INF 5490 RF MEMS

LN04: RF circuit design challenges

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Lecture overview INF5490

- Basic topics
  - LN01: Introduction. MEMS in RF
  - LN02: Fabrication
  - LN03: Modeling, design and analysis (part 1, 2 of 3)

- Main topic of today's lecture:
  - Modeling, 3: Analysis using **F**inite **E**lement **M**ethods
  - **Some characteristics and challenges of RF circuit design**

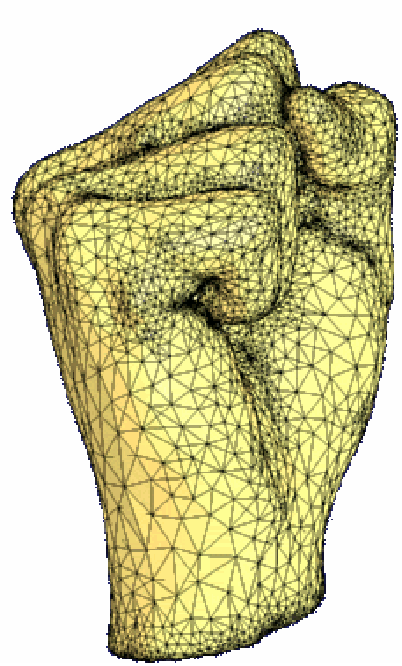
3. Finite Element Method analysis

- Simple mathematical models are approximations
  - Not accurate enough for complex structures
    - Ex. Beam deflection: non-uniform charge distribution ←→ force

- FEM characteristics
  - Build 3D model
  - **Mesh** the 3D model into smaller elements
  - Solve mathematical equations for interaction between elements
  - → Many iterations needed before a stable solution is obtained

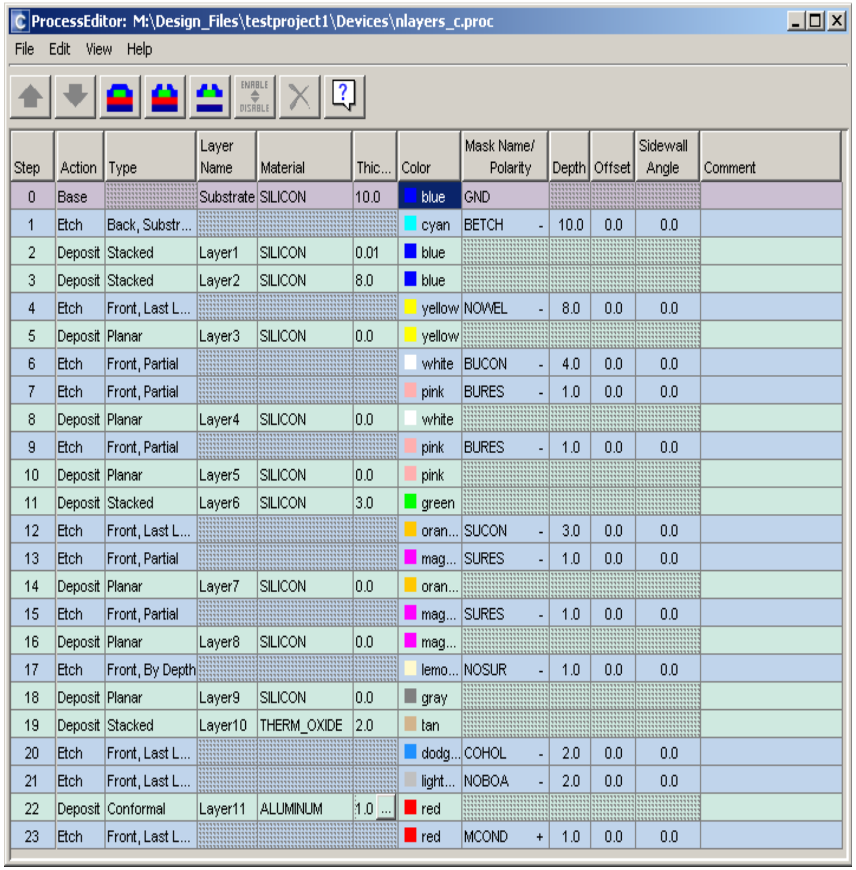
- Tool for FEM-simulations
  - CoventorWare, CW
  - Examples of bulk process modeling in CW →

Finite Element Methods



- Features
  - + Good precision
  - + Coupled electrostatic/ mech. interaction
  - + Can cope with irregular topologies
  - - Insight into parameters influence is lost
  - - Only small parts are practical
- Critical issues
  - Proper system selection, building the 3D model
  - Partitioning (precision of meshing)
  - Simulation parameters

