INF5490 RF MEMS

L3: Modeling, design and analysis

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Today's lecture

- MEMS - function
  - Transducer principles
  - Sensor principles

- Methods for RF MEMS modeling
  - **1. Simple mathematical models**
  - **2. Convert to electrical equivalents**
  - **(3. Analyzing using Finite Element Methods)**
    - → L4

Transducers for (RF) MEMS

- **Electromechanical transducers**
  - Transforming
    electrical energy ←→ mechanical energy
- Transducer principles
  - Electrostatic
  - Electromagnetic
  - Electro thermal
  - Piezoelectric

Transducer principles

- **Electrostatic** transducers
  - Principle: Forces between electric charges
    - ”Coulombs law”
  - Stored energy when mechanical or electrical work is performed on the unit can be **converted** to the other form of energy
  - The most used form of electromechanical energy conversion
  - Fabrication is simple

  - Often implemented using a **capacitor** with movable plates
    - **Vertical** movement: parallel plates
    - **Horizontal** movement: Comb structures

Electrostatic transducers

- + Beneficial due to simplicity
- + Actuation controlled by voltage
  - voltage → charge → attractive force → movement
- + Movement gives current
  - movement → variable capacitor → current when voltage is constant
- ÷ Need environmental protection (dust)
  - Packaging required (vacuum)
- ÷ Transduction mechanism is non-linear
  - Gives distortions (force is not proportional to voltage)
  - Solution: small signal variations around a DC voltage

Transducer principles, contd.

- **Electromagnetic** transducers
  – Magnetic winding pulls the element

- **Electro thermal** actuator
  – Different thermal expansion on different locations due to temperature gradients
    - Large deflections can be obtained
    - Slow

Transducer principles, contd.

- **Piezoelectric** transducers
  - In some **anisotropic** crystalline materials the charges will be displaced when **stressed** → electric field
    - stress = ”mechanical stress”
  - Similarly, **strain** results when an electric field is applied
    - strain = ”mechanical strain”
  - Ex. PZT (lead zirconate titanates) – ceramic materials

- *(Electrostrictive transducers*
  - *Mechanical deformation by electric field*
- *Magnetostrictive transducers*
  - *Deformation by magnetic field)*

Comparing different principles

Table 1.4 Comparison of electromechanical transducers

Actuator	Fractional stroke (%)	Maximum energy density ($J\,cm^{-3}$)	Efficiency	Speed
Electrostatic	32	0.004	High	Fast
Electromagnetic	50	0.025	Low	Fast
Piezoelectric	0.2	0.035	High	Fast
Magnetostrictive	0.2	0.07	Low	Fast
Electrostrictive	4	0.032	High	Fast
Thermal	50	25.5	Low	Slow

Source: Wood, Burdess and Hariss, 1996.

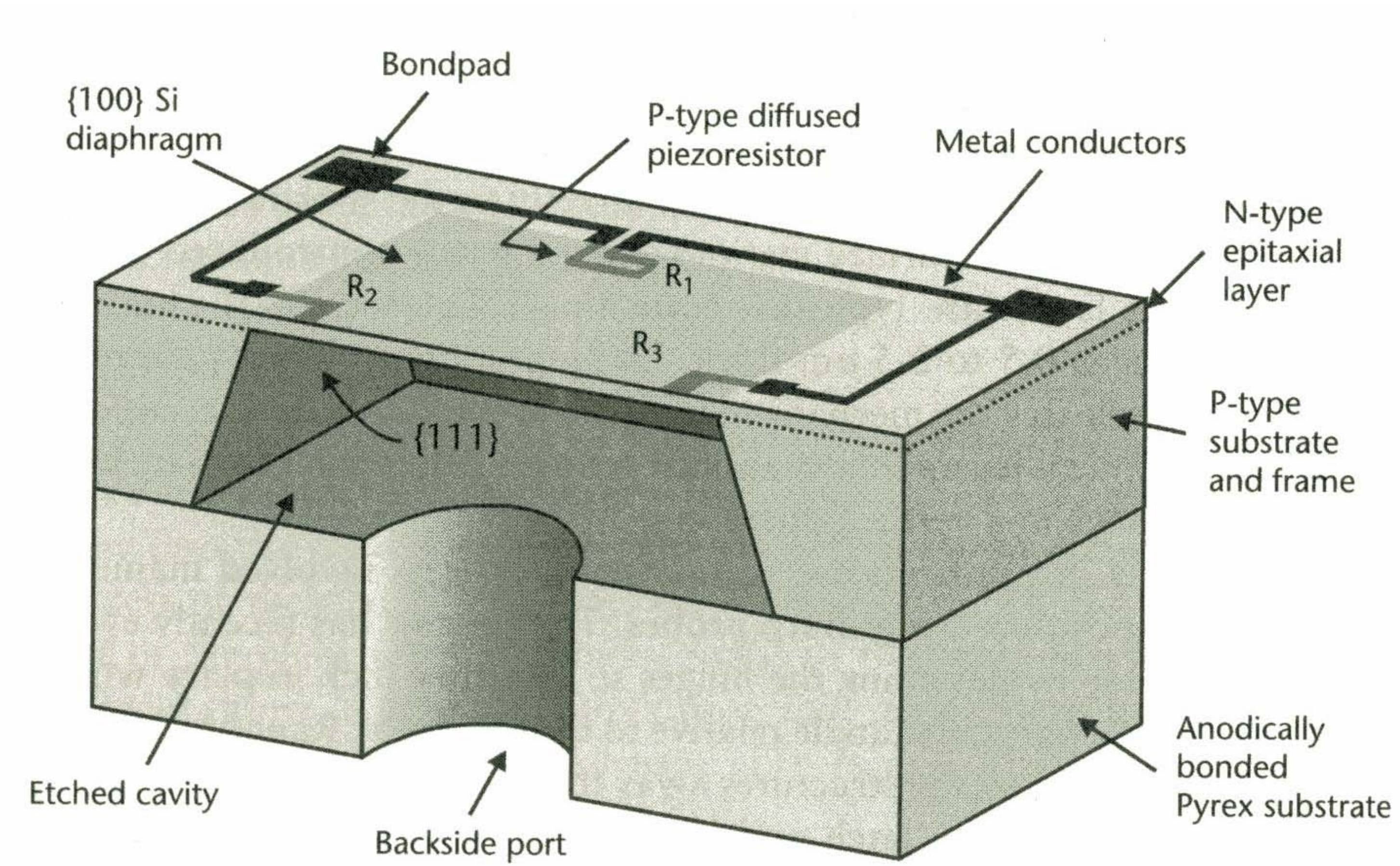
Sensor principles

- Piezoresistive detection
- Capacitive detection
- Piezoelectric detection
- Resonance detection

Sensor principles

- **Piezoresistive** detection
  - Resistance varies due to external pressure/stress
  - Used in pressure sensors
    - deflection of membrane
  - Piezo-resistors placed on membrane where strain is maximum
  - Resistor value is proportional to strain
  - Performance of piezoresistive micro sensors are temperature dependent

Pressure sensor



Senturia

Sensor principles, contd.

