INF5490 RF MEMS

L11: RF MEMS resonatorer III

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Dagens forelesning

- Vertikalt vibrerende resonatorer
  - Clamped-clamped beam (c-c beam)
    - Virkemåte
    - → **Detaljert modellering**
  - free-free beam (f-f beam)
- Andre typer resonatorer
  - Tuning fork
  - Bjelke med lateral bevegelse
  - Disk resonatorer

Beam-resonator

- Det er ønskelig med en høyere resonansfrekvens enn kam-strukturen kan oppnå
  - Massen må reduseres mer-> **beam resonator**

- Fordeler ved beam-resonatorer
  - Mindre dimensjoner
  - Høyere resonansfrekvens
  - Enkel
  - Kan ha mange frekvens-referanser på en chip
  - Mer lineær frekvensvariasjon mhp temp over et større område
  - Mulighet for integrering med elektronikk → lavere kostnader

Beam-resonator

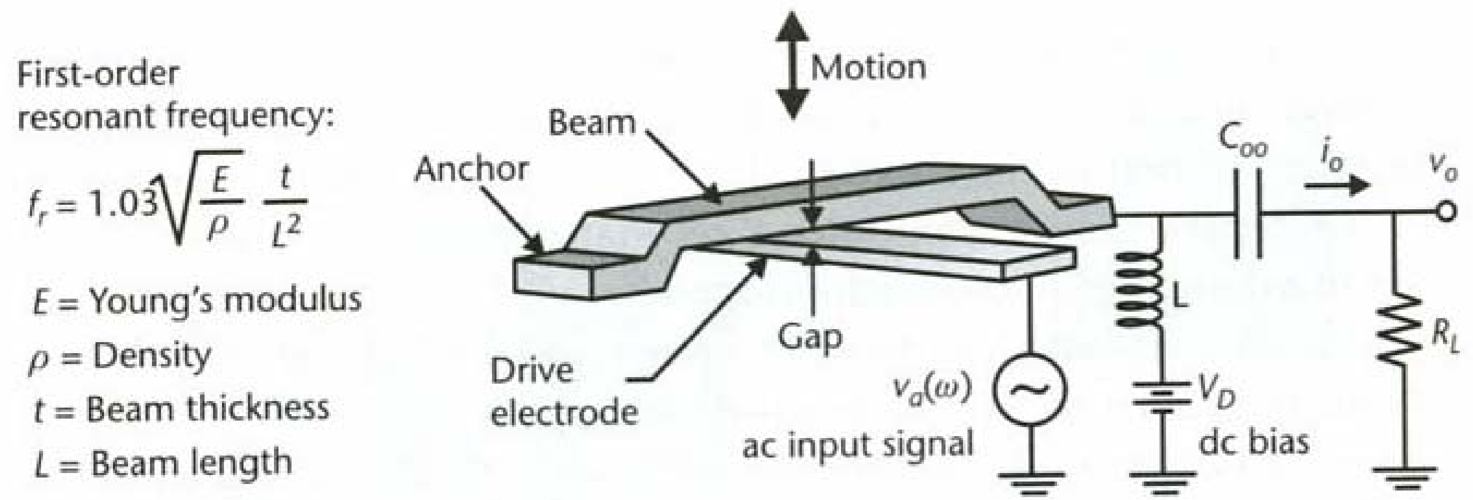


Figure 7.10 Illustration of a beam resonator and a typical circuit to measure the signal. The beam is clamped on both ends by anchors to the substrate. The capacitance between the resonant beam and the drive electrode varies with the deflection.

”En-port”-realisering

Utgangskrets

- Resonator er en tidsvarierende kapasitans C(ω)
- Enkel elektrisk utgangskrets
  - L = shunt blokkerende induktor: Åpen ved høye frekvenser
  - C_∞ = serie blokkerende kapasitans: Kortsluttet ved høye frekvenser
  - Når Vd er en høy DC-spenning, så er den dominerende utgangs-strømmen ved inngangsfrekvens ω: i0 = Vd * dC/dt
  - Ved høye frekvenser er i0 strømmen gjennom R_L
    - Kan være inngangsimpedans i måleutrustningen. Kan erstattes av transimp.-forsterker

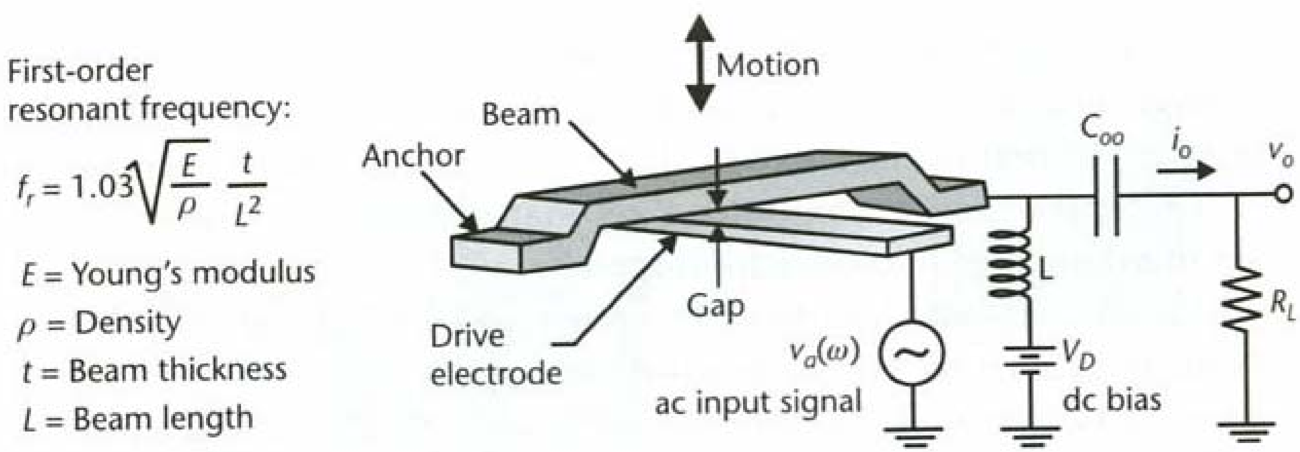


Figure 7.10 Illustration of a beam resonator and a typical circuit to measure the signal. The beam is clamped on both ends by anchors to the substrate. The capacitance between the resonant beam and the drive electrode varies with the deflection.

Mekanisk resonans-frekvens

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\kappa\sqrt{\frac{E}{\rho}\frac{h}{L_r^2}}[1 - g(V_P)]^{1/2}, \qquad (12.2)$$

- Parametre
  - E = Youngs modul
  - ρ = tettheten av materialet
  - h = tykkelsen av beam
  - Lr = lengde av beam
  - g modellerer effekten av en **elektrisk fjærstivhet k_e**
    - Gjør seg gjeldende når en setter spenning på elektrodene
    - Subtraheres fra den mekaniske fjærstivheten, k_m (beam-softening)
  - κ =skaleringsfaktor (effekten av overflatens topografi, typisk 0.9)
  - V_p = DC spenning på ledende beam
  - k_r = effektiv resonator fjærstivhet
  - m_r = effektiv masse
- **NB! E og ρ inngår + fjærstivhetsledd**

Typiske frekvenser

TABLE 12.1. μMechanical Resonator Frequency Design[a]

Frequency (MHz)	Material	Mode	h_r (μm)	W_r (μm)	L_r (μm)
70	Silicon	1	2	8	14.54
110	Silicon	1	2	8	11.26
250	Silicon	1	2	4	6.74
870	Silicon	2	2	4	4.38
870	Diamond	2	2	4	8.88
1800	Silicon	3	1	4	3.09
1800	Diamond	3	1	4	6.16

[a] Determined for free-free beams using Timoshenko methods that include the effects of finite h and W_r [11].

”Beam-softening”

- DC-spenningen, Vd, forårsaker en nedoverrettet elektrostatisk kraft
- Kraften virker mot den mekaniske gjenopprettelses-kraften i bjelken
- Dette gjør den effektive mekaniske fjærkonstanten til systemet mindre
  - Resonansfrekvensen faller med en gitt faktor $\sqrt{1 - C \cdot {V_P}^2 / (k \cdot g^2)}$
  - → **resonansfrekvensen kan tunes elektrisk!**

Detaljert modellering

- **c-c beam** modelleres med referanse til boka
  - *T. Itoh et al: RF Technologies for Low Power Wireless Communications", kap. 12: "Transceiver Front-End Architectures Using Vibrating Micromechanical Signal Processors", by Clark T.-C. Nguyen*
  - *(+ oppsummering av resultater fra diverse publikasjoner)*

Clamped-clamped beam

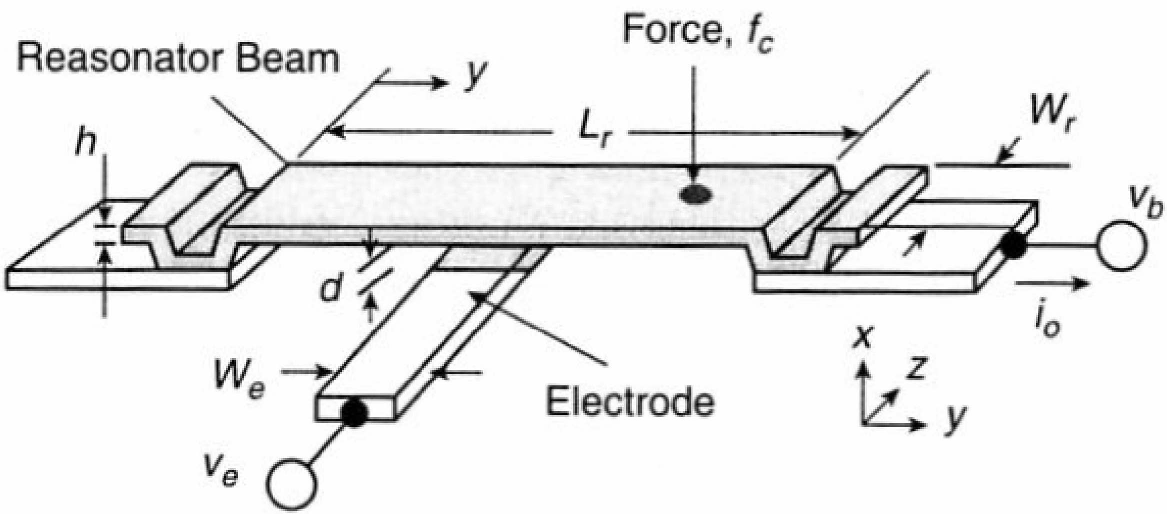


Figure 12.4. Perspective-view schematic of a clamped–clamped beam μmechanical resonator in a general bias and excitation configuration.

Beregning av elektrisk eksitasjon

- 2 påtrykte spenninger
- A) Beregn først potensiell energi
- B) Deretter beregnes kraften →

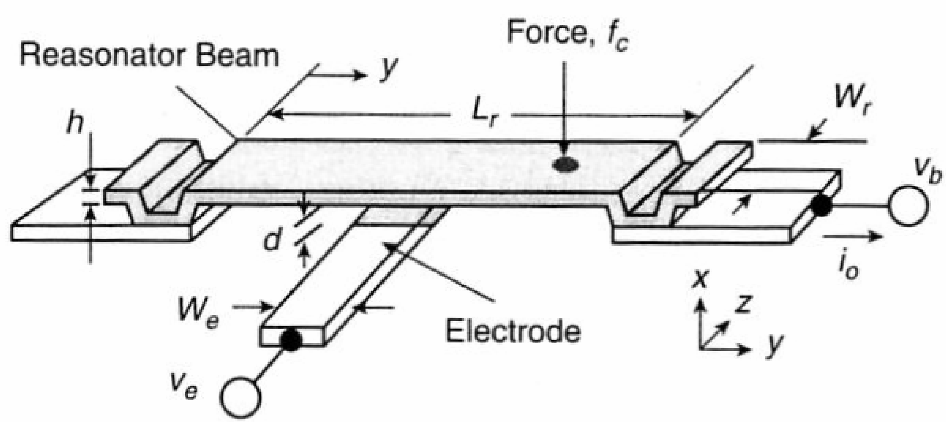


Figure 12.4. Perspective-view schematic of a clamped–clamped beam μmechanical resonator in a general bias and excitation configuration.