Ex. 3D model building: process specification

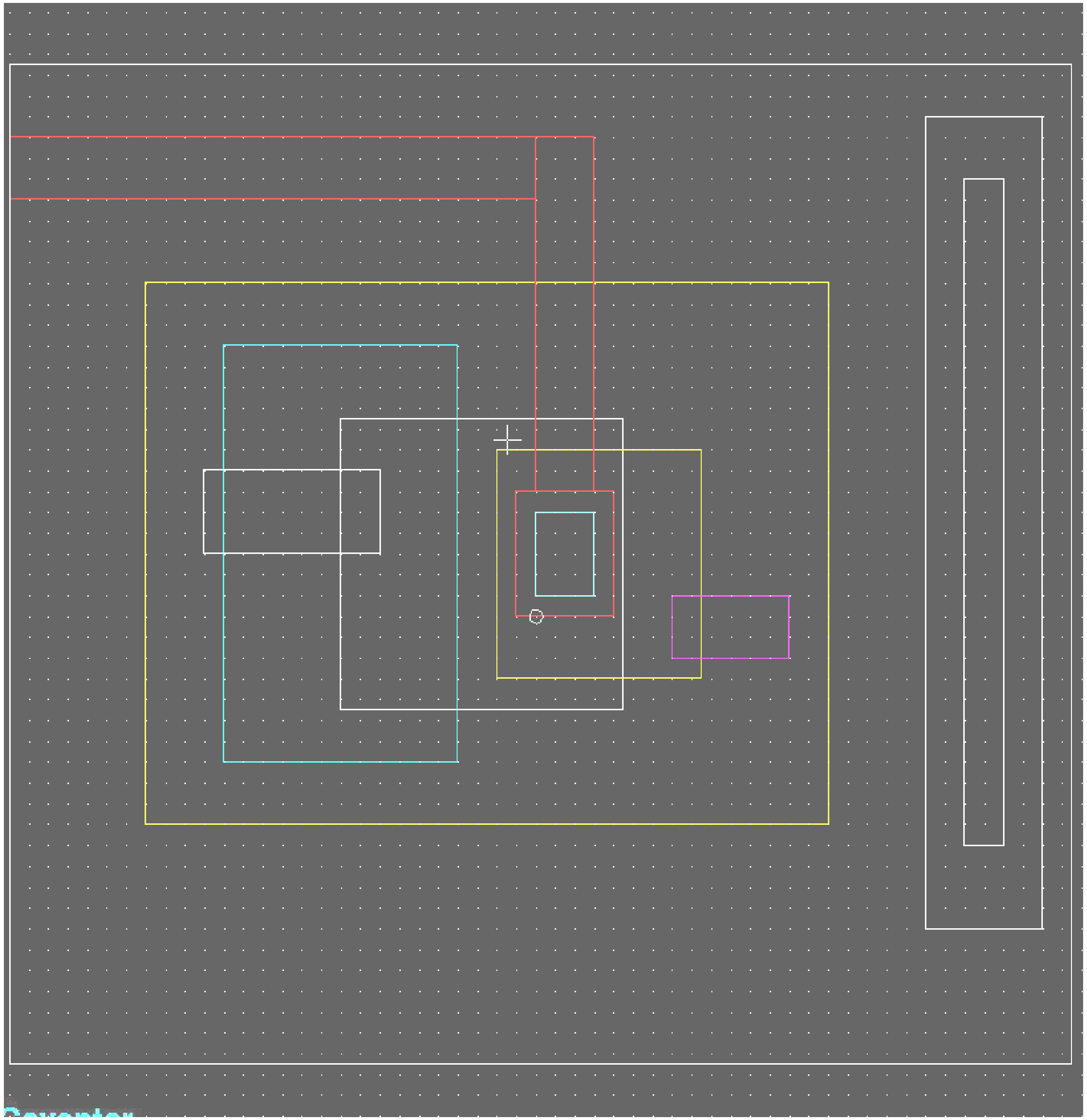


ProcessEditor: M:\Design_Files\testproject1\Devices\nlayers_c.proc

Step	Action	Type	Layer Name	Material	Thic...	Color	Mask Name/ Polarity	Depth	Offset	Sidewall Angle	Comment
0	Base		Substrate	SILICON	10.0	blue	GND				
1	Etch	Back, Substr...				cyan	BETCH -	10.0	0.0	0.0	
2	Deposit	Stacked	Layer1	SILICON	0.01	blue					
3	Deposit	Stacked	Layer2	SILICON	8.0	blue					
4	Etch	Front, Last L...				yellow	NOWEL -	8.0	0.0	0.0	
5	Deposit	Planar	Layer3	SILICON	0.0	yellow					
6	Etch	Front, Partial				white	BUCON -	4.0	0.0	0.0	
7	Etch	Front, Partial				pink	BURES -	1.0	0.0	0.0	
8	Deposit	Planar	Layer4	SILICON	0.0	white					
9	Etch	Front, Partial				pink	BURES -	1.0	0.0	0.0	
10	Deposit	Planar	Layer5	SILICON	0.0	pink					
11	Deposit	Stacked	Layer6	SILICON	3.0	green					