- **Capacitive** detection
  - Exploiting capacitance variations
  - Pressure → electric signal
    - Detected by change in oscillation frequency, charge, voltage (V)
  - Potentially higher performance than piezoresistive detection
    - + Better sensitivity
    - + Can detect small pressure variations
    - + High stability

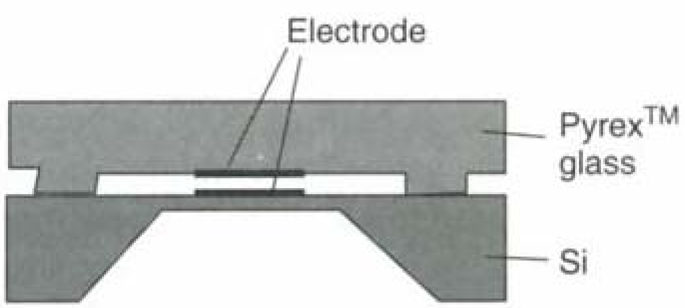


Figure 1.19 Capacitive sensing structure

Sensor principles, contd.

- **Piezoelectric** detection
  - Electric charge distribution changed due to external force → electric field → current

- **Resonance** detection
  - Analogy: stress variation on a string gives strain and is changing the **“natural” resonance frequency**

Methods for modeling RF MEMS

- **1. Simple mathematical models**
  - Ex. parallel plate capacitor
- **2. Converting to electrical equivalents**
- **3. Analysis using Finite Element Methods**

1. Simple mathematical models

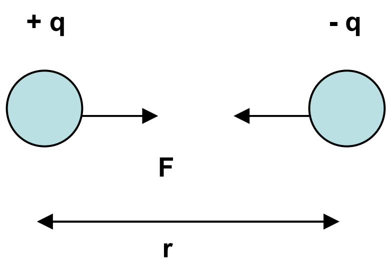
- Equations, formulas describing physical phenomena
  - Simplification, approximations
  - Explicit solutions for simple problems
    - linearization around a bias point
  - Numerical solution of the set of equations
    - Typical differential equations
- + Gives the designer insight/ understanding
  - How the performance changes by parameter variations
  - May be used for initial estimates

Ex. On mathematical modeling

- Important equatins for many RF MEMS components:
  - → Parallel plate capacitor!
  - **Electrostatic** actuation of the capacitor with one spring-suspended plate
  - Calculating **”pull-in”**
    - Formulas and figures →

Electrostatics

Electric force between charges: **Coulombs law**



$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

Electric field = force pr. unit charge

$$\overline{E} = \frac{\overline{F}}{q_0}$$

Work done by a force = change in potential energy

$$W_{a\to b} = \int_a^b \overline{F} \cdot d\overline{l} = U_a - U_b$$

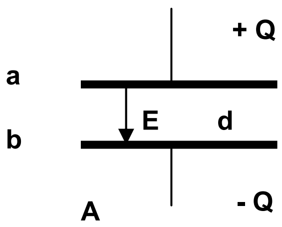
Potential, V = potential energy pr. unit charge

$$V = \frac{U}{q_0}$$

Voltage = potential difference

$$V_a - V_b = \int_a^b \overline{E} \cdot d\overline{l}$$

Capacitance



V_{ab}

Definition of capacitance $$C = \frac{Q}{V_{ab}}$$

Surface charge density = σ

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A} \cdot \frac{1}{\varepsilon_0}$$

Voltage

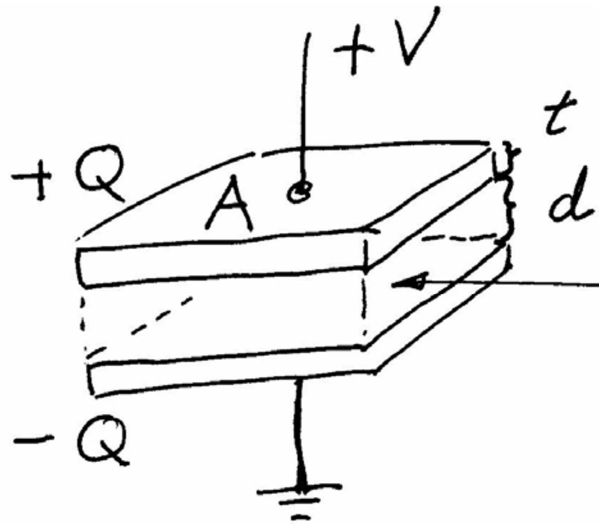
$$V_{ab} = E \cdot d = \frac{Q}{A\varepsilon_0} \cdot d$$

$$C = \frac{Q}{V_{ab}} = \varepsilon_0 \frac{A}{d}$$

Energy stored in a capacitor, C,
that is charged to a voltage V_0 at a current $i = \dot{Q} = C\frac{dV}{dt}$

$$U = \int v \cdot i \cdot dt = \int v \cdot C\frac{dv}{dt} \cdot dt = C\int_0^{V_0} v \cdot dv = \frac{1}{2}CV_0^{\,2} = \frac{\varepsilon_0 A}{2d}V_0^{\,2}$$