A. Electrical exitation

$$v_e = \text{input on electrode}$$
$$v_b = \text{input on beam}$$
$$v_e - v_b = \text{effective voltage}$$
$$U = \frac{1}{2}CV^2 = \frac{1}{2}C(v_e - v_b)^2 = \text{potential energy}$$

$$F_d = \frac{\partial U}{\partial x} = \frac{1}{2}(v_e - v_b)^2 \frac{\partial C}{\partial x}$$

B. Force is change of potential energy vs. x

$$= \frac{1}{2}(v_b{}^2 - 2v_b v_e + v_e{}^2)\frac{\partial C}{\partial x}$$
$$C = \frac{\varepsilon_0 A}{d_0} = \varepsilon_0 \frac{W_e W_r}{d_0}$$
$$W_e = \text{electrode width}, \mathrm{W_r} = \text{beam width}$$
$$d_0 = \text{electrode} - \text{resonator gap (static, non-resonance)}$$
$$\varepsilon_0 = \text{permittivity in vacuum}$$

Prosedyre, forts.

- C) Sett på DC-spenning, Vp
- D) Regn videre på ligningen for kraften
- E) Diskusjon av ulike bidrag
  - Off-resonans DC-kraft
  - Kraft i takt med inngangsspenning
  - Dobbeltfrekvens-ledd

C. A DC voltage is applied to the beam

$$v_b = V_P, \;\; v_e = v_i = V_i \cos \omega_i t$$

D.

$$F_d = \frac{1}{2}(V_P{}^2 - 2V_P V_i \cos \omega_i t + V_i{}^2 \cos^2 \omega_i t)\frac{\partial C}{\partial x}$$

Observe that:

$$\cos^2 \omega_i t = \frac{1}{2}(1 + \cos 2\omega_i t)$$

$$V_i{}^2 \cos^2 \omega_i t = \frac{V_i{}^2}{2}(1 + \cos 2\omega_i t)$$

Then

$$F_d = (\frac{1}{2}V_P{}^2 - V_P V_i \cos\omega_i t + \frac{1}{2}\frac{V_i^2}{2} + \frac{1}{2}\frac{V_i^2}{2}\cos 2\omega_i t)\frac{\partial C}{\partial x}$$

E.

$$F_d = \frac{\partial C}{\partial x}(\frac{V_P{}^2}{2} + \frac{V_i^2}{4}) - V_P\frac{\partial C}{\partial x}V_i\cos\omega_i t + \frac{\partial C}{\partial x}\frac{V_i^2}{4}\cos 2\omega_i t$$

Off-resonance DC force
Static bending of beam

Force driven by the input frequency,
amplified by V_P

$$\frac{\partial C}{\partial x}\frac{V_i^2}{4}\cos 2\omega_i t$$

This term can drive the beam into vibrations at

$$2\omega_i = \omega_0, \ \ and \ \ \omega_i = \frac{\omega_0}{2}$$

The term can usually be neglected

Prosedyre, forts.

- → Kraftens hovedbidrag er prop med cos
  - Driver beam inn i resonans
- F) Kraftien gir ”displacement” (x-variasjon)
  - Den lokale fjærstivheten varierer over bredden av drive-elektroden
  - Lokalt displacement er avhengig av posisjon y
- G) Utledning av et uttykk for displacement, x(y), som funksjon av fjærstivheten i posisjonen y

Topologi

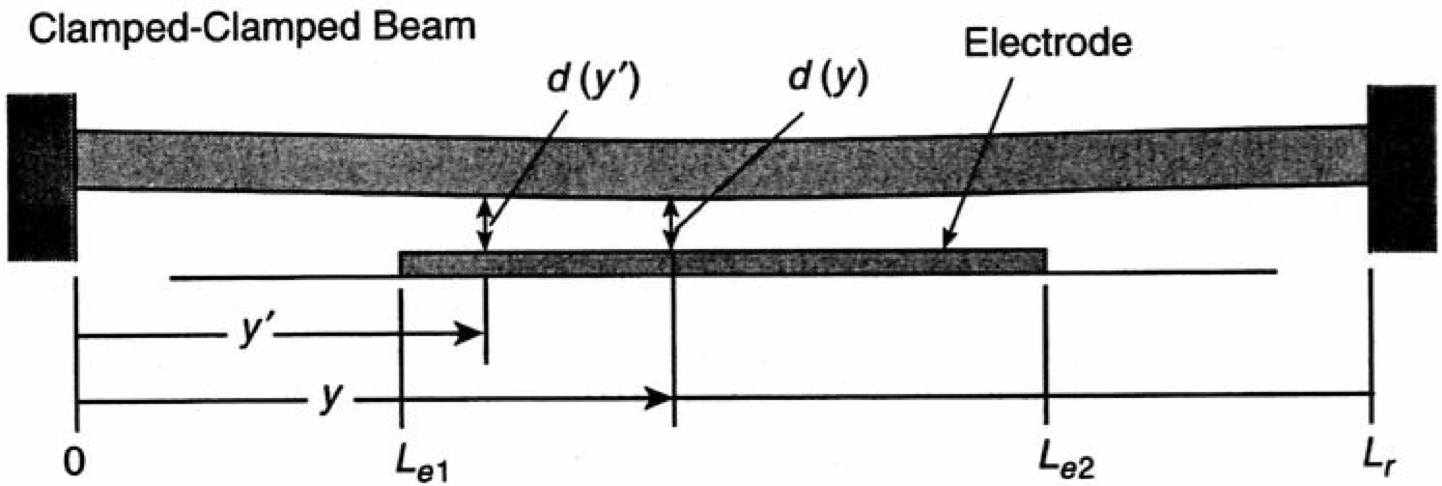


Figure 12.9. Resonator cross-sectional schematic for frequency-pulling and impedance analysis.

The main contribution to the force:

$$-V_P \frac{\partial C}{\partial x} V_i \cos \omega_i t$$

At resonance the force will be:

$$F_d = -V_P \frac{\partial C}{\partial x} v_i(\omega_0)$$

The force will give a varying displacement, since the distance between the beam and electrode is dependent on y-position

Generally : $F = k \cdot x,$ static!

$k(y) = k_{reff}(y)$ = effective beam stiffness in y

Dynamic performance of a mechanical system:

F.

$$H(s) = \frac{x}{F} = \frac{displacement}{force} = \frac{1/m}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

$$H'(s) = \frac{kx}{F} = \frac{k/m}{s^2 + \frac{b}{m}s + \frac{k}{m}} = \frac{\omega_0{}^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0{}^2}$$

$$H'(j\omega_0) = \frac{\omega_0{}^2}{-\omega_0{}^2 + j\frac{\omega_0}{Q}\omega_0 + \omega_0{}^2} = \frac{Q}{j}$$

$kx = F \cdot \frac{Q}{j}$, at resonance (generally)

G. In our case:

$$x(y) = +\frac{Q}{j}\frac{F_d}{k_{reff}(y)} = -\frac{Q}{jk_{reff}} \cdot V_P \cdot \frac{\partial C}{\partial x} \cdot v_i$$

Force and displacement in opposite directions

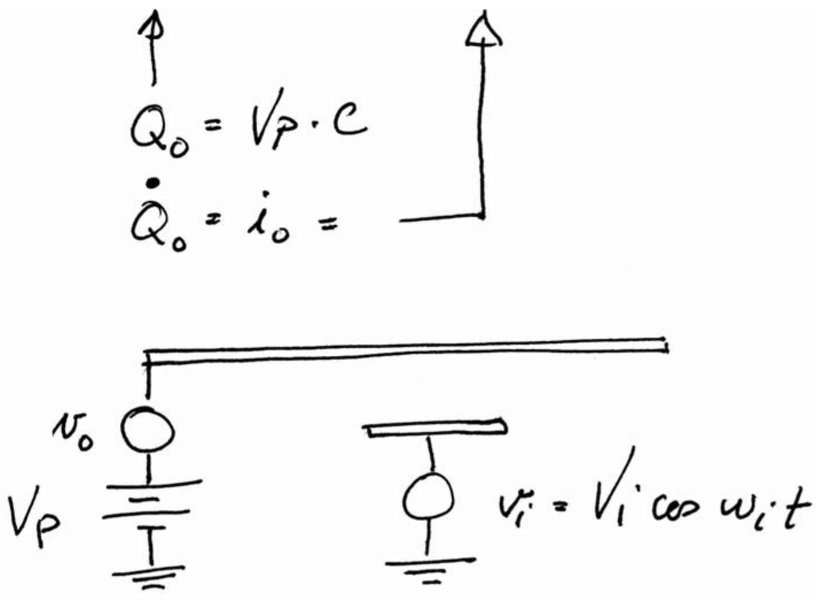
Prosedyre, forts

- Når bjelken beveger seg, dannes det en tidsvarierende kapasitans mellom elektroden og resonatoren
- H) Dette fører til en utgangsstrøm som er ”DC-biased” via Vp
  - dC/dx er her et ulineært ledd
  - dx/dt er hastigheten

When the beam moves, a time dependent capacitance between the electrode and resonator will be created, giving an output current:

H.

$$i_O = -V_P \frac{\partial C}{\partial x}\frac{\partial x}{\partial t} = \dot{Q}_O, \text{ where } Q_O = V_P C$$



Frekvenskarakteristikk

- Typiske parametre, Q, vakuum
  - Gir båndpassfilter, Q ~ 10,000
  - Egner seg for referanse oscillatorer og filtre med lave tap
- Q ~ noen hundre ved atmosfære-trykk

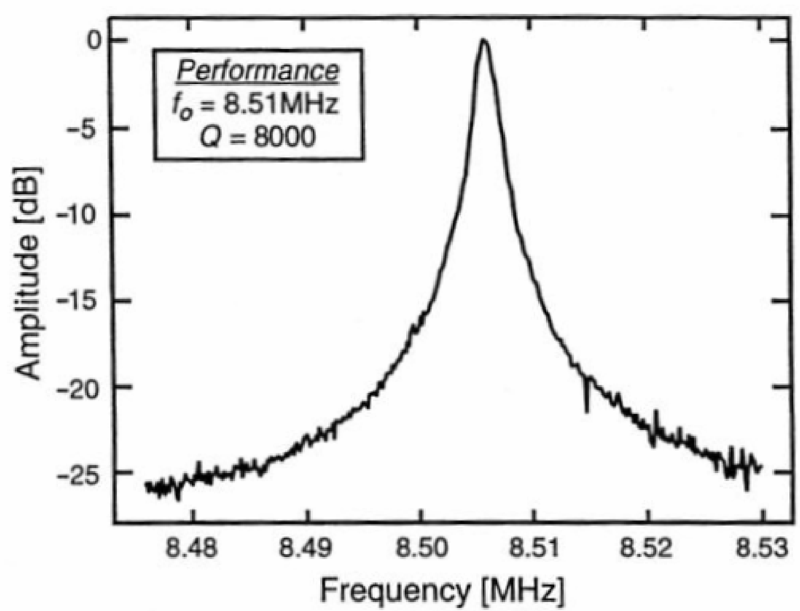


Figure 12.7. Frequency characteristic for an 8.5 MHz clamped–clamped beam polysilicon µmechanical resonator measured under 70 mtorr vacuum using a dc-bias voltage $V_P = 10$ V, a drive voltage of $v_i = 3$ mV, and a transresistance amplifier with a gain of 33 KΩ to yield an output voltage v_o. Amplitude $= v_o/v_i$. (From reference [18])

Prosedyre, forts.

- Overføring til mekanisk ekvivalentkrets:
  - ”mass-spring-damper”-krets
  - NB! Befinner oss fortsatt i mekanisk domene
- Bjelken beskrives ved ”lumped elements”
- Elementverdiene er avhengig av **HVOR** på bjelken en betrakter, - avhengig av y

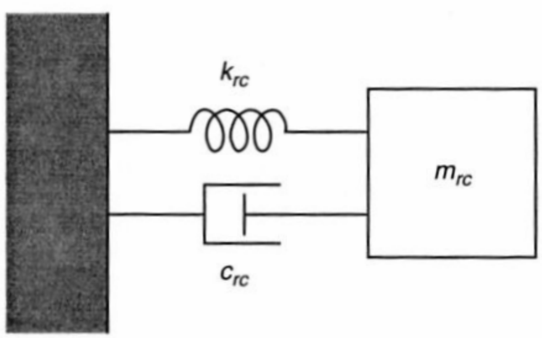


Figure 12.8. Lumped-parameter mechanical equivalent circuit for the micromechanical resonator of Figure 12.4.

