INF5490 RF MEMS

L9: RF MEMS resonatorer III

Dagens forelesning

- Vertikalt vibrerende resonatorer
  - Clamped-clamped beam (c-c beam)
    - Virkemåte
    - → **Detaljert modellering**
  - free-free beam (f-f beam)
- Andre typer resonatorer
  - Tuning fork
  - Bjelke med lateral bevegelse
  - Disk resonatorer

Beam-resonator

- Ønsker høyere resonansfrekvens enn kam-strukturen
  - Massen må reduseres mer-> **beam resonator**
- Fordeler ved beam-resonatorer
  - Mindre dimensjoner, enkel
  - Høyere resonansfrekvens
  - Kan ha mange frekvens-referanser på en enkel chip
  - Mer lineær frekvensvariasjon mhp temp over et større område
  - Mulighet for integrering med elektronikk → lavere kostnader

Beam-resonator

First-order resonant frequency:

$$f_r = 1.03\sqrt{\frac{E}{\rho}}\,\frac{t}{L^2}$$

E = Young's modulus

ρ = Density

t = Beam thickness

L = Beam length

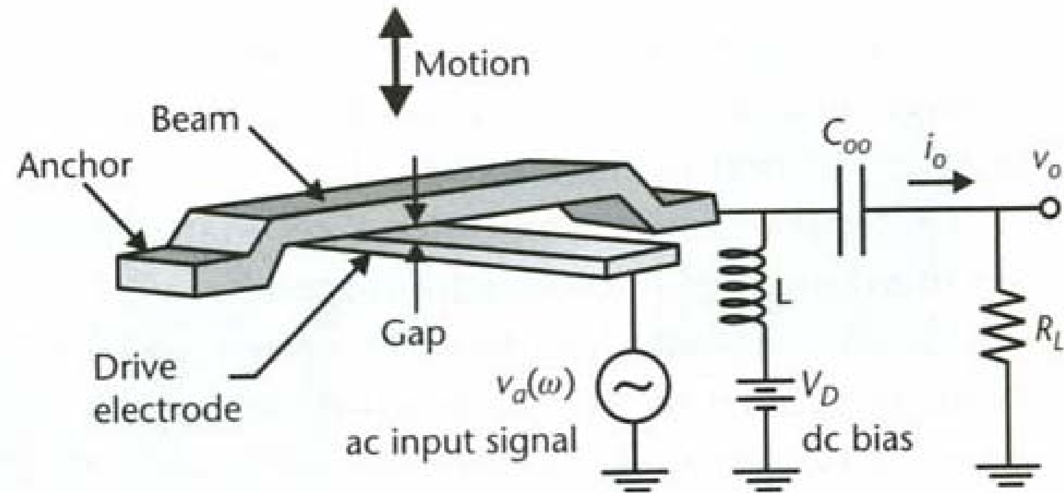


Figure 7.10 Illustration of a beam resonator and a typical circuit to measure the signal. The beam is clamped on both ends by anchors to the substrate. The capacitance between the resonant beam and the drive electrode varies with the deflection.

Utgangskrets

- Resonator er en tidsvarierende kapasitans C(ω)
- Enkel elektrisk utgangskrets
  - L = shunt blokkerende induktor: Åpen ved høye frekvenser
  - C_∞ = serie blokkerende kapasitans: Kortsluttet ved høye frekvenser
  - Når Vd er en høy DC-spenning, så er den dominerende utgangs-strømmen ved inngangsfrekvens ω: i0 = Vd * dC/dt
  - Ved høye frekvenser er i0 strømmen gjennom R_L
    - Kan være inngangsimpedans i måleutrustningen. Kan erstattes av transimp.-forsterker

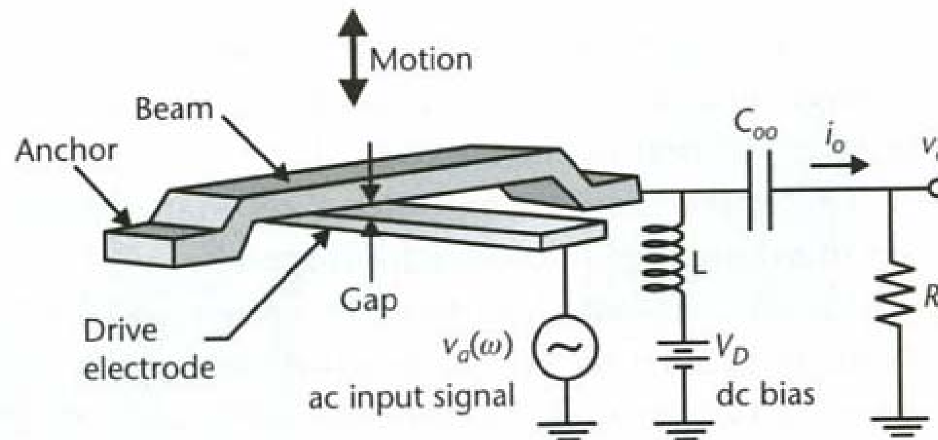


Figure 7.10 Illustration of a beam resonator and a typical circuit to measure the signal. The beam is clamped on both ends by anchors to the substrate. The capacitance between the resonant beam and the drive electrode varies with the deflection.

Mekanisk resonans-frekvens

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\kappa\sqrt{\frac{E}{\rho}\frac{h}{L_r^2}}\left[1 - g(V_P)\right]^{1/2}, \qquad (12.2)$$

- Parametre
  - E = Youngs modul
  - ρ = tettheten av materialet
  - h = tykkelsen av beam
  - Lr = lengde av beam
  - g modellerer effekten av en elektrisk fjærstivhet k_e
    - Gjør seg gjeldende når en setter spenning på elektrodene
    - Subtraheres fra den mekaniske fjærstivheten, k_m (beam-softening)
  - κ =skaleringsfaktor
  - V_p = DC spenning på ledende beam
  - k_r = effektiv resonator fjærstivhet
  - m_r = effektiv masse
- NB! E og ρ inngår + fjærstivhetsledd

Forventede frekvenser

TABLE 12.1. μMechanical Resonator Frequency Design[a]

Frequency (MHz)	Material	Mode	h_r (μm)	W_r (μm)	L_r (μm)
70	Silicon	1	2	8	14.54
110	Silicon	1	2	8	11.26
250	Silicon	1	2	4	6.74
870	Silicon	2	2	4	4.38
870	Diamond	2	2	4	8.88
1800	Silicon	3	1	4	3.09
1800	Diamond	3	1	4	6.16

[a] Determined for free-free beams using Timoshenko methods that include the effects of finite h and W_r [11].

”Beam-softening”

- DC-spenningen, Vd, forårsaker en nedoverrettet elektrostatisk kraft
- Kraften virker mot den mekaniske gjenopprettelses-kraften i bjelken
- Dette gjør den effektive mekaniske fjærkonstanten til systemet mindre

$$k_r = k_m - k_e$$

  - k_e endres med Vd
  - Resonansfrekvensen faller med en gitt faktor

  - **→ Resonansfrekvensen kan tunes elektrisk!**

Detaljert modellering

- **c-c beam** modelleres med referanse til boka
  - *T. Itoh et al: RF Technologies for Low Power Wireless Communications", kap. 12: "Transceiver Front-End Architectures Using Vibrating Micromechanical Signal Processors", by Clark T.-C. Nguyen*
  - *(+ oppsummering av resultater fra diverse publikasjoner)*

Clamped-clamped beam

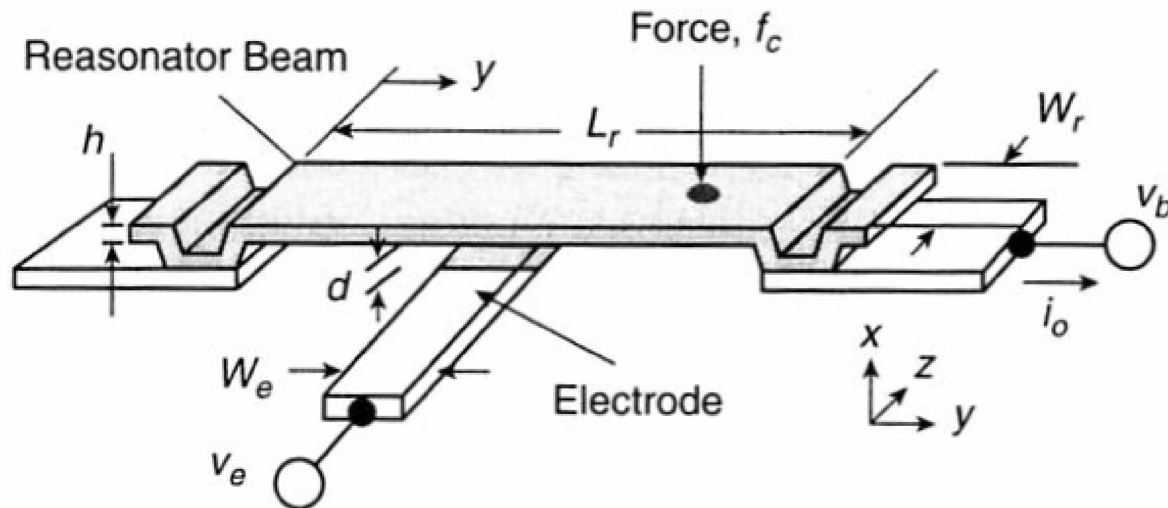


Figure 12.4. Perspective-view schematic of a clamped–clamped beam μmechanical resonator in a general bias and excitation configuration.

Beregning av elektrisk eksitasjon

- 2 påtrykte spenninger
- A) Beregn først potensiell energi
- B) Deretter beregnes kraften →

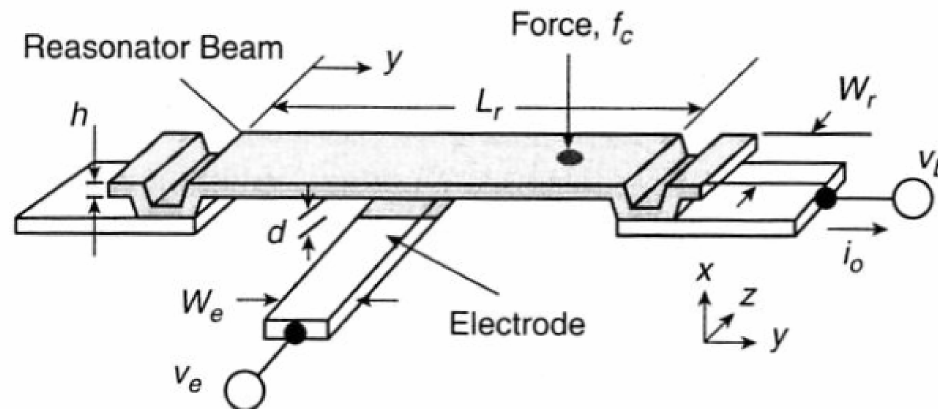


Figure 12.4. Perspective-view schematic of a clamped–clamped beam μmechanical resonator in a general bias and excitation configuration.

Elektrisk eksitasjon

v_e = input på elektrode
v_b = input på beam

$v_e - v_b$ = effektiv spenning elektrode – bjelke

A

$$U = \frac{1}{2} C V^2 = \frac{1}{2} C \left(v_e - v_b\right)^2 = \text{pot. energi}$$

Kraft = endring mhp. x av pot energi

B

$$F_d = \frac{\partial U}{\partial x} = \frac{1}{2}\left(v_e - v_b\right)^2 \frac{\partial C}{\partial x} = \text{kraften}$$

$$= \frac{1}{2}\left(v_b^2 - 2 v_b \cdot v_e + v_e^2\right)\frac{\partial C}{\partial x}$$

$$C = \frac{\varepsilon_0 A}{d_0} = \varepsilon_0 \frac{W_e \cdot W_r}{d_0}$$

W_e = elektrode - bredde
W_r = beam - bredde
d_0 = elektrode - resonator gap (statiske betingelser. Ikke - resonans)
ε_0 = permittivitet i vakuum

Prosedyre, forts.