Parallel plate capacitor



$$C = \frac{Q}{V}$$

ε = dielektrisk permittivitet

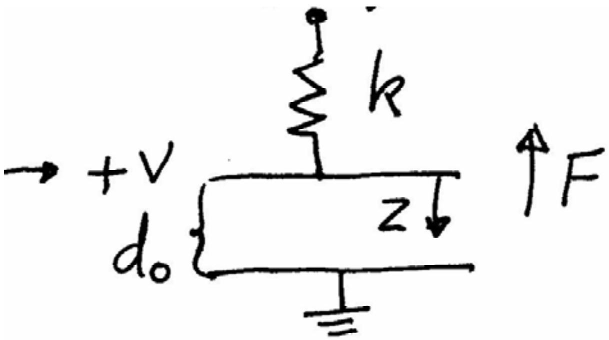
Attractive force between plates

$$F = -\frac{\partial U}{\partial d} = -\frac{\partial}{\partial d}\left(\frac{\varepsilon A}{2d}V^2\right) = \frac{\varepsilon A V^2}{2d^2}$$

Movable capacitor plate

- Assumptions for calculations:
  - Suppose air between plates
  - Spring attached to upper plate
    - Spring constant: k
  - Voltage is turned on
    - Electrostatic attraction
  - At equilibrium
    - **Forces up and forces down are in balance →**

Force balance



k = spring constant

$$F_{spring} = k \cdot x$$

deflection from start position

d0 = gap at 0V and zero spring strain

d = d0 – z

z=d0 – d

Force on upper plate at V and d:

$$F_{net} = -\frac{\varepsilon A V^2}{2d^2} + k(d_o - d)$$

= 0 at equilibrium

Two equilibrium positions

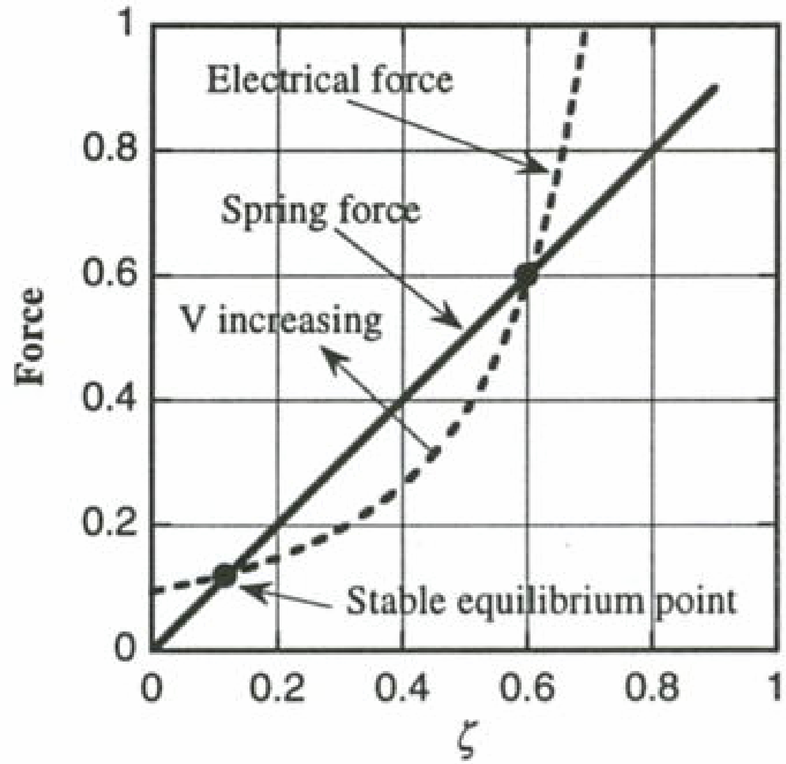


Figure 6.7. Electrical and spring forces for the voltage-controlled parallel-plate electrostatic actuator, plotted for $V/V_{PI} = 0.8$.

$\varsigma = 1 - d/d0$

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Stability

- How the forces develop when d decreases
  - Suppose a small perturbation in the gap at constant voltage

$$\delta F_{net} = \left.\frac{\partial F_{net}}{\partial d}\right|_{V} \cdot \delta d$$

$$\delta F_{net} = \left(\frac{\varepsilon A V^2}{d^3} - k\right)\delta d$$

Suppose the gap decreases $\delta d < 0$

If the upward force also deceases, the system is **UNSTABLE!** $\delta F_{net} < 0,$

Stability, contd.

Stability condition: $$\left.\frac{\partial F_{net}}{\partial d}\right|_{V} < 0$$

$$k > \frac{\varepsilon A V^2}{d^3}$$

Pull-in when: $$\boxed{k = \frac{\varepsilon A V_{PI}^2}{d_{PI}^3}}$$

Pull-in

$$F_{net} = 0$$

$$\frac{\varepsilon A V_{PI}^2}{2 d_{PI}^2} = k \left(d_o - d_{PI} \right)$$

$$= \frac{\varepsilon A V_{PI}^2}{d_{PI}^3}$$

Pull-in when:

$$d_{PI} = \frac{2}{3} d_o$$

$$\boxed{V_{PI} = \sqrt{\frac{8 k d_o^3}{27 \varepsilon A}}}$$

Pull-in

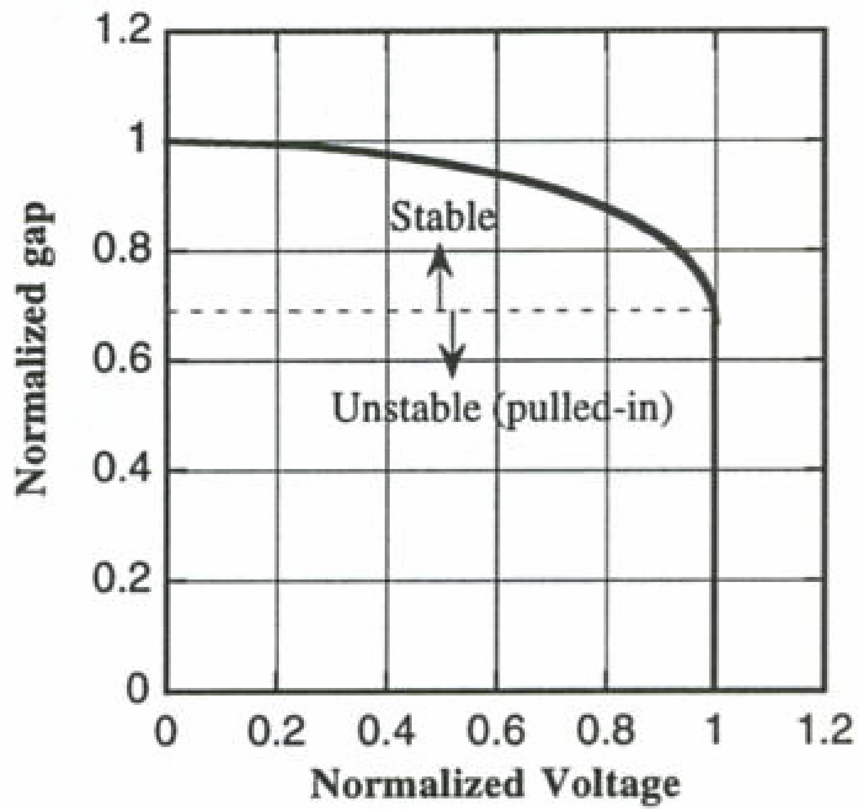

Figure 6.8. Normalized gap as a function of normalized voltage for the electrostatic actuator.

