Optical MEMS in Communication and Sensing

Fabrication, Design, and Scaling of Optical Microsystems

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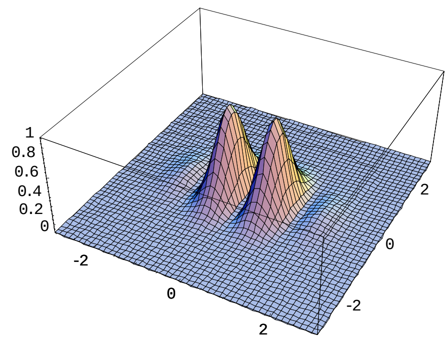
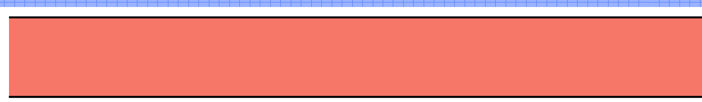
Section 2:

- Scaling of Microoptics
  - Diffraction made simple - Gaussian Beam propagation
  - Resolution of microscanners and Spatial Light Modulator (projection displays)
  - Surface quality



Diffraction



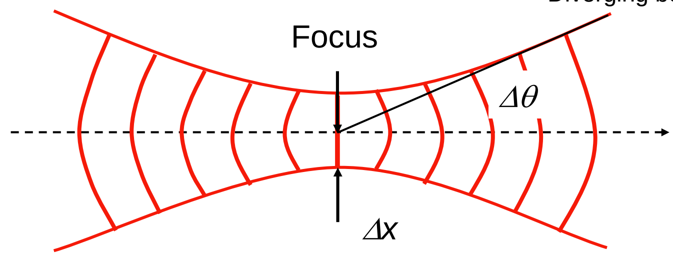
Light ray or collimated (perfectly parallel) optical beam that propagates over infinite distance without changing its cross section

- What is wrong with the concept of a light ray or collimated beam?
  - It is inconsistent with Maxwell's Equations and Huygen's principle
  - It violates the second law of thermodynamics
  - It even violates energy conservation!
- Conclusion: All wave phenomena has diffraction, i.e. spreading of beams to larger cross sections as they propagate over long distances (over short distances, focusing is possible)

Focused beams

Concave wavefronts => Converging beam

Convex wavefronts => Diverging beam



- Any physically realizable beam will have converging, and diverging regions, i.e. they go through a focus. (Plane waves and Bessel beams are not physical)
- What makes Gaussian beams special:
  - Lasers and waveguides have modes that can be approximated as Gaussian
  - The fundamental Gaussian beam has the smallest $\Delta x \cdot \Delta\theta$ product
  - After long propagation, all beams are fundamental Gaussians
  - Any beam can be expressed as a sum of fundamental and higher order Gaussians
  - The mathematical description of Gaussian beam propagation is simple

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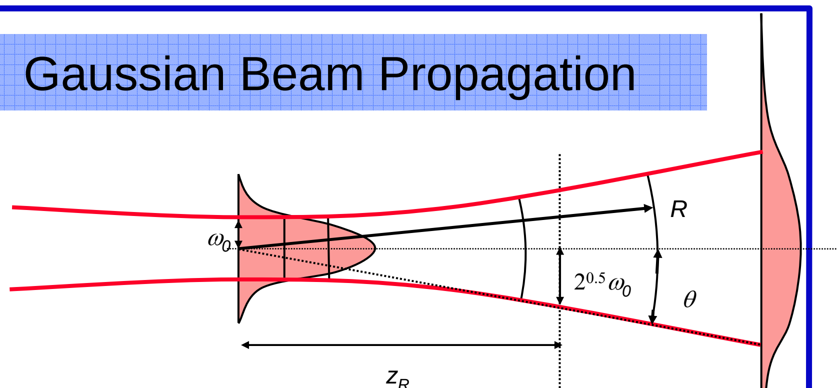
Gaussian Beam Propagation



$$\theta = \frac{\lambda}{\pi \cdot \omega_0}$$

Beam radius: $$\omega^2(z) = \omega_0^2 \left[1 + \left(\frac{\lambda \cdot z}{\pi \cdot \omega_0^2}\right)^2\right]$$