- C) Sett på DC-spenning, Vp
- D) Regn videre på ligningen for kraften
- E) Diskusjon av ulike bidrag
  - Off-resonans DC-kraft
  - Kraft i takt med inngangsspenning
  - Dobbeltfrekvens-ledd

Ved bruk som resonator eller filter:
DC-spenning settes på beam: V_P

C

$$v_b = V_P \quad \text{beam}$$

$$v_e = v_i = V_i \cos \omega_i t \quad \text{elektrode}$$

D

$$F_d = \frac{1}{2}\left(V_P^2 - 2 V_P V_i \cos \omega_i t + V_i^2 \cos^2 \omega_i t\right) \frac{\partial C}{\partial x}$$

Mellomregning:

$$\cos^2 \omega_i t = 1 - \sin^2 \omega_i t = \frac{1}{2}\left(2 - 2 \sin^2 \omega_i t\right)$$

$$= \frac{1}{2}\left(1 + 1 - \sin^2 \omega_i t - \sin^2 \omega_i t\right)$$

$$= \frac{1}{2}\left(1 + \sin^2 \omega_i t + \cos^2 \omega_i t - \sin^2 \omega_i t - \sin^2 \omega_i t\right)$$

$$= \frac{1}{2}\left(1 + \cos^2 \omega_i t - \sin^2 \omega_i t\right)$$

$$= \frac{1}{2}\left(1 + \cos 2 \omega_i t\right)$$

d.v.s

$$V_i^2 \cos^2 \omega_i t = \frac{V_i^2}{2}\left(1 + \cos 2 \omega_i t\right)$$

E

Innsetting:

$$F_d = \left(\frac{1}{2}V_p^2 - V_p V_i \cos\omega_i t + \frac{1}{2}\cdot\frac{V_i^2}{2} + \frac{1}{2}\frac{V_i^2}{2}\cos 2\omega_i t\right)\cdot\frac{\partial C}{\partial x}$$

$$F_d = \frac{\partial C}{\partial x}\left(\frac{V_p^2}{2} + \frac{V_i^2}{4}\right) - V_p\frac{\partial C}{\partial x}V_i\cos\omega_i t + \frac{\partial C}{\partial x}\frac{V_i^2}{4}\cos 2\omega_i t$$

Første ledd	Andre ledd
Off-resonans DC-kraft som statisk bøyer beam. Har liten effekt på signalprosesseringsfunksjon	Kraft i takt med input-frekvensen, forsterket med DC-spenningen V_p

3.dje ledd:

$$\frac{\partial C}{\partial x}\frac{V_i^2}{4}\cos 2\omega_i t$$

Kan drive beam inn i vibrasjoner ved

$$2\omega_i = \omega_0$$

$$\omega_i = \frac{\omega_0}{2}$$

Hvis $V_p \gg V_i$, så blir dette leddet lite

[Kan gi problemer i visse tilfelle, feks. ved BP-filter hvor en får inn kraftig interferens ved halve passbånd-frekvensen.

Prosedyre, forts.

- Kraftens hovedbidrag er prop med cos
  - Driver beam inn i resonans
- F) Kraften gir ”displacement” (x-variasjon)
  - Den lokale fjærstivheten varierer over bredden av drive-elektroden
  - Derved er også displacement avhengig av posisjon y
- G) Utledning av et uttykk for displacement, x(y), som funksjon av fjærstivhetens avhengighet av y

Topologi

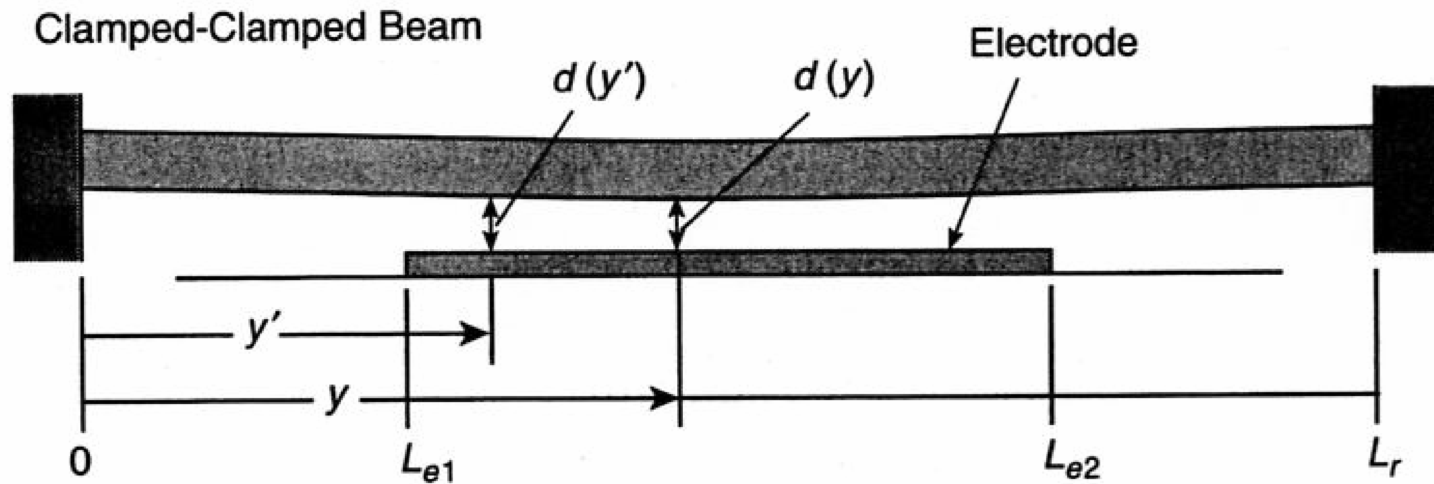


Figure 12.9. Resonator cross-sectional schematic for frequency-pulling and impedance analysis.

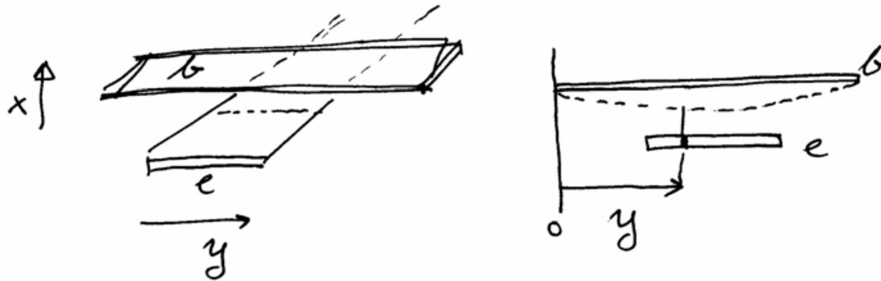
Hovedbidraget for høy-Q-tank og filter-anvendelser:

$$-V_P \frac{\partial c}{\partial x} V_i \cos \omega_i t$$

Når $\omega_i = \omega_0$ vil denne kraften drive beam inn i <u>resonans</u>

$$F_d = -V_P \frac{\partial c}{\partial x} \cdot v_i(\omega_0)$$, x pos oppover

Avstanden mellom beam og elektrode er avhengig av y (= displacement)



Generelt: $F = k \cdot x$

Den lokale fjærstivheten varierer med y

$$k(y) = k_{reff}(y)$$

k_{reff} = effektiv fjærstivhet i y

F

For mekaniske systemer gjelder

$$H(s) = \frac{x}{F} = \frac{\text{displacement}}{\text{force}} = \frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

$$H'(s) = \frac{kx}{F} = \frac{k/m}{s^2 + \frac{b}{m}s + \frac{k}{m}} = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H'(j\omega_0) = \frac{\omega_0^2}{-\omega_0^2 + j\frac{\omega_0}{Q}\cdot\omega_0 + \omega_0^2} = \frac{Q}{j}$$

Generelt: $kx = F \cdot \frac{Q}{j}$

i dette tilfelle:

G

$$x(y) = -\frac{Q \cdot F_d}{j\, k_{eff}(y)} = -\frac{Q}{j\, k_{eff}(y)} \cdot V_P \cdot \frac{\partial C}{\partial x} \cdot v_i$$

(Kraft og displacement i motsatt retning)

Prosedyre, forts

- Når bjelken beveger seg, dannes det en tidsvarierende kapasitans mellom elektroden og resonatoren
- H) Dette fører til en utgangsstrøm som er DC-biased via Vp
  - dC/dx er her et ulineært ledd
  - dx/dt er hastigheten

Når beam beveger seg, dannes det en tidsvarierende kapasitans mellom elektrode og resonator

Dette fører til en <u>utgangsstrøm</u> gitt av

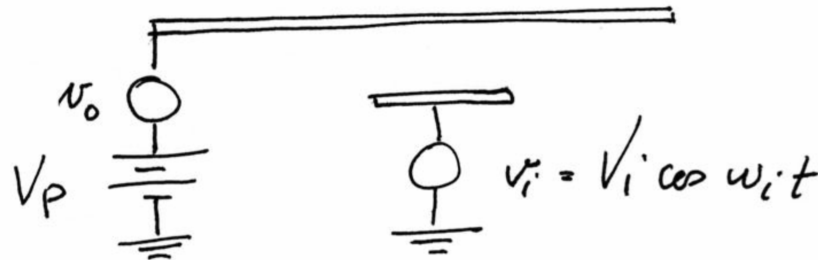
H

$$i_o = -V_P \frac{\partial C}{\partial x} \cdot \frac{\partial x}{\partial t}$$

(V_P er DC-bias på beam)

$$Q_o = V_P \cdot C$$

$$\dot{Q}_o = i_o =$$



Frekvenskarakteristikk

- Typiske parametre, Q, vakuum
  - Gir båndpassfilter, Q ~ 10,000
  - Egner seg for referanse osciallatorer og filtre med lave tap
- Q ~ noen hundre ved atmosfære-trykk

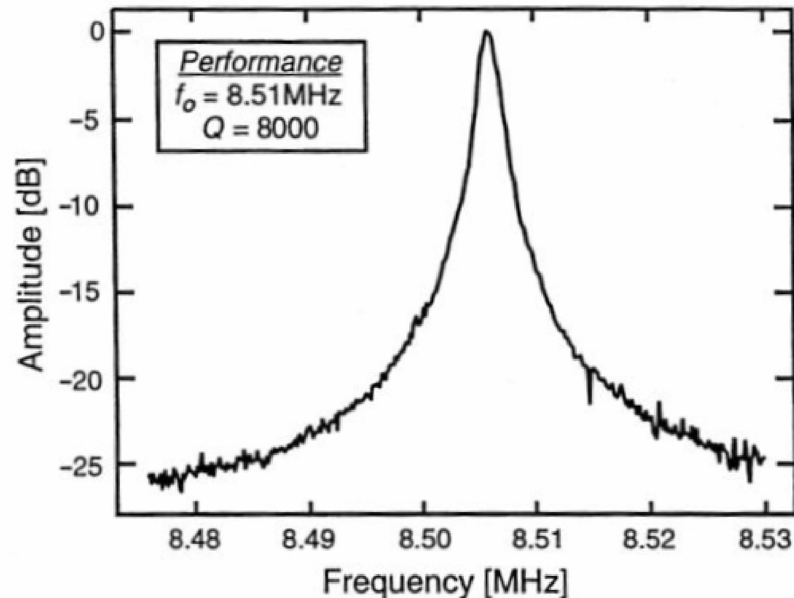


Figure 12.7. Frequency characteristic for an 8.5 MHz clamped–clamped beam polysilicon μmechanical resonator measured under 70 mtorr vacuum using a dc-bias voltage $V_P = 10$ V, a drive voltage of $v_i = 3$ mV, and a transresistance amplifier with a gain of 33 KΩ to yield an output voltage v_o. Amplitude $= v_o/v_i$. (From reference [18])

Prosedyre, forts.

- Overføring til mekanisk ekvivalentkrets:
  - ”mass-spring-damper”-krets
  - NB! Befinner oss fortsatt i mekanisk domene

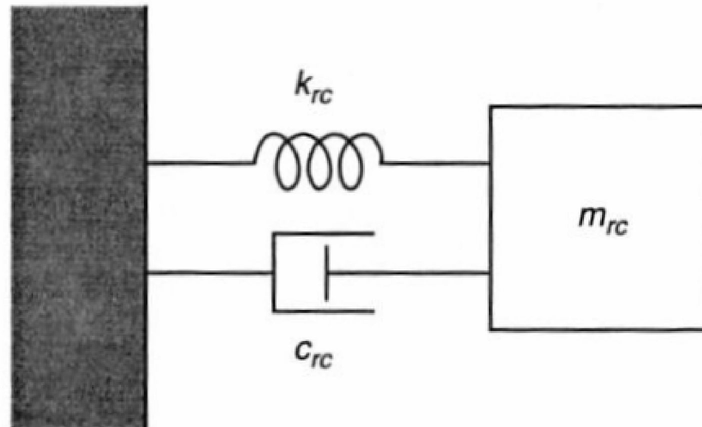


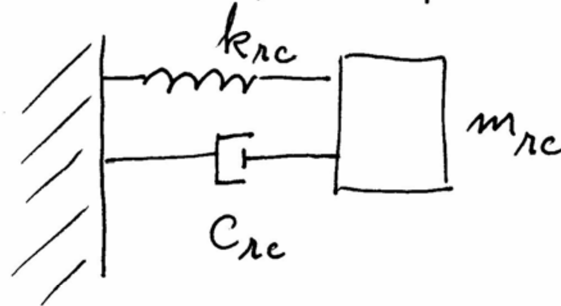
Figure 12.8. Lumped-parameter mechanical equivalent circuit for the micromechanical resonator of Figure 12.4.