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2. Converting to electrical equivalents

- Mechanical behavior can be modeled using **electrical circuit elements**
  - Mechanical structure → simplifications → equivalent electrical circuit
    - ex. spring/mass → R, C, L
  - Possible to “interconnect” electrical and mechanical energy domains
    - Simplified modeling and co-simulation of electronic and mechanical parts of the system
  - Proper **analysis-tools** can be used
    - Ex. SPICE

Converting to electrical equivalents, contd.

- We will discuss:
  - Needed circuit theory
  - Conversion principles
    - **effort - flow**
  - Example of conversion
    - Mechanical resonator
  - In a future lecture:
    - Co-existence and coupling between various energy domains

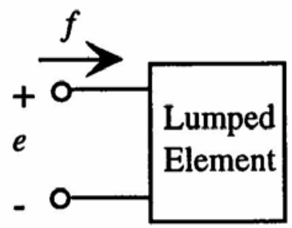
Circuit theory

- Basic circuit elements: R, C, L
- Current and voltage equations for basic elements (low frequency)
  - Ohms law, C and L-equations
    - **$V = RI$, $I = C\, dV/dt$, $V = L\, dI/dt$**
  - **Laplace** transformation
    - From differential equations to algebraic (s-polynomial)
    - → Complex impedances: **R, 1/sC, sL**
- Kirchhoffs equations
  - **Σ current** into nodes = 0, Σ **voltage** in a loop = 0

Effort - flow

- Electrical circuits are described by a **set of variables:** *conjugate power variables*
  - Voltage V: **across** or **effort** variable
  - Current I: **through** or **flow** variable
  - *An effort variable drives a flow variable through an impedance, Z*

- Circuit element is modeled as a **1-port** with terminals
  - Same current (f = flow) in and out and through the element
    - ***Positive flow*** *into a terminal defining a* ***positive effort***



Energy-domains, analogies

- Various energy domains exist
  - Electric, elastic, thermal, for liquids etc.

- ***For every energy domain it is possible to define a set of conjugate power variables that may be used as basis for lumped component modeling using equivalent circuits elements***

- Table 5.1 Senturia ->

Ex. of conjugate power variables

Energy Domain	Effort	Flow	Momentum	Displacement
Mechanical translation	Force F	Velocity $\dot{x}, v$	Momentum p	Position x
Fixed-axis rotation	Torque τ	Angular velocity ω	Angular momentum J	Angle θ
Electric circuits	Voltage V, v	Current I, i	...	Charge Q
Magnetic circuits	Magnetomotive force MMF	Flux rate $\dot{\phi}$	...	Flux ϕ
Incompressible fluid flow	Pressure P	Volumetric flow Q	Pressure momentum Γ	Volume V
Thermal	Temperature T	Entropy flow rate $\dot{S}$	...	Entropy S

Conjugate power variables: e,f

- Assume conversion between energy domains were the **energy is conserved!**
- Properties
  - **e * f = power**
  - **e / f = impedance**
- Generalized **displacement** represents the state, f. ex. position or charge

$$f(t) = \dot{q}(t) \qquad\qquad q(t) = \int_{t_0}^{t} f(t)dt + q(t_0)$$

  - **e * q = energy**

Generalized momentum

$$p(t) = \int_{t_0}^{t} e(t)dt + p(t_0)$$

– Mechanics: impulse
  - F*dt = mv – mv0

– General: **p * f = energy**

Ex.: Mechanical energy domain

$e = F$ (kraft) — force

$f = v, \dot{x}$ (hastighet) — velocity

$q = x$ (posisjon) $= \int \dot{x}\, dt$ — position

$p = p$ (momentum) $= \int F\, dt$ — momentum

(kraft $\times$ tid) — force x time

$e \cdot f \rightarrow F \cdot \dot{x} = \frac{F \Delta x}{\Delta t} = \frac{\text{arbeid}}{\text{tid}} = \text{effekt}$ — work/time = power

$e \cdot q \rightarrow F \cdot x = \text{kraft} \times \text{vei} = \text{arbeid} = \text{energi}$ — force*distance = work = energy

$p \cdot f \rightarrow p \cdot \dot{x} = mv \cdot v = mv^2 = \text{energi}$ — energy

Ex.: Electrical energy domain

$e = V$	(spenning)	voltage
$f = I$	(strøm)	current
$q = \int I\,dt = Q$	(ladning)	charge
$p = $ n.a.		

$e \cdot f \rightarrow V \cdot I = $ effekt	power
$e \cdot q \rightarrow V \cdot Q = V \int I\,dt = $ energi	energy

e → V - convention

- **Senturia** and **Tilmans** use the

  e→V –convention

- Ex. electrical and mechanical circuits
  - e → V (voltage)   equivalent to F (force)
  - f → I (current)   equivalent to v (velocity)
  - q→ Q (charge)   equivalent to x (position)
  - e * f = ”power” injected into the element

H. Tilmans, Equivalent circuit representation of electromagnetical transducers:
I. Lumped-parameter systems, J. Micromech. Microeng., Vol. 6, pp 157-176, 1996