Radius of curvature: $$R(z) = z\left[1 + \left(\frac{\pi \cdot \omega_0^2}{\lambda \cdot z}\right)^2\right]$$

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Beam Waist and Beam Parameter

- The Gaussian beam narrows to a minimum radius, ω_0, called the beam waist
- The the beam radius, radius of curvature, and far-field diffraction angle are expressed in terms of ω_0, and z, the distance from the waist

$$\omega^2(z) = \omega_0^2\left[1+\left(\frac{\lambda \cdot z}{\pi \cdot \omega_0^2}\right)^2\right] \qquad R(z) = z\left[1+\left(\frac{\pi \cdot \omega_0^2}{\lambda \cdot z}\right)^2\right] \qquad \theta = \frac{\lambda}{\pi \cdot \omega_0}$$

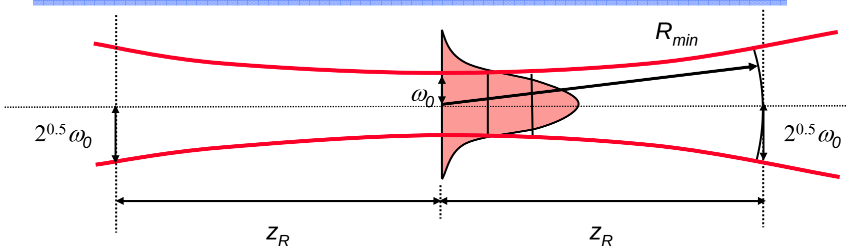
- Gaussian beams are characterized by the beam parameter

$$\frac{1}{q} = \frac{1}{R} - j\frac{\lambda}{\pi \cdot \omega^2}$$

- At the waist, the beam parameter is purely imaginary

$$q_0 = j\frac{\pi \cdot \omega_0^2}{\lambda} \qquad q(z) = q_0 + z = j\frac{\pi \cdot \omega_0^2}{\lambda} + z$$

Rayleigh Length



Rayleigh length: $z_R = \frac{\pi \cdot \omega_0^2}{\lambda}$ Confocal parameter: 2 z_R

Beam radius: $\omega(z) = \omega_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}$ Diff. angle: $\theta = \frac{\lambda}{\pi \cdot \omega_0} = \frac{\omega_0}{z_R}$

Radius of curvature: $R(z) = Z + \frac{Z_R^2}{Z}$

Gaussian Beam Profile

Field distribution :

$$u(r,z) = \sqrt{\frac{2}{\pi}} \frac{1}{\omega(z)} \exp\left[-jkz + j\psi(z) - \frac{r^2}{\omega^2(z)} - jk\frac{r^2}{2R(z)} \right]$$

$$\omega(z) = \omega_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2} \qquad R(z) = Z + \frac{Z_R^2}{Z}$$

$$\psi(z) = \tan^{-1}\left(\frac{z}{z_R}\right) \qquad z_R = \frac{\pi\omega_0^2}{\lambda} \quad \text{Rayleigh length}$$

Intensity : $I(r,z) = I(0,z) \cdot \exp\left[-2\frac{r^2}{\omega^2(z)} \right]$

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Gaussian Beams of Order m,l

$$u(r,z) = H_m H_l \frac{\omega_0}{\omega(z)} \exp\left[-jkz + j\phi(z) - \frac{r^2}{\omega^2(z)} - jk\frac{r^2}{2R(z)} \right]$$

$$\omega(z) = \omega_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2} \qquad R(z) = z + \frac{z_R^2}{z}$$

$$\phi(z) = (m + l + 1)\tan^{-1}\left(\frac{z}{z_R}\right) \qquad z_R = \frac{\pi\omega_0^2}{\lambda} \quad \text{Rayleigh length}$$

- H_m is a Hermite polynomial of order *m*
- Some low order Hermite polynomials
- Cylindrical coordinates gives Laguerre polyn.