I. Beregning av ”ekvivalent masse” som funksjon av y

Utfyllende fra R. A. Johnson: ”Mechanical Filters in Electronics”, Wiley, 1983

Forenklet utledning av utbøyningsligning

Form på ”fundamentalmoden”

Hvert punkt, y, har en gitt effektiv masse, en gitt hastighet
og en gitt fjærkonstant

Lavest ”masse” midt på, der hastigheten er høyest

$$m_r(y) = \frac{KE_{tot}}{\frac{1}{2}\left[v(y)\right]^2}$$

Den ekvivalente massen ved en lokasjon y

$KE_{tot} =$ peak kinetic energy of the system

$v(y) =$ velocity at location y

Flexural mode resonator: beam

$w =$ width, u = displacement in x – direction

$E =$ elastic modulus, $\rho =$ density

$$I = \frac{wt^3}{12} = \text{moment of inertia}$$

The beam equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{EI}{\rho A} \cdot \frac{\partial^4 u}{\partial y^4}, \text{ where } u = u_1 e^{j\omega t}$$

$$\Rightarrow \frac{\partial^4 u}{\partial y^4} = (\omega^2 \frac{\rho A}{EI}) u$$

Trial solution:

here: k is the ”wave number”

$$u(y) = A \cosh ky + B \sinh ky + C \cos ky + D \sin ky$$

A, B, C, D can be found from initial conditions

Mode shape for fundamental frequency, c - c beam:

$$u(y) = \xi(\cos ky - \cosh ky) + (\sin ky - \sinh ky)$$

(From ”Johnson”)

Velocity in y - direction (along the beam)

$$v(y) = \dot{u}(y) = \frac{\partial}{\partial t}(u_1 e^{j\omega t}) = j\omega \cdot u(y)$$

Equivalent mass :

$$M_{eq}(y) = \frac{KE_{tot}}{\frac{1}{2}v^2(y)} = \frac{\frac{1}{2}\rho A \int_0^l v^2(y')dy'}{\frac{1}{2}v^2(y)}$$

$$M_{eq}(y) = \frac{\frac{1}{2}\rho A(-\omega^2)\int_0^l u^2(y')dy'}{\frac{1}{2}(-\omega^2)u^2(y)} = \frac{\rho wt \int_0^l [X_{\mathrm{mod}e}(y')]^2 dy'}{[X_{\mathrm{mod}e}(y)]^2}$$

Xmode is the **”shape” of the fundamental mode**
= displacement as a function of y

$$X_{\text{mode}} = \text{shape of the fundmental mode}$$
$$= \text{displacement as a function of y}$$
$$X_{\text{mode}}(y) = \xi(\cos\beta y - \cosh\beta y) + (\sin\beta y - \sinh\beta y)$$
$$\beta = 4.730 / L_r, \text{"wave number"}$$
$$\xi = -1.01781$$

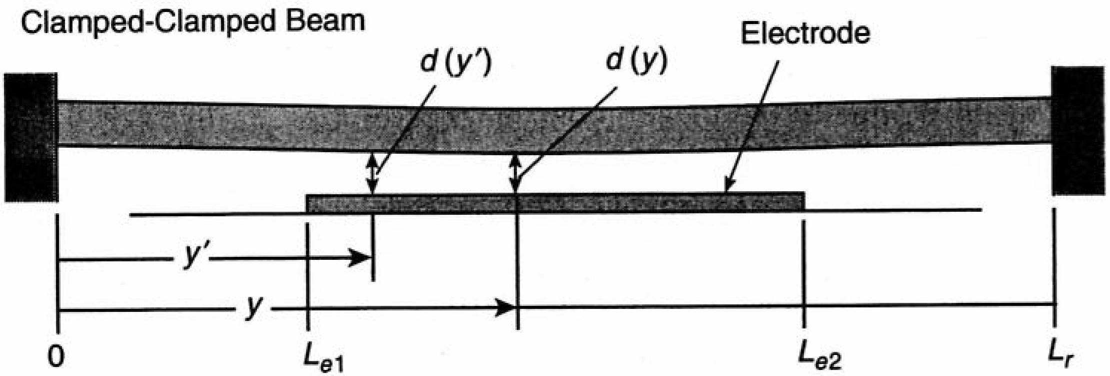


Figure 12.9. Resonator cross-sectional schematic for frequency-pulling and impedance analysis.

Prosedyre, forts.

- J) Når en har beregnet ekvivalent masse som funksjon av (y), kan en beregne ekvivalent fjærstivhet k_r (y) og dempefaktor c_r (y)
  - k_r = ”ekvivalent”, dvs. med innvirkning både fra mekaniske og elektriske effekter

Resonance frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}},\ \ \omega_0^{\ 2} = \frac{k_r}{m_r}$$

Equivalent spring stiffness

J.

$k_r(y) = \omega_0^{\ 2} \cdot m_r(y)$, where $m_r(y)$ is the equivalent mass

The damping factor $c_r(y)$:

$$s^2 + \frac{c}{m}s + \frac{k}{m} = s^2 + \frac{\omega_0}{Q}s + \omega_0^{\ 2}$$

$$c = m\frac{\omega_0}{Q} = \frac{m\sqrt{k/m}}{Q} = \frac{\sqrt{km}}{Q}$$

By just looking at the mechanical contribution:
A certain frequency, ω_nom, and a corresponding Q-factor, Q_nom are obtained:

The mechanical spring constant : $k_m(y)$

gives the nominal values : $\omega_{nom},\ Q_{nom}$

The damping is only dependent on the mechanical factors :

$$c_r(y) = b = \frac{\sqrt{k_m(y)\cdot m_r(y)}}{Q_{nom}},\ \text{where}\ k_m(y) = \omega_{nom}{}^2 \cdot m_r(y)$$

K.

$$c_r(y) = \frac{\omega_{nom}\cdot m_r(y)}{Q_{nom}} = \frac{k_m(y)}{\omega_{nom} Q_{nom}}$$

Q_nom is the Q-factor of the resonator without the effect of the applied voltage
$k_m(y)$ is the mechanical stiffness without being influenced by the applied voltage and electrodes

Tunbar elektrisk fjærstivhet

- Fjærstivheten kan tunes ved Vp
  – Resultanten er avhengig av forholdet mellom k_e og k_m

- L) Beregn hvordan k_e avhenger av lokasjon y

The resonance frequency can be tuned by Vp

The electrically tunable spring constant, k_e, is subtracted from the mechanical one

The electrostatic beam - softening will change the spring stiffness

The resulting spring constant will be decreased :

$k_r = k_m - k_e$, mechanical minus electrical

The resonance frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_m - k_r}{m_r}} = \frac{1}{2\pi}\sqrt{\frac{k_m}{m_r}(1 - \frac{k_e/m_r}{k_m/m_r})} = \frac{1}{2\pi}\sqrt{\frac{k_m}{m_r}}(1 - \langle\frac{k_e}{k_m}\rangle)^{1/2}$$

$$f_0 = 1.03\chi\sqrt{\frac{E}{\rho}}\cdot\frac{h}{L^2}(1 - \langle\frac{k_e}{k_m}\rangle)^{1/2}$$

The relation is changed along the y-direction and has to be ”summed” in an integral

k_e is dependent on the capacitance $C(y')$ which is dependent on the gap $d(y')$ caused by V_P

By equating the potential energy to the work :

$$U = \frac{1}{2}k_e \cdot d^2 = \frac{1}{2}CV_P{}^2 = \frac{1}{2}V_P{}^2\frac{\varepsilon_0 A}{d}$$

$$k_e = V_P{}^2\frac{C}{d^2} = V_P{}^2\frac{\varepsilon_0 A}{d^3}$$

L. A contribution to the total spring stiffness from an element at the location y' and with a small electrode width dy'

$$dk_e(y') = V_P{}^2\frac{\varepsilon_0 W_r dy'}{[d(y')]^3}$$

The local spring stiffness is dependent on the gap!

(d is the displacement from an equilibrium position)

The gap, d(y), has to be computed:

A force of F will give a displacement, d, from the equilibrium position where $V_P = 0$:

$$F = \frac{1}{2} V_P{}^2 \frac{\varepsilon_0 A}{d^2} = k \cdot \text{"displacement"}$$

$$d(y) = d_0 - \frac{1}{2} V_P{}^2 \varepsilon_0 W_r \int_{L_{e1}}^{L_{e2}} \frac{1}{k_m(y')\left[d(y')\right]^2} \cdot \frac{X_{sh}(y)}{X_{sh}(y')} dy'$$

The equation must be solved iteratively

Static bending shape due to the distributed DC force

When $d(y)$ has been found, then $dk_e(y')$ can be computed:

$$dk_e(y') = V_P{}^2 \frac{\varepsilon_0 W_r dy'}{\left[d(y')\right]^3}$$

Then

$$\langle \frac{k_e}{k_m} \rangle = g(d, V_P) = \int_{L_{e1}}^{L_{e2}} \frac{dk_e(y')}{k_m(y')} dy'$$

Forenklet betraktning (De Los Santos):

Regner bjelken flat over elektroden

Potensiell energi pga. påsatt spenning

$$U_1 = \frac{1}{2} C V_P^{\,2}$$

Det arbeidet som utføres ved å forflytte bjelken en avstand g, MOT kraften som skyldes den elektriske fjærstivheten k_e
(Forutsetter at fjærstivheten er konstant i hvert punkt, y´)

$$U_2 = \int_0^g k_e \cdot x \cdot dx = \frac{1}{2} k_e \cdot g^2$$

Energiene kan settes lik hverandre

$$\frac{1}{2} k_e \cdot g^2 = \frac{1}{2} C \cdot V_P^{\,2}$$

Forenklet uttrykk for elektrisk fjærstivhet

$$k_e = \frac{C \cdot V_P^{\,2}}{g^2}$$

Forenklet uttrykk for frekvensen:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}} = \frac{1}{2\pi}\sqrt{\frac{k_m}{m_r}\left(1 - \frac{k_e}{k_m}\right)}$$

$$= \frac{1}{2\pi}\sqrt{\frac{k_m}{m_r}} \cdot \sqrt{1 - \frac{k_e}{k_m}} = f_{nom} \cdot \sqrt{1 - \frac{C \cdot V_P{}^2}{k_m \cdot g^2}}$$

Innsatt for C:

$$C = \varepsilon_0 \cdot \frac{A}{g}$$

$$f = f_{nom} \cdot \sqrt{1 - \frac{\varepsilon_0 \cdot A \cdot V_P{}^2}{k_m \cdot g^3}}$$

Dette harmonerer med de tidligere detaljerte beregningene

$$k_e = \varepsilon_0 \cdot \frac{A \cdot V_P{}^2}{g^3}$$

$$dk_e(y') = V_P{}^2 \cdot \frac{\varepsilon_0 \cdot W_r \cdot dy'}{[d(y')]^3}$$

Differential electrical spring stiffness in location y´ and with an electrode width dy´

Beam-softening

- Resonansfrekvensen faller med en gitt faktor

$$\sqrt{1 - C_0 \cdot V_P{}^2 / (k_m \cdot g^2)}$$

  - → **resonansfrekvensen kan tunes elektrisk!**

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\kappa\sqrt{\frac{E}{\rho}\frac{h}{L_r^2}}\left[1 - g(V_P)\right]^{1/2}, \qquad (12.2)$$

Småsignal-ekvivalent

- En elektrisk ekvivalentkrets trengs for å modellere og simulere impedansoppførselen til denne mikro-mekaniske resonatoren i en **felles** elektromekanisk krets

$$L_x = \frac{m_{re}}{\eta_e^2}, \qquad C_x = \frac{\eta_e^2}{k_{re}}, \qquad R_x = \frac{\sqrt{k_{re} m_{re}}}{Q\eta_e^2} = \frac{C_{re}}{\eta_e^2}, \qquad (12.17)$$