Other conventions

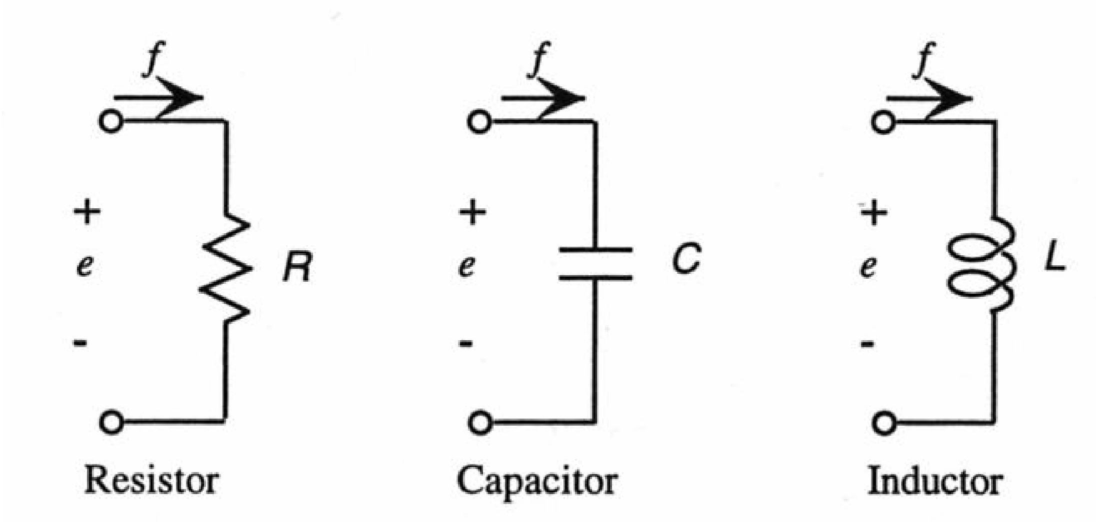
- Different conventions exist for defining **through-** or **across-variables**

Table 5.2. Different conventions for assigning circuit variables.

Convention		Across Variable	Through Variable	Product	Principal Use
$e \rightarrow V$	*	e	f	power	electric circuit elements
$f \rightarrow V$	alternativt	f	e	power	mechanical circuit elements
Thermal		T	$\dot{Q}$	Watt-Kelvin	thermal circuits
HDL		q	e	energy	HDL circuit representation of mechanical elements

Generalized circuit elements

- **One-port** circuit elements
  - R, dissipating element
  - C, L, energy-storing elements
  - Elements can have a **general function!**
    - **Can be used in various energy domains**



Generalized capacitance

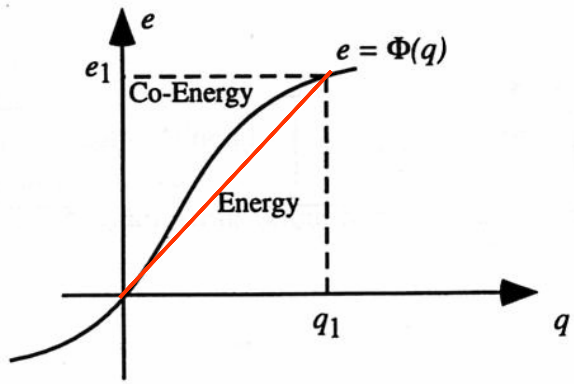


Figure 5.5. Illustrating energy and co-energy for a generalized capacitor.

Compare with a **simplified case:**
- a **linear** capacitor

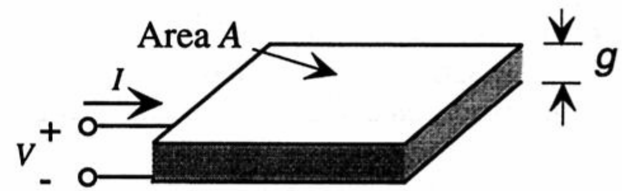
$$Q = V \cdot C$$

definisjon av C

$$V = \frac{1}{C} \cdot Q$$

$$\Downarrow$$

$$e = \frac{1}{C} \cdot q$$

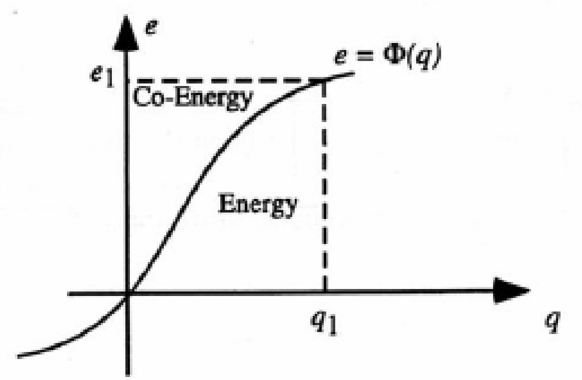


$$C = \frac{\varepsilon A}{g}$$

Generalized capacitance, contd.

Capacitance is associated with stored **potential energy**

$$\mathcal{W}(q_1) = \int_0^{q_1} e\,dq = \int_0^{q_1} \Phi(q)\,dq \qquad (5.10)$$



Co-energy:

$$\mathcal{W}^*(e) = eq - \mathcal{W}(q) \qquad (5.11)$$

$$\mathcal{W}^*(e_1) = \int_0^{e_1} q\,de = \int_0^{e_1} \Phi^{-1}(e)\,de \qquad (5.12)$$

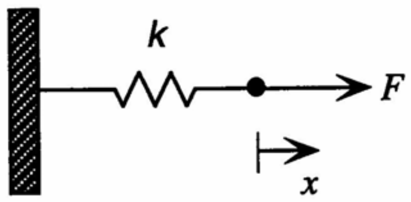
Energy stored in parallel plate capacitor

Energy: $$W(Q) = \int_0^Q e \cdot dq = \int_0^Q \frac{q}{C} \cdot dq = \frac{Q^2}{2C}$$

Co-energy: $$W^*(V) = \int_0^V q \cdot de = \int_0^V C \cdot v \cdot dv = \frac{CV^2}{2}$$

$W^*(V) = W(Q)$ for linear capacitance

Mechanical spring



Hook´s law: $F = k \cdot x$

Stored energy $$\mathcal{W}(x_1) = \int_0^{x_1} F(x)dx = \frac{1}{2}kx_1^2 \quad (5.18)$$

Compare with capacitor $$W(Q) = \frac{1}{2} \cdot \frac{1}{C} \cdot Q^2$$

Q displacement

x1 displacement

→ 1/C equivalent to k

”Compliance”

