INF5490 RF MEMS

LN08: RF MEMS resonators II

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Today's lecture

- Lateral vibrating resonator:

  Comb resonator

  - **Working principle**
  - Detailed **modeling**
    - A) ”phasor”-modeling
    - B) modeling by converting between mechanical and electrical energy domains

Lateral and vertical movement

- Lateral movement in the resonator
  - Parallel to substrate
  - Folded beam comb structure
- Vertical movement (next lecture)
  - Vertical to substrate
  - Clamped-clamped beam (c-c beam)
  - free-free beam (f-f beam)

Comb resonator

- Fixed comb + movable, suspended comb
- Suspended by folded springs, compact layout
- Total capacitance between the combs can be varied
- Applied bias (+ or -) generates an electrostatic force between left anchor-comb and ”shuttle”-comb. Shuttle pulled to the left in the plane

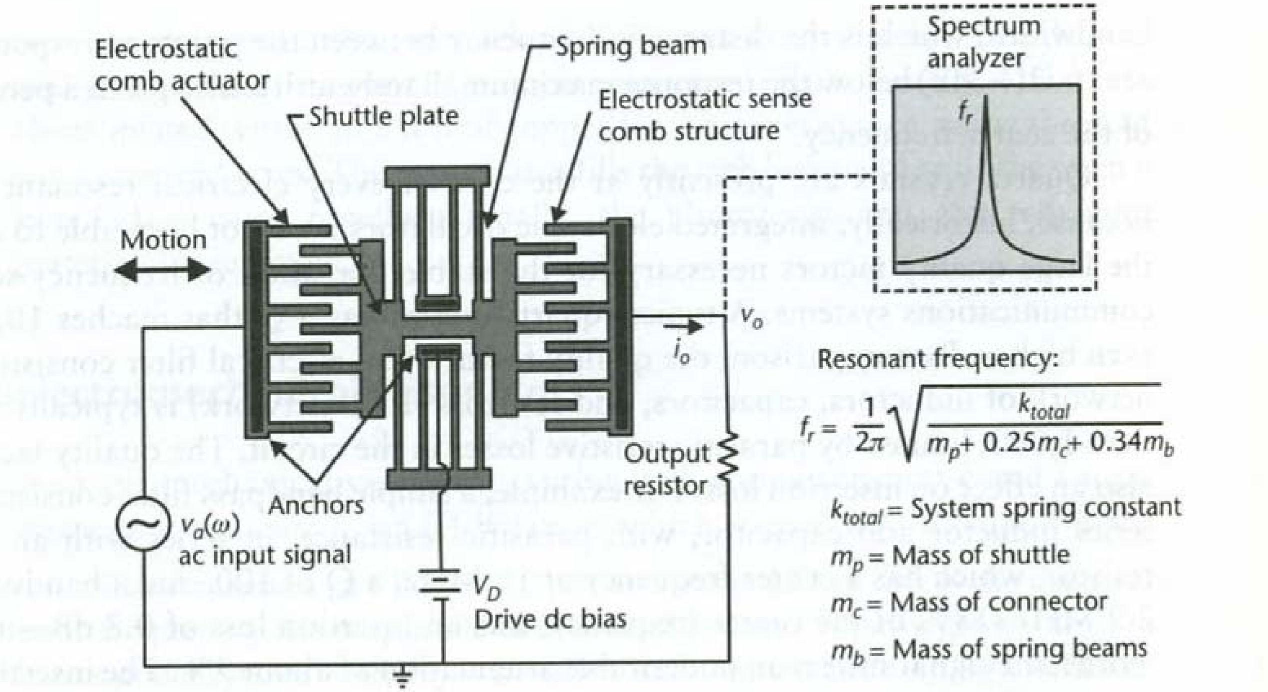


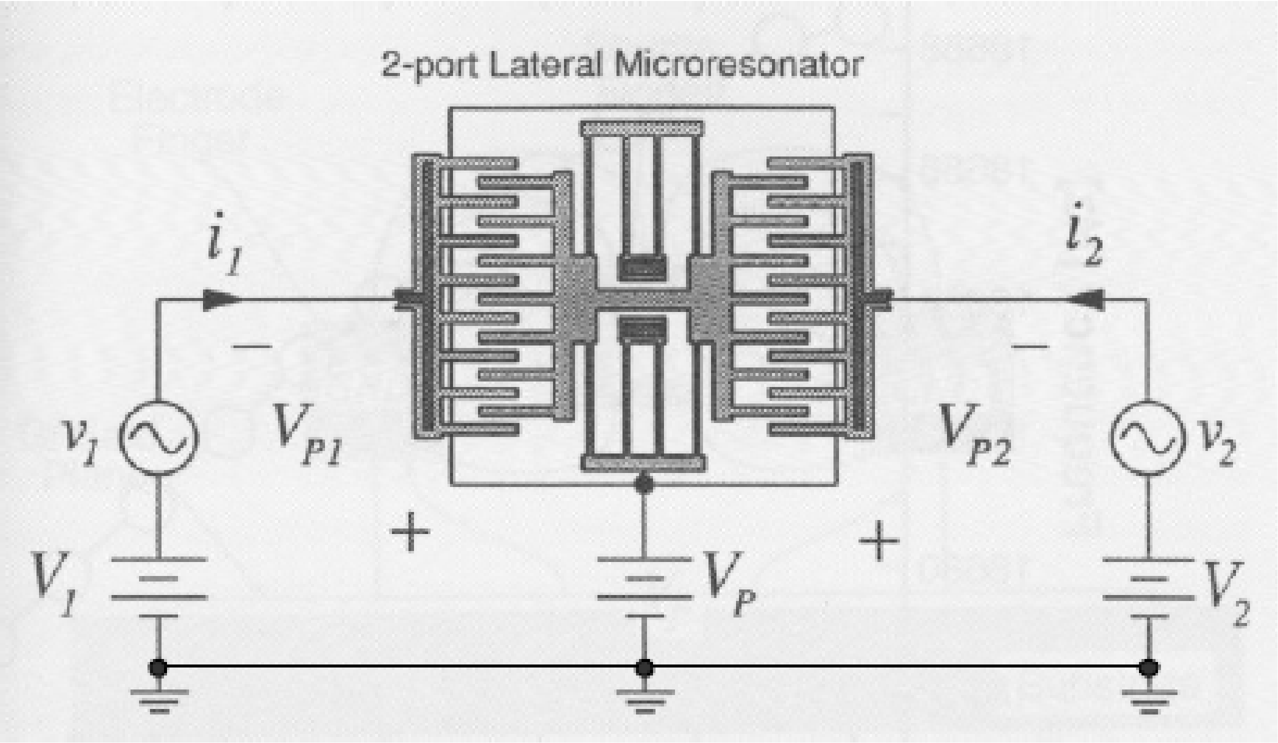
Figure 7.9 Illustration of a micromachined folded-beam comb-drive resonator. The left comb drive actuates the device at a variable frequency ω. The right capacitive-sense-comb structure measures the corresponding displacement by turning the varying capacitance into a current, which generates a voltage across the output resistor. There is a peak in displacement, current, and output voltage at the resonant frequency.

Detailed modeling

- Modeling of **lateral comb structure**
  - ”Phasor”-modeling ala UoC, Berkeley
    - Detailed calculations included
  - Conversion between energy domains
    - Material from UCLA

- In next lecture, LN09, the **c-c beam** will be modeled with reference to the book
  - T. Itoh et al: RF Technologies for Low Power Wireless Communications”, chap. 12: ”Transceiver Front-End Architectures Using Vibrating Micromechanical Signal Processors”, by Clark T.-C. Nguyen

The Lateral Resonator as a “Two-Port”



C. T.-C. Nguyen, Ph.D. Thesis, EECS Dept., UC Berkeley, 1994

Calculation procedure

- **A.** Model the comb as a two-port. Analyze first the input port
- **B.** When the comb moves the input capacitance will have a static and a variable component
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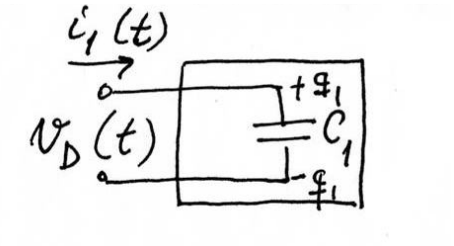
A. Model the comb as a two-port. Analyze first the input port

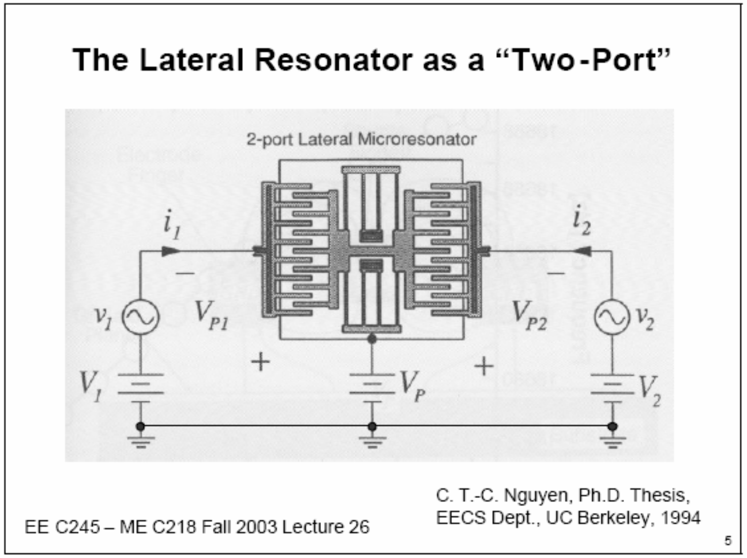


$$q_1 = C_1 v_D$$

$$\dot{q}_1(t) = i_1(t) = C_1 \frac{dv_D}{dt} + v_D \frac{dC_1}{dt}$$

$$v_D(t) = V_1 + v_1(t) - V_P = -V_{P1} + v_1 \cos \omega t$$

$$V_{P1} = V_P - V_1$$



The Lateral Resonator as a “Two-Port”

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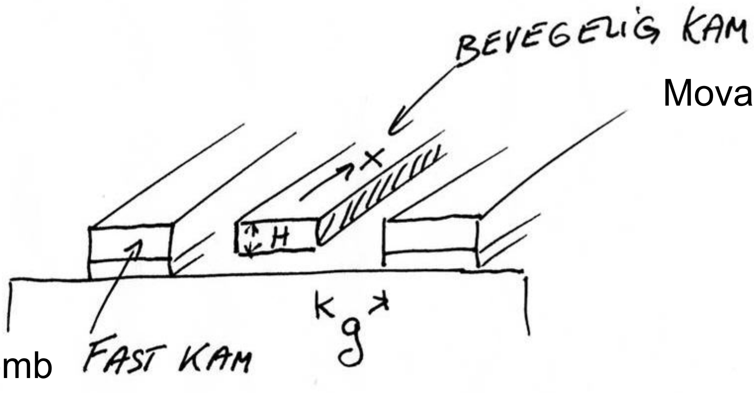
V_{P1} = positive when $V_P > V_1$