$$H_0 = 1 \qquad H_1 = 2x$$

$$H_2 = 4x^2 - 2 \qquad H_3 = 8x^3 - 12x$$

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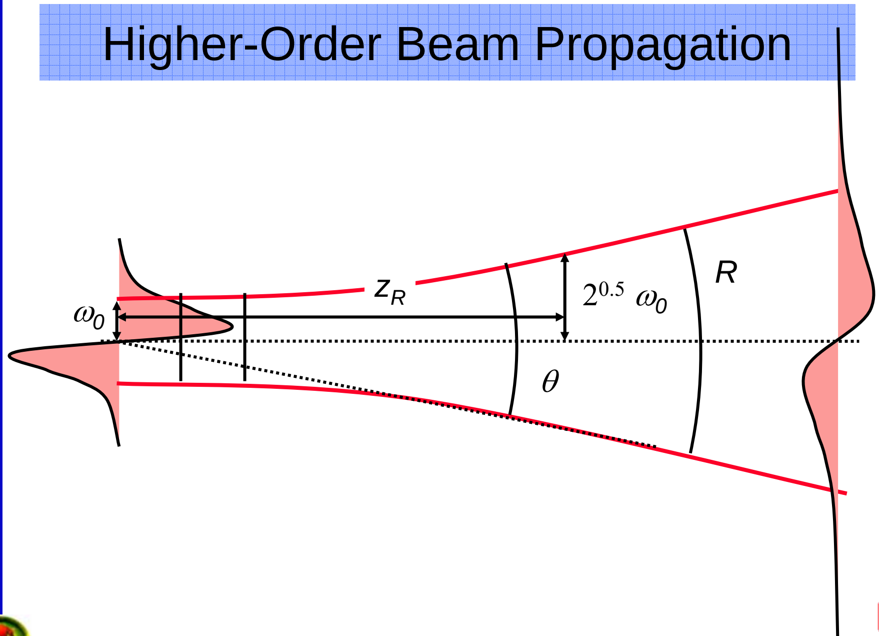
Higher-Order Beam Propagation



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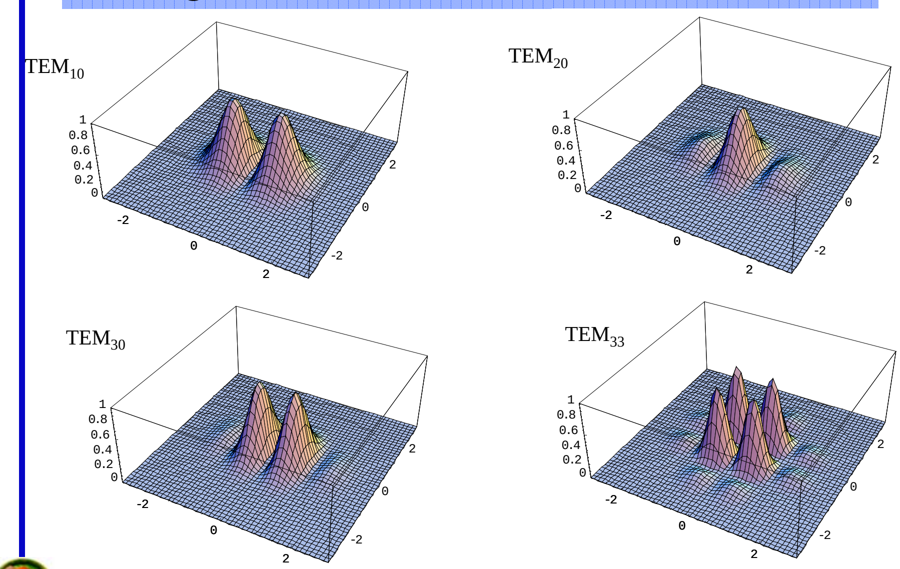
Higher-Order Gaussian Profiles



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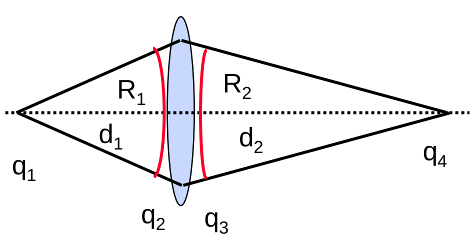
Hermite Gaussian Modes