- ”Compliance” = ”inverse stiffness”

$$C_{spring} = \frac{1}{k}$$

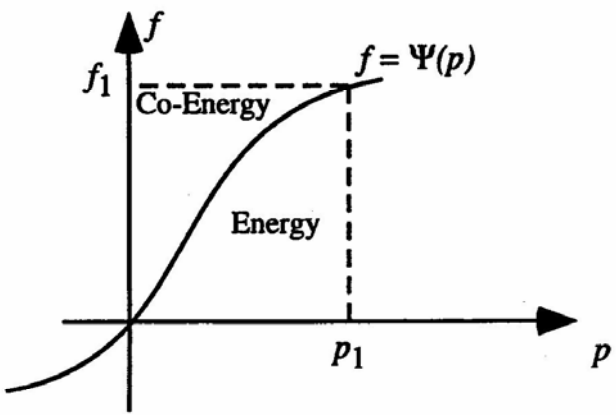
- Stiff spring → small capacitor
- Soft spring → large capacitor

Generalized inductance

Energy also defined as:

$$f \cdot p = \text{flow} \times \text{momentum}$$

$$p = \int e\,dt \qquad \text{flow} = v \qquad \text{momentum} = m \cdot v$$



Energy = stored kinetic energy

$$W(p_1) = \int_0^{p_1} f(p)\,dp$$

Ex.: Electrical inductor

Co-energy: $$W^*(f) = \int_0^{f_1} p(f)\, df$$

$$V = L \frac{dI}{dt}$$

$$p = \int e\, dt = \int V\, dt = \int L \frac{dI}{dt}\, dt = \int L\, dI$$

$$p(f) = p(I) = L I$$

$$W^*(f_1) = W^*(I_1) = \int_0^{I_1} L \cdot I \cdot dI = \frac{1}{2} L I_1^{\,2}$$

Analogy between mass (mechanical inertance) and inductance L

A mechanical system has **linear momentum**: p = mv

Flow:

$$f = v = \frac{p}{m}$$

$$W(p_1) = \int_0^{p_1} f(p)\,dp = \int_0^{p_1} \frac{p}{m}\,dp = \frac{p_1^2}{2m}$$

Co-energy:

$$W^*(v_1) = \int_0^{v_1} p(v)\,dv = \int_0^{v_1} (mv)\,dv = \frac{1}{2} m v_1^2$$

Analogy between m and L

$$W^*(f_1) = W^*(I_1) = \int_0^{I_1} L \cdot I \cdot dI = \frac{1}{2} L I_1^2$$

Compare with: $W^*(v_1) = \qquad = \frac{1}{2} m v_1^2$

I_1 = flow

v_1 = — " —

L is equivalent to **m**

m = L inertance

Mechanical inertance = mass m
is analog to inductance L

Interconnecting elements

- e → V follows two basic principles
  - Elements that share a common flow , and hence a common variation of displacement, are connected in **series**
  - Elements that share a common effort are connected in **parallel**

Ex. of interconnection:

”Direct transformation”

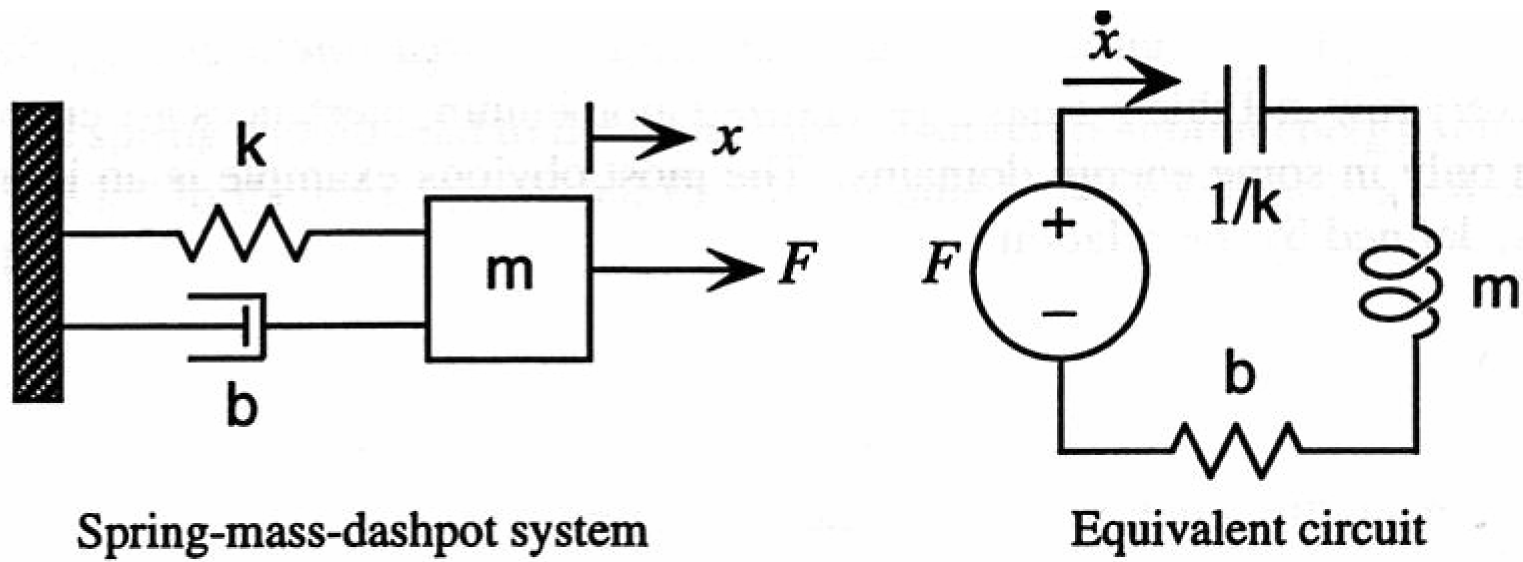
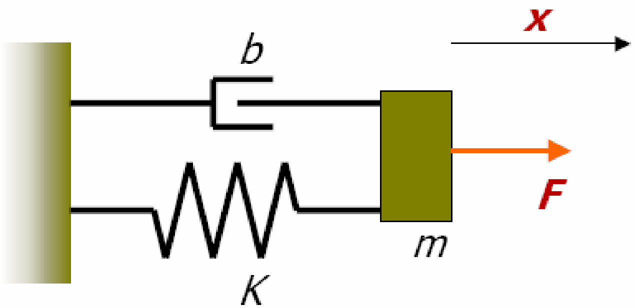
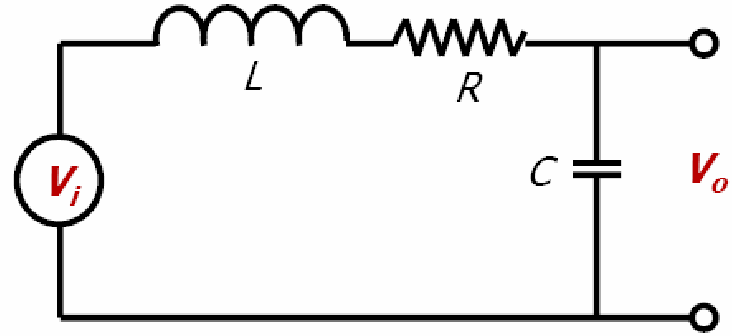


Figure 5.9. Translating mechanical to electrical representations.