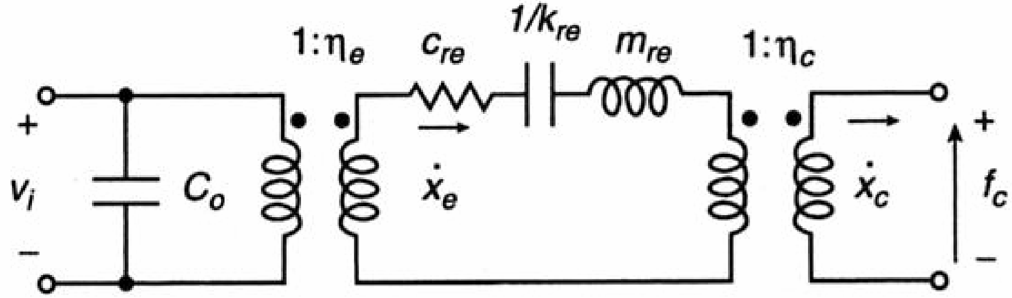


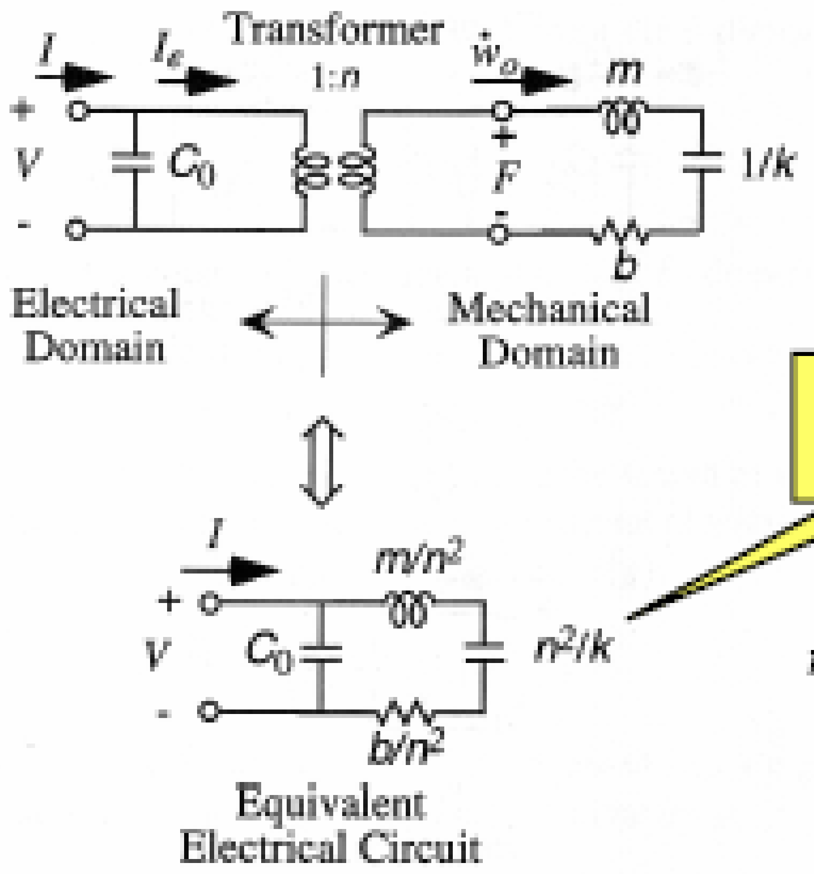
Figure 12.10. Equivalent circuit for a μmechanical resonator with both electrical (voltage v_i) and mechanical (force f_c) inputs and outputs.

Koblingskoeffisient

- Hvis en ser inn i kretsen fra venstre
- En ser en en transformert LCR-krets med nye elementverdier gitt av (12.17)
  - Elektromekanisk koblingskoeffisient = ”transformer turns ratio”

- Koblingskoeffisienten er utmeislet i notater fra UCLA
  - Tatt i forbindelse med 2-port lateral comb-drive actuator (L10)

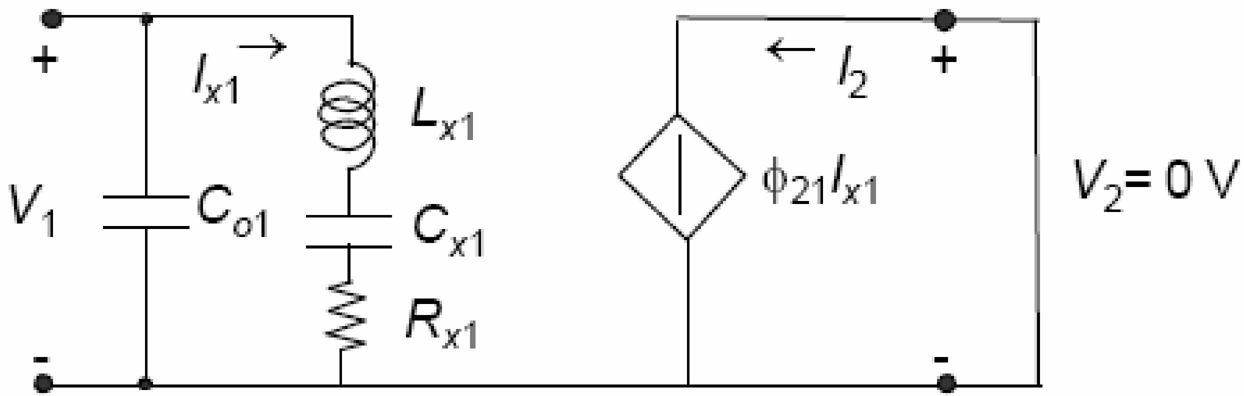
Small Signal Equivalent Circuit of Microresonators



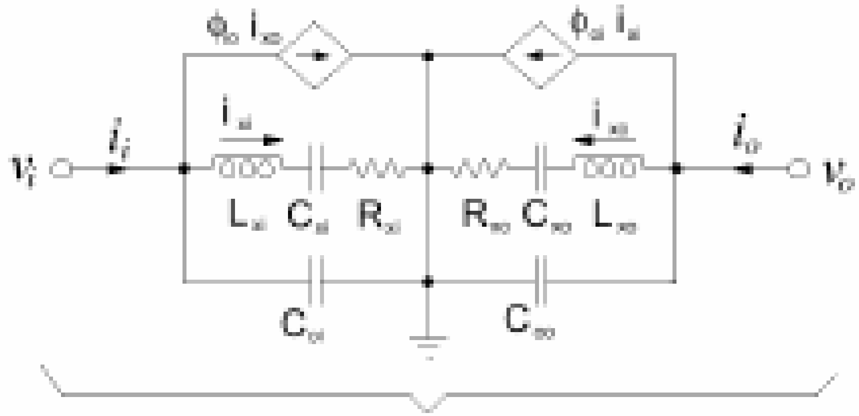
Unit of n^2/k is Farad

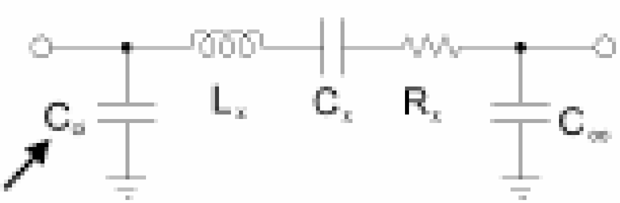
$$n = V_{dc} \frac{\partial C}{\partial x}$$

Two-Port Equivalent Circuit ($v_2 = 0$)



Equivalent Circuit of 2-Port Resonator
(in Electrical Domain)





$C_x = 0.5$ fF

$L_x = 200$ kH

$R_x = 500$ kΩ

$C_{o1}, C_{o2} = 15$ fF

Fixed electrical Capacitance Between fixed comb And ground plane

$$C_{xn} = \frac{\eta_n^2}{k} \qquad R_{xn} = \frac{\sqrt{km}}{Q\eta_n^2} \qquad \eta_n = V_{Pn}\frac{\partial C_n}{\partial x}$$

$$L_{xn} = \frac{m}{\eta_n^2} \qquad \phi_{mn} = \frac{\eta_m}{\eta_n}$$

C. T.-C. Nguyen, "Micromechanical resonators for oscillators and filters," *Proceedings* IEEE International Ultrasonics Symposium, Seattle, WA, pp. 489-498, Nov. 7-10, 1995

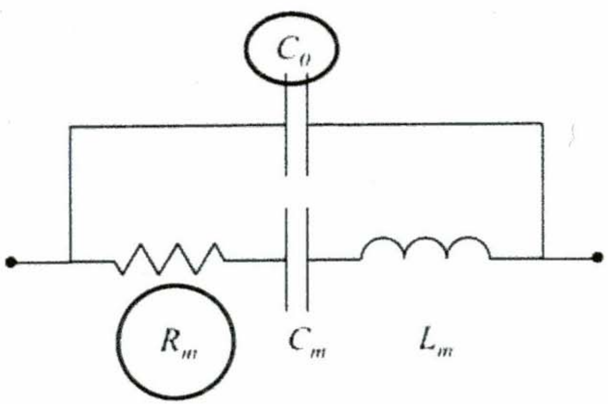
Diskusjon:

FSRM

Resonator equivalent circuit

Two types of currents possible:

- **from resonator motion** (should dominate!)
- from electrodes and resonator acting as pure electrical structure (from feedthrough capacitance)



Admittance at resonance is

$$Y_{in} = \frac{1}{R_m} + j\omega_o C_o$$

where we want to minimize the motional resistance, R_m :

$$R_m = \frac{\sqrt{k * m}}{Q\eta^2} \qquad \eta = V_{DC}\frac{dC}{dg}$$

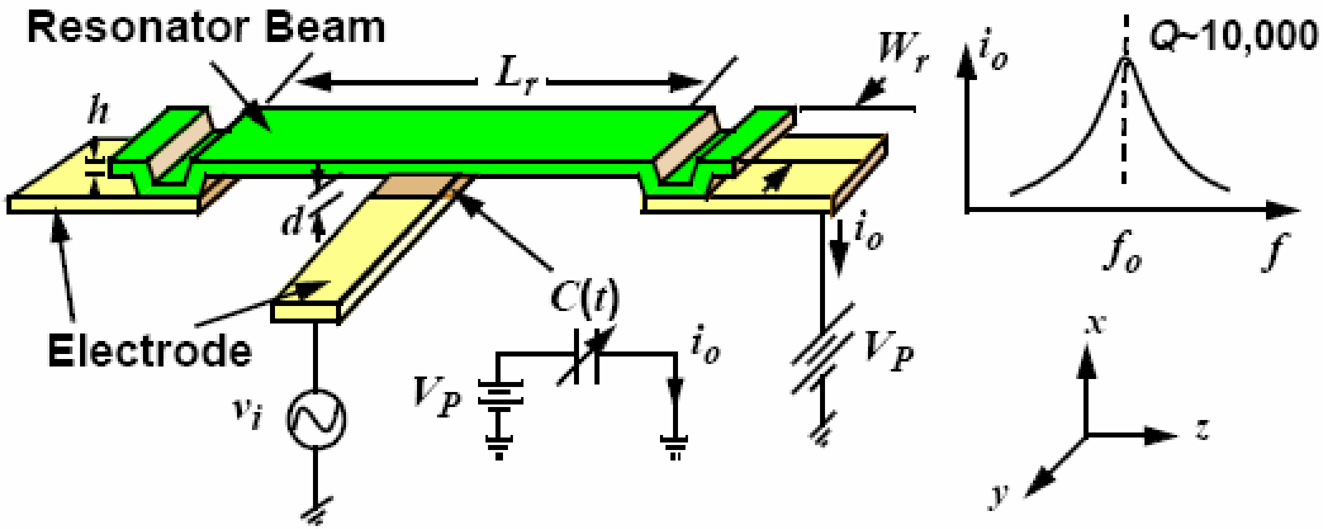
- Need:
  - ➢High Q
  - ➢High coupling (high voltage or small gap)
  - ➢Low mass
  - ➢Low stiffness (!)