Prosedyre, forts.

- ”Lumped parameter” elementverdiene er avhengig av hvor på bjelken en betrakter
- I) Beregning av ”ekvivalent masse” som funksjon av y
  - *Utfyllende fra R. A. Johnson: ”Mechanical Filters in Electronics”, Wiley, 1983*
    - Forenklet utledning av utbøyningsligning
    - Form på ”fundamentalmoden”

Mekanisk modellering

Mass-spring-damper-krets defineres



Elementverdiene varierer mhp. <u>lokasjon</u>, dvs. hvor på beamen en betrakter

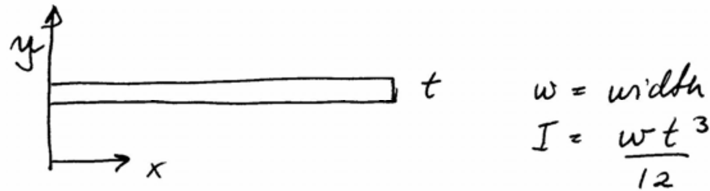
Den ekvivalente massen ved en lokasjon y på resonatoren er gitt av:

$$m_r(y) = \frac{KE_{tot}}{\frac{1}{2}\left[v(y)\right]^2}$$

KE_{tot} = peak kinetisk energi i systemet

$v(y)$ = hastighet i lokasjon y

Flexural-mode resonators



w = width

$I = \frac{wt^3}{12}$

u = displacement i y-retning
E = elastisk modulus
ρ = tetthet

Bølgeligningen:

$$\frac{\partial^2 u}{\partial t^2} = \frac{E \cdot I}{\rho \cdot A} \cdot \frac{\partial^4 u}{\partial x^4}$$

$$u = u_1 e^{j\omega t}$$

$$\Rightarrow \quad \frac{\partial^4 u}{\partial x^4} = \left(\omega^2 \frac{\rho A}{EI}\right) u$$

Har løsning:

$u(x) = A \cosh kx + B \sinh kx + C \cos kx + D \sin kx$

A, B, C, D bestemmes av grensebetingelsene

For fundamental-frekvensen:

"Mode shape" = $u(x) = \xi \left(\cos kx - \cosh kx \right) + \left(\sin kx - \sinh kx \right)$

Velocity i x-retning (langs beam)

$$v(x) = \dot{u}(x) = \frac{d}{dt}\left(u_1 e^{j\omega t}\right) = j\omega\, u(x)$$

Ekvivalent masse

$$M_{eq}(x) = \frac{KE}{\frac{1}{2} v^2(x)} = \frac{\frac{1}{2}\rho A \int_0^{\ell} v^2(x')\,dx'}{\frac{1}{2}\, v^2(x)}$$

$$M_{eq}(x) = \frac{\frac{1}{2}\rho A(-\omega^2)\int_0^{\ell} u^2(x')\,dx'}{\frac{1}{2}(-\omega^2)\, u^2(x)}$$

$$= \frac{\rho\, w \cdot t \int_0^{\ell} \left[X_{mode}(x')\right]^2 dx'}{\left[X_{mode}(x)\right]^2}$$

Anta y-retning langs bjelken

Ekvivalent masse

$$m_r(y) = \frac{KE_{tot}}{\frac{1}{2}[v(y)]^2}$$

$$= \frac{\rho W_r \cdot h \int_0^{L_r} [X_{mode}(y')]^2 dy'}{[X_{mode}(y)]^2}$$

der

$$X_{mode}(y) = \xi(\cos\beta y - \cosh\beta y) + (\sin\beta y - \sinh\beta y)$$

$\beta = 4.730/L_r$

$\xi = -1.01781$ (fundamental modus)

Dimensjoner: fig 12.9

Topologi

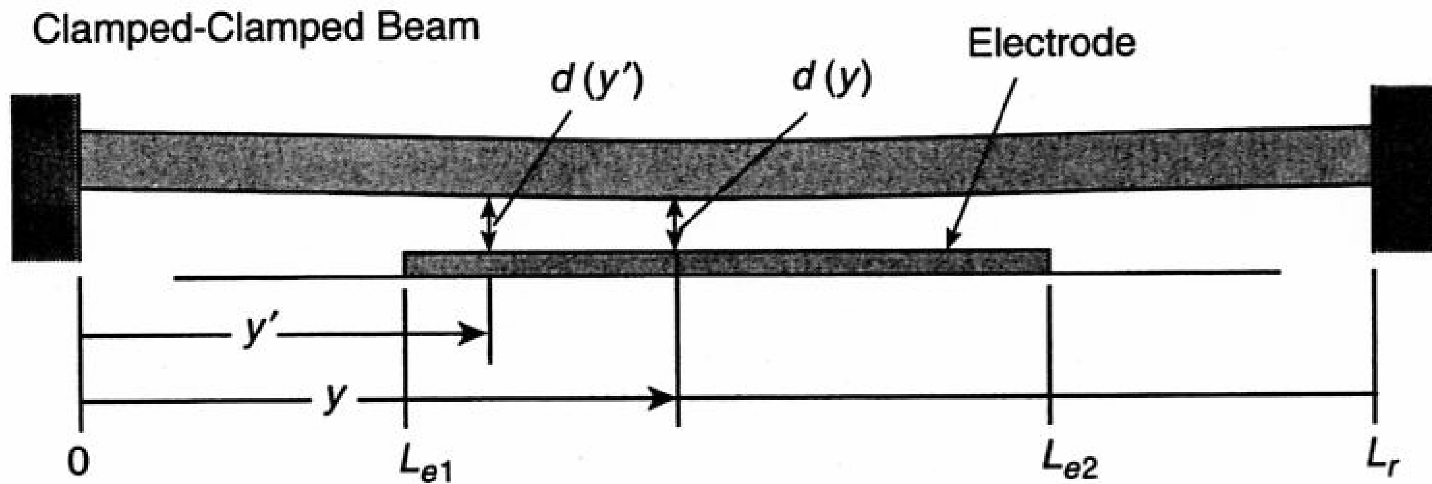


Figure 12.9. Resonator cross-sectional schematic for frequency-pulling and impedance analysis.

Prosedyre, forts.

- J) Når en har beregnet ekvivalent masse som funksjon av (y), kan en beregne ekvivalent fjærstivhet k_r (y) og dempefaktor c_r (y)
  - ω_0 er radian resonans-frekvensen
- K) Den mekaniske stivheten av resonatoren alene uten innflytelse av påtrykt spenning og elektroder finnes

J

Resonans-frekvens gitt av tidligere ligning

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} \quad , \quad \omega_0^2 = \frac{k_r}{m_r}$$

Ekvivalent fjærstivhet

$$k_r(y) = \omega_0^2 \cdot m_r(y)$$

↑ ekvivalent masse

Dempefaktoren $C_r(y)$:

$$s^2 + \frac{b}{m}s + \frac{k}{m} = s^2 + \frac{\omega_0}{Q}s + \omega_0^2$$

b = dempefaktor

$$b = m \cdot \frac{\omega_0}{Q} = \frac{m \cdot \sqrt{\frac{k}{m}}}{Q} = \frac{\sqrt{k \cdot m}}{Q}$$

Her: $$C_r(y) = b = \frac{\sqrt{k_m(y) \cdot m_r(y)}}{Q_{nom}}$$

$$k_m(y) = \omega_{nom}^2 \cdot m_r(y)$$

$$\Rightarrow \quad C_r(y) = \frac{\omega_{nom} \cdot m_r(y)}{Q_{nom}} = \frac{k_m(y)}{\omega_{nom} \cdot Q_{nom}}$$

K

$$k_m(y) = \omega_{nom}^2 \cdot m_r(y)$$

⇑ Den <u>mekaniske</u> stivhet av resonatoren alene <u>uten</u> innflytelse av påtrykt spenning og elektroder

Q_{nom} = Q-faktoren til resonatoren <u>uten</u> innflytelse av påtrykt spenning og elektroder

Tunbar elektrisk fjærstivhet

- Fjærstivheten kan tunes ved Vp
  - Resultanten er avhengig av forholdet mellom k_e og k_m
- L) Beregn hvordan k_e avhenger av lokasjon y

Voltage-tunbar elektrisk stivhet

- Resonans-frekvens kan tunes ved V_P
- Gir en V_P-avhengig <u>elektrisk</u> fjærkonstant k_e som subtraheres fra den <u>mekaniske</u> fjærkonstanten k_m
- Den resulterende fjærkonstanten senkes

$$k_r = k_m - k_e$$

(k_m: mekanisk; k_e: elektrisk)

Resonansfrekvensen

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}} = \frac{1}{2\pi}\sqrt{\frac{k_m}{m_r}\left(1 - \frac{k_e/m_r}{k_m/m_r}\right)}$$

i realiteten

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_m}{m_r}}\left(1 - \left\langle\frac{k_e}{k_m}\right\rangle\right)^{1/2}$$

$\frac{k_m}{m_r}$: Verdier i senter-lokasjon

$\left\langle\frac{k_e}{k_m}\right\rangle$: Finnes ved integrasjon over elektrode-bredden W_e

$$f_0 = 1.03\,\varkappa \sqrt{\frac{E}{\rho}} \cdot \frac{h}{L^2} \left(1 - \left\langle \frac{k_e}{k_m} \right\rangle \right)^{1/2}$$

k_e avhenger av kapasitansen $C(x)$ som igjen er avhengig av beam – elektrode-gapet som oppstår pga. V_P

$d(y')$ ← avhengig av avstanden y'

↑ gapet

Generelt:

Pot. energi $U = \frac{1}{2} V_P^2 C = \frac{1}{2} V_P^2 \frac{\varepsilon_0 A}{d}$

(Kraft $F = -\frac{1}{2} V_P^2 \frac{\varepsilon_0 A}{d^2}$)

$$U = \int_0^d k \cdot x \, dx = \frac{1}{2} k d^2 = \frac{1}{2} V_P^2 \frac{\varepsilon_0 A}{d}$$

Har: $k = V_P^2 \frac{\varepsilon_0 A}{d^3}$

↓

$$dk_e(y') = V_P^2 \frac{\varepsilon_0 \cdot W_r \cdot dy'}{[d(y')]^3}$$

↑ Differensiell elektrisk fjærstivhet i lokasjon y' og ved en en liten elektrode-bredde dy'

L

M

Må finne et uttrykk for $d(y)$:

Anta at kraft ved en forskyvning ut fra likevektstilling ($V_p = 0$), er:

$$F = -\frac{1}{2} V_p^2 \frac{\varepsilon_0 A}{d^2} = k \cdot \text{"displacement"}$$

Gap-avstanden kan uttrykkes ved:

$$d(y) = d_0 - \frac{1}{2} V_p^2 \varepsilon_0 W_r \int_{L_{e1}}^{L_{e2}} \frac{1}{k_m(y') \left[d(y')\right]^2} \cdot \frac{X_{stat}(y)}{X_{stat}(y')} dy'$$

d_0: gap ved $V_p=0$

X_{stat}: Statisk "bending shape" pga. den distribuerte dc-kraften

$d(y)$-ligningen må løses iterativt

Dermed finnes $dk_e(y')$ av $V_p^2 \frac{\varepsilon_0 W_r \, dy'}{\left[d(y')\right]^3}$

$$g\left(\frac{k_e}{k_m}\right) = g(d, V_p) = \int_{L_{e1}}^{L_{e2}} \frac{dk_e(y')}{k_m(y')}$$

Forenklet betraktning (De Los Santos):

Potensiell energi pga. påsatt spenning

$$U_1 = \frac{1}{2} C V_P^{\ 2}$$

Det arbeidet som utføres ved å forflytte bjelken en avstand g, MOT kraften som skyldes den elektriske fjærstivheten k_e
(Forutsetter at fjærstivheten er konstant i hvert punkt, y´)

$$U_2 = \int_0^g k_e \cdot x \cdot dx = \frac{1}{2} k_e \cdot g^2$$

Energiene kan settes lik hverandre

$$\frac{1}{2} k_e \cdot g^2 = \frac{1}{2} C \cdot V_P^{\ 2}$$

Forenklet uttrykk for elektrisk fjærstivhet

$$k_e = \frac{C \cdot V_P^{\ 2}}{g^2}$$

Forenklet uttrykk for frekvensen:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}} = \frac{1}{2\pi}\sqrt{\frac{k_m}{m_r}\left(1 - \frac{k_e}{k_m}\right)}$$

$$= \frac{1}{2\pi}\sqrt{\frac{k_m}{m_r}} \cdot \sqrt{1 - \frac{k_e}{k_m}} = f_{nom} \cdot \sqrt{1 - \frac{C \cdot V_P{}^2}{k_m \cdot g^2}}$$

Innsatt for C:

$$C = \varepsilon_0 \cdot \frac{A}{g}$$

$$f = f_{nom} \cdot \sqrt{1 - \frac{\varepsilon_0 \cdot A \cdot V_P{}^2}{k_m \cdot g^3}}$$