- The Hermite Gaussian modes is a complete, orthogonal set of functions
- Higher order modes have the same beam radius and curvature as the fundamental (but different phase)
- Higher order modes occupy larger areas (same ω, but multiplied by higher order polynomial)
- All modes propagate according to the simple rule
  - $q_2 = q_1 + z$
- The difference in phase means that different modes will have different frequencies in a laser cavity
- An arbitrary optical field can be expanded on the Gaussians. Propagating the Gaussians and summing lets us find the effect of propagation on the original field

Beam transformation in lenses

- An ideal lens does not change the transverse distribution of an optical field, so a Gaussian will remain in the same order after passing through a lens
- *R(z)* and ω*(z)* does change when passing through a lens
- Only *R(z)* changes when passing through a ***thin*** lens



$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{f} \Rightarrow$$

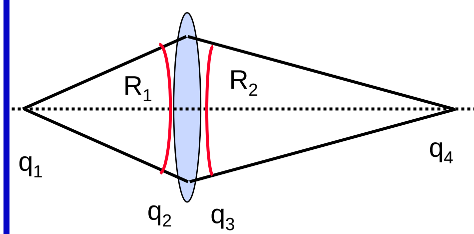
$$\frac{1}{q_1} - \frac{1}{q_2} = \frac{1}{f}$$

Gaussians in Lens Systems

- Repeated application of the propagation and lens laws
- allow us to find the transformation of the beam through any lens system

$$q_1 = q_2 + z$$

$$\frac{1}{q_1} - \frac{1}{q_2} = \frac{1}{f}$$



$$q_2 = q_1 + d_1$$

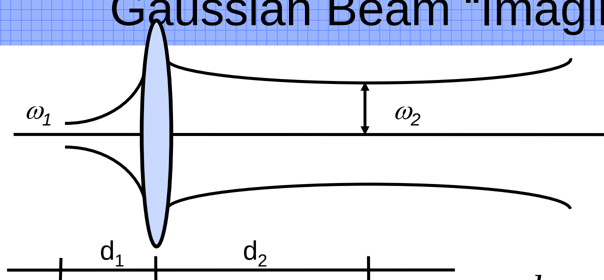
$$\frac{1}{q_3} = \frac{1}{q_2} - \frac{1}{f} \Rightarrow q_3 = \frac{fq_2}{f - q_2}$$

$$q_4 = q_3 + d_2$$

$$\Rightarrow q_4 = \frac{f(q_1 + d_1)}{f - q_1 - d_1} + d_2$$

$$= \frac{fq_1 + fd_1 + d_2 f - d_2 q_1 - d_2 d_2}{f - q_1 + d_1}$$

Gaussian Beam “Imaging”



$$z_R = \frac{\pi \cdot \omega_1^2}{\lambda}$$

$$d_1 = f \Rightarrow d_2 = f$$

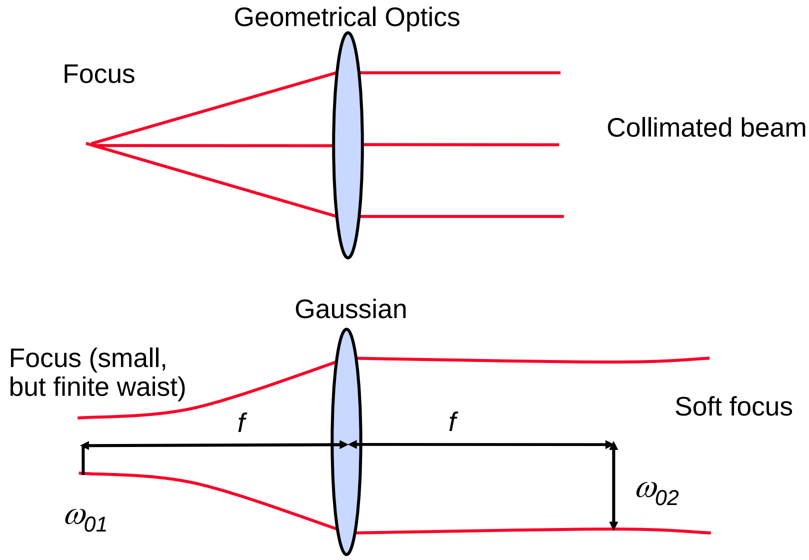
$$\omega_2^2 = \frac{\omega_1^2}{(1 - d_1/f)^2 + z_R^2/f^2}$$