95

Ionescu, EPFL

Vertically-Driven Micromechanical Resonator

- To date, most used design to achieve VHF frequencies



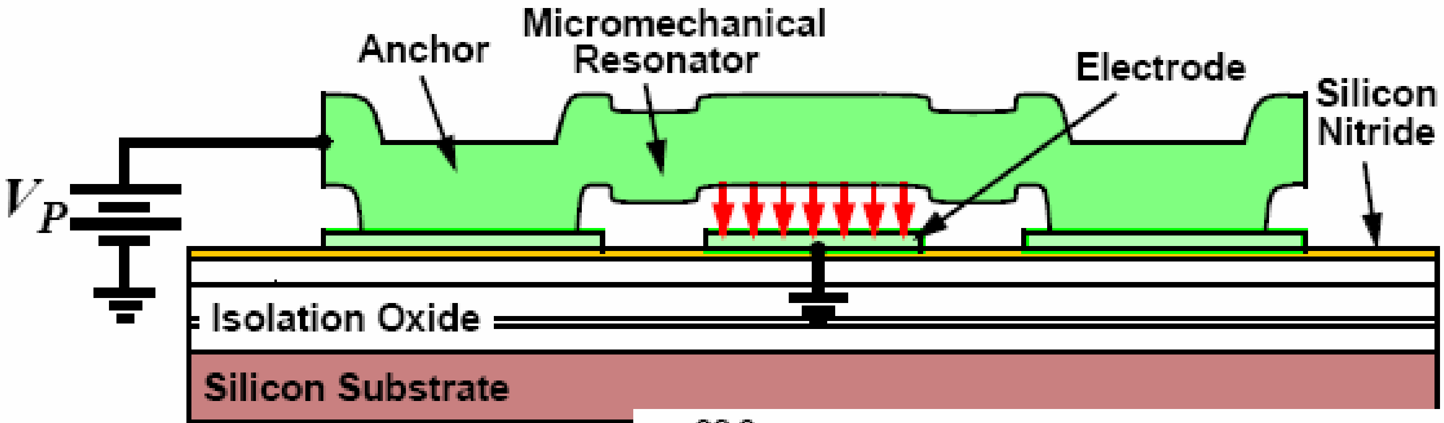
$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L_r^2}$$

(e.g. m_r=10^{-13}kg)

E = Youngs Modulus
ρ=density

- Smaller mass ➠ higher frequency range and lower series R_x

Voltage-Controllable Center Frequency



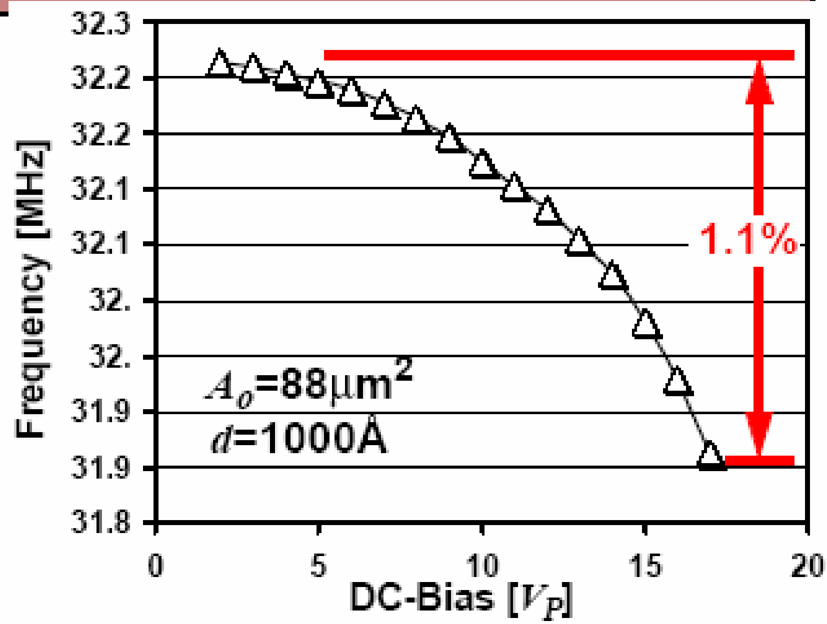
- Quadrature force ➠ voltage-controllable electrical stiffness:

$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

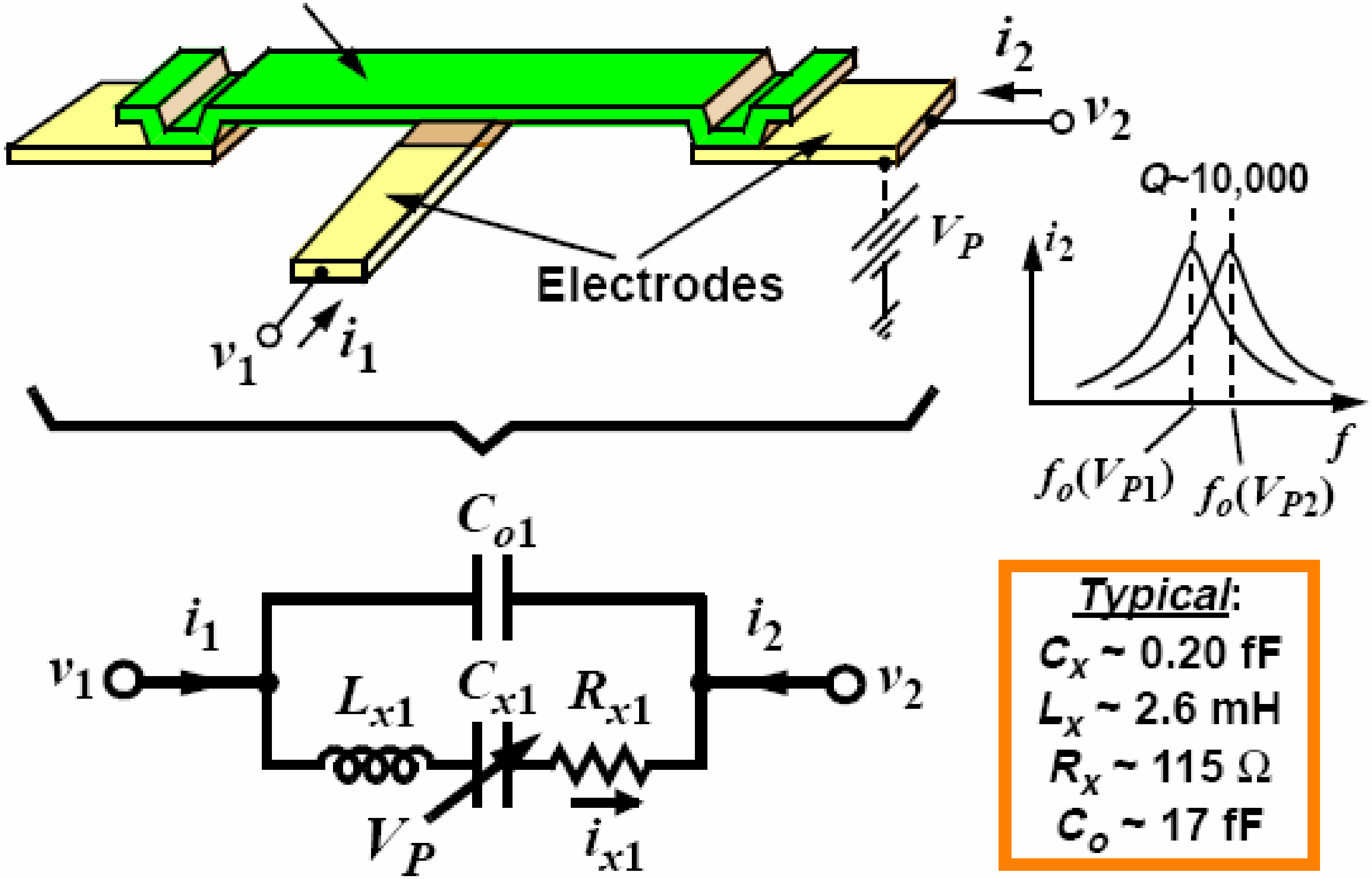
Electrode Overlap Area

Finger Gap

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$



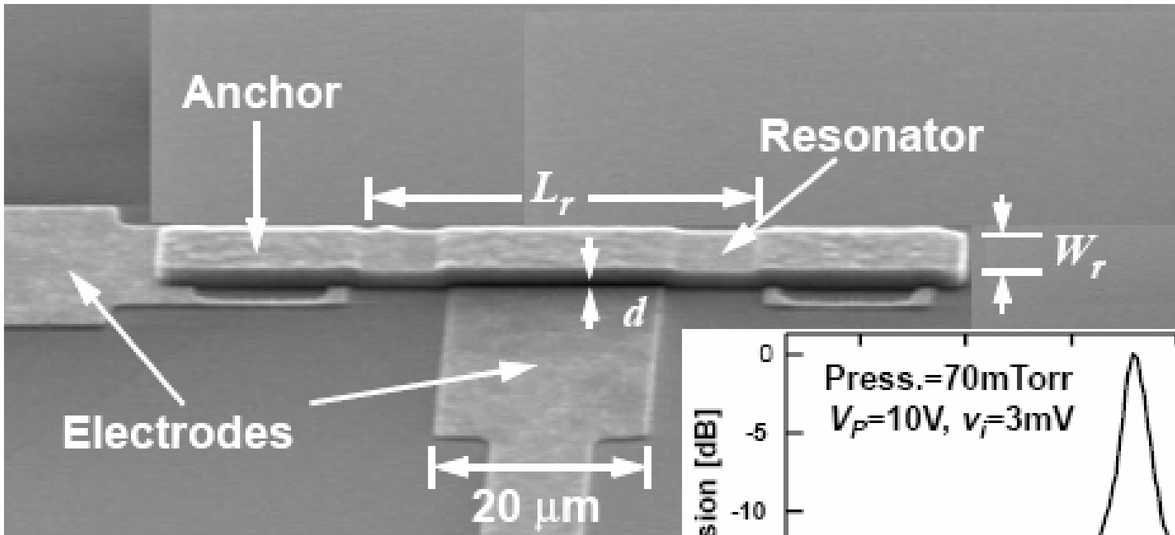
Micromechanical Resonator Equivalent Circuit



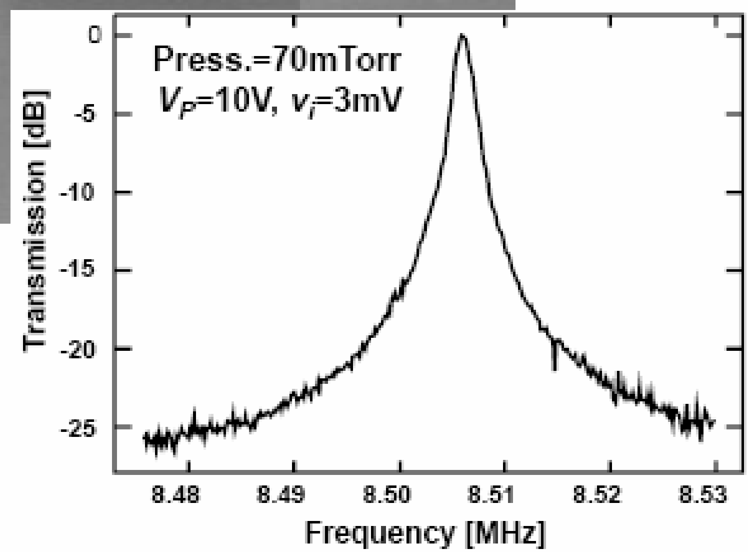
Typical:
- C_x ~ 0.20 fF
- L_x ~ 2.6 mH
- R_x ~ 115 Ω
- C_o ~ 17 fF

Fabricated HF μMechanical Resonator

- Surface-micromachined, $POCl_3$-doped polycrystalline silicon



L_r=40.8 μm, W_r=8 μm, h=2 μm, d=0.1μm



- Extracted Q = 8,000 (vacuum)
- Freq. influenced by dc-bias and anchor effects

Tap, c-c-beam

- Stivheten til en gitt resonator-bjelke øker i takt med økende resonans-frekvens
  - Mer energi pr sykel går inn i substratet via ankere

- c-c-beam har tap gjennom ankerfestene
  - → Q-faktoren går ned når frekvensen øker
  - c-c-beam er ikke den beste strukturen ved de høyeste frekvensområdene!
  - Eks. Q = 8,000 ved 10 MHz, Q = 300 ved 70 MHz
- c-c beam kan brukes til referanse-oscillator eller HF/VHF filter/mikser