12	Etch	Front, Last L...				oran...	SUCON -	3.0	0.0	0.0	
13	Etch	Front, Partial				mag...	SURES -	1.0	0.0	0.0	
14	Deposit	Planar	Layer7	SILICON	0.0	oran...					
15	Etch	Front, Partial				mag...	SURES -	1.0	0.0	0.0	
16	Deposit	Planar	Layer8	SILICON	0.0	mag...					
17	Etch	Front, By Depth				lemo...	NOSUR -	1.0	0.0	0.0	
18	Deposit	Planar	Layer9	SILICON	0.0	gray					
19	Deposit	Stacked	Layer10	THERM_OXIDE	2.0	tan					
20	Etch	Front, Last L...				dodg...	COHOL -	2.0	0.0	0.0	
21	Etch	Front, Last L...				light...	NOBOA -	2.0	0.0	0.0	
22	Deposit	Conformal	Layer11	ALUMINUM	1.0 ...	red					
23	Etch	Front, Last L...				red	MCOND +	1.0	0.0	0.0	

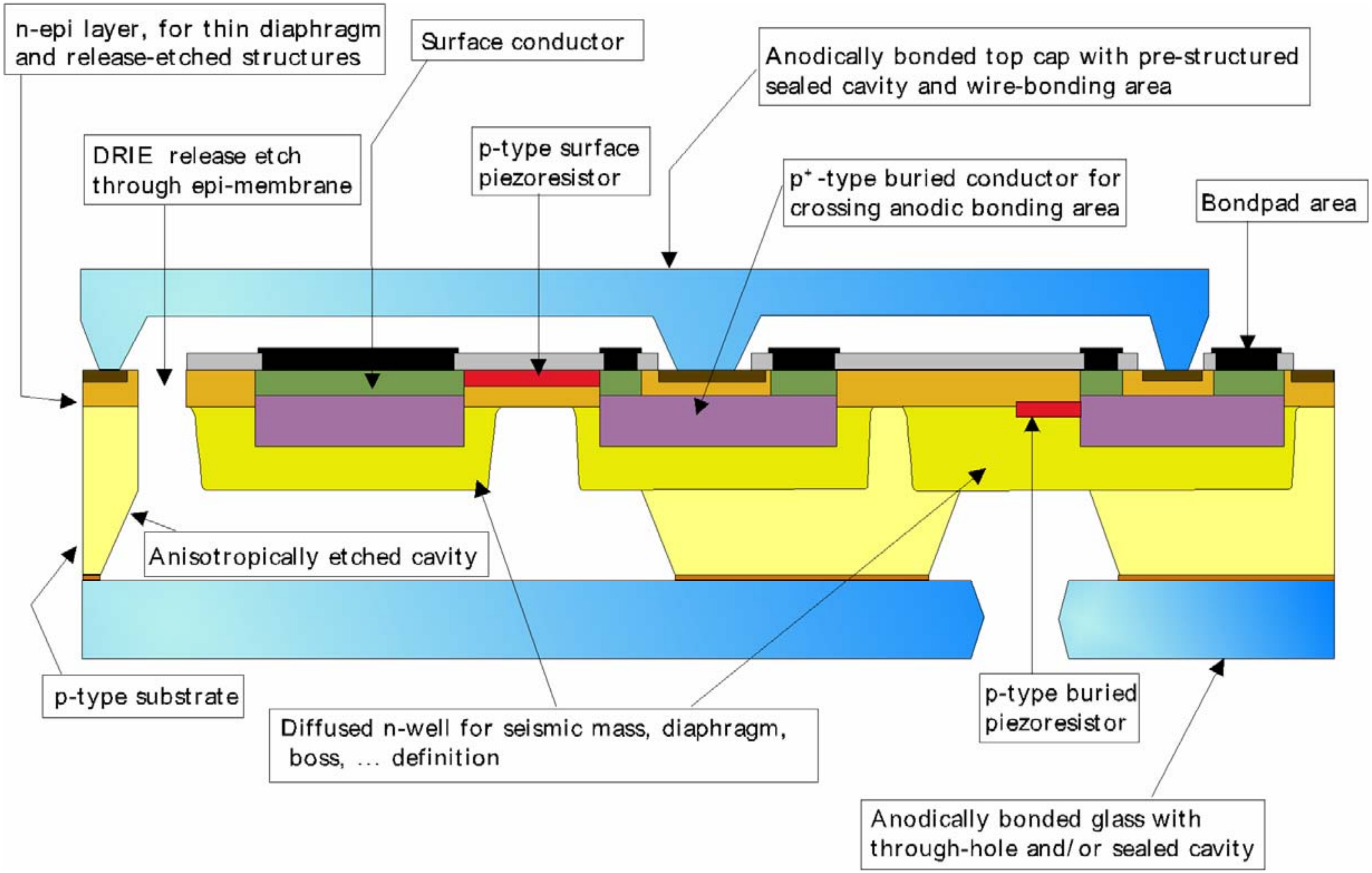
- Specify a **process file** which matches an actual foundry process
  - – simplifications
  - – realistic: essential process features included
- → **pseudo layers**