B. When the comb moves the input capacitance will have a static and variable component



Movable comb

Fixed comb

$$C_1(t) = C_{01} + C_{m1}(t)$$

$$C_1 = \frac{\varepsilon_0 A}{g} = \frac{\varepsilon_0 \cdot x \cdot 2H \cdot n}{g}$$

$$C_1(t) = C_{01}(\mathit{fixed}) + C_{m1}(\mathit{prop\_with\_}x(t))$$

$$C_1(t) = C_{01} + \frac{\partial C_1}{\partial x} \cdot x(t)$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \ldots$$

← general formula

C. Find the input current versus displacement, X

$$i_1(t) = C_1 \frac{dv_D}{dt} + v_D \frac{dC_1}{dt}$$

$$= C_1 \frac{dv_1(t)}{dt} + (-V_{P1} + v_1(t)) \frac{\partial C_1}{\partial x} \frac{\partial x}{\partial t} = \left[C_{01} + \frac{\partial C_1}{\partial x} \cdot x(t) \right] \frac{dv_1(t)}{dt} + \ldots$$

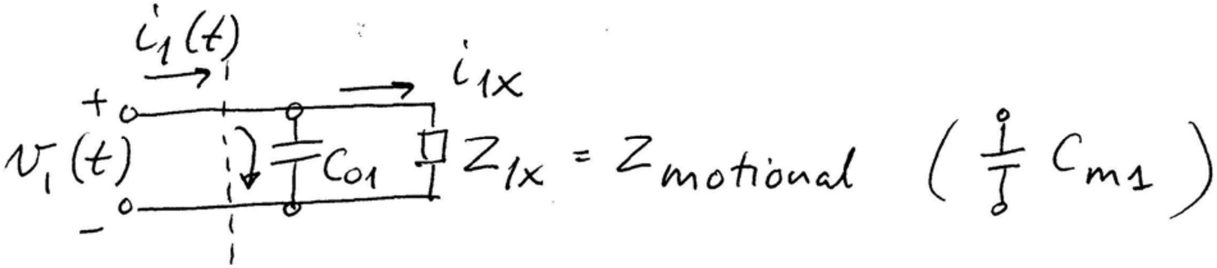
$$= C_{01} \frac{dv_1(t)}{dt} + \frac{\partial C_1}{\partial x} \cdot x(t) \cdot \frac{\partial v_1(t)}{\partial t} - V_{P1} \frac{\partial C_1}{\partial x} \frac{\partial x}{\partial t} + v_1(t) \frac{\partial C_1}{\partial x} \frac{\partial x}{\partial t}$$

$$= C_{01} \frac{dv_1(t)}{dt} + \frac{\partial C_1}{\partial x} \underbrace{(x \cdot \frac{\partial v_1}{\partial t} + v_1 \frac{\partial x}{\partial t})}_{\frac{\partial}{\partial t}(x \cdot v_1),\_where\_v_1 = v_0 \cos \omega t,\_x = x_0 \cos \omega t} - V_{P1} \frac{\partial C_1}{\partial x} \frac{\partial x}{\partial t}$$

$$(x \cdot v_1) \cong \cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$$

double frequency, small contribution

$$i_1(t) \approx \underbrace{C_{01}\frac{\partial v_1(t)}{\partial t}}_{\text{Current into the DC-capacitance}} - \underbrace{V_{P1}\frac{\partial C_1}{\partial x}\frac{\partial x(t)}{\partial t}}_{\text{"motional current"}}$$



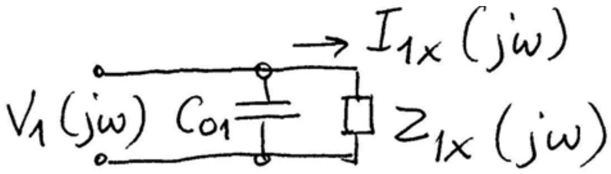
$$i_{1x}(t) = -V_{P1}\frac{\partial C_1}{\partial x}\frac{\partial x(t)}{\partial t} = (-V_{P1}\frac{\partial C_1}{\partial t})$$ ”motional current”

$$I_{1x}(j\omega) = -V_{P1}\frac{\partial C_1}{\partial x}\cdot j\omega\cdot X(j\omega)$$ phasor-form of ”motional current”

= current as function of movement (“displacement”)

D. Calculate the input admittance, Y (”motional admittance”)

- **D1.** Find Y versus X



$$Y_{1x}(j\omega)=\frac{I_{1x}(j\omega)}{V_1(j\omega)}=-V_{P1}\frac{\partial C_1}{\partial x}\cdot j\omega\cdot\frac{X(j\omega)}{V_1(j\omega)}$$

displacement

voltage

- **D2.** X depends on the electrostatic force, F, and m, b and k

$$Y_{1x}(j\omega)=-V_{P1}\frac{\partial C_1}{\partial x}\cdot j\omega\cdot\frac{X(j\omega)}{F_d(j\omega)}\cdot\frac{F_d(j\omega)}{V_1(j\omega)}$$

F_d depends of m,b og k

voltage V_1 creates an electrostatic force F_d

D3. F depends on the applied bias, V

Relationship between force and voltage can be found from:

$$U = \frac{1}{2} C_1 v_D{}^2(t)$$

Potential energy, v_D is independent of x

$$F = \frac{\partial U}{\partial x} = \frac{1}{2} v_D{}^2(t) \cdot \frac{\partial C_1}{\partial x}$$

non-linear relation

$$F = F_0 + f \cos \omega t, \;\; v_D = -V_{P1} + v_1 \cos \omega t$$

Linearizing around a DC-point

$$F_0 + f \cos \omega t = \frac{1}{2}(-V_{P1} + v_1 \cos \omega t)^2 \cdot \frac{\partial C_1}{\partial x}$$

Substitute

$$= \frac{1}{2}(V_{P1}{}^2 - 2 \cdot V_{P1} \cdot v_1 \cos \omega t + v_1{}^2 \underbrace{\cos^2 \omega t}_{\cos 2\omega t - term}) \cdot \frac{\partial C_1}{\partial x}$$

$$f \cos \omega t = -V_{P1} \cdot v_1 \cos \omega t \cdot \frac{\partial C_1}{\partial x}$$

Comparing AC-terms

$$f_{d,\omega} = -V_{P1} \frac{\partial C_1}{\partial x} v_1(t)$$

← LINEAR RELATION!

$$F_d(j\omega) = -V_{P1}\frac{\partial C_1}{\partial x}\cdot V_1(j\omega)$$

In phasor-form

$$\frac{F_d(j\omega)}{V_1(j\omega)} = -V_{P1}\frac{\partial C_1}{\partial x}$$

Relation between displacement and force:

$$\frac{X(s)}{F_d(x)} = \frac{1}{ms^2+bs+k} = \frac{1}{k}\frac{k/m}{s^2+\frac{b}{m}s+\frac{k}{m}}$$

D2. X depends on the electrostatic force, F, and m, b and k

$$\omega_0{}^2 = k/m,\ \ b/m = \omega_0/Q$$

$$Q = \frac{\sqrt{k/m}}{b/m} = \frac{\sqrt{km}}{b}$$

Substitute

$$\frac{X(s)}{F_d(s)} = \frac{1}{k}\cdot\frac{\omega_0{}^2}{s^2+\frac{\omega_0}{Q}s+\omega_0{}^2} \rightarrow_{s=j\omega} \frac{1}{k}\cdot\frac{\omega_0{}^2}{(\omega_0{}^2-\omega^2)+j\frac{\omega_0\omega}{Q}}$$

$$\frac{X(j\omega)}{F_d(j\omega)} = \frac{1}{k}\cdot\frac{1}{\left[1-(\omega/\omega_0)^2\right]+j\frac{\omega}{Q\omega_0}}$$

E. Find an expression for Y (dynamic behavior)

$$Y_{1x}(j\omega) = -V_{P1}\frac{\partial C_1}{\partial x}\cdot j\omega\cdot\frac{X(j\omega)}{F_d(j\omega)}\cdot\frac{F_d(j\omega)}{V_1(j\omega)}$$

$$= -V_{P1}\frac{\partial C_1}{\partial x}\cdot j\omega\cdot\frac{1/k}{\left[1-(\omega/\omega_0)^2\right]+j\dfrac{\omega}{\omega_0 Q}}\cdot(-V_{P1}\frac{\partial C_1}{\partial x})$$

$$\eta = V_{P1}\frac{\partial C_1}{\partial x}$$

← η defined

$$Y_{1x}(j\omega) = \eta^2\cdot j\omega\cdot\frac{1/k}{\left[1-(\omega/\omega_0)^2\right]+j\dfrac{\omega}{\omega_0 Q}}$$

$$I_{1x}(j\omega) = [\ldots]\cdot V_1(j\omega)$$

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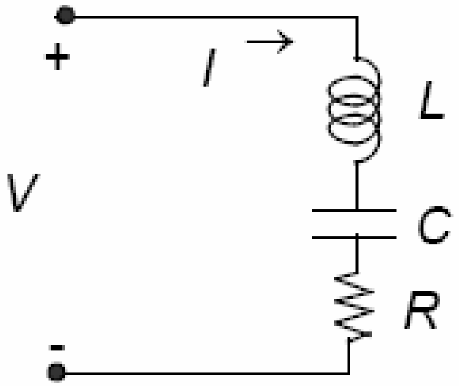
F.