- **”free-free beam” kan brukes for å minske tapet gjennom ankerene i substratet!**

free-free-beam

- Gunstig når det gjelder tap til substratet gjennom ankerfestene
- f-f-beam er opphengt ved 4 support-bjelker i bredde-retningen
  - Torsjons-oppheng
  - Oppheng festet ved nodepunktene for ”flexural mode”
- Support-dimensjonene tilsvarer en kvart-bølgelengde av f-f-bjelkens resonans-frekvens
    - Impedansen som bjelken erfarer fra support nulles ut
    - Bjelken blir fri til å vibrere som om den ikke hadde noe oppheng
- Høyere Q kan oppnås
  - Eks. Q= 20,000 ved 10 – 200 MHz
  - Anvendes i referanse-oscillatorer, HF/VHF-filtre/miksere

free-free beam

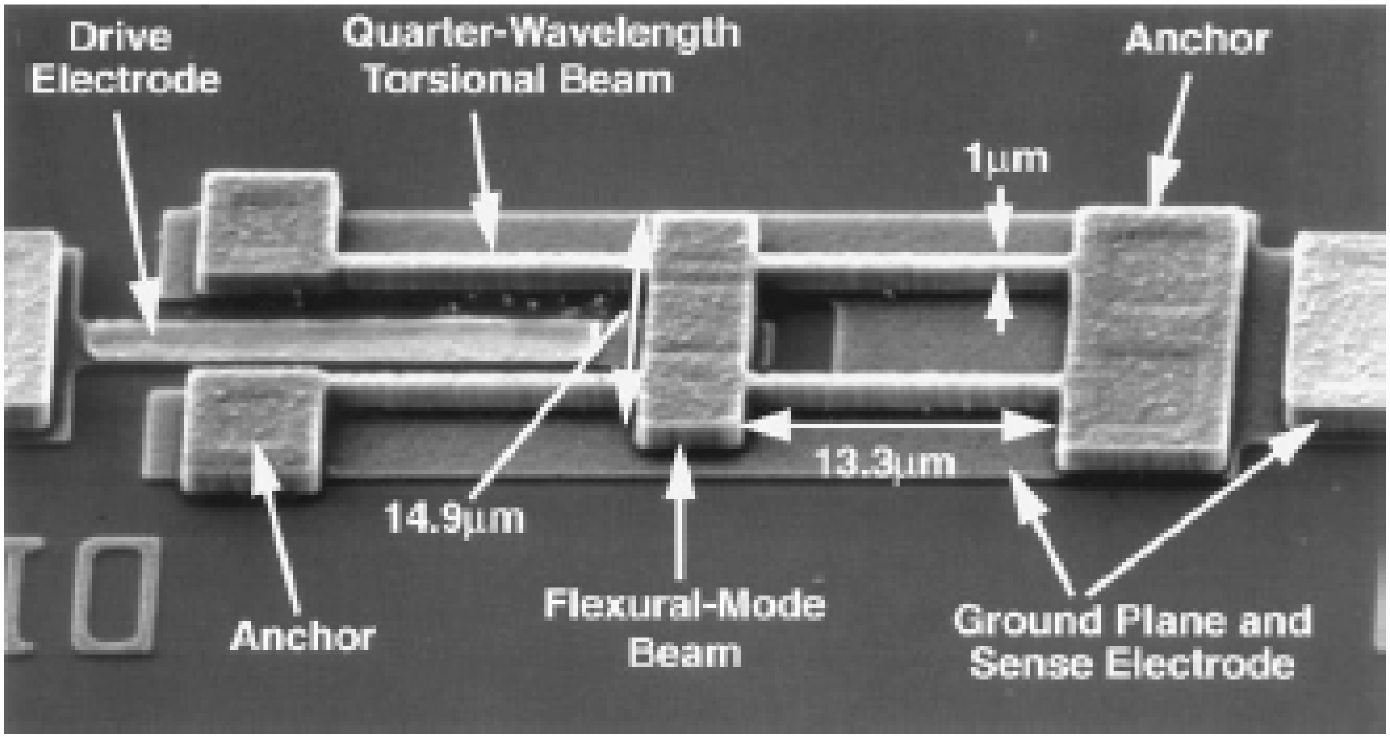
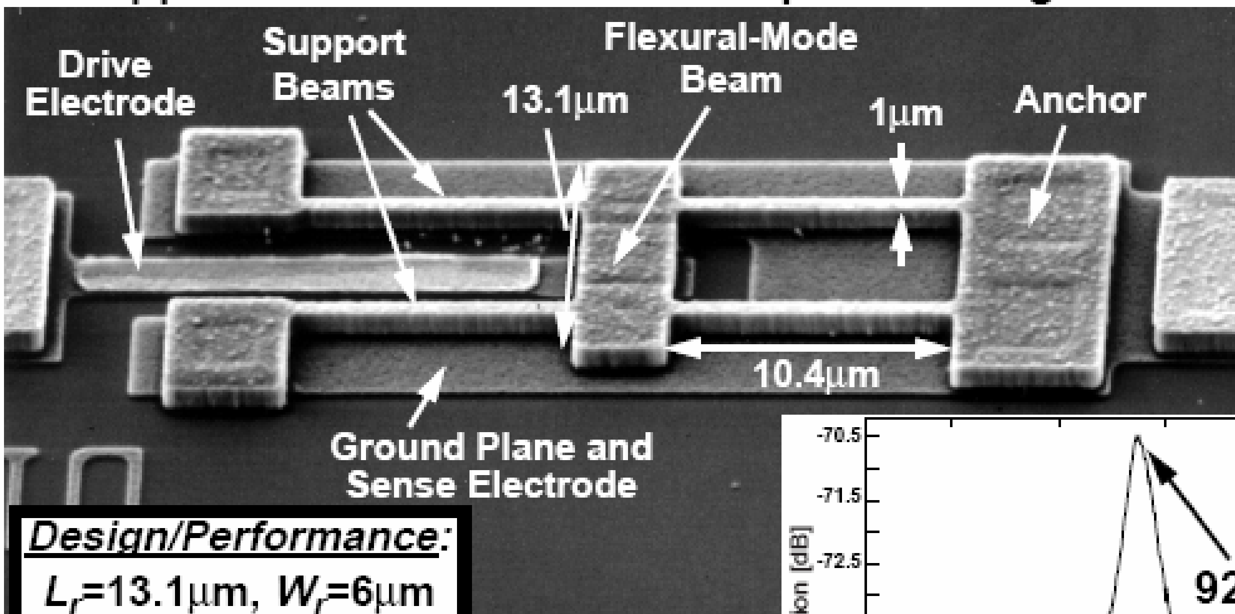


Fig. 29. SEM of free–free beam virtually levitated micromechanical resonator with relevant dimensions for $f_o = 71$ MHz.

Nguyen, 1999

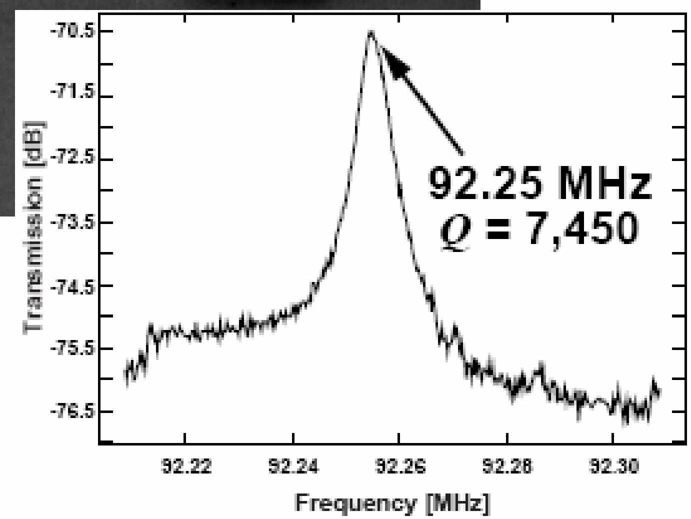
92 MHz Free-Free Beam μResonator

- Free-free beam μmechanical resonator with non-intrusive supports ➠ reduce anchor dissipation ➠ higher *Q*

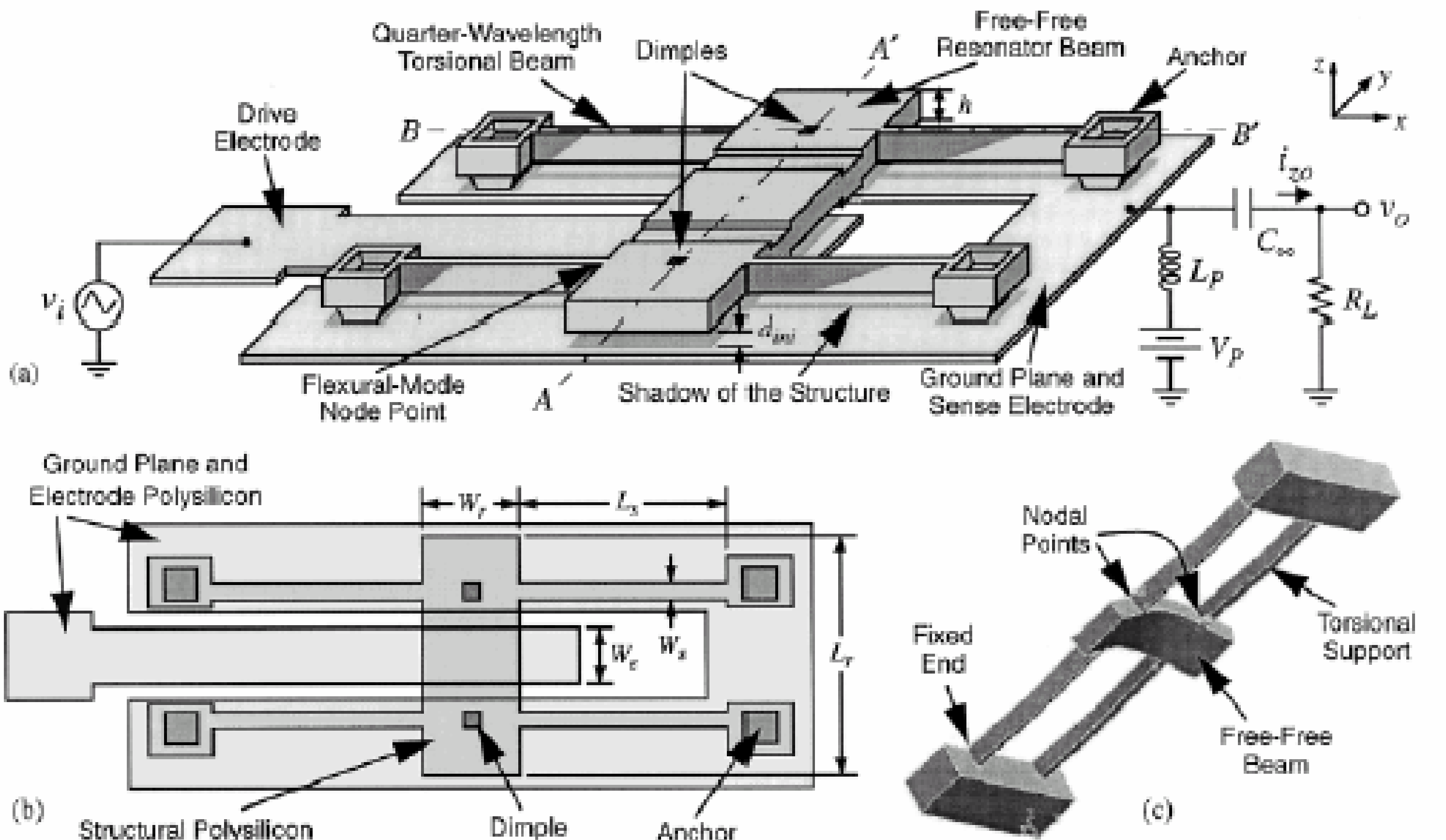


Design/Performance:
L_r=13.1μm, W_r=6μm
h=2μm, d=1000Å
V_P=28V, W_e=2.8μm
f_o~92.25MHz
Q~7,450 @ 10mTorr

[Wang, Yu, Nguyen 1998]