$$\omega_{x1,y1} = \frac{f\lambda}{\pi \cdot \omega_{x2,y2}}$$

$$\frac{1}{d_2} + \frac{1}{d_1}\frac{1}{1 + \frac{z_R^2}{d_1(d_1 - f)}} = \frac{1}{f}$$

$$z_R = 0 \Rightarrow \frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

Gaussian vs. Geometrical optics



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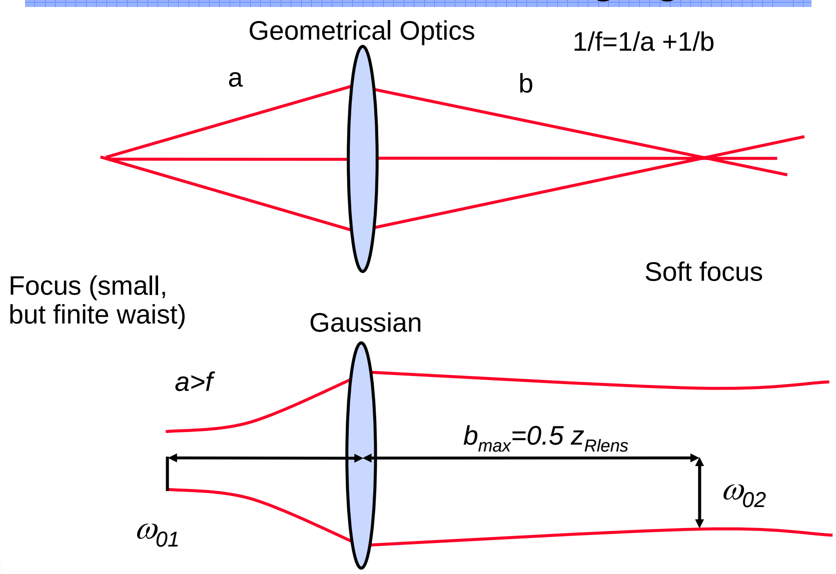
Gaussian Beam Imaging



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Summary

- Gaussian beams are solutions to the paraxial wave equation
- The diffraction angle is given by: $\theta = \frac{\lambda}{\pi \cdot \omega_0}$
- Higher-order Gaussian beams (also solutions to the paraxial wave eq.)
  - Same beam radius and curvature, but different phase shift compared to zero order
  - Provide means to solve general diffraction problems (not always practical)
- Gaussian beam propagation through lenses
  - Same as for Geometrical optics, but consequences are dramatically different!
  - Collimation: No such thing, just a “soft” focus
  - Focusing: Maximum lens-waist distance
  - Imaging: Imaging and FT regimes

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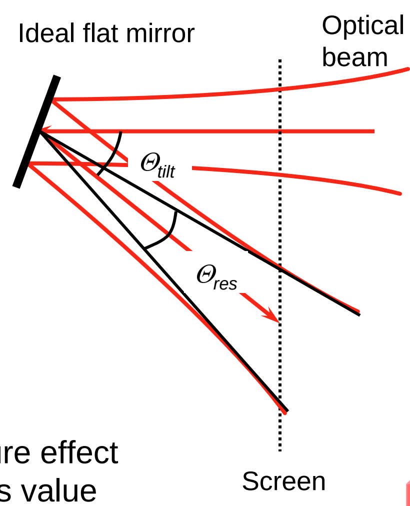
Mirror Resolution (far field)

- The number of resolvable spots on the screen is given by the ratio of the range of tilt angles to the diffraction angle of the beam
- The diffraction angle depends on the (application specific) definition of resolvability