Dette harmonerer med de tidligere detaljerte beregningene

$$k_e = \varepsilon_0 \cdot \frac{A \cdot V_P^{\,2}}{g^3}$$

$$dk_e(y') = V_P^{\,2} \cdot \frac{\varepsilon_0 \cdot W_r \cdot dy'}{\left[d(y')\right]^3}$$

Ha:

$$k = V_P^2 \frac{\varepsilon_0 A}{d^3}$$

↓

$$dk_e(y') = V_P^2 \frac{\varepsilon_0 \cdot W_r \cdot dy'}{\left[d(y')\right]^3}$$

↑ Differensiell elektrisk fjærstivhet i lokasjon y' og ved en en liten elektrode-bredde dy'

L

Beam-softening

- Resonansfrekvensen faller med en gitt faktor
  - sqrt [1 – C0 * Vp exp2 / (k_m *d exp2)]
  - **→ resonansfrekvensen kan tunes elektrisk!**

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\kappa\sqrt{\frac{E}{\rho}\frac{h}{L_r^2}}\left[1 - g(V_P)\right]^{1/2}, \qquad (12.2)$$

Småsignal-ekvivalent

- En elektrisk ekvivalentkrets trengs for å modellere og simulere impedansoppførselen til denne mikro-mekaniske resonatoren i en **felles** elektromekanisk krets

$$L_x = \frac{m_{re}}{\eta_e^2}, \qquad C_x = \frac{\eta_e^2}{k_{re}}, \qquad R_x = \frac{\sqrt{k_{re} m_{re}}}{Q \eta_e^2} = \frac{C_{re}}{\eta_e^2}, \qquad (12.17)$$

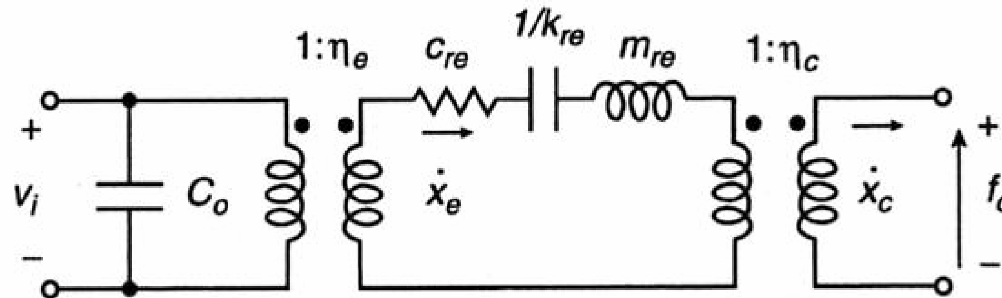


Figure 12.10. Equivalent circuit for a μmechanical resonator with both electrical (voltage v_i) and mechanical (force f_c) inputs and outputs.

Småsignal elektrisk ekvivalent

Fig 12.10

Omforme systemet til elektrisk ekvivalent-krets

- Kombinere elektriske og mekaniske innganger/utganger
- Bruk av "lumped elements"

Transformatorer modellerer både el. og mek. koblinger til/fra resonatoren

- Resonator LCR-system = el. analogi til masse-fjær-demping-systemet

Benytte verdiene funnet for

$m_r(y) \rightarrow m_{re}$

$k_r(y) \rightarrow k_{re}$

$c_r(y) \rightarrow c_{re}$

Den elektromekaniske analogien som benyttes er vist i Table 2.2

Transformasjons-analogi

TABLE 12.2. Mechanical-to-Electrical Correspondence in the Current Analogy

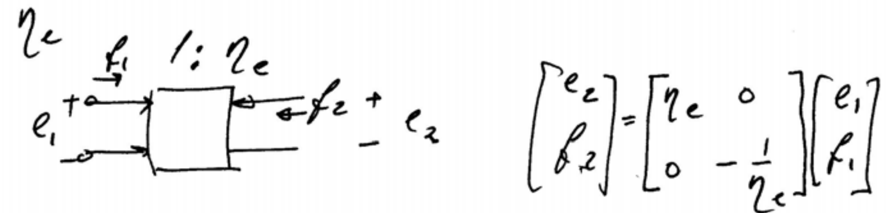
Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

Ekvivalent-krets

- ”Lumped element” modell foretrekkes
  - Kretsen må kunne inkludere både elektriske og mekaniske faktorer
- Modellen er **linearisert** slik at en transformator kan benyttes for å modelleres el-mek koblinger til og fra resonatoren
- Resonatoren selv er modellert i det mekaniske domenet med en LCR-krets som er den elektriske analogi til ”mass-string-damper” systemet
- Elementverdiene til den resulterende elektriske kretsen svarer til verdiene gitt i figur 12.10
  - Beregnes i det følgende →

LCR-kretsen transformeres til nye elementverdier gitt ved koblingskoeffisienten

η_e



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta_e & 0 \\ 0 & -\frac{1}{\eta_e} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Hvis snur saus

$$\left. \begin{aligned} e_2 &= \eta_e \cdot e_1 \\ f_2 &= \frac{1}{\eta_e} f_1 \end{aligned} \right\} \quad Z_{in}(s) = \frac{e_1}{f_1} = \frac{1}{n^2} \cdot \frac{e_2}{f_2} = \frac{Z_2(s)}{n^2}$$

Nye verdier av LRC-nettet

$$sL_x = \frac{sL_{re}}{\eta_e^2} = \frac{s \cdot m_{re}}{\eta_e^2} \Rightarrow L_x = \frac{m_{re}}{\eta_e^2}$$

$$\frac{1}{sC_x} = \frac{1/sC_{re}}{\eta_e^2} = \frac{k_{re}/s}{\eta_e^2} \Rightarrow C_x = \frac{\eta_e^2}{k_{re}}$$

$$R_x = \frac{R_{re}}{\eta_e^2} = \frac{c_{re}}{\eta_e^2} = \frac{\sqrt{k_{re} \cdot m_{re}}}{Q \cdot \eta_e^2}$$

Ekvivalentkrets

$$L_x = \frac{m_{re}}{\eta_e^2}, \qquad C_x = \frac{\eta_e^2}{k_{re}}, \qquad R_x = \frac{\sqrt{k_{re} m_{re}}}{Q\eta_e^2} = \frac{C_{re}}{\eta_e^2}, \qquad (12.17)$$

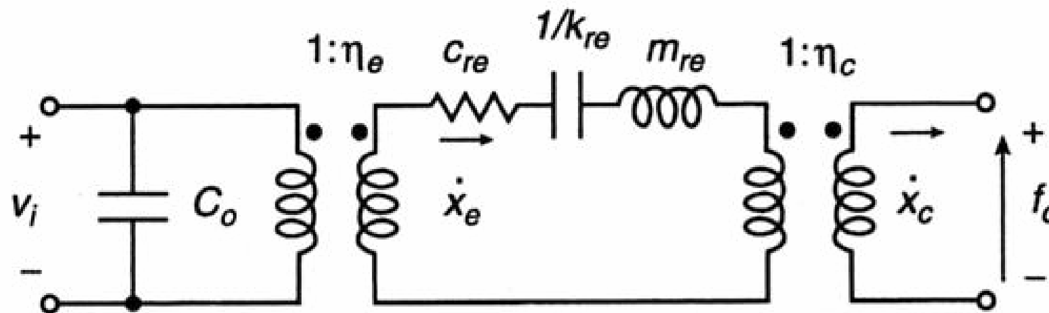
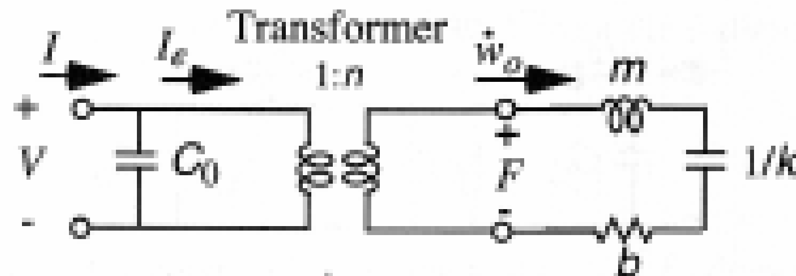


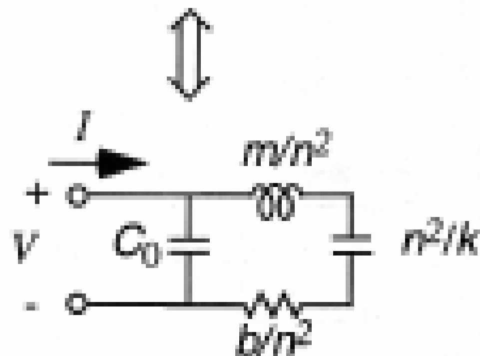
Figure 12.10. Equivalent circuit for a μmechanical resonator with both electrical (voltage v_i) and mechanical (force f_c) inputs and outputs.

Koblingskoeffisient

- Hvis en ser inn i kretsen fra venstre
  - En ser en en transformert LCR-krets med nye elementverdier gitt av (12.17)
  - Elektromekanisk koblingskoeffisient = ”transformer turns ratio”
    - Koblingskoeffisienten er utmeislet i notater fra UCLA
    - Tatt i forbindelse med 2-port ”lateral comb-drive actuator” (L8)

Small Signal Equivalent Circuit of Microresonators

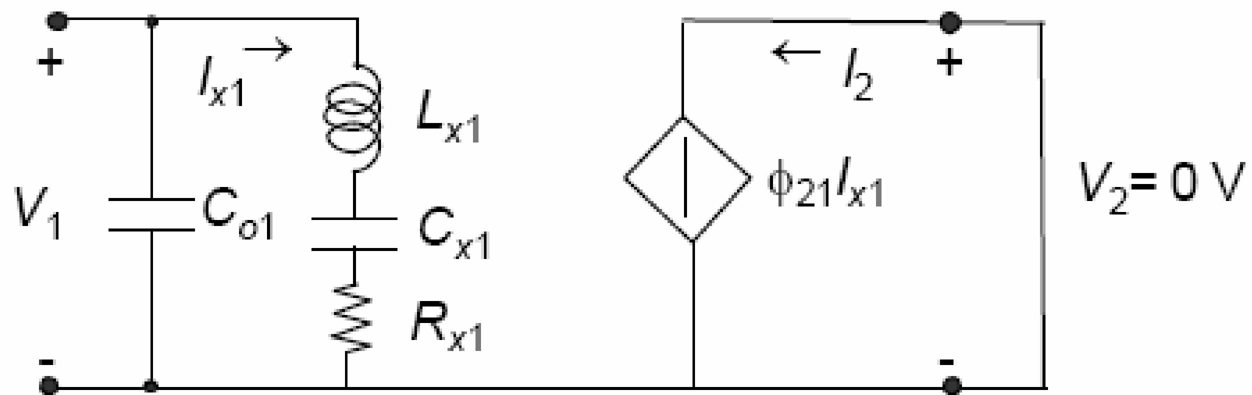




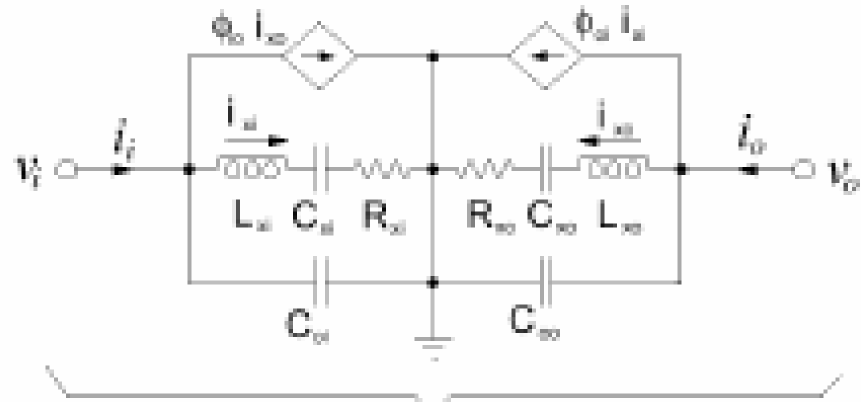
Unit of n^2/k is Farad

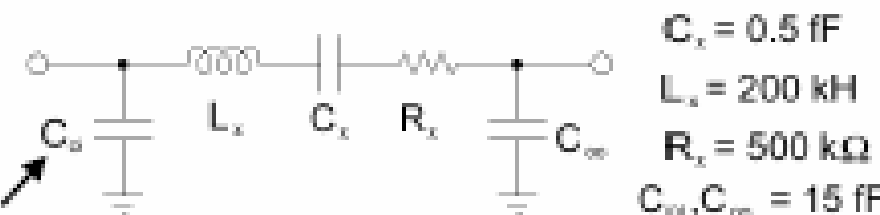
$$n = V_{dc} \frac{\partial C}{\partial x}$$