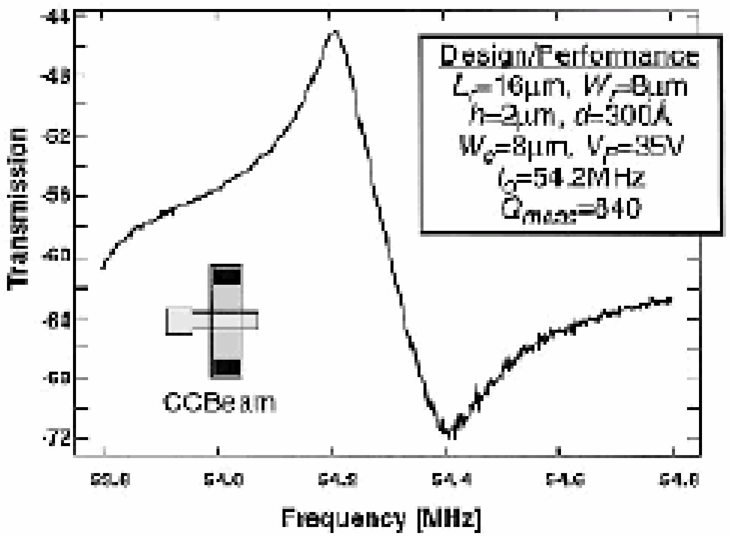
VHF Free-Free Beam High-Q Micromechanical Resonator

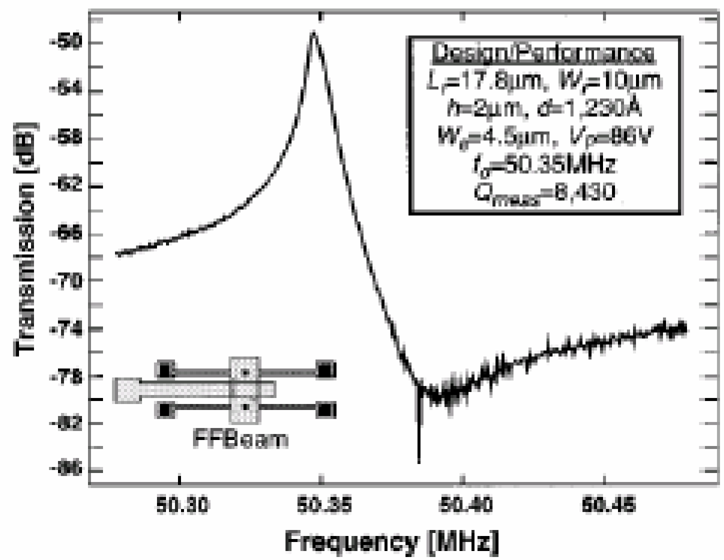


(determined the gap)

J. MEMS, Vol. 9, No. 3, 2000, C. T. –C. Nguyen, et al.

Comparison of Frequency Characteristics





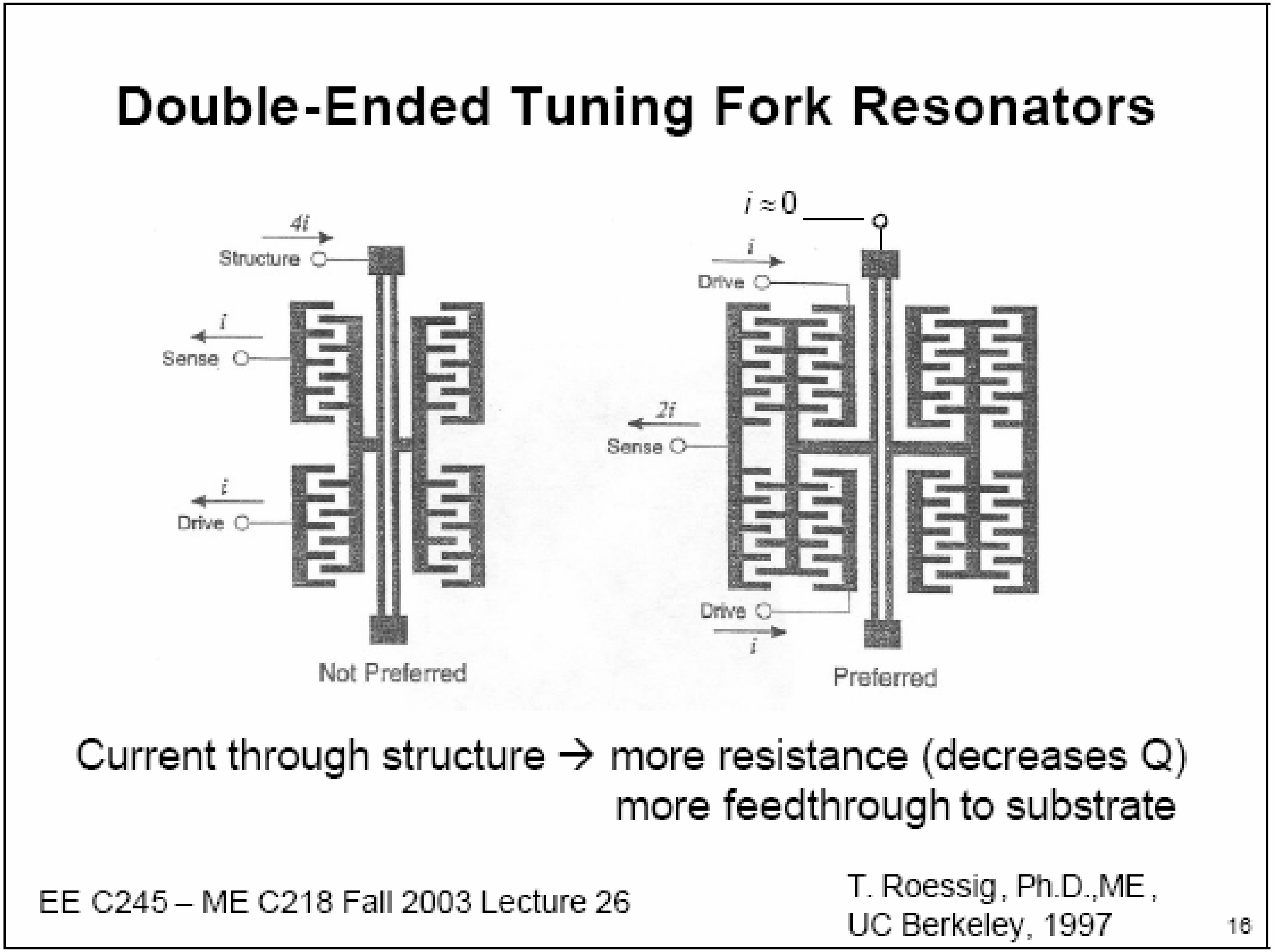
Clamped-clamped beam

- L_r=16 µm, d=0.03 µm
- V_p=35 V, f_0=54.2 MHz
- Q=840

Free-free beam

- L_r=17.8 µm, d=0.12 µm
- V_p=86 V, f_0=50.35 MHz
- Q=8,430

Andre typer resonatorer



"Stemmegaffel" → balansert!

Scaling of Lateral Micromechanical Resonators

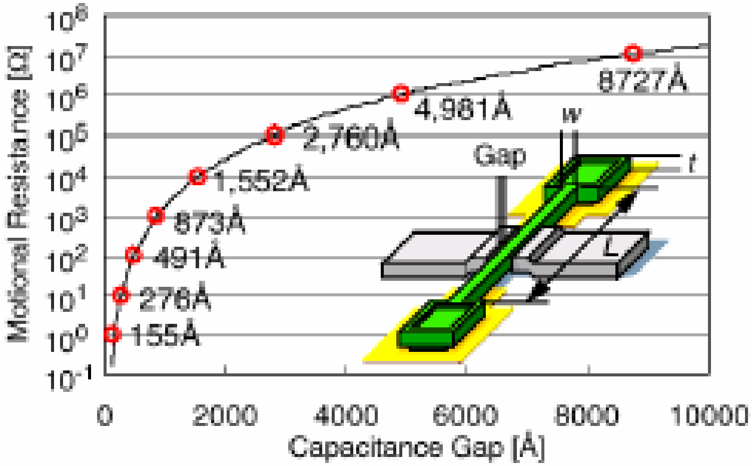


Fig. 1: Simulated plot of motional resistance versus electrode-to-resonator gap for a 40μm-long, 2μm-wide, 3μm-thick, lateral clamped-clamped beam μmechanical resonator.

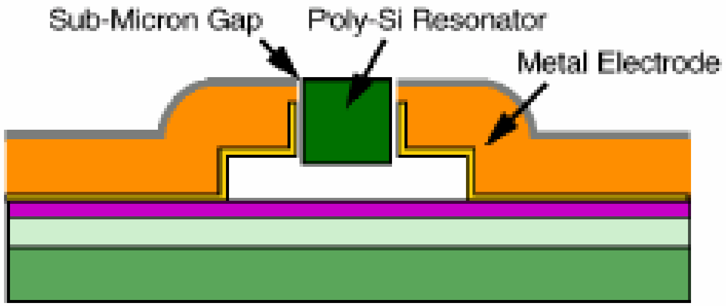
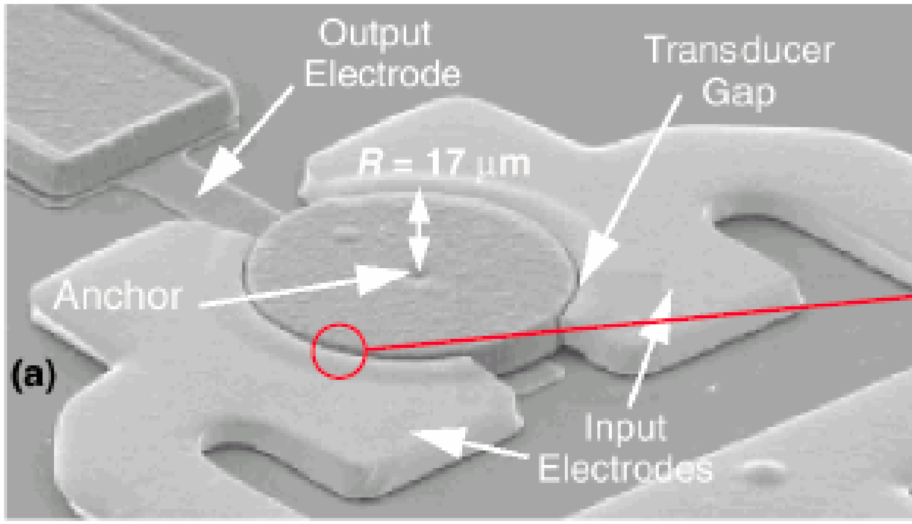


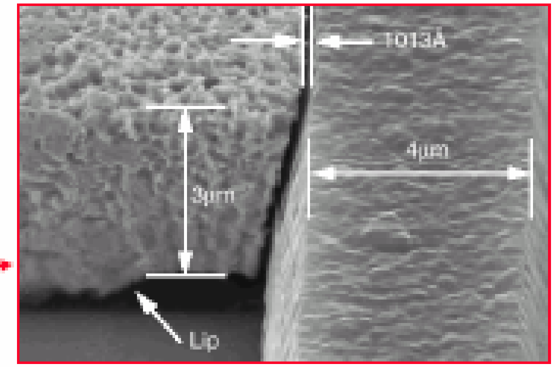
Fig. 2: Cross-section of the described sub-μm electrode-to-resonator gap process for lateral μstructures with metal electrodes.

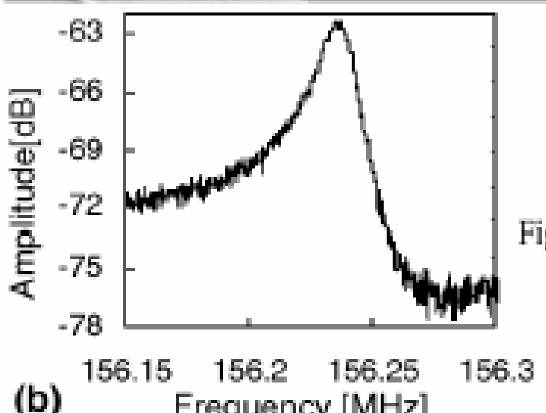
- Advantages of lateral resonator
  - Wider variety of resonant modes
  - Balanced resonators (push-pull)
  - More design flexibility
- As frequency scales up
  - Resonator size shrinks
  - Capacitive transducer gaps must also shrink (to sub-100 nm for VHF)
  - High aspect ratio structures
- Combine Poly-Si (high-Q structural materials) with metal electrode (high conductivity)
  - Self-aligned process

Hsu, Clark, Nguyen, "A sub-micron capacitive gap process for multiple-metal-electrode lateral micromechanical resonators," MEMS 2001, p. 349

Radial Contour-Mode Disk μ-mechanical Resonator







Data:
R=17μm, h=2μm
d=1,000Å, V_P=35V
f_o=156.23MHz, Q=9,400

Fig. 5: SEM and measured frequency characteristic for a 156.23 MHz contour-mode disk μmechanical resonator fabricated via the process of Fig. 3.