Series *L-C-R* Admittance

The current through an *L-C-R* branch is:

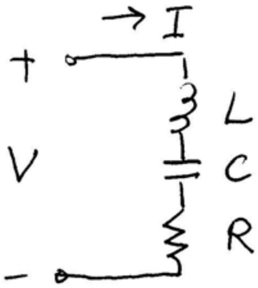


$$\frac{I(j\omega)}{V(j\omega)} = \frac{j\omega C}{1-(\omega/\omega_o)^2 + j(\omega RC)}$$

$$\omega_o^{-2} = LC$$

Match terms in motional admittance → find equivalent elements

Current through the L-C-R-circuit



$$V = I(sL + 1/sC + R)$$

$$\frac{I(s)}{V(s)} = \frac{sC}{s^2LC + sRC + 1}$$

$$Y(j\omega) = \frac{I(j\omega)}{V(j\omega)} = \frac{j\omega C}{-\omega^2 LC + j\omega RC + 1}$$

Introduce

$$\omega_0{}^2 = \frac{1}{LC},\ \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Y(j\omega) = \frac{j\omega C}{\left[1-(\omega/\omega_0)^2\right] + j\omega RC} = \frac{j\omega C}{\left[...\right] + j\dfrac{\omega}{\omega_0 Q}}$$

$$RC = \frac{1}{\omega_0 Q},\ Q = \frac{1}{\omega_0 RC} = \frac{\sqrt{LC}}{RC} = \sqrt{\frac{L}{C}} \cdot \frac{1}{R}$$

Which gives:

$$Y(j\omega) = \frac{j\omega C}{\left[1-(\omega/\omega_0)^2\right] + j\frac{\omega}{\omega_0 Q}}$$

Compare to

$$Y_{1x}(j\omega) = \eta^2 \cdot \frac{j\omega \cdot 1/k}{\left[1-(\omega/\omega_0)^2\right] + j\frac{\omega}{\omega_0 Q}}$$

This results in:

$$C_{x1} = \eta^2 / k$$

$$\omega_0{}^2 = k/m = 1/LC \Rightarrow L_{x1} = \frac{1}{C}\cdot\frac{m}{k} = \frac{k}{\eta^2}\cdot\frac{m}{k} = \frac{m}{\eta^2}$$

$$RC = \frac{1}{Q\omega_0} = \frac{1}{Q\sqrt{k/m}} \Rightarrow R_{x1} = \frac{1}{C}\cdot\frac{1}{Q\sqrt{k/m}} = \frac{k}{\eta^2}\frac{\sqrt{m}}{Q\sqrt{k}} = \frac{\sqrt{km}}{Q\eta^2}$$

η = Electromagnetic coupling coefficient

$$I_{x1}(\omega_0) = \frac{V_1(\omega_0)}{R_{x1}}$$

At resonance the impedances from L and C cancel

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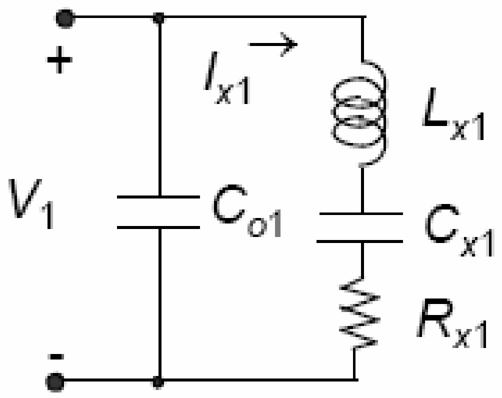
G. Equivalent Circuit for Input Port

A series L-C-R circuit results in the identical expression → find equivalent values L_{x1}, C_{x1}, and R_{x1}

$$L_{x1} = \frac{m}{\eta^2} \qquad C_{x1} = \frac{\eta^2}{k} \qquad R_{x1} = \frac{\sqrt{km}}{Q\eta^2} \qquad \eta = V_{P1}\frac{\partial C_1}{\partial x} =$$ electromechanical coupling coefficient



At resonance, the impedances of the inductance and the capacitance *cancel out* →

$$I_{x1} = \frac{V_1}{R_{x1}}$$

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H. Find the output current for a given input

$$i_{1x}(t) = -V_{P1}\frac{\partial C_1}{\partial t}$$

This displacement causes the output capacitance C2 also to change.
Output current due to displacement (v_2 = 0V, short-circuited):

$$i_2(t) = -V_{P2}\frac{\partial C_2}{\partial t} = -V_{P2}\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$$

$$I_2(j\omega) = -V_{P2}\frac{\partial C_2}{\partial x}\cdot j\omega\cdot X(j\omega)$$

In phasor-form

$$X(j\omega) = \frac{1/k}{\left[1-(\omega/\omega_0)^2\right]+j\dfrac{\omega}{\omega_0 Q}}\cdot F_d(j\omega)$$

$$F_d(j\omega) = -V_{P1}\frac{\partial C_1}{\partial x}\cdot V_1(j\omega)$$

voltage → force → displacement → current

$$\Rightarrow I_2(j\omega) = \frac{V_{P1}V_{P2}\dfrac{\partial C_1}{\partial x}\dfrac{\partial C_2}{\partial x}}{\left[1-(\omega/\omega_0)^2\right]+j\dfrac{\omega}{\omega_0 Q}}\cdot j\omega\cdot(1/k)\cdot V_1(j\omega)$$

I. Calculate the ratio between the output and input currents (”forward current gain”)

”Forward current gain”

$$\Phi_{21}=\frac{I_2(j\omega)}{I_{x1}(j\omega)}=\frac{-V_{P2}\dfrac{\partial C_2}{\partial x}\cdot j\omega\cdot X(j\omega)}{-V_{P1}\dfrac{\partial C_1}{\partial x}\cdot j\omega\cdot X(j\omega)}=\frac{V_{P2}}{V_{P1}}\frac{\dfrac{\partial C_2}{\partial x}}{\dfrac{\partial C_1}{\partial x}}$$

$$I_2(j\omega)=\Phi_{21}\cdot I_{x1}(j\omega),\ V_2=0$$

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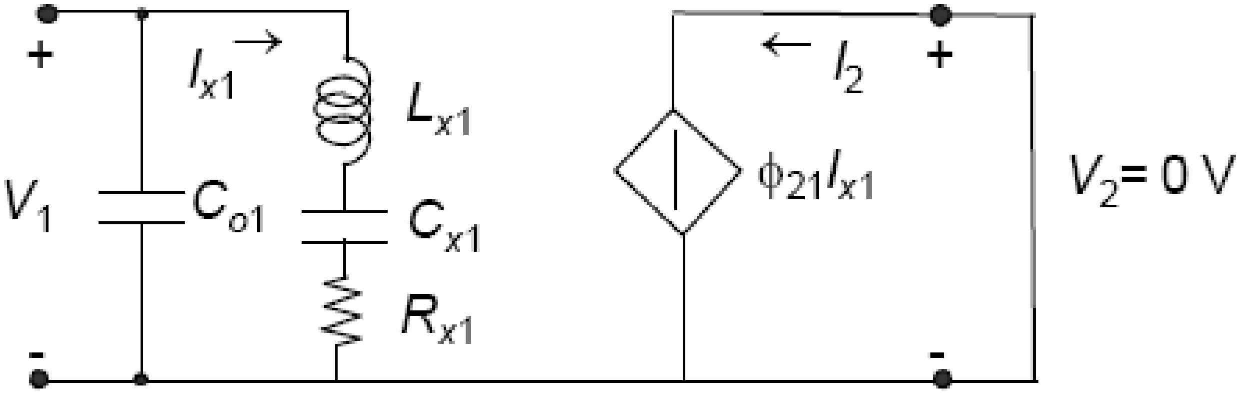
Two-Port Equivalent Circuit ($v_2 = 0$)



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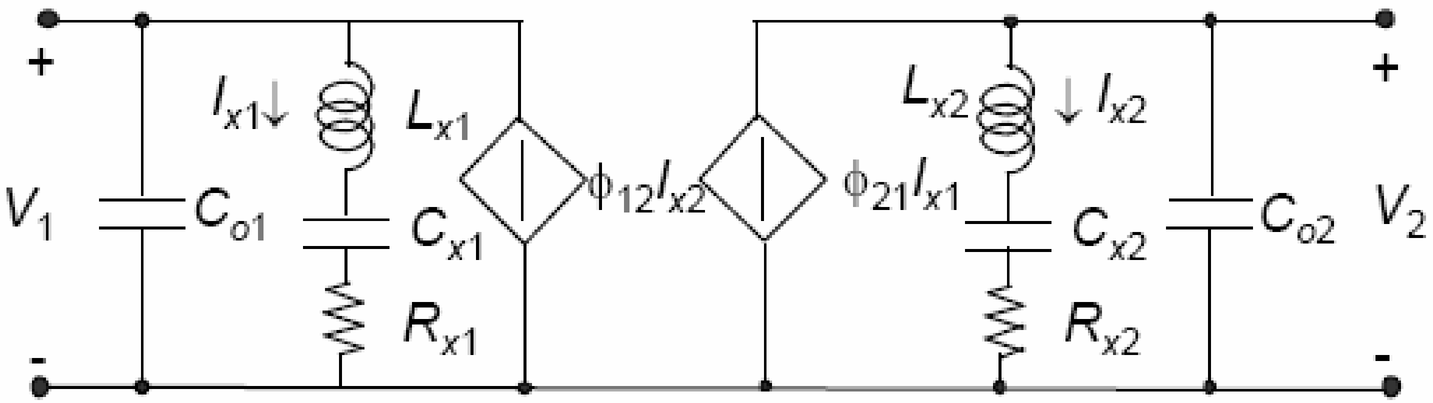
K.