Mechanical / Electrical Systems





Input : external force F

Output : displacement x

$m\ddot{x}(t) + b\dot{x}(t) + Kx(t) = F$

m mass, b damping, K stiffness

Transfer function :

$$H(s) = \frac{x}{F} = \frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{K}{m}}$$

Input : voltage V_i

Output : voltage V_o

$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = V_i$

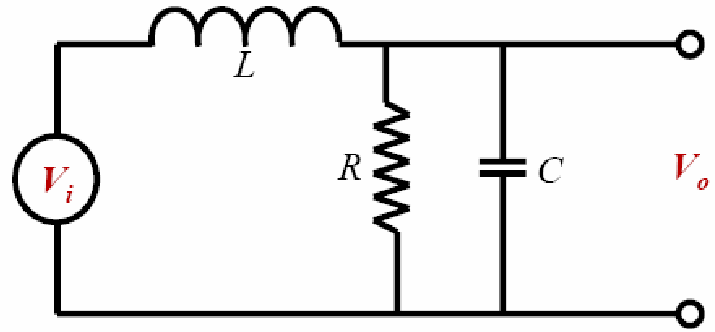
L induct., R resist., C capacit.

Transfer function :

$$H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Mechanical / Electrical Systems

Alternative circuit:



Input : voltage V_i

Output : voltage V_o

$$L\ddot{q}(t) + \tfrac{L}{RC}\dot{q}(t) + \tfrac{1}{C}q(t) = V_i$$

L inductance , R resistance , C capacitance

Transfer function :

$$H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Resonators

- Analogy between mechanical and electrical system:
  - Mass m - inductivity L
  - Spring K - capacitance C
  - Damping b - resistance R (depending where R is placed in circuit)

- Solution to 2nd order differential equation:

$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

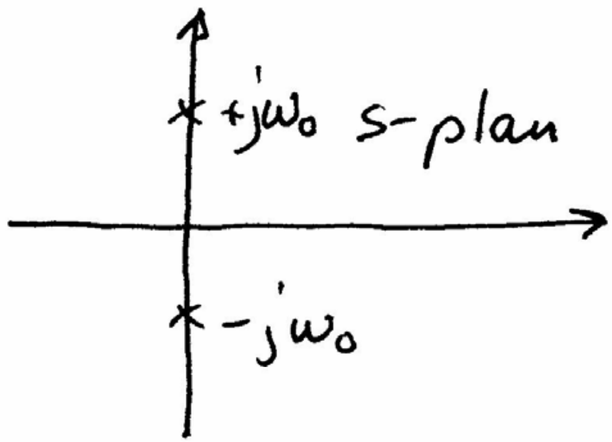
$\omega_0 = 2\pi f_0$ natural frequency

$\omega_0 = \sqrt{\frac{K}{m}}$ mechanical system, $\omega_0 = \sqrt{\frac{1}{LC}}$ electrical system

Q quality factor

System without damping (b=0, R=0)

$$H(s) = \frac{\omega_0^2}{s^2 + \omega_0^2} = \frac{\omega_0^2}{(s + j\omega_0)(s - j\omega_0)}$$



$$|H(j\omega_0)| = \infty$$

$$H(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

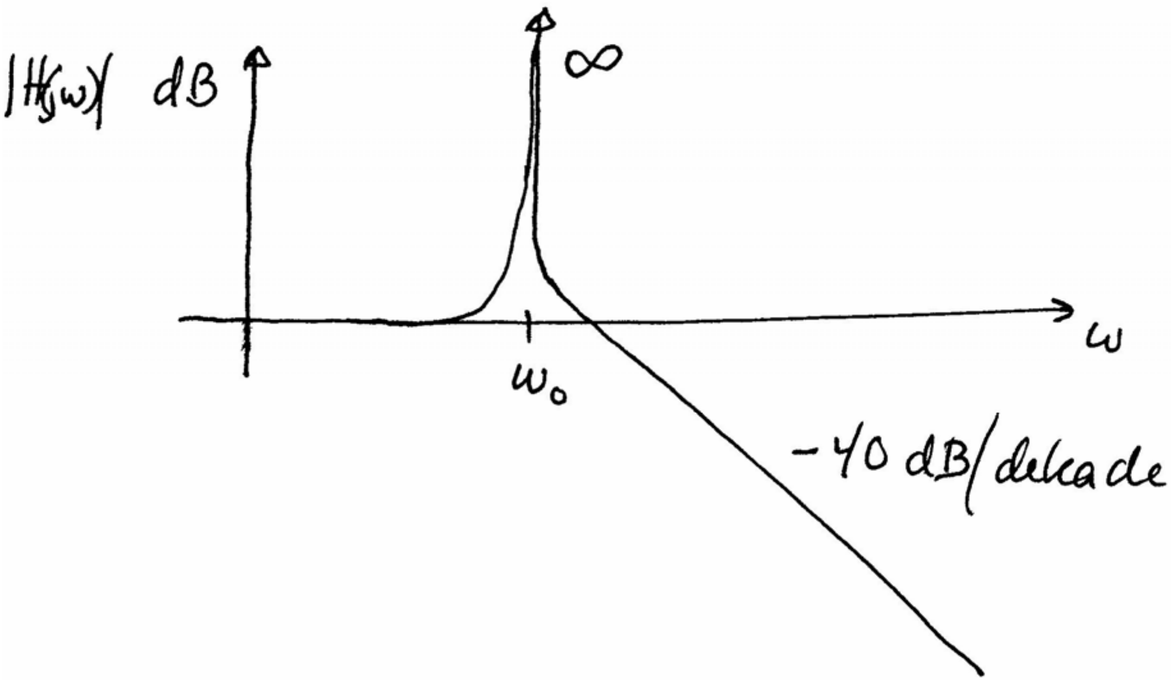
$$\omega_0 = \sqrt{\frac{1}{LC}}, \omega_0 = \sqrt{\frac{k}{m}}$$