- Radial contour mode allows high resonant frequency without requiring sub-micron structures
- Place anchor at disk center – nodal point of contour mode
  ➔ Reduce mechanic loss and increase Q

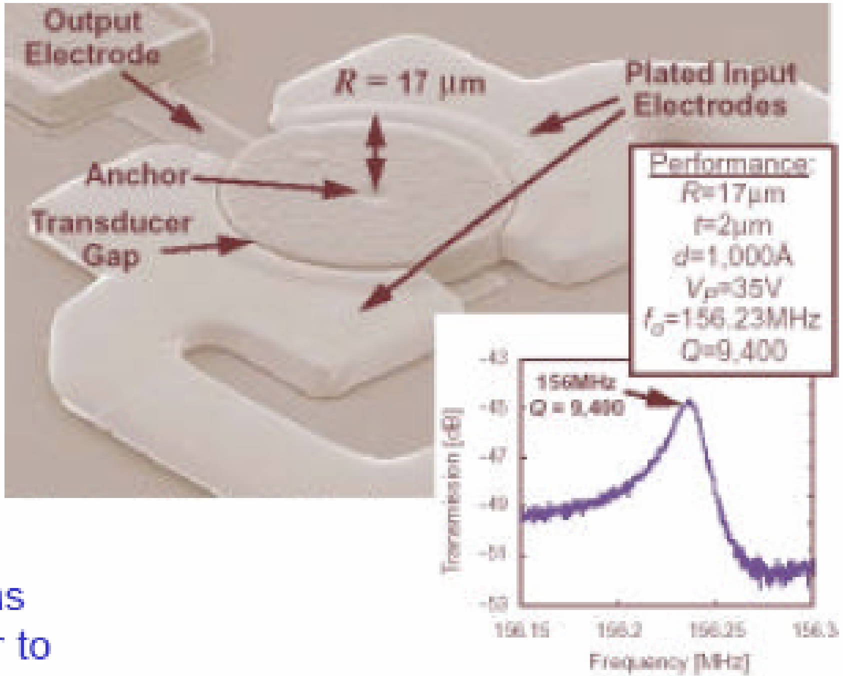
Hsu, Clark, Nguyen, "A sub-micron capacitive gap process for multiple-metal-electrode lateral micromechanical resonators," MEMS 2001, p. 349

Disk resonatorer

- Fordeler av disker framfor bjelker
  - Redusert luft-demping
    - Vakuum trengs ikke for måling av Q-faktor
  - Høyere stivhet
    - Frekvensen er høyere for gitte dimensjoner
  - Større volum
    - Høyere Q fordi mer energi er lagret
    - Mindre problemer med termisk støy
- Periferien av disken kan ha ulike bevegelsesmønstre
  - Radial, wine-glass

Increasing the Resonant Frequency

option 2. spring rate → ∞



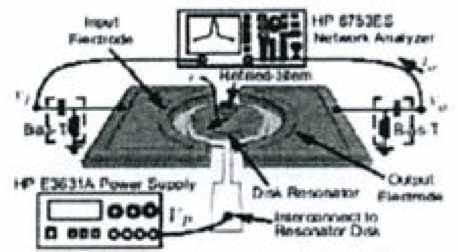
Clark Nguyen, Michigan

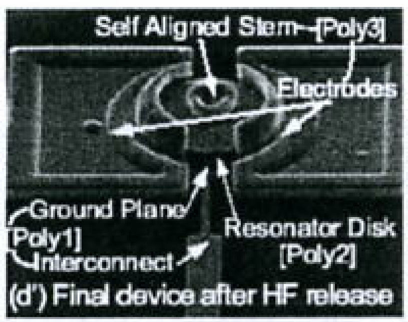
Motivation: keep mass as large as possible in order to improve precision of fab, power handling

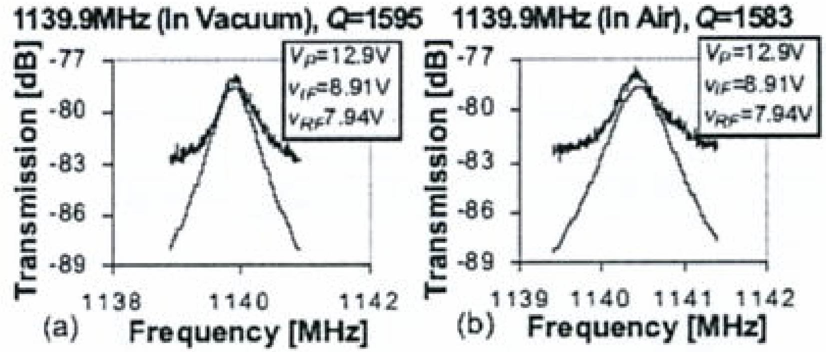
IEEE IEDM 2000.

FSRM

Bulk contour-mode resonators







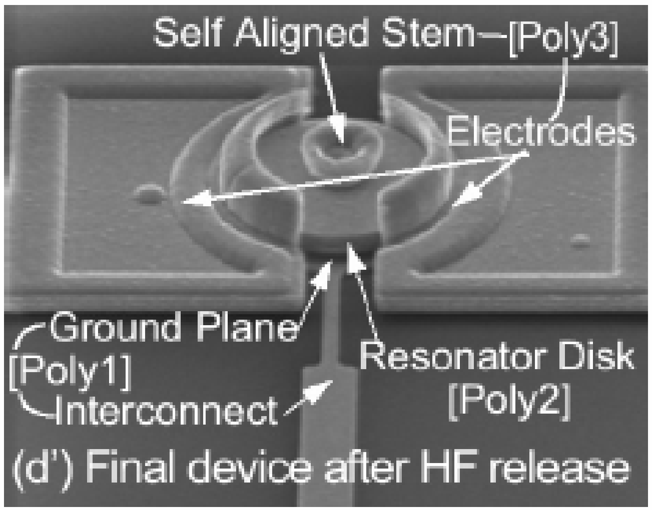
- > **1GHz** resonance frequency demonstrated
- **Q > 1'500** in both vaccum and air
- **Tcoeff ~-15ppm/°C**

J. Wang et al, Transducers 2003.

- **Bulk acoustic mode resonators / contour-mode disk resonators**
- **Frequency range: tens of kHz to GHz**
- **Quality factors > 10'000 for single crystal silicon demonstrated**
- **Further developments: process with nano-gaps → GHz frequency**

100

1.14 GHz Poly-Si Disk Resonator



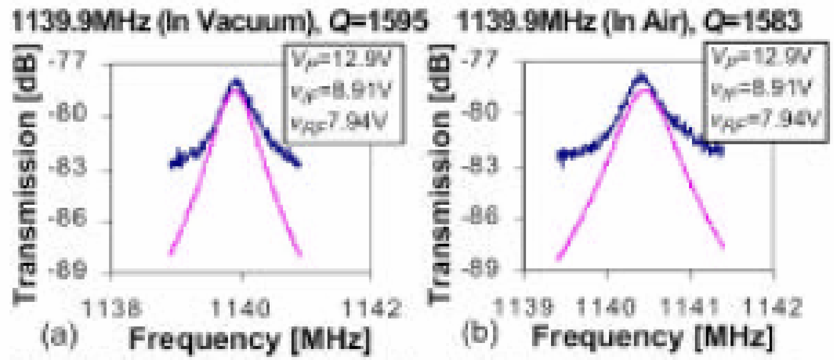


Fig. 7: Measured (dark) and predicted (light) frequency characteristics for a 1.14-GHz, 3rd mode, 20μm-diameter disk resonator measured in (a) vacuum and (b) in air, using a mixing measurement setup.

* Note *Q* in vacuum and in air is the same: little energy loss to ambient; however, energy loss through anchor ("stem") is significant

* EAM-like technique is used to extract the motional current.

EAM = Electromechanical Amplitude Modulation (sinus også på "shuttle")

Begrensninger i mikromekaniske resonatorer

- Frekvens-begrensninger
  - Redusere m for å oppnå høyere frekvens
  - Vil gi fluktuasjoner i frekvensen
    - ”mass loading”: utveksling av molekyler mot omgivelsene
    - Luft-gass-molekyler utøver Brownske bevegelser (kraft)
- Energi-begrensninger
  - Q avhenger av energitap fra demping
    - Viskøs demping
    - Vertikal bevegelse: squeezed-film damping
    - Horisontal bevegelse: Stokes- eller Couette-type demping

Begrensninger, forts.

- Temperaturavhengighet
  - Resonansfrekvensen endres pga. temperaturøkning og aldring
  - Økt temperatur fører til redusert frekvens
    - Analog eller digital kompensasjon (feedback)
    - Mekanisk kompensasjon
      - Benytte strukturer med deler som har både kompressivt og tensilt stress: motvirkende effekter →

Temperatur-kompensasjon

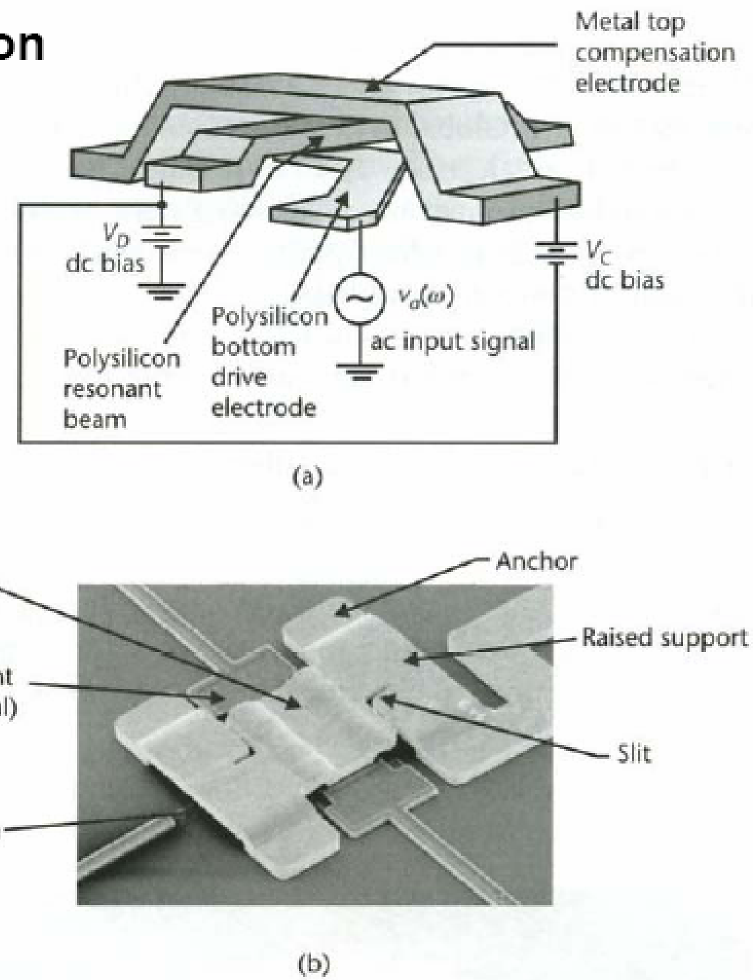


Figure 7.11 Illustration of the compensation scheme to reduce sensitivity in a resonant structure to temperature. A voltage applied to a top metal electrode modifies through electrostatic attraction the effective spring constant of the resonant beam. Temperature changes cause the metal electrode to move relative to the polysilicon resonant beam, thus changing the gap between the two layers. This reduces the electrically induced spring constant opposing the mechanical spring while the mechanical spring constant itself is falling, resulting in their combination varying much less with temperature. (a) Perspective view of the structure [23], and (b) scanning electron micrograph of the device. (*Courtesy of:* Discera, Inc., of Ann Arbor, Michigan.)

Temperatur-kompensasjon, forts.

- Topp-elektroden **reduserer** effektiv fjærkonstant ved at Vc gir en elektrostatisk tiltrekning

- Topp-elektroden hever seg fysisk pga. temperaturøkning → fører til en mindre reduksjonen av fjærkonstanten

- Generelt sett faller den mekaniske fjærkonstanten med økende temperatur. Men reduksjonen blir mindre enn den skulle ha blitt pga. at topp-elektroden sin effekt avtar (dvs. ”beam softening”-effekten blir mindre)!