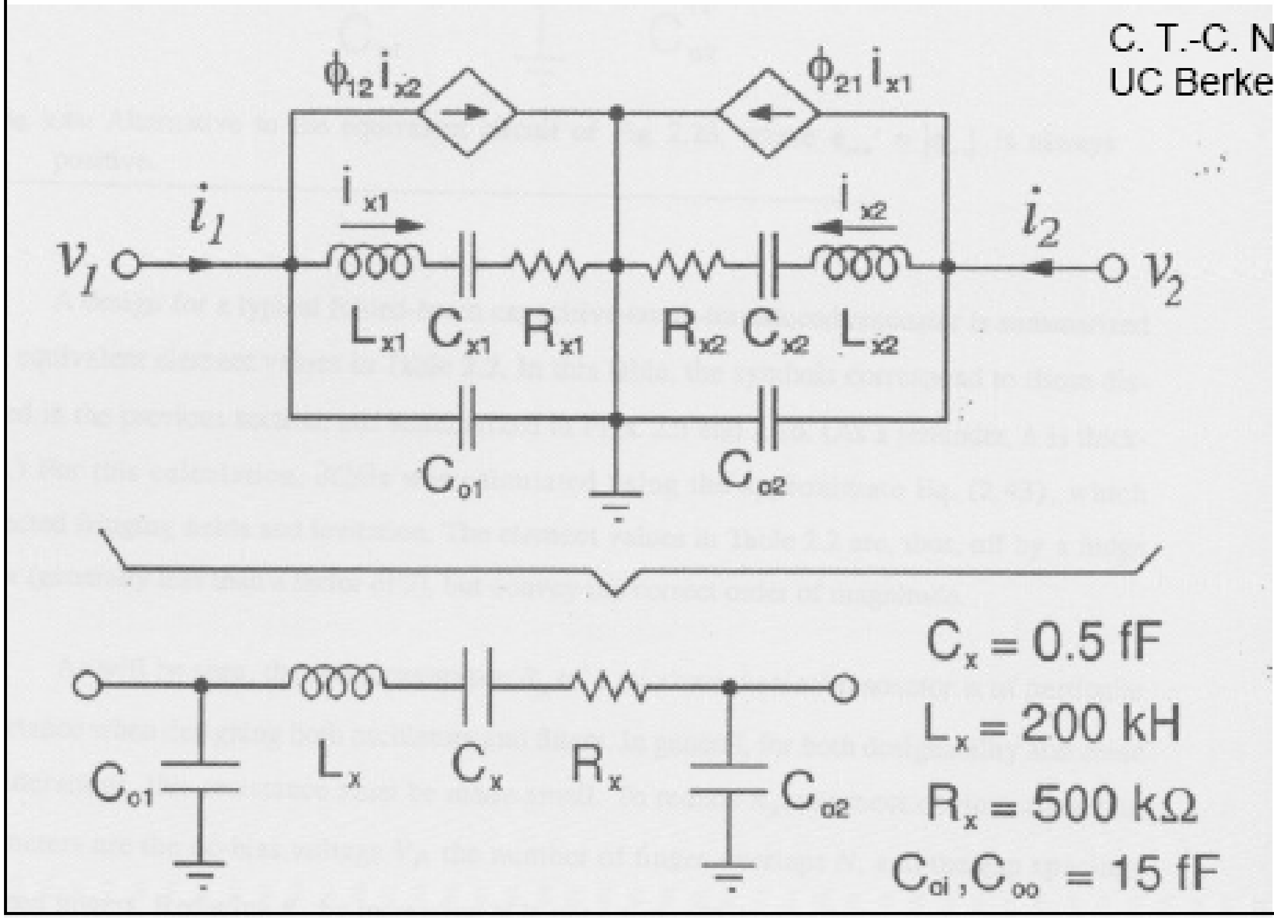
Complete Two-Port Model



Symmetry implies that modeling can be done from port 2, with port 1 shorted → superimpose the two models

Equivalent Circuit for Symmetrical Resonator ($\phi_{21} = \phi_{12} = 1$)

C. T.-C. Nguyen, Ph.D.,
UC Berkeley, 1994



$C_x = 0.5$ fF

$L_x = 200$ kH

$R_x = 500$ kΩ

$C_{oi}, C_{oo} = 15$ fF

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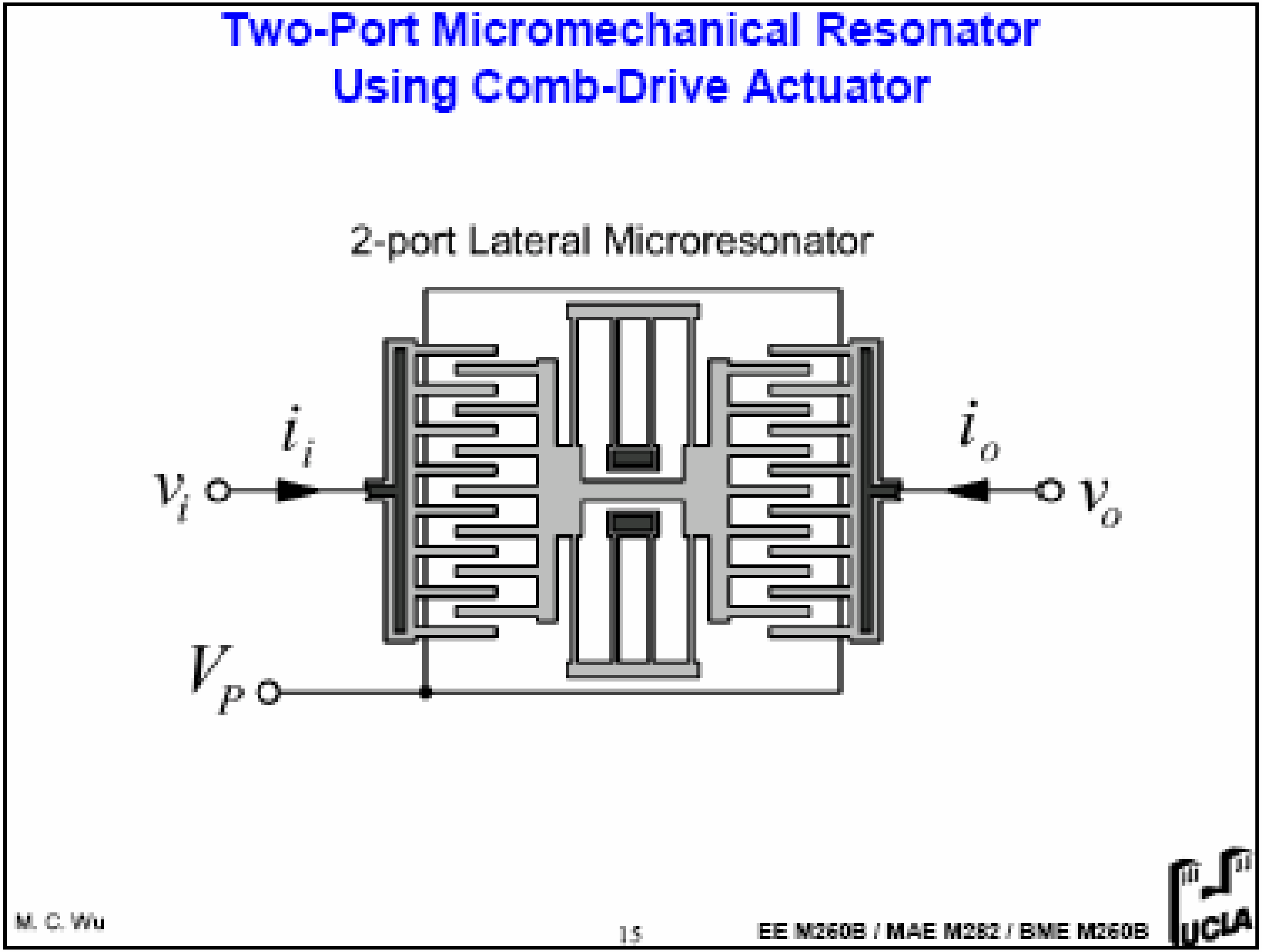
Alternative modeling

- Exploit conversion between mechanical and electrical energy domains
  - Slides from UCLA

- Supported by lecture notes →



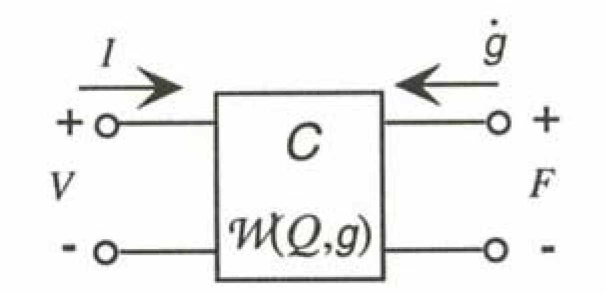
Two-Port Micromechanical Resonator Using Comb-Drive Actuator

2-port Lateral Microresonator

Conversion between energy domains

- Both vertical and lateral resonator structures may be described by a **generalized non-linear capacitance, C**, interconnecting energy-domains

Electrical domain



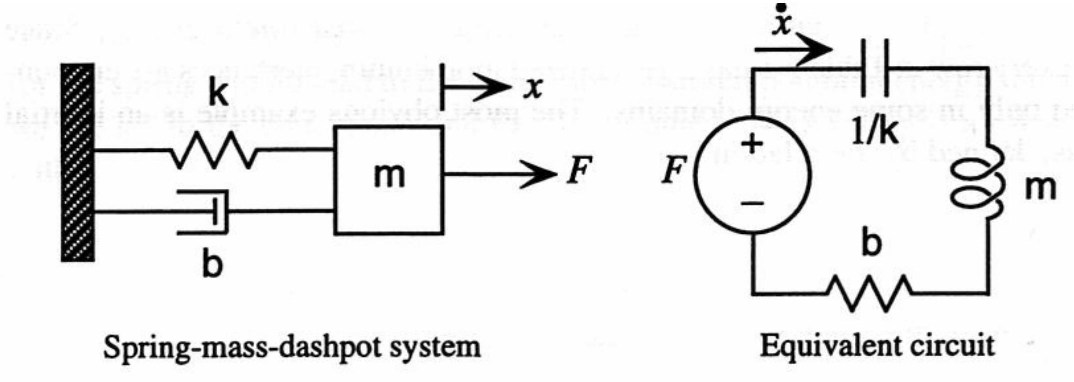
Mechanical domain

Transducer

Interconnecting where there is **no energy loss**

Procedure

- First, transform the mechanical domain impedances to an electrical representation
  - The mechanical components are modeled as lumped electrical components
- NB! You are still in the mechanical domain!
  - C = 1/k
  - L = m
  - R = b



Spring-mass-dashpot system

Equivalent circuit

- Power-variables
  - Effort = force → voltage
  - Flow = velocity → current

Interconnecting different energy domains

- 1. Each energy domain is transformed to its electrical equivalent
- 2. Domains are interconnected by a generalized non-linear capacitance, C
- 3. Transformer and gyrator may be used for **interconnecting** if a **linear** **relationship** exists between the power-variables!
  - Problem: Transducer C is generally **NOT** a linear 2-port
- 4. Then, must **linearize** the 2-port transducer to be able to substitute it with a **transformer**
- 5. The transformer can ”be removed” by recalculating the component **values** to **new** ones
  - → **Electromechanical coupling coefficient used! =** turn ratio
  - → Results in a common circuit diagram

Interaction between energy domains

- Suppose **linear** relation between power variables
  - A linear 2-port element can be used:
  - Use a transformer or gyrator

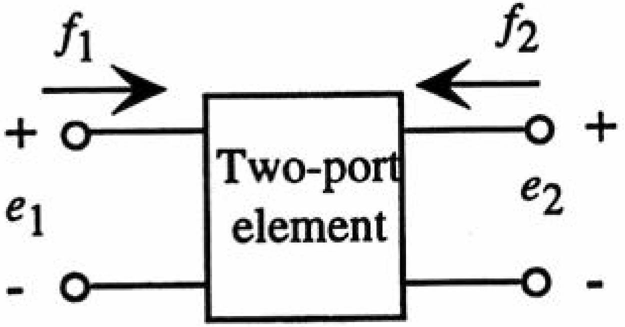
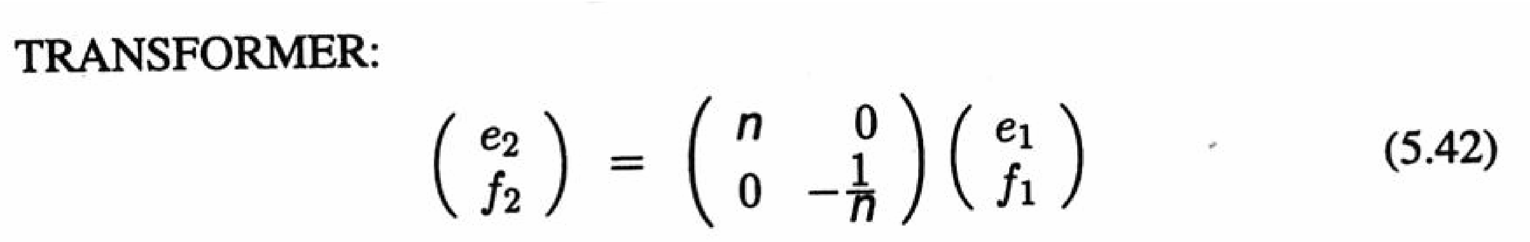