$$N = \frac{\theta_{range}}{\theta_{res}} + 1$$

Mirror curvature and the aperture effect reduce the resolution below this value



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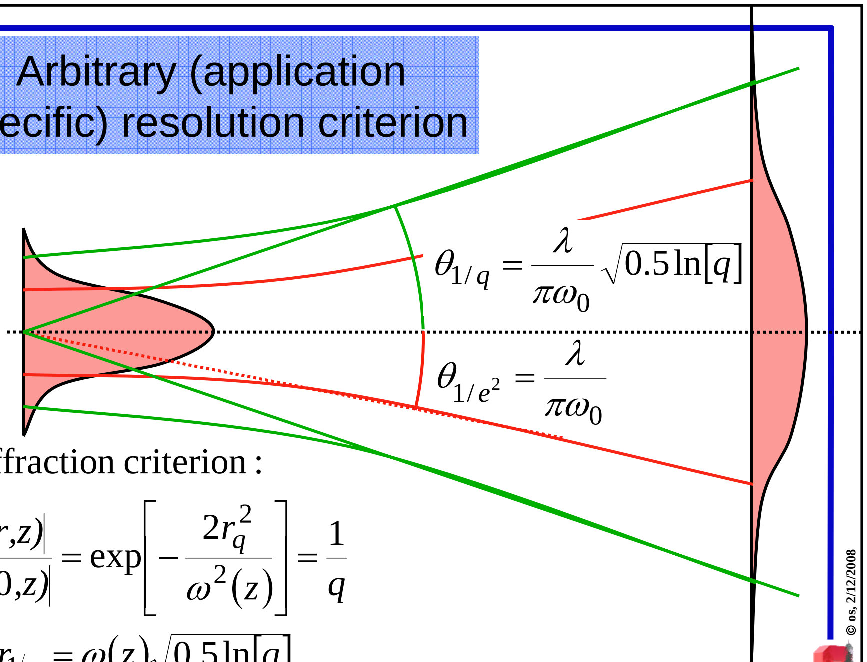
Arbitrary (application specific) resolution criterion



Diffraction criterion:

$$\frac{|u(r,z)|}{|u(0,z)|} = \exp\left[-\frac{2r_q^2}{\omega^2(z)}\right] = \frac{1}{q}$$

$$\Rightarrow r_{1/q} = \omega(z)\sqrt{0.5\ln[q]}$$

Diffraction (half) angle for arbitrary resolution criterion

Combine $r_{1/q} = \omega(z)\sqrt{0.5\ln[q]}$ with

$$\omega(z) = \omega_0\sqrt{1+\left(\frac{z}{z_R}\right)^2} \qquad z_R = \frac{\pi\omega_0^2}{\lambda}$$

to find the diffraction angle:

$$\theta_{1/q} = \lim_{z\to\infty}\frac{r_{1/q}}{z} = \lim_{z\to\infty}\frac{\omega(z)\sqrt{0.5\ln[q]}}{z}$$

$$\theta_{1/q} = \frac{\omega_0 z\sqrt{0.5\ln[q]}}{z z_R} = \frac{\lambda\sqrt{0.5\ln[q]}}{\pi\omega_0}$$

Mirror resolution (near field)

- The resolution depends on
  - Wavelength
  - mirror size
  - resolution criterion
- The resolution does NOT depend on
  - radius of curvature of the beam at the mirror (caused by either curvature of the mirror or a converging/diverging incident optical beam)
  - lenses placed after the mirror
- These two effects change the screen position that gives optimum resolution

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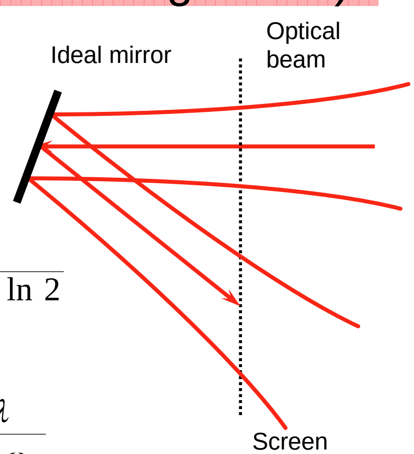
Example: Resolution of Scanning Displays (CRTs or scanning lasers)

In displays we say that two pixels are resolved when they are separated by their FWHM



Diffraction angle (FWHM):

$$e^{-2\frac{r_{HM}^2}{\omega^2}} = \frac{1}{2} \Rightarrow r_{FWHM} = \omega\sqrt{0.5\ln 2}$$

$$FWHM = 2r_{HM} = 1.18\cdot\omega \Rightarrow$$

$$\theta_{diff} = \lim_{z\to\infty}\frac{2r_{HM}}{z} = 1.18\frac{\lambda}{\pi\cdot\omega_0}$$