Two-Port Equivalent Circuit (v_2 = 0)



Equivalent Circuit of 2-Port Resonator
(in Electrical Domain)





$C_x = 0.5$ fF

$L_x = 200$ kH

$R_x = 500$ kΩ

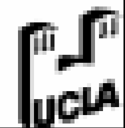
$C_{oi}, C_{oo} = 15$ fF

Fixed electrical Capacitance Between fixed comb And ground plane

$$C_{xn} = \frac{\eta_n^2}{k} \qquad R_{xn} = \frac{\sqrt{km}}{Q\eta_n^2} \qquad \eta_n = V_{Pn}\frac{\partial C_n}{\partial x}$$

$$L_{xn} = \frac{m}{\eta_n^2} \qquad \phi_{mn} = \frac{\eta_m}{\eta_n}$$

C. T.-C. Nguyen, "Micromechanical resonators for oscillators and filters," *Proceedings IEEE International Ultrasonics Symposium, Seattle, WA, pp. 489-498, Nov. 7-10, 1995*

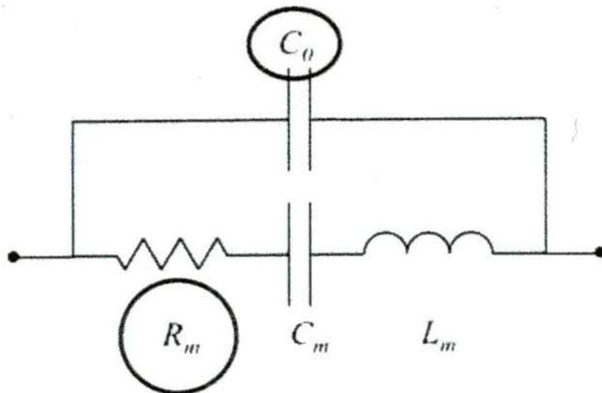


FSRM

Resonator equivalent circuit

Two types of currents possible:

- **<u>from resonator</u> motion** (should dominate!)
- from electrodes and resonator acting as pure electrical structure (from feedthrough capacitance)



Admittance at resonance is

$$Y_{in} = \frac{1}{R_m} + j\omega_o C_o$$

where we want to minimize the motional resistance, R_m :

$$R_m = \frac{\sqrt{k * m}}{Q\eta^2} \qquad \eta = V_{DC}\frac{dC}{dg}$$

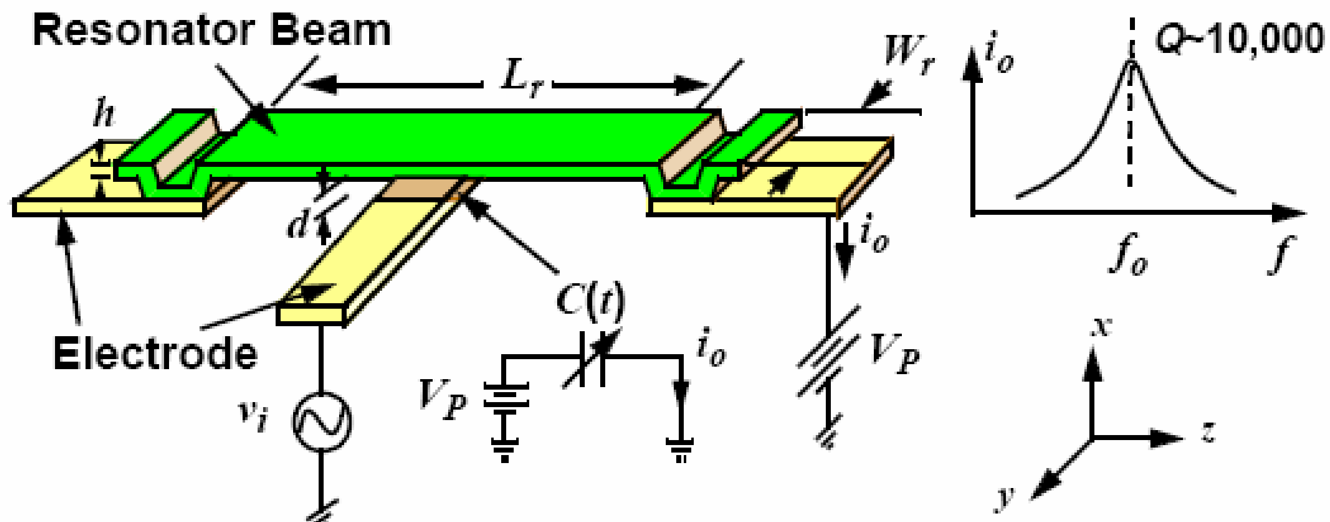
- Need:
  - ➢High Q
  - ➢High coupling (high voltage or small gap)
  - ➢Low mass
  - ➢Low stiffness (!)

95

Ionescu, EPFL

Vertically-Driven Micromechanical Resonator

- To date, most used design to achieve VHF frequencies



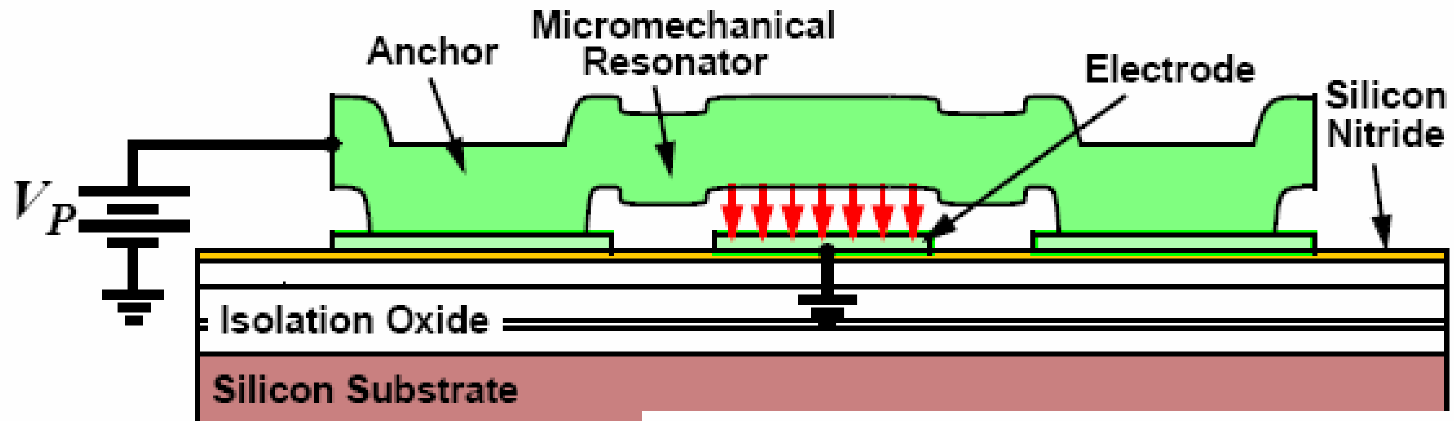
$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L_r^2}$$

(e.g. m_r=10^{-13}kg)

E = Youngs Modulus
ρ=density

- Smaller mass ➠ higher frequency range and lower series R_x

Voltage-Controllable Center Frequency



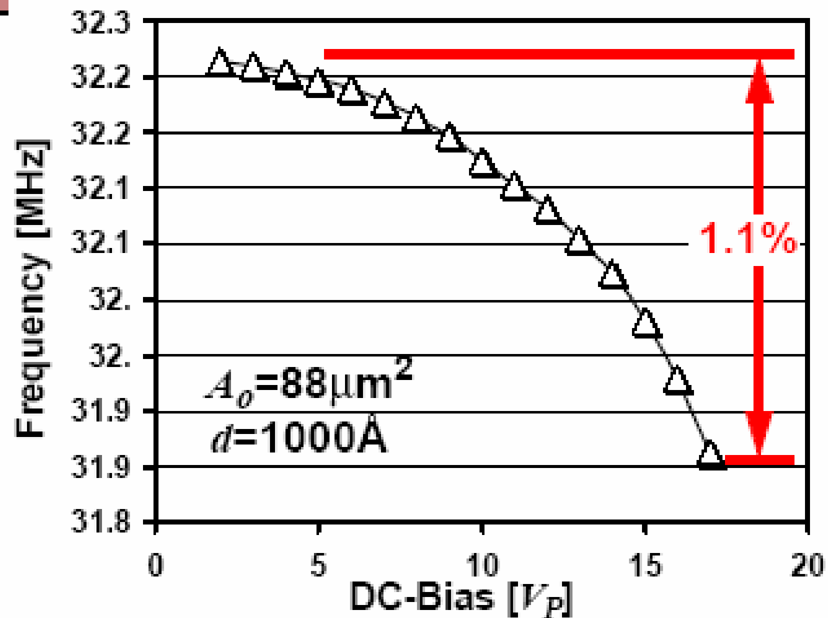
- Quadrature force ➠ voltage-controllable electrical stiffness:

$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

Electrode Overlap Area

Finger Gap

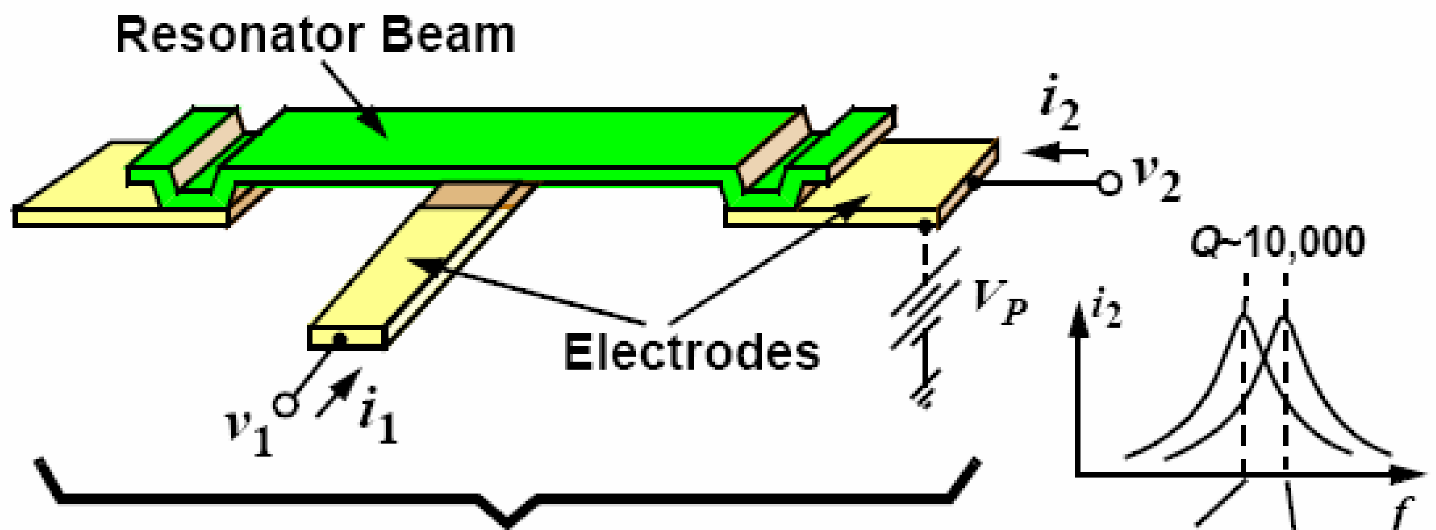
$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$

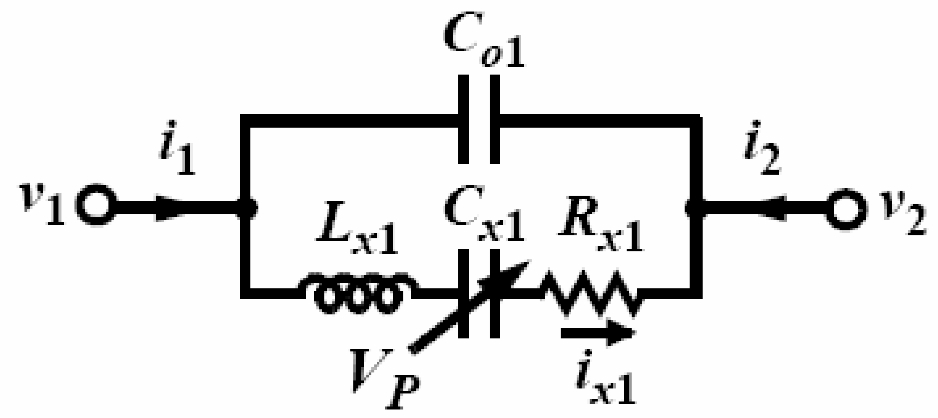


C. T.-C. Nguyen

Univ. of Michigan

Micromechanical Resonator Equivalent Circuit





Typical:

C_x ~ 0.20 fF

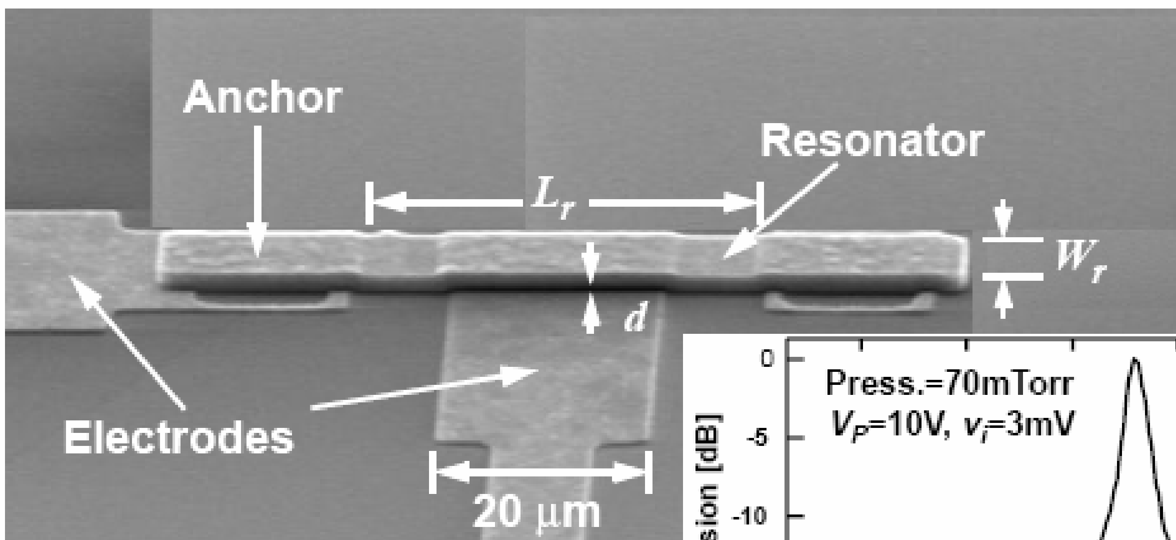
L_x ~ 2.6 mH

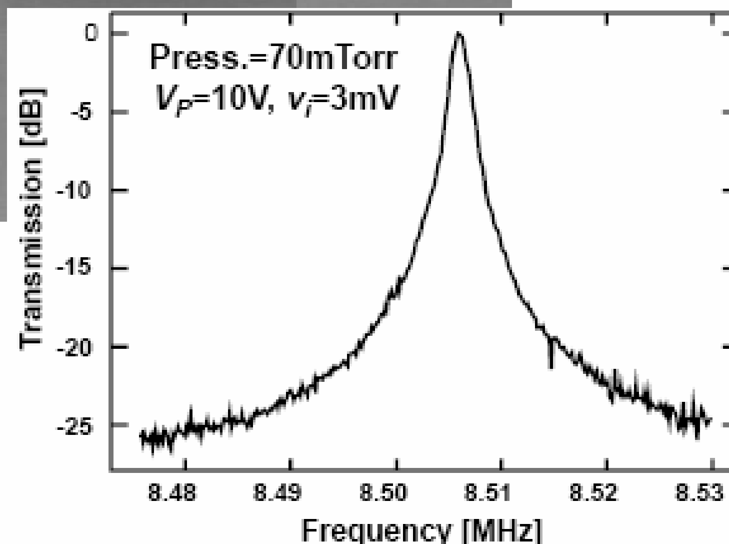
R_x ~ 115 Ω

C_o ~ 17 fF

Fabricated HF μMechanical Resonator

- Surface-micromachined, $POCl_3$-doped polycrystalline silicon





L_r=40.8 μm, W_r=8 μm, h=2 μm, d=0.1μm

- Extracted Q = 8,000 (vacuum)
- Freq. influenced by dc-bias and anchor effects

free-free-beam

- Gunstig når det gjelder tap til substratet gjennom ankerfestene
- f-f-beam er opphengt ved 4 support-bjelker i bredde-retningen
  - Torsjons-oppheng
  - Oppheng festet ved nodepunktene for ”flexural mode”
- Support-dimensjonene tilsvarer en kvart-bølgelengde av f-f-bjelkens resonans-frekvens
    - Impedansen som bjelken erfarer fra support nulles ut
    - Bjelken blir fri til å vibrere som om den ikke hadde noe oppheng
- Høyere Q kan oppnås
  - Eks. Q= 20,000 ved 10 – 200 MHz
  - Anvendes i referanse-oscillatorer, HF/VHF-filtre/miksere

free-free beam

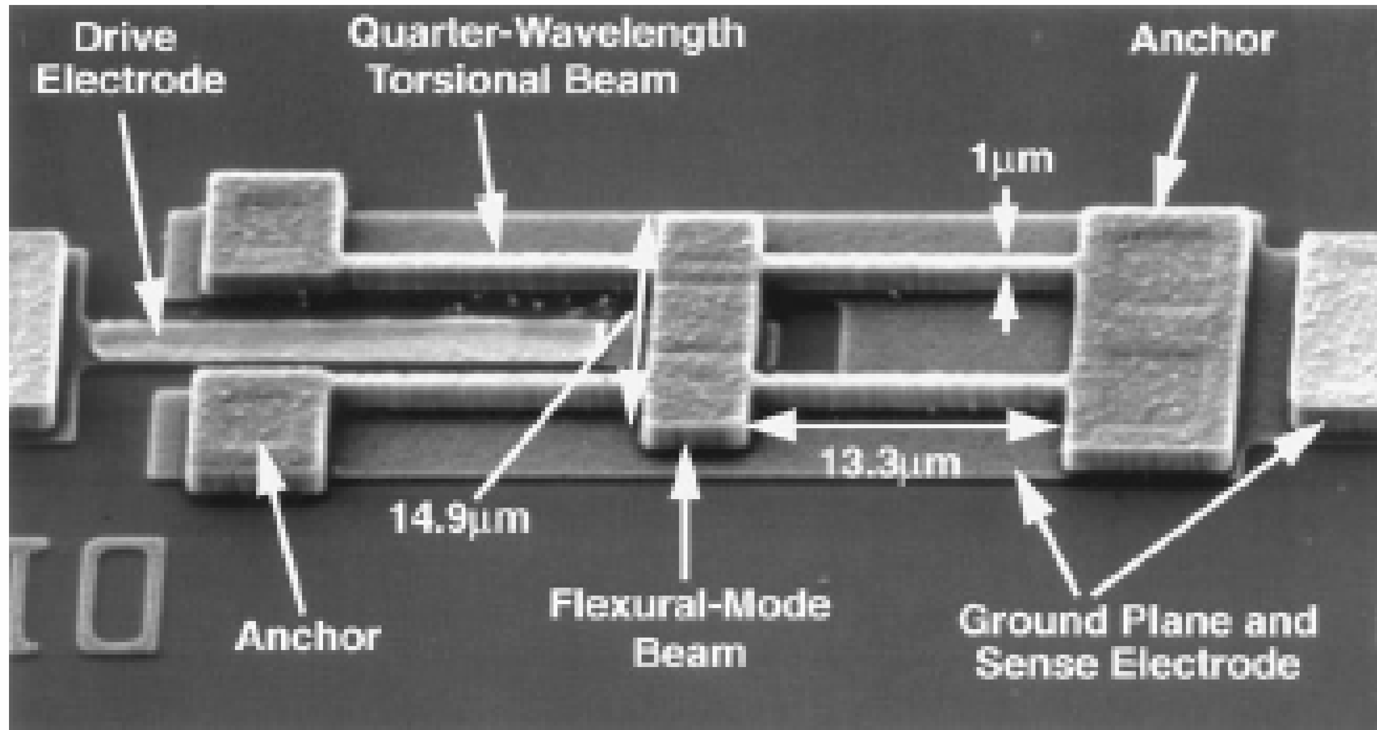
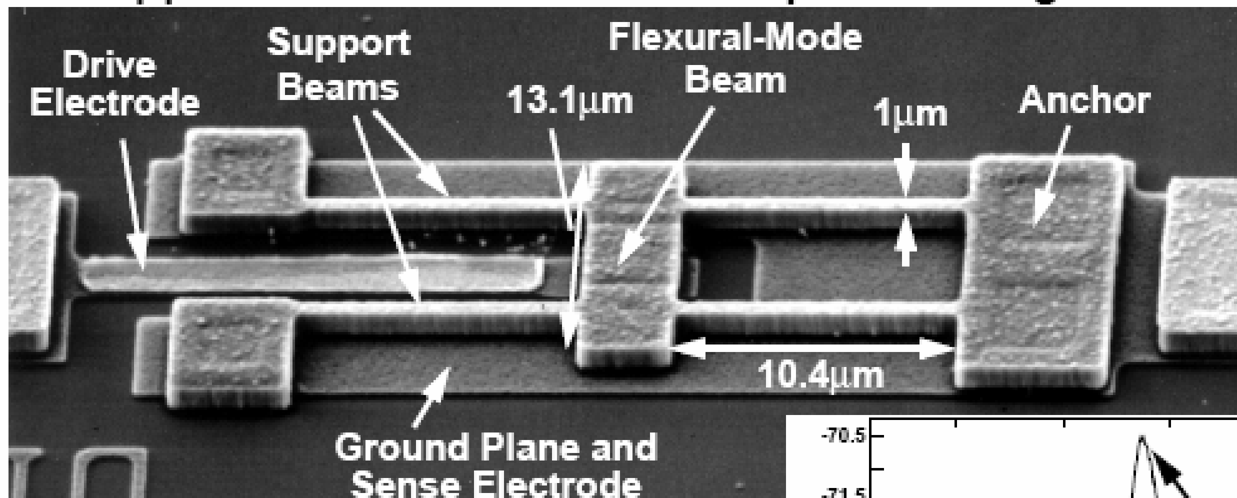


Fig. 29. SEM of free–free beam virtually levitated micromechanical resonator with relevant dimensions for $f_o = 71$ MHz.

Nguyen, 1999

92 MHz Free-Free Beam μResonator

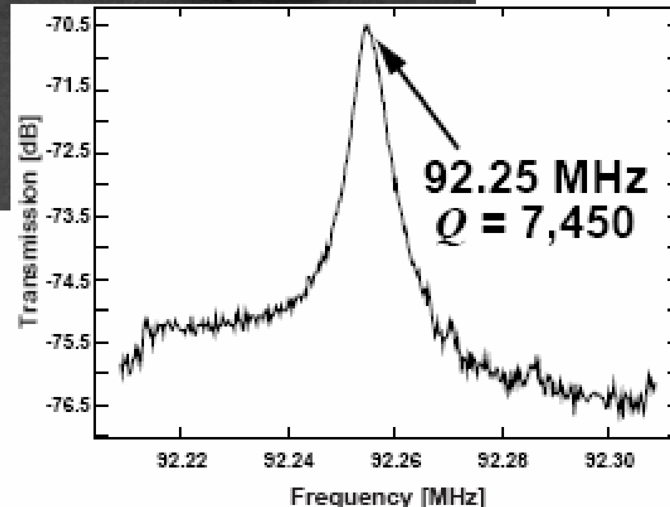
- Free-free beam μmechanical resonator with non-intrusive supports ➠ reduce anchor dissipation ➠ higher Q



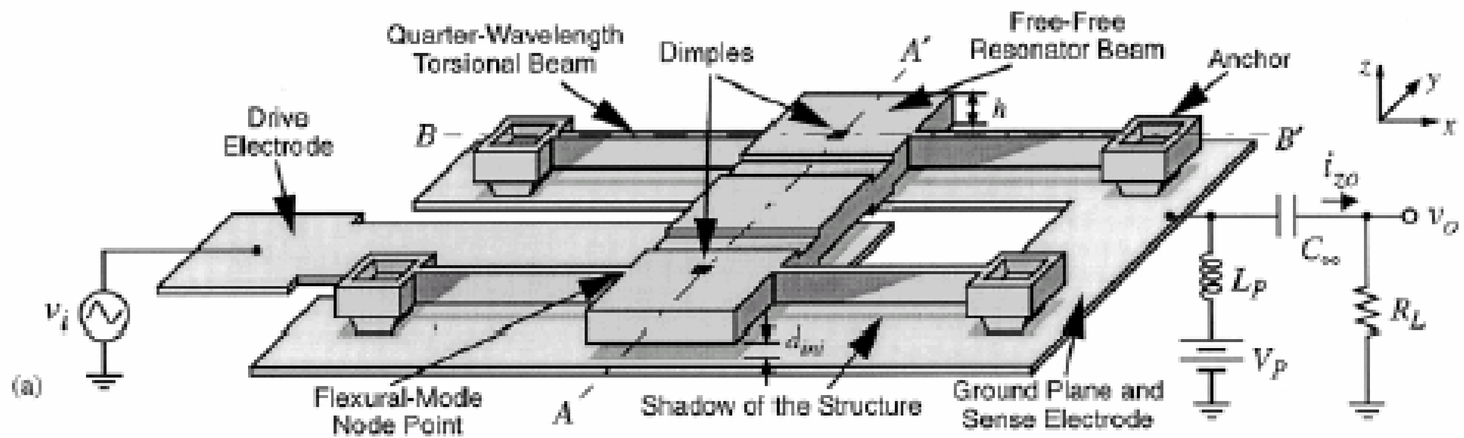
Design/Performance:

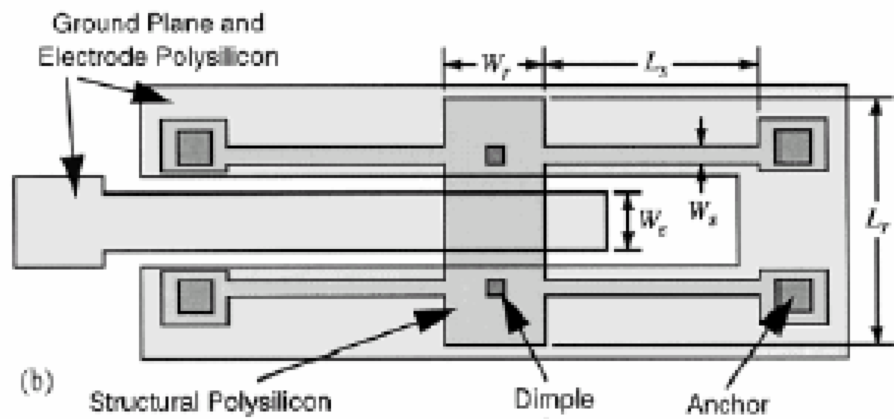
L_r=13.1μm, W_r=6μm
h=2μm, d=1000Å
V_P=28V, W_e=2.8μm
f_o~92.25MHz
Q~7,450 @ 10mTorr

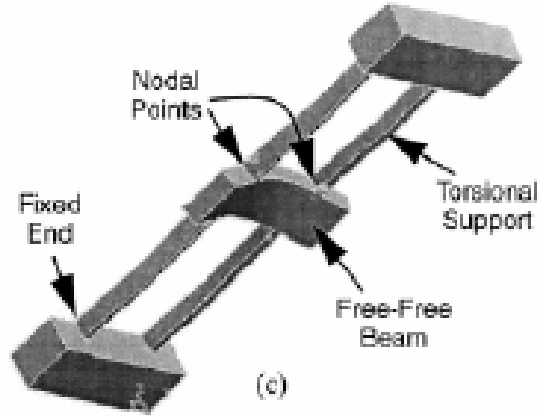
[Wang, Yu, Nguyen 1998]



VHF Free-Free Beam High-Q Micromechanical Resonator





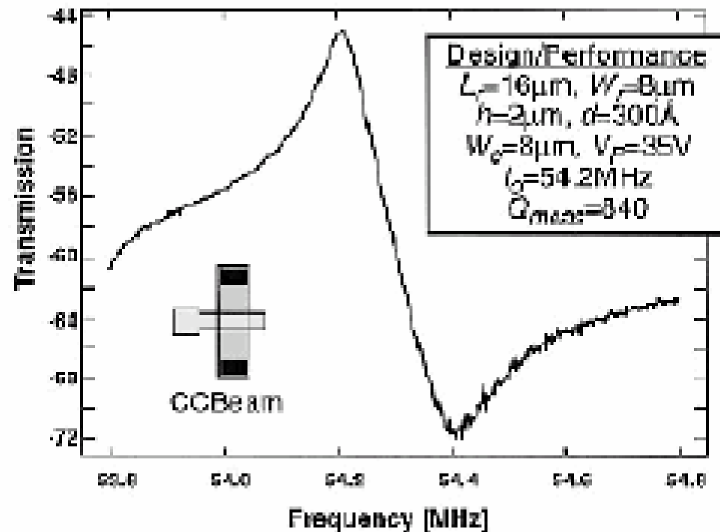


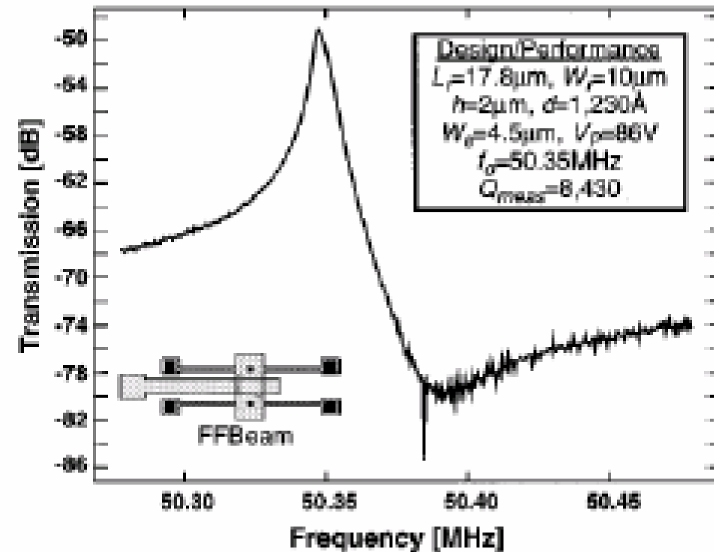
(determined the gap)

M. C. Wu

J. MEMS, Vol. 9, No. 3, 2000, C. T. –C. Nguyen, et al.

Comparison of Frequency Characteristics





Clamped-clamped beam

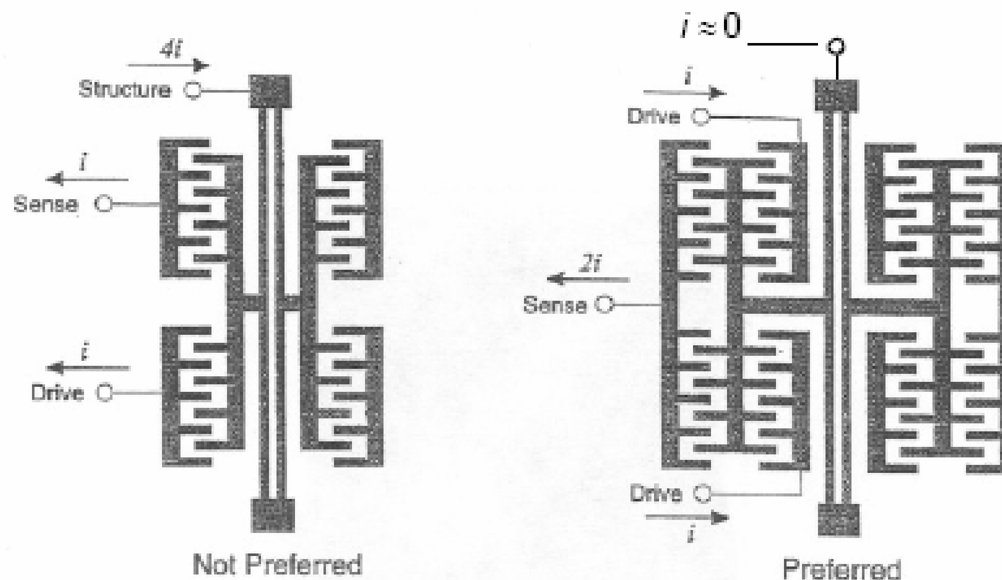
- L_r=16 μm, d=0.03 μm
- V_p=35 V, f_0=54.2 MHz
- Q=840

Free-free beam

- L_r=17.8 μm, d=0.12 μm
- V_p=86 V, f_0=50.35 MHz
- Q=8,430

Andre typer resonatorer

Double-Ended Tuning Fork Resonators



Current through structure → more resistance (decreases Q)
more feedthrough to substrate

EE C245 – ME C218 Fall 2003 Lecture 26

T. Roessig, Ph.D.,ME,
UC Berkeley, 1997

16

Scaling of Lateral Micromechanical Resonators

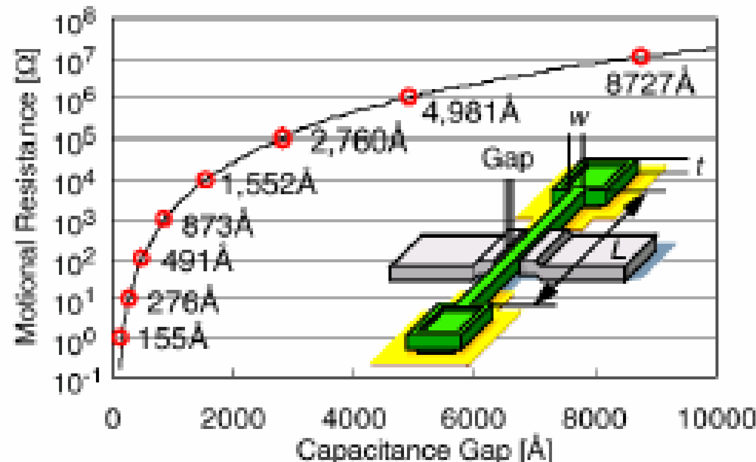


Fig. 1: Simulated plot of motional resistance versus electrode-to-resonator gap for a 40μm-long, 2μm-wide, 3μm-thick, lateral clamped-clamped beam μmechanical resonator.

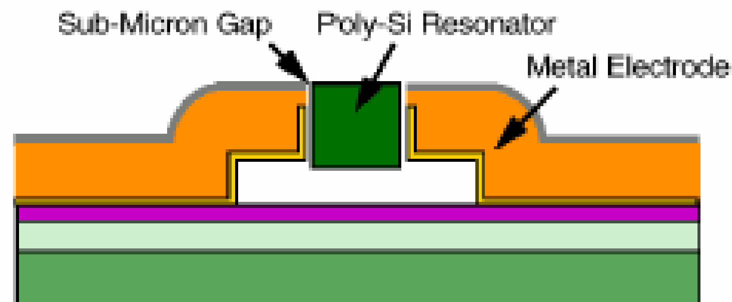
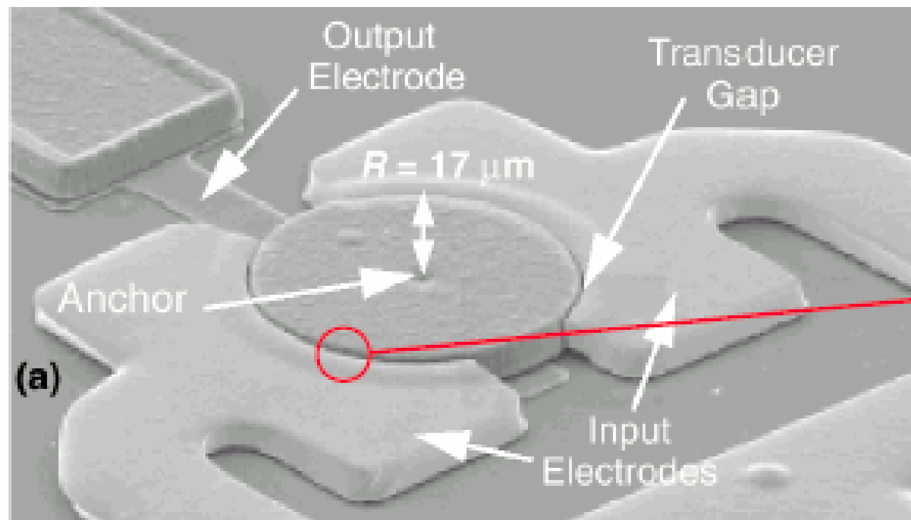


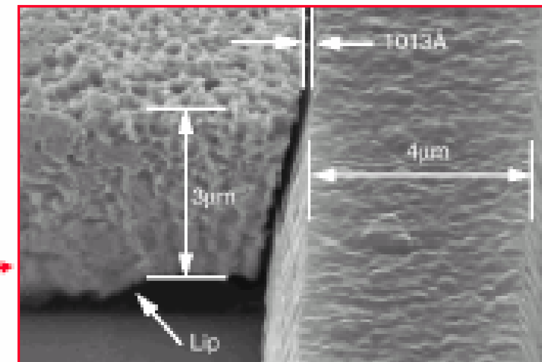
Fig. 2: Cross-section of the described sub-μm electrode-to-resonator gap process for lateral μstructures with metal electrodes.