Figure 5.11. General two-port element.

power in = power out NO POWER LOSS

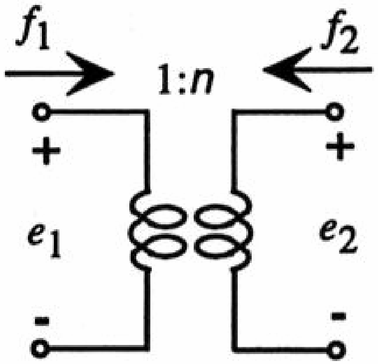
$$e_1 f_1 + e_2 f_2 = 0 \tag{5.41}$$

Transformer



TRANSFORMER:

$$\begin{pmatrix} e_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \begin{pmatrix} e_1 \\ f_1 \end{pmatrix} \tag{5.42}$$



Transformer

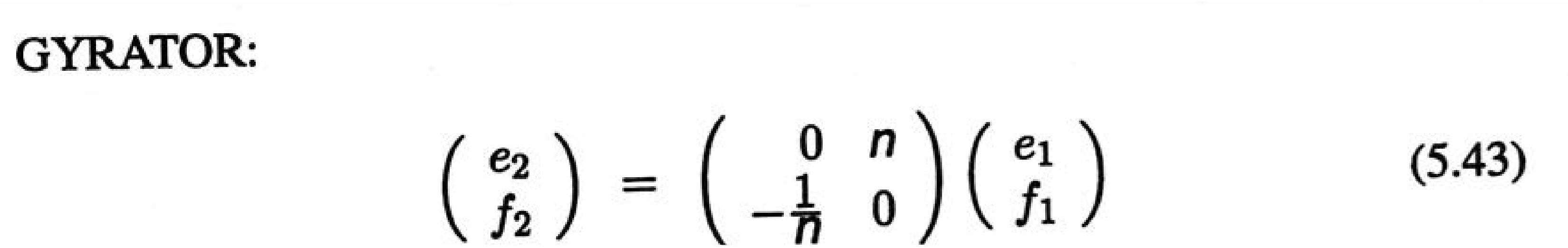
$$e_2 = n \cdot e_1$$

$$f_2 = -\frac{1}{n} f_1$$

n = ”turns ratio”

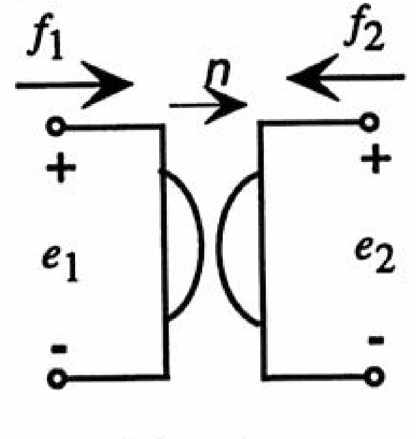
Ex. V and F can be interconnected

Gyrator



GYRATOR:

$$\begin{pmatrix} e_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} 0 & n \\ -\frac{1}{n} & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ f_1 \end{pmatrix} \qquad (5.43)$$



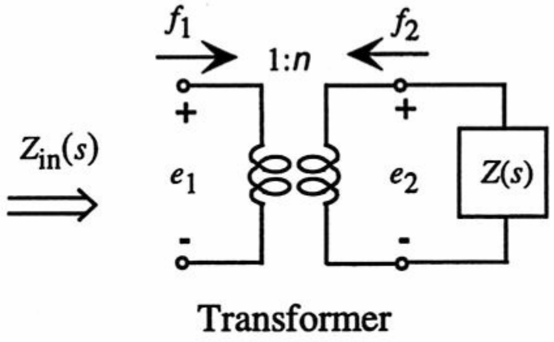
Gyrator

$$e_2 = n \cdot f_1$$

$$f_2 = -\frac{1}{n} e_1$$

The impedances can be transformed

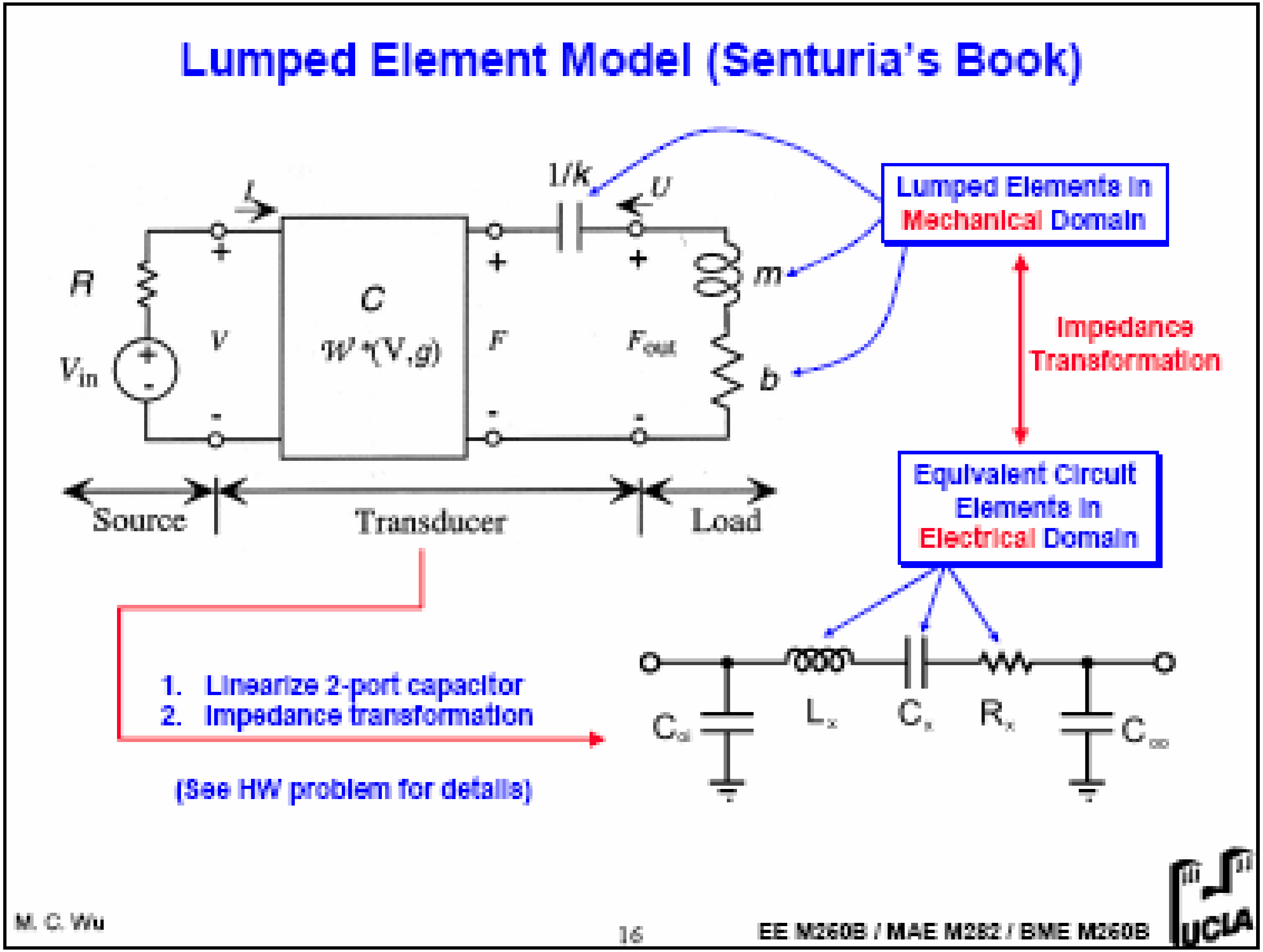
$$Z_{in}(s) = \frac{Z(s)}{n^2}$$



n = coupling coefficient between energy domains

$$Z_{in}(s) = \frac{e_1}{f_1}$$

$$Z(s) = \frac{e_2}{-f_2} = \frac{n \cdot e_1}{\frac{1}{n} \cdot f_1} = n^2 \cdot \frac{e_1}{f_1} = n^2 \cdot Z_{in}(s)$$



Lumped Element Model (Senturia's Book)