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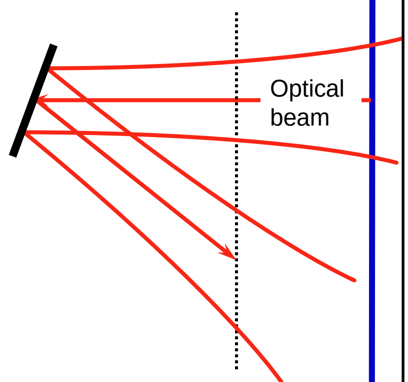
Scanner Resolution

For a transverse Gaussian profile on an ideal flat, infinite mirror, the number of resolvable spots (pixel count) is:

$$N = \frac{\Delta\theta_{tilt}}{\theta_{diff}} + 1$$

$$\Delta\theta_{tilt} \quad \frac{\pi}{1.18}\frac{\omega_m}{\lambda} + 1$$



It is reasonable to assume that our lens technology can support angles up to 0.7 radians

$$N \approx 0.7 \cdot \frac{\pi}{1.18}\frac{\omega_m}{\lambda} + 1$$

$$\approx 1.86 \cdot \frac{\omega_m}{\lambda} + 1$$

In the visible (λ=500nm), we see that a beam radius of 270 micron is sufficient for HDTV resolution! A micromirror of a diameter of about 800 micron can support this size beam

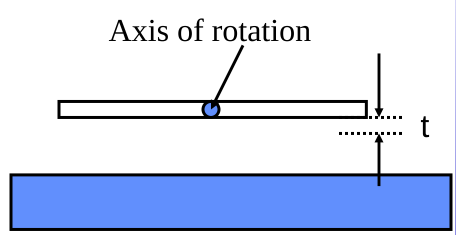
Traditional Micromirror Design

Number of resolvable spots:

$$N = \Delta\theta \cdot \frac{\pi}{1.18}\frac{\omega_m}{\lambda} + 1 \quad \&$$

$$\Delta\theta = 2\frac{t}{\omega_m} \quad \text{(one-sided motion)}$$

$$\Rightarrow N = \frac{2\pi}{1.18}\frac{t}{\lambda} + 1 \approx 5.3 \cdot \frac{t}{\lambda} + 1$$



MUMPs
sacrificial layer = 2.75 µm
N < 5 for t = 0.92 µm

Scanning Display $(\lambda = 0.5\mu\text{m})$:

$$N \approx 5.3 \cdot \frac{t}{\lambda} \approx 10 \cdot t \quad \mu m^{-1}$$

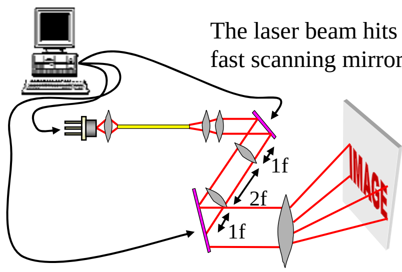
Challenges:

- Electrostatic instability
- Pure rotation

Video Display System Based on Microscanners (TV on a chip)

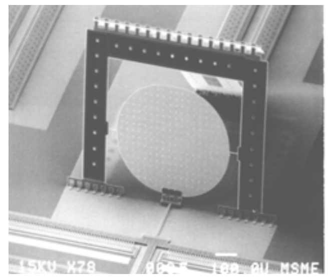
Computer controls the laser diode and both scanning mirrors

The laser beam hits the fast scanning mirror,



... is imaged onto the slow scanning mirror,

…and the image is projected onto the screen



Surface micromachined, flip-up scanning mirror

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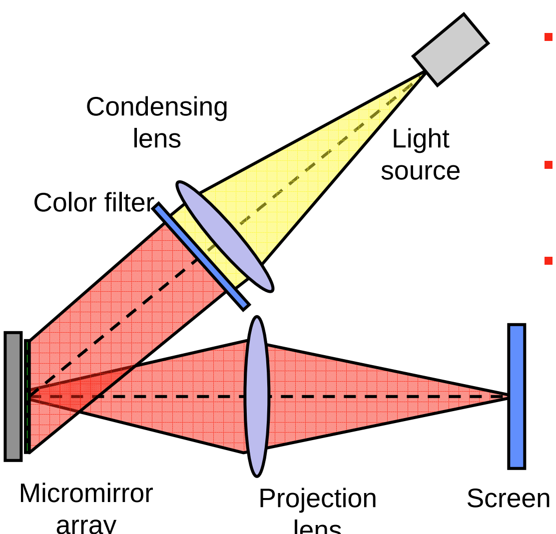
Projection Display