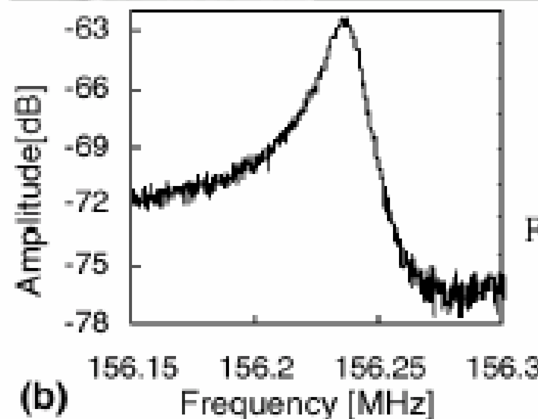
- Advantages of lateral resonator
  - Wider variety of resonant modes
  - Balanced resonators (push-pull)
  - More design flexibility
- As frequency scales up
  - Resonator size shrinks
  - Capacitive transducer gaps must also shrink (to sub-100 nm for VHF)
  - High aspect ratio structures
- Combine Poly-Si (high-Q structural materials) with metal electrode (high conductivity)
  - Self-aligned process

Hsu, Clark, Nguyen, "A sub-micron capacitive gap process for multiple-metal-electrode lateral micromechanical resonators," MEMS 2001, p. 349

Radial Contour-Mode Disk μ-mechanical Resonator







Data:
R=17μm, h=2μm
d=1,000Å, V_P=35V
f_o=156.23MHz, Q=9,400

Fig. 5: SEM and measured frequency characteristic for a 156.23 MHz contour-mode disk μmechanical resonator fabricated via the process of Fig. 3.

- Radial contour mode allows high resonant frequency without requiring sub-micron structures
- Place anchor at disk center – nodal point of contour mode
  ➔ Reduce mechanic loss and increase Q

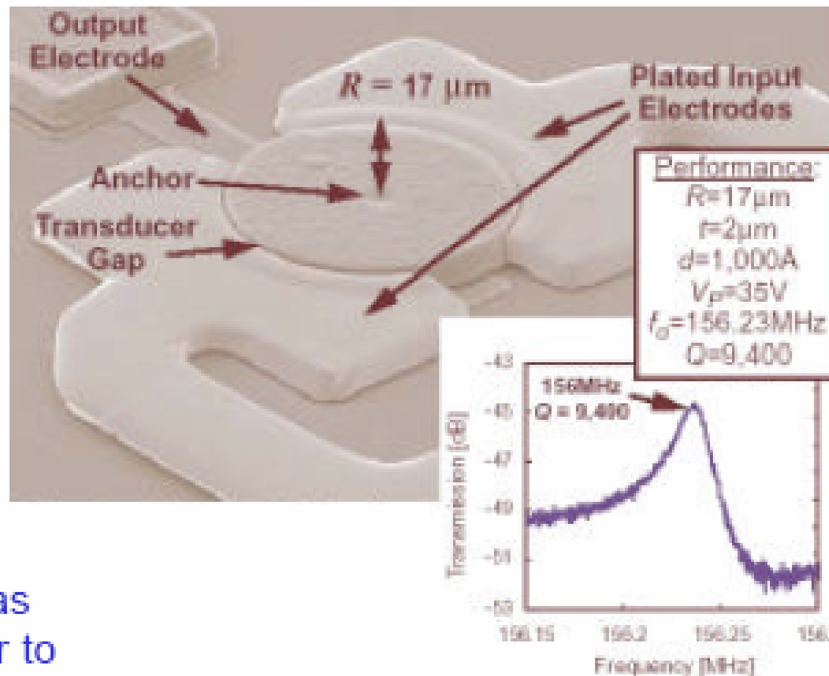
Hsu, Clark, Nguyen, "A sub-micron capacitive gap process for multiple-metal-electrode lateral micromechanical resonators," MEMS 2001, p. 349

Disk resonatorer

- Fordeler av disker framfor bjelker
  - Redusert luft-demping
    - Vakuum trengs ikke for måling av Q-faktor
  - Høyere stivhet
    - Frekvensen er høyere for gitte dimensjoner
  - Større volum
    - Høyere Q fordi mer energi er lagret
    - Mindre problemer med termisk støy

Increasing the Resonant Frequency

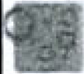
option 2. spring rate → ∞



Clark Nguyen, Michigan

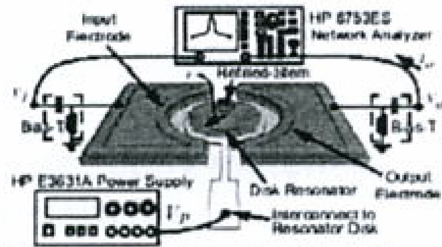
Motivation: keep mass as large as possible in order to improve precision of fab, power handling

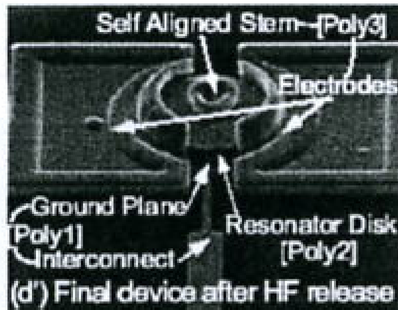
IEEE IEDM 2000.

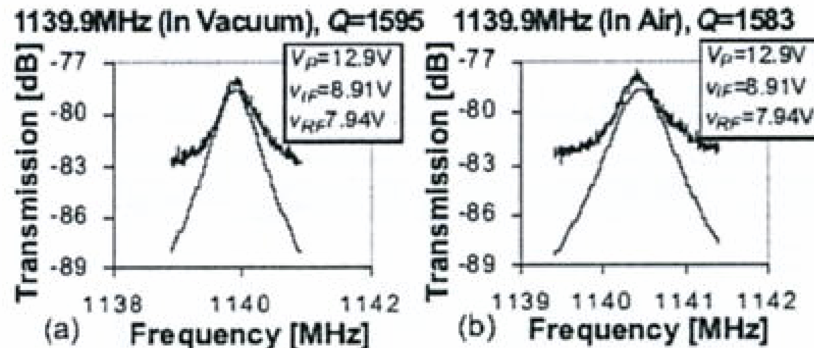


FSRM

Bulk contour-mode resonators







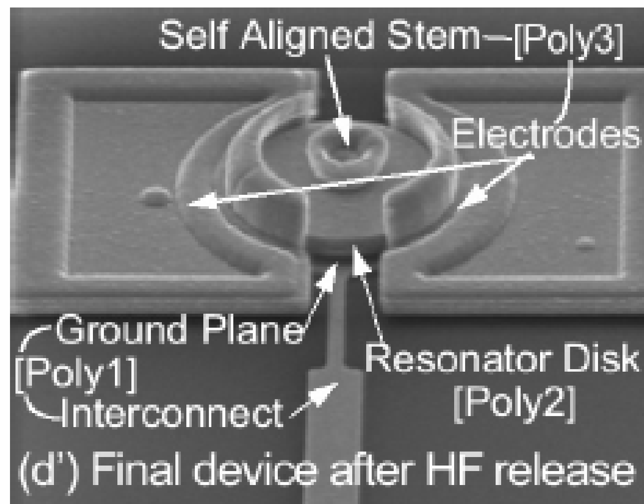
- > **1GHz** resonance frequency demonstrated
- **Q > 1'500** in both vaccum and air
- **Tcoeff ~-15ppm/°C**

J. Wang et al, Transducers 2003.

- **Bulk acoustic mode resonators / contour-mode disk resonators**
- **Frequency range: tens of kHz to GHz**
- **Quality factors > 10'000 for single crystal silicon demonstrated**
- **Further developments: process with nano-gaps → GHz frequency**

100

1.14 GHz Poly-Si Disk Resonator



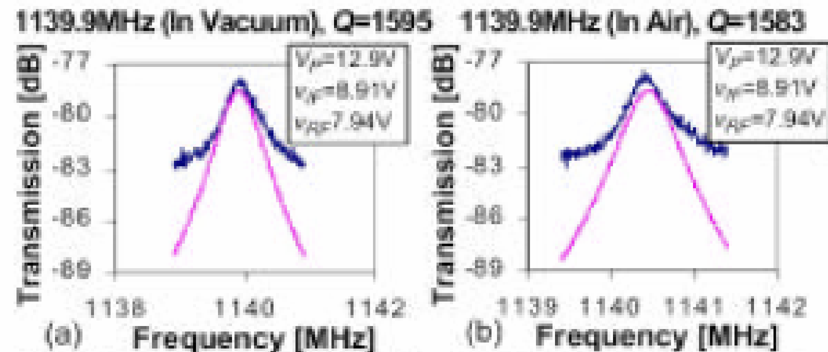


Fig. 7: Measured (dark) and predicted (light) frequency characteristics for a 1.14-GHz, 3rd mode, 20μm-diameter disk resonator measured in (a) vacuum and (b) in air, using a mixing measurement setup.

* Note *Q* in vacuum and in air is the same: little energy loss to ambient; however, energy loss through anchor ("stem") is significant
* EAM-like technique is used to extract the motional current.

Transducers '03, Boston

EAM = Electromechanical Amplitude Modulation (sinus også på "shuttle")

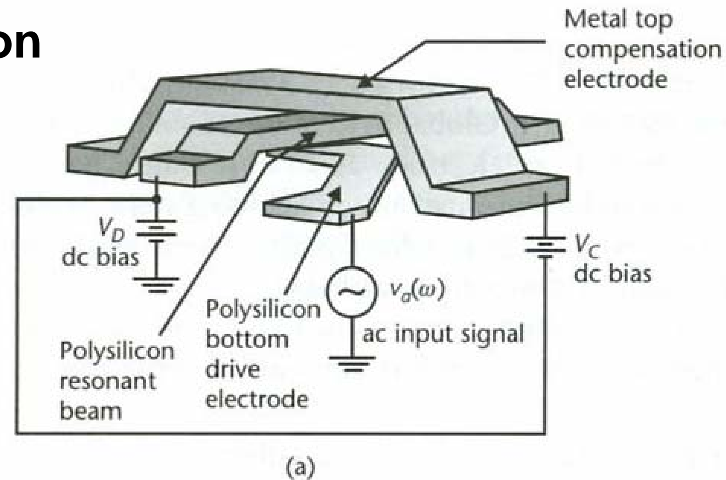
Begrensninger i mikromekaniske resonatorer

- Frekvens-begrensninger
  - Kan redusere m for å oppnå høyere frekvenser
  - Små masser vil kunne gi fluktuasjoner i frekvensen
    - ”mass loading”: utveksling av molekyler mot omgivelsene
    - Luft-gass-molekyler utøver Brownske bevegelser (kraft)
- Energi-begrensninger
  - Q avhenger av energitap
    - Viskøs demping
    - Vertikal bevegelse: squeezed-film damping
    - Horisontal bevegelse: Stokes- eller Couette-type demping

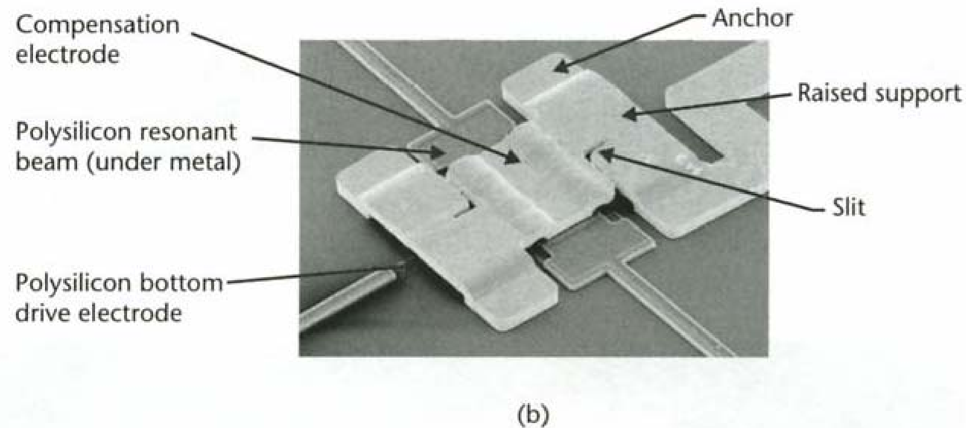
Begrensninger, forts.

- Temperaturavhengighet
  - Resonansfrekvensen endres pga. temperaturøkning og aldring
  - Økt temperatur fører til redusert frekvens
    - Analog eller digital kompensasjon (feedback)
    - Mekanisk kompensasjon
      - Benytte strukturer med deler som har både kompressivt og tensilt stress: motvirkende effekter

Temperatur-kompensasjon



(a)



(b)

Figure 7.11 Illustration of the compensation scheme to reduce sensitivity in a resonant structure to temperature. A voltage applied to a top metal electrode modifies through electrostatic attraction the effective spring constant of the resonant beam. Temperature changes cause the metal electrode to move relative to the polysilicon resonant beam, thus changing the gap between the two layers. This reduces the electrically induced spring constant opposing the mechanical spring while the mechanical spring constant itself is falling, resulting in their combination varying much less with temperature. (a) Perspective view of the structure [23], and (b) scanning electron micrograph of the device. (*Courtesy of:* Discera, Inc., of Ann Arbor, Michigan.)