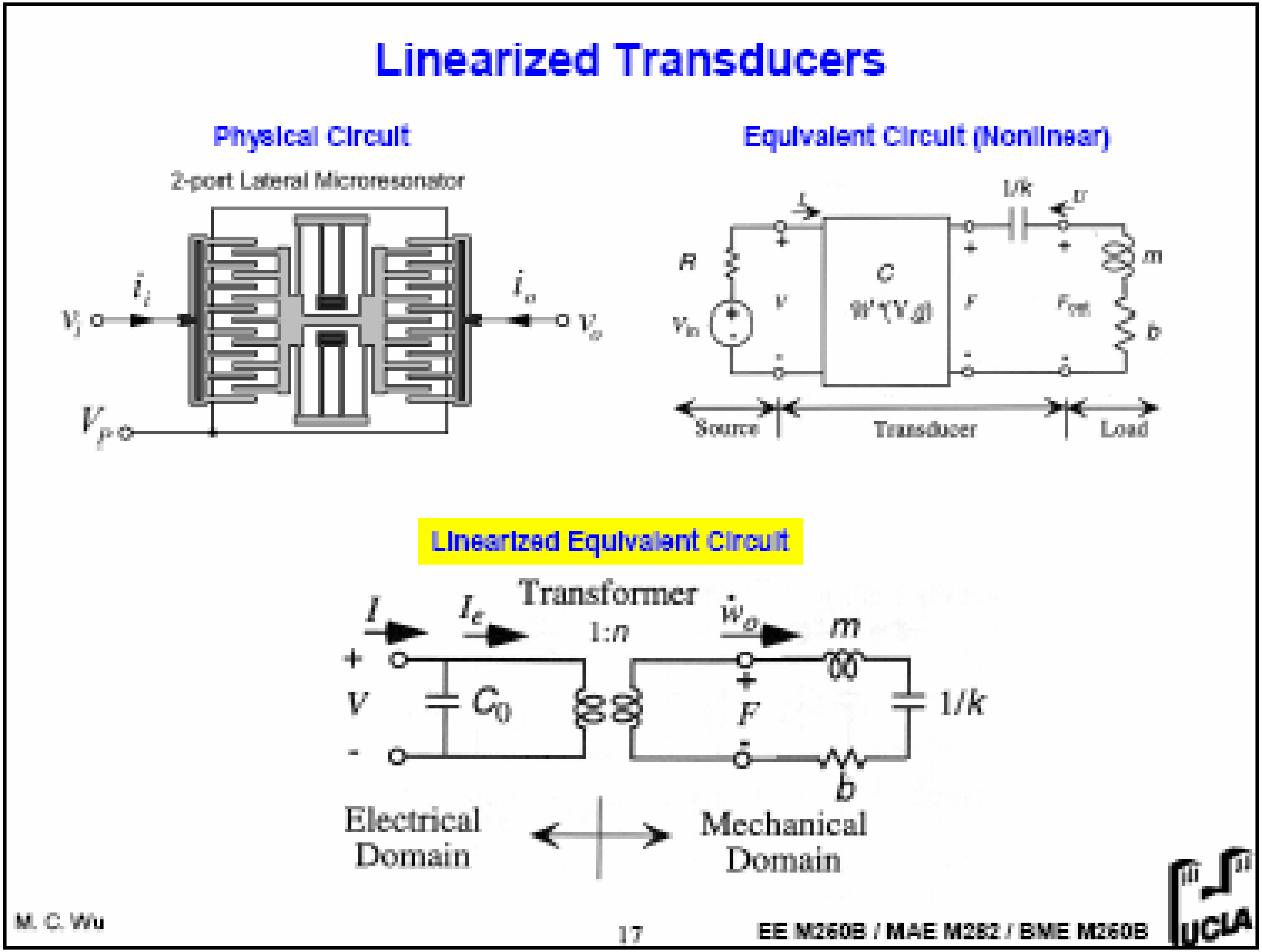
Lumped Elements in Mechanical Domain

Impedance Transformation

Equivalent Circuit Elements in Electrical Domain

1. Linearize 2-port capacitor
2. Impedance transformation

(See HW problem for details)



Linearized Transducers

Physical Circuit

2-port Lateral Microresonator

Equivalent Circuit (Nonlinear)

Linearized Equivalent Circuit

Procedure

- Investigate relation between ”efforts” and ”flows” in the 2 domains
- **Efforts:** calculation procedure
  - – 1. Start with an expression for potential energy
  - – 2. Calculate force
  - – 3. Look at perturbations around the DC-bias
  - – 4. Find the relationship between AC-terms
    - → A linear relationship is obtained!

Relation between ”efforts”

$$F = \frac{\partial W^*}{\partial x} = \frac{1}{2} V^2 \frac{\partial C}{\partial x}$$

$$F = F_{dc} + f \cdot \sin(\omega t)$$

$$V = V_{dc} + v \cdot \sin(\omega t)$$

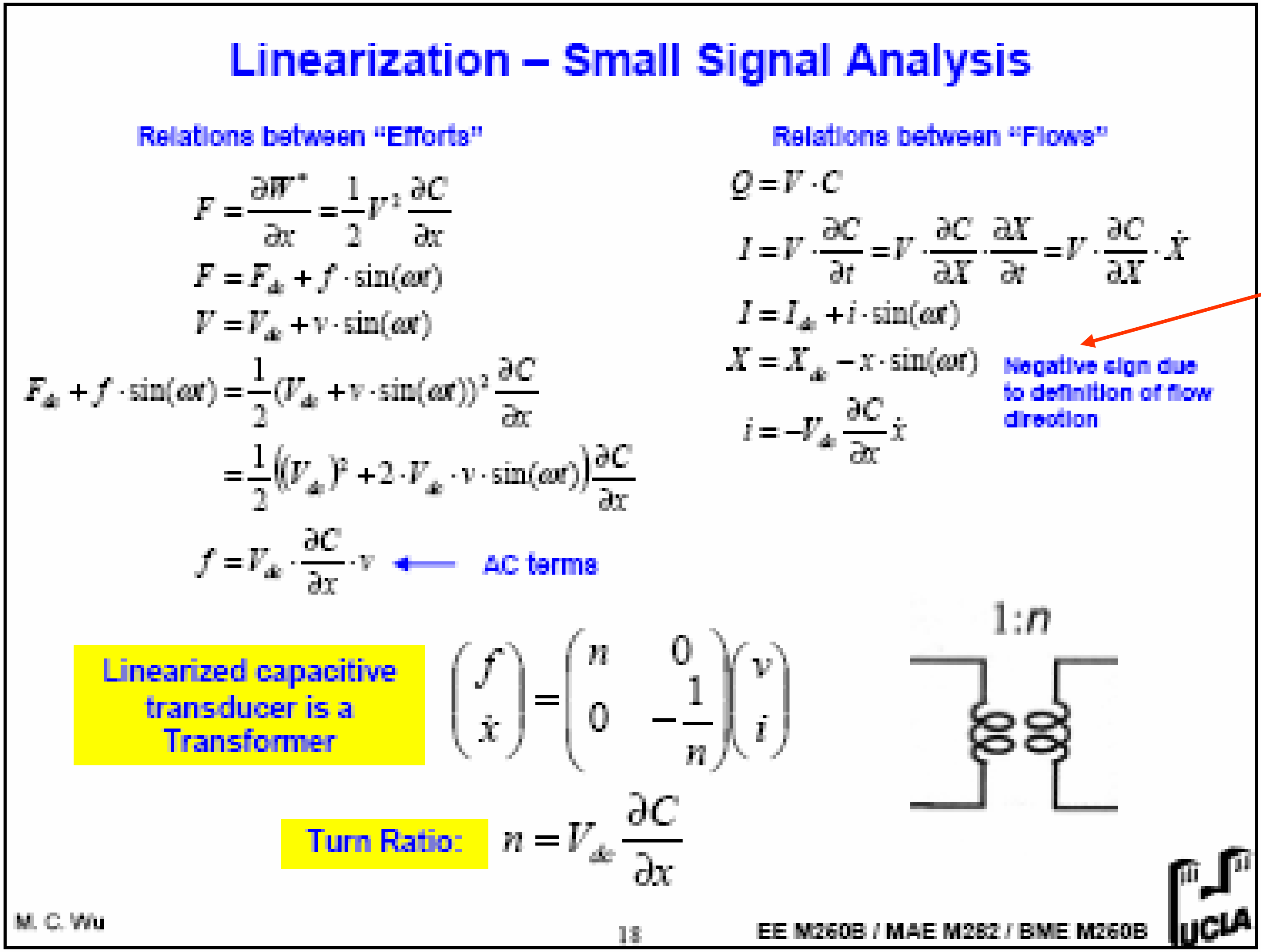
$$F_{dc} + f \cdot \sin(\omega t) = \frac{1}{2}(V_{dc} + v \cdot \sin(\omega t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2}\left((V_{dc})^2 + 2 \cdot V_{dc} \cdot v \cdot \sin(\omega t)\right)\frac{\partial C}{\partial x}$$

$$f = V_{dc} \cdot \frac{\partial C}{\partial x} \cdot v \quad \longleftarrow \quad \text{AC terms}$$

effort (mechanical domain) = const. * effort (electrical domain)

Similarly for relationship between FLOWS:



Linearization – Small Signal Analysis

Relations between "Efforts"

$$F = \frac{\partial W^*}{\partial x} = \frac{1}{2}V^2\frac{\partial C}{\partial x}$$

$$F = F_{dc} + f\cdot\sin(\omega t)$$

$$V = V_{dc} + v\cdot\sin(\omega t)$$

$$F_{dc} + f\cdot\sin(\omega t) = \frac{1}{2}(V_{dc} + v\cdot\sin(\omega t))^2\frac{\partial C}{\partial x}$$

$$= \frac{1}{2}\left((V_{dc})^2 + 2\cdot V_{dc}\cdot v\cdot\sin(\omega t)\right)\frac{\partial C}{\partial x}$$

$$f = V_{dc}\cdot\frac{\partial C}{\partial x}\cdot v \longleftarrow$$ AC terms

Relations between "Flows"

$$Q = V\cdot C$$

$$I = V\cdot\frac{\partial C}{\partial t} = V\cdot\frac{\partial C}{\partial X}\cdot\frac{\partial X}{\partial t} = V\cdot\frac{\partial C}{\partial X}\cdot\dot{X}$$

$$I = I_{dc} + i\cdot\sin(\omega t)$$

$$X = X_{dc} - x\cdot\sin(\omega t)$$

$$i = -V_{dc}\frac{\partial C}{\partial x}\dot{x}$$

Negative sign due to definition of flow direction

Linearized capacitive transducer is a Transformer

$$\begin{pmatrix} f \\ \dot{x} \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix}\begin{pmatrix} v \\ i \end{pmatrix}$$

Turn Ratio: $n = V_{dc}\frac{\partial C}{\partial x}$

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flow (electrical domain) = - const. * flow (mechanical domain)

Current direction, mechanical domain

- Flow in the mechanical domain is defined as positive **into** the 2-port transducer
- Choose the current to go **out of** 2-port C. Then we have:
  - Current goes **into** the electrical domain
  - → creates an attractive force on the comb
  - → spring stretches
  - → potential energy is built up
  - → equivalent to charging of an 1/k-capacitor

  - →Current increases → charge on the capacitor increases → attractive force increases → displacement (x) decreases