- Bistable micromirrors are used in projection displays (TI's DLP technology)
- Similar optical configurations are used in mask less lithography
- Other applications include spectroscopy and confocal microscopy in which the mirror array is used as a programmable spatial mask

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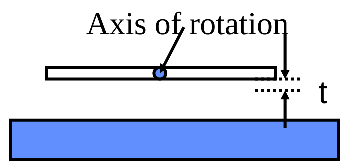
Projection Display Resolution

- In projection displays the contrast requirements are between -20 and -30 dB
- Conservatively:

$$d = 2r = 4\omega \Rightarrow$$

$$\theta_{diff} = 4\frac{\lambda}{\pi \cdot \omega_0}$$



Projection Display $(\lambda = 0.5\mu\text{m}, \text{N} = 2)$:

$$N = \pi\frac{t}{\lambda} + 1 \Rightarrow \text{t} \geq \frac{\lambda}{\pi} = 160\text{nm}$$

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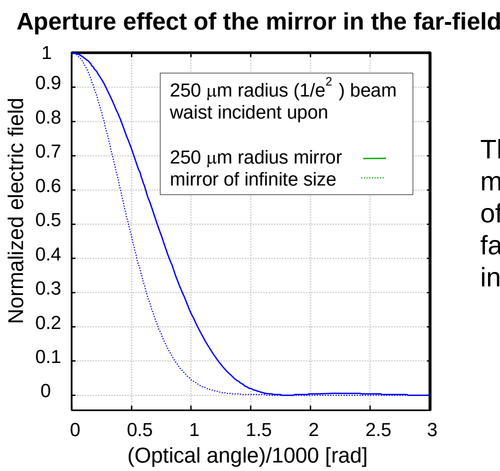
Aperture effect reduces resolution



The finite size of the mirror causes expansion of the optical beam in the far-field compared to an infinite-sized mirror.

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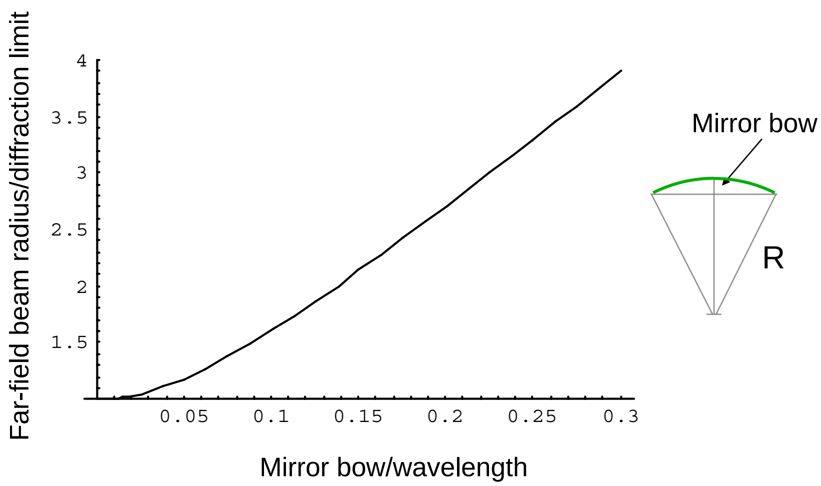
Resolution vs. Mirror Bow



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Micromirror status and prospects

- Micromirrors have unmatched resolution!
  - Challenges: Mirror quality (flatness, reproducibility), accuracy, speed, alignment (fiber coupling), packaging
  - High resolution requires high quality (DRIE of SOI)
  - Applications: Integrated scanners, TV on a chip, Fiber switches
- Micromirror arrays are well suited for traditional optical systems
  - Challenges: Multiplexing, Integration of MEMS & electronics
  - Applications: Projection displays, spectrometers, microscopy, lithography…..
- With more MEMS foundries, applications will explode!

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