System without damping, contd.

$$H(j\omega) = \frac{1}{1-\left(\frac{\omega}{\omega_0}\right)^2}$$

$|H(j\omega)| = 1$ når $\omega \ll \omega_0$ 0 dB

$|H(j\omega)| = -\left(\frac{\omega_0}{\omega}\right)^2$ når $\omega \gg \omega_0$ -40 dB/dekade

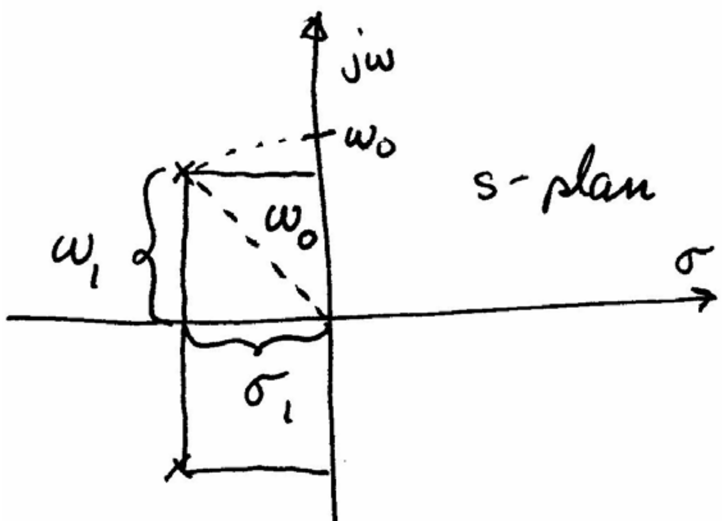


With damping

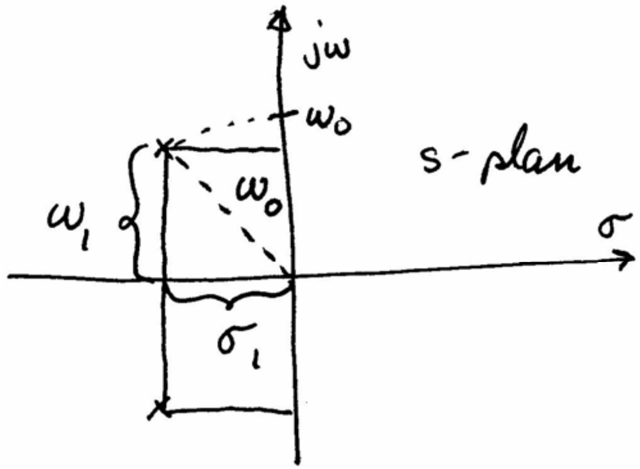
$$s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0$$

$$s = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1 - \frac{1}{4Q^2}}$$

$$= -\sigma_1 \pm j\omega_1$$



Damped system, contd.



$$\frac{\omega_0}{Q} = \frac{b}{m} = \frac{1}{\tau} \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

$$\sigma_1 = \frac{1}{2\tau} = \frac{b}{2m}$$

$$\omega_1 = \omega_0\sqrt{1 - \frac{1}{4\omega_0^2\tau^2}} = \omega_0\sqrt{1 - \frac{b^2}{4km}}$$

$$\omega_1^2 + \sigma_1^2 = \omega_0^2$$

Mechanical Resonator

- Frequency and phase shift under damping:

$$x(t) = Ae^{-t/2\tau}\cos(\omega_1 t + \varphi)$$

$\tau = m/b$ damping time

$$\omega_1 = \omega_0\sqrt{1 - \frac{1}{4\omega_0^2\tau^2}} = \omega_0\sqrt{1 - \frac{b^2}{4Km}}$$

φ phase shift

- Energy dissipation:

$$E(t) = E_0 e^{-t/\tau}$$

What is the meaning of "damping time"?

τ = damping time

$$e^{-t/2\tau}\Big|_{t=\tau} = e^{-1/2} = \frac{1}{\sqrt{e}}$$

Power

Effekten

$$|x(t)|^2\Big|_{t=\tau} = \frac{1}{e}$$

$$\boxed{x(t) = A e^{-t/2\tau} \cos(\omega_1 t + \varphi)}$$

$$x(0) = A \cdot \cos\varphi$$ initialbetingelser

initial conditions

Q-factor and damping time

Generell ligning

General equation

$$s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0$$

$$\Rightarrow \quad s^2 + \frac{1}{\tau} s + \omega_0^2 = 0$$

$$\boxed{Q = \omega_0 \tau}$$

$\tau = \frac{m}{b}$ mekanisk

mechanical

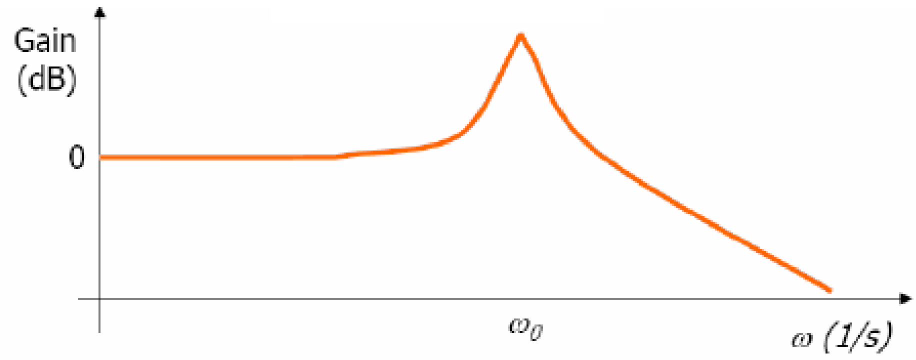
$\tau = \frac{L}{R}$ elektrisk

electrical

$$Q_{mek} = \frac{\omega_0 m}{b}$$

$$Q_{el} = \frac{\omega_0 L}{R}$$

Amplitude at resonance for forced vibrations



$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H(j\omega) = \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + j\,\frac{\omega\omega_0}{Q}}$$

$$|H(j\omega_0)| = \left|\frac{\omega_0^2}{0 + j\,\frac{\omega_0^2}{Q}}\right| = Q$$