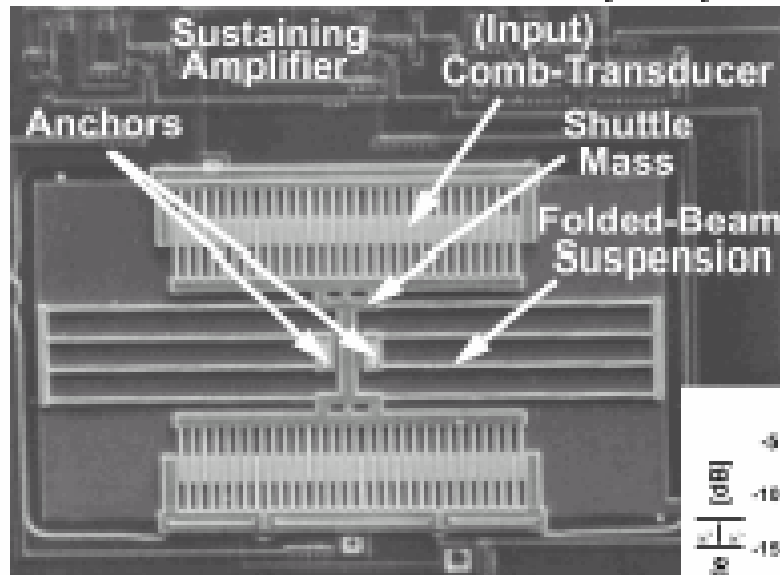
$$L_{sM} = \frac{M}{\eta_n^2} \quad \phi_{sM} = \frac{\eta_{sM}}{\eta_n}$$

C. T.-C. Nguyen, "Micromechanical resonators for oscillators and filters," Proceedings IEEE International Ultrasonics Symposium, Seattle, WA, pp. 489-496, Nov. 7-10, 1986



Comb-Transduced Folded-Beam Microresonator

- Micromachined from *in situ* phosphorous-doped polysilicon



- At right: $Q = 50,000$ measured at 20 mTorr pressure
- ($Q = 27$ at atmospheric pressure)
- Problems: large mass \Rightarrow limited to low frequencies; low coupling

Kam-resonator, oppsummering

- $F_e = \frac{1}{2} \frac{dC}{dx} V \exp^2$ (alltid tiltrekkende)
 - Inngangs-signal $V_a \cdot \cos(\omega t)$
 - $F_e \sim V_a \exp^2 \cdot \frac{1}{2} [1 + \cos(2\omega t)]$
 - Kraften som driver er 2x inngangs-frekvensen + DC:
UØNSKET
- Legge til DC forspenning, V_d
 - $F_e \sim V_d \exp^2 + 2 V_d \cdot V_a \cdot \cos \omega t$ + neglisjerbart ledd
 - Linearisert AC kraft-komponent $\sim V_d \cdot V_a$, med V_a sin frekvens
- C øker når finger-overlappingen øker
 - $\epsilon \cdot A/d$ (A = kam-tykkelse * overlapps-lengde)
- $dC/dx =$ konstant for et gitt design

Kam-resonator, utgangsstrøm

- Det dannes en tidsvarierende kapasitans ved utgangsomformeren
 - Beregning av utgangsstrøm
 - $I_0 = d/dt (Q) = d/dt (C \cdot V) = V_d \cdot dC/dt = V_d \cdot dC/dx \cdot dx/dt$
 - $I_0 = V_d \cdot dC/dx \cdot \omega \cdot x_{\max}$
 - I_0 plottet mot frekvens, viser BP-karakteristikk
 - BP = båndpass

Kam-resonator, fjærstivhet

- Fjærstivhet til enkel c-c beam som bøyes til siden
 - $k_{\text{beam}} = \text{const} * E * t * (w/L) \text{ exp}^3$
 - $E = \text{Youngs modul}$, $t = \text{tykkelse}$, $w = \text{bredde}$, $L = \text{lengde}$
- For eksemplet i figur 7.9
 - $\text{const} = 1$
 - $k_{\text{total}} = 2 * k_{\text{beam}}$

Design-parametre

- For å få høyere resonans-frekvens
 - Total **fjærstivhet** må økes
 - Eller **bevegelses-massen** må avta
 - Vanskelig å få til fordi det trengs et visst antall fingre
 - For å få god nok innkobling
 - Oppløsningen i prosessen spiller inn ved laterale strukturer
 - Eks. $L = 10\mu\text{m}$, $w = 0.2\mu\text{m}$, enkrystall Si-bjelke
 - $f_r = 14\text{ MHz}$
 - Frekvens kan økes ved å bruke et materiale med større **E/ρ** enn Si
 - E/ρ sier noe om fjærstivheten i forhold til tyngden
 - Aluminium og titan har E/ρ lavere enn for Si
 - Si carbide, poly diamond har E/ρ høyere enn for Si (poly diamond er forskningstema)