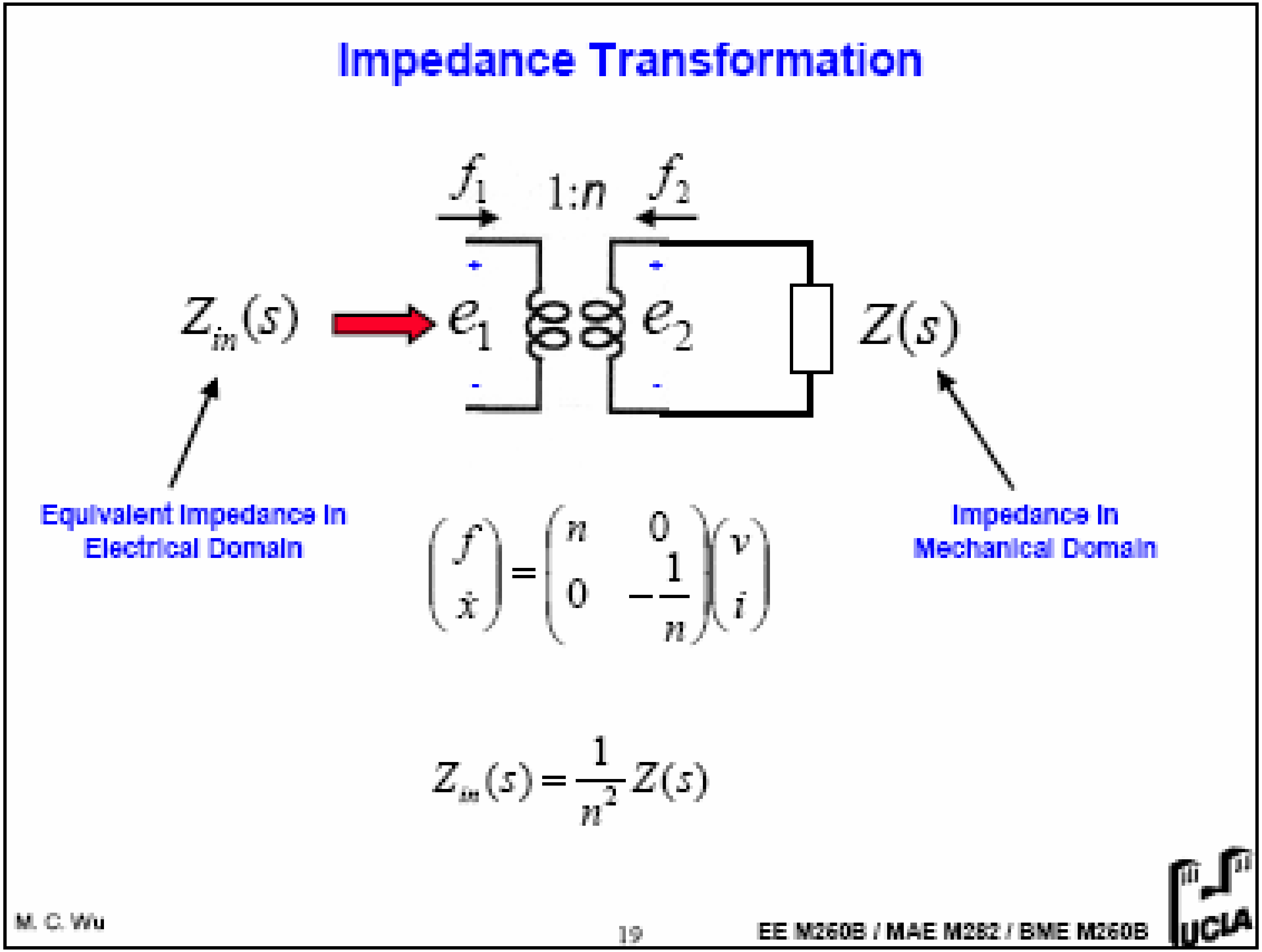
Compatible relations both between ”efforts” and ”flows”

$$f = V_{dc} \cdot \frac{\partial c}{\partial x} \cdot v = n \cdot v \qquad \text{där} \quad n = V_{dc} \cdot \frac{\partial c}{\partial x}$$

$$i = -V_{dc} \cdot \frac{\partial c}{\partial x} \cdot \dot{x} = -n \cdot \dot{x} \quad \Rightarrow \quad \dot{x} = -\frac{1}{n} \cdot i$$

- effort (mechanical domain) = n * effort (electrical domain)
- flow (mechanical domain) = -1/n * flow (electrical domain)

- A linearized capacitive transducer implemented as a **transformer** can be used!



Impedance Transformation

Equivalent Impedance In Electrical Domain

Impedance In Mechanical Domain

$$\begin{pmatrix} f \\ \dot{x} \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \begin{pmatrix} v \\ i \end{pmatrix}$$

$$Z_{in}(s) = \frac{1}{n^2} Z(s)$$

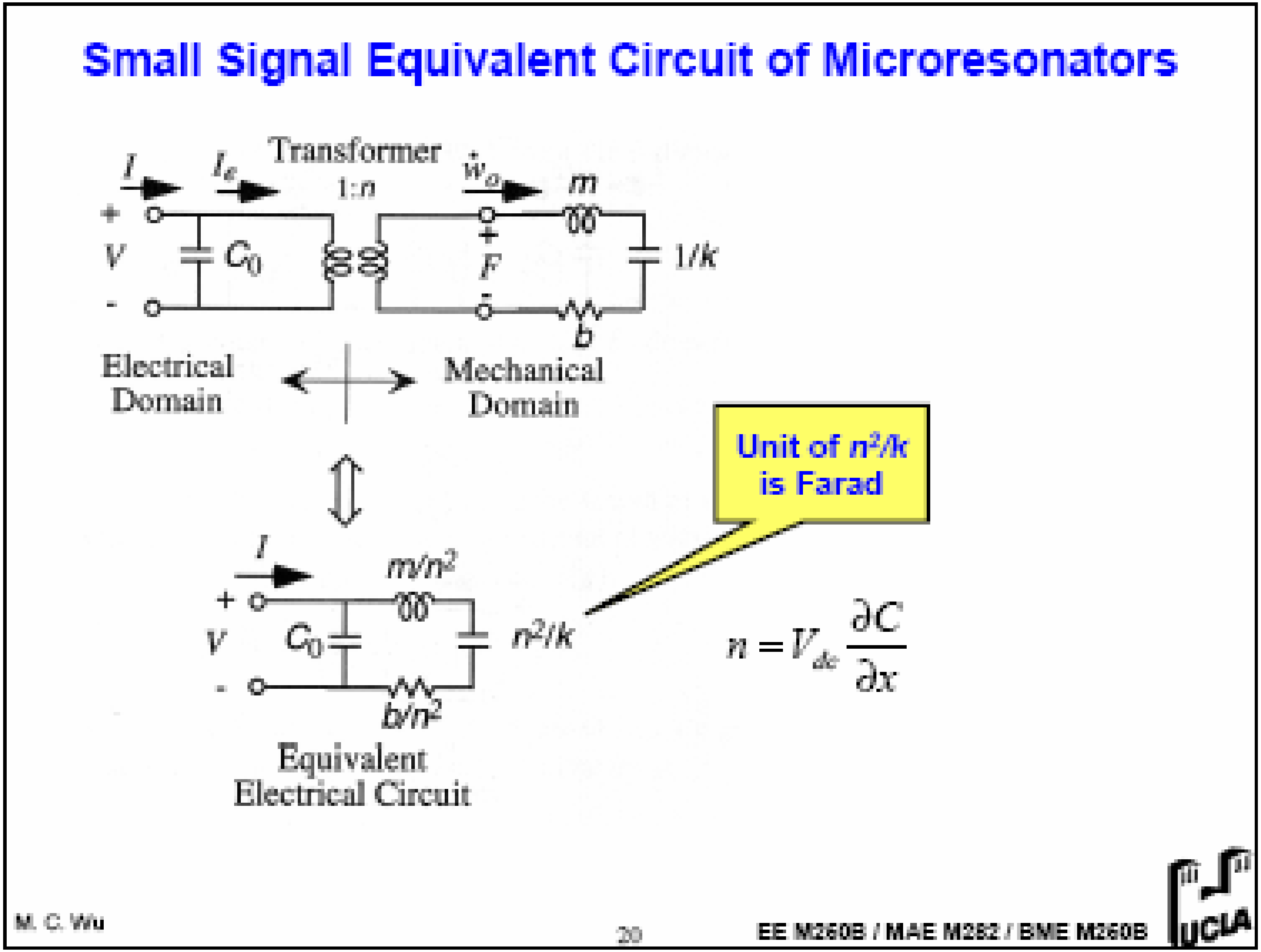
Transformation of impedances

$$z_{el} = \frac{1}{n^2} \cdot z_{mek}$$

Inductor $$sL_{el} = \frac{1}{n^2} \cdot sL_{mek} = \frac{sm}{n^2} \Rightarrow L_{el} = \frac{m}{n^2}$$

Resistor $$R_{el} = \frac{1}{n^2} \cdot R_{mek} = \frac{b}{n^2}$$

Capacitor $$\frac{1}{sC_{el}} = \frac{1}{n^2} \cdot \frac{1}{sC_{mek}} = \frac{1}{n^2} \cdot \frac{k}{s} \Rightarrow C_{el} = \frac{n^2}{k}$$

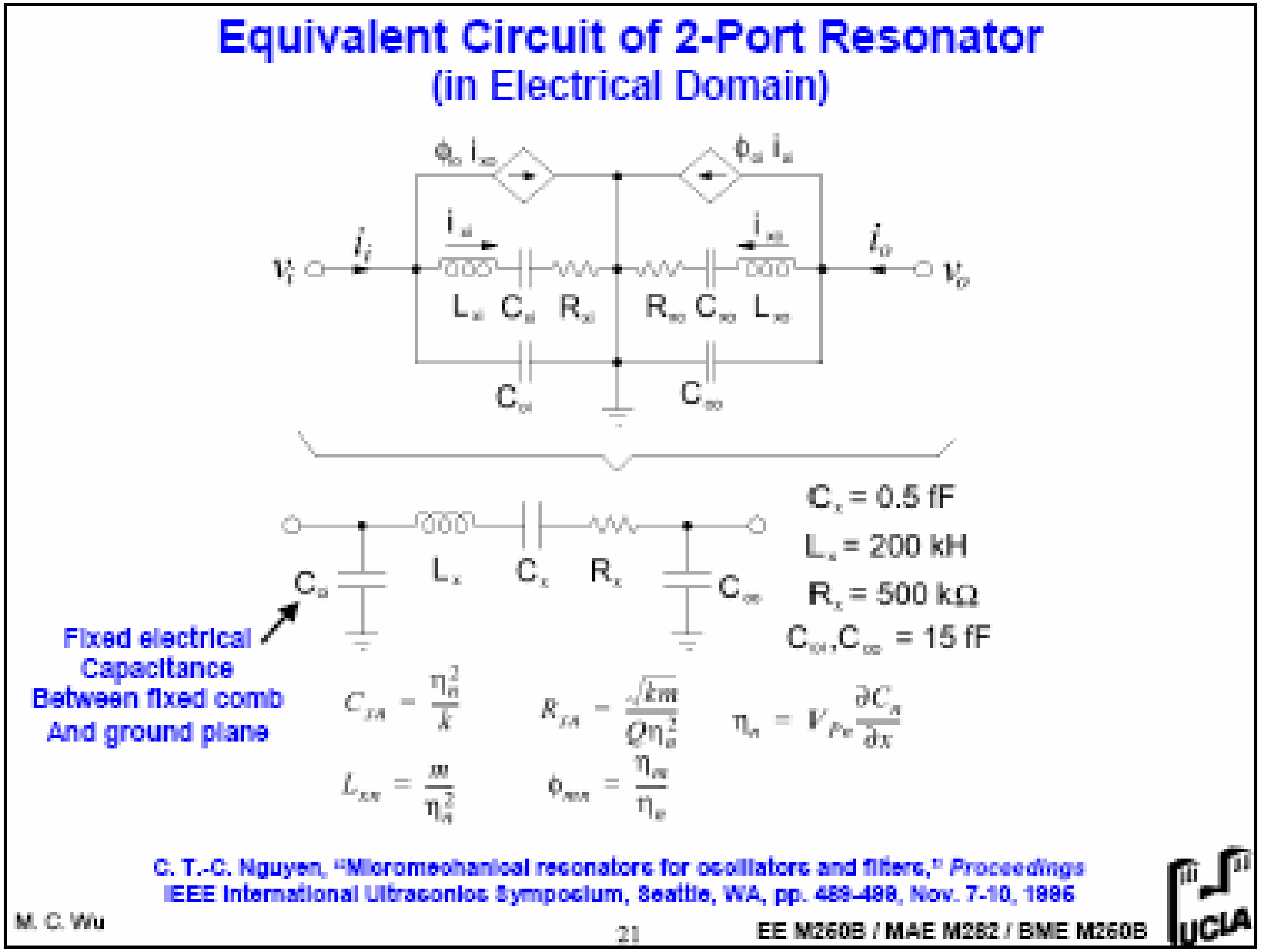


Small Signal Equivalent Circuit of Microresonators

Unit of n^2/k is Farad

$$n = V_{dc}\frac{\partial C}{\partial x}$$

Both methods result in the same equivalent circuit:



Equivalent Circuit of 2-Port Resonator

(in Electrical Domain)

C_x = 0.5 fF

L_x = 200 kH

R_x = 500 kΩ

C_{oi},C_{oo} = 15 fF

Fixed electrical Capacitance Between fixed comb And ground plane

$$C_{xn} = \frac{\eta_n^2}{k} \qquad R_{xn} = \frac{\sqrt{km}}{Q\eta_n^2} \qquad \eta_n = V_{Pn}\frac{\partial C_n}{\partial x}$$

$$L_{xn} = \frac{m}{\eta_n^2} \qquad \phi_{mn} = \frac{\eta_m}{\eta_n}$$

C. T.-C. Nguyen, "Micromechanical resonators for oscillators and filters," *Proceedings* IEEE International Ultrasonics Symposium, Seattle, WA, pp. 489-498, Nov. 7-10, 1995

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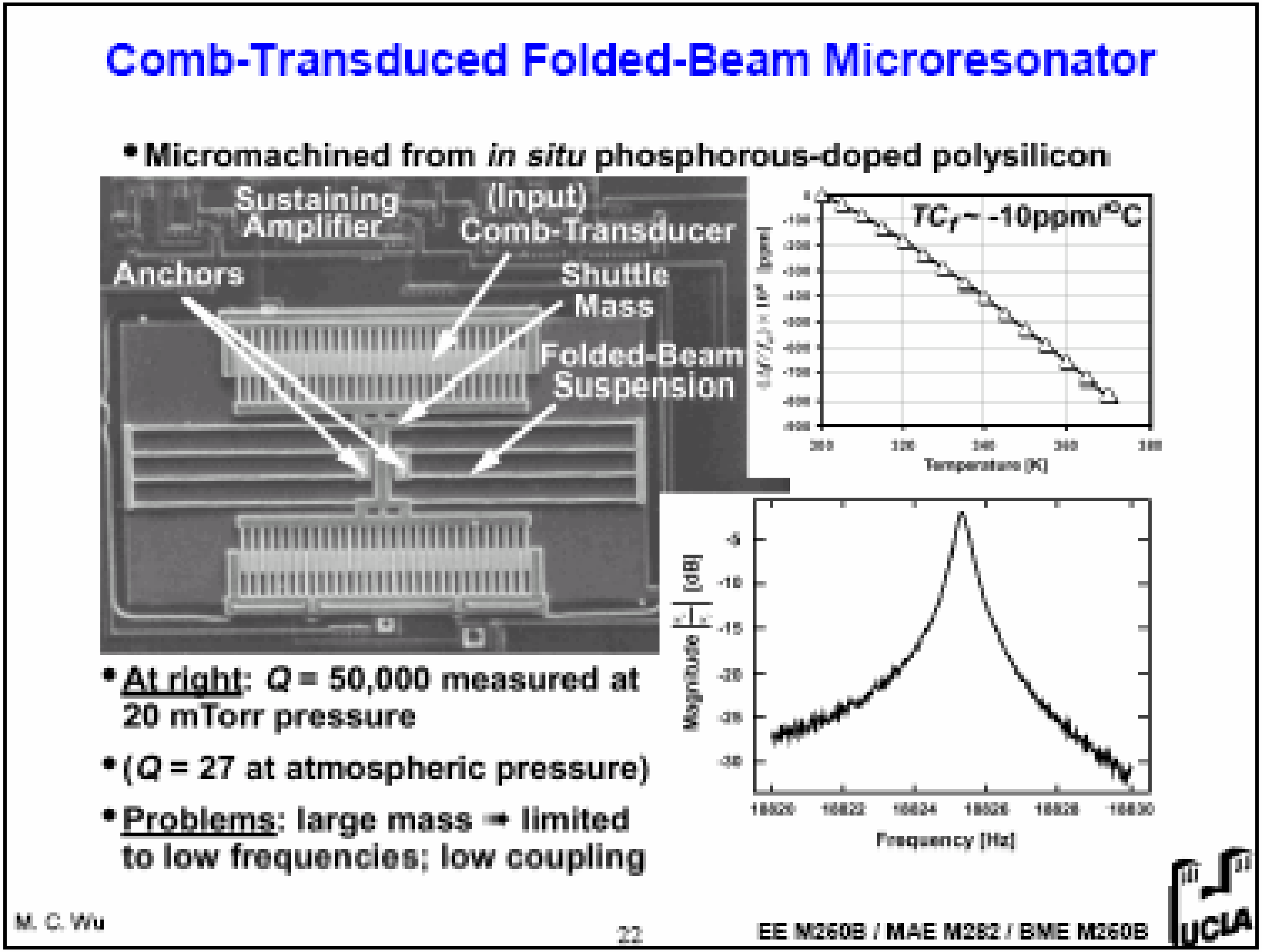


Comb-Transduced Folded-Beam Microresonator

- Micromachined from *in situ* phosphorous-doped polysilicon

- <u>At right</u>: $Q = 50{,}000$ measured at 20 mTorr pressure
- ($Q = 27$ at atmospheric pressure)
- <u>Problems</u>: large mass → limited to low frequencies; low coupling

Comb resonator, summary

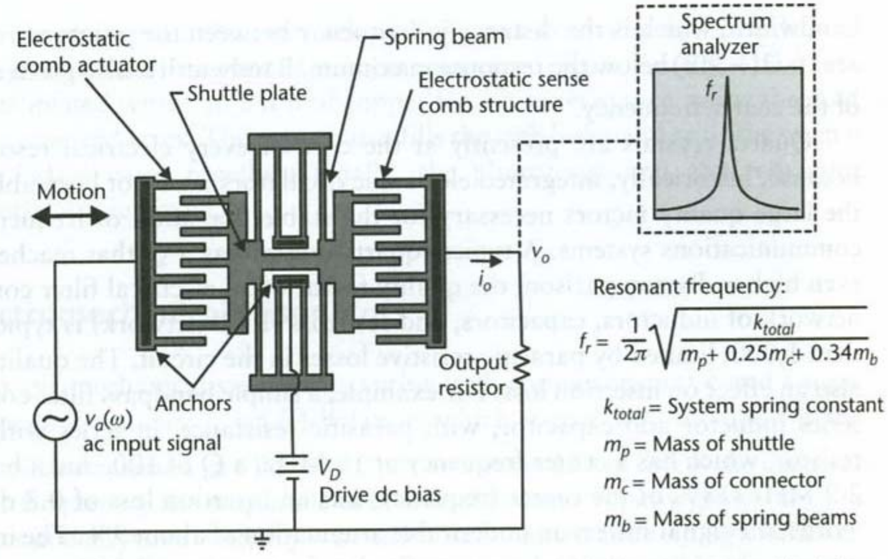
- Summary of modeling:
- Force: Fe = **½ dC/dx V ^2** (force is always attractive)
  - Input signal Va * cos (ωt)
  - Fe ~ Va^2 * ½ [1 + cos (2ωt)]
  - Driving force is 2x input-frequency + DC: NOT DESIRABLE
- Add DC bias, Vd
  - Fe ~ Vd ^2 + 2 Vd * Va * cos ω t + negligible term (2ωt)
  - Keep linearized AC force-component **~ Vd * Va**, which oscillates with the same frequency as Va: ω
- C increases when finger-overlap increases (comb moves)
  - ε * A/d (A = comb-thickness * overlap-length)
- dC/dx = constant for a given design (linear change, C is proportional to length-variation)

Comb-resonator, output current

- A time varying capacitance is established at the output comb
  - Calculate output current when V_d is kept constant and C is varying
    - I_0 = d/dt (Q) = d/dt (C*V) = V_d * dC/dt = V_d * dC/dx * dx/dt
    - I_0 = Vd * dC/dx * ω * x_max
    - I_0 plotted versus frequency, shows a BP-characteristic

Comb-resonator, spring constant

- Spring constant for simple beam deflected to the side
  - k_beam = const * E * t * (w/L) exp3
    - E = Youngs modul, t = thickness, w = width, L = length
- Example in figure 7.9:
  - const = 1 = 4 * ¼
  - k_total = 2 * k_beam



Resonant frequency:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k_{total}}{m_p + 0.25m_c + 0.34m_b}}$$

k_{total} = System spring constant

m_p = Mass of shuttle

m_c = Mass of connector

m_b = Mass of spring beams

Figure 7.9 Illustration of a micromachined folded-beam comb-drive resonator. The left comb drive actuates the device at a variable frequency ω. The right capacitive-sense-comb structure measures the corresponding displacement by turning the varying capacitance into a current, which generates a voltage across the output resistor. There is a peak in displacement, current, and output voltage at the resonant frequency.

Design parameters

- To obtain a **higher resonance-frequency:**
- Total spring constant must increase
- Dynamic mass must decrease
  - Difficult to achieve because a minimum number of fingers are needed
    - To have good electrostatic coupling (voltage → force)
  - Process resolution determines how small the lateral structures can be fabricated (geometrical design rules)

- Frequency can be increased by using another material with larger E/ρ than Si
  - E/ρ is a measure of the spring constant relative to weight
    - Elastic modulus versus material density
  - Aluminum and titanium has E/ρ lower than Si
  - Si carbide, poly diamond has E/ρ higher than for Si (poly diamond is a research topic)