3D model building: layout



Make accompanying **layout**

MultiMEMS, typical features



n-epi layer, for thin diaphragm and release-etched structures

Surface conductor

Anodically bonded top cap with pre-structured sealed cavity and wire-bonding area

DRIE release etch through epi-membrane

p-type surface piezoresistor

p⁺-type buried conductor for crossing anodic bonding area

Bondpad area

Anisotropically etched cavity

p-type substrate

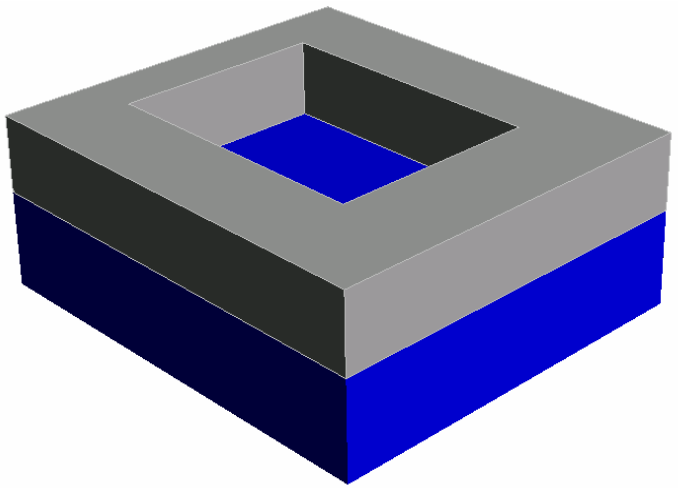
Diffused n-well for seismic mass, diaphragm, boss, ... definition

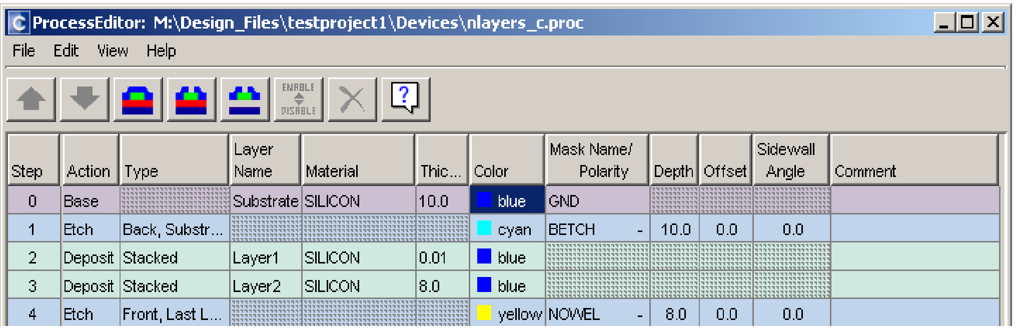
p-type buried piezoresistor

Anodically bonded glass with through-hole and/or sealed cavity

How to model the MultiMEMS bulk process in CoventorWare?

- Problem:
  - The process is not based on “stacking layers”
- Create a **pseudo process**!
  - Simplified, but matching
  - Transfer to a procedure of **stacking layers**
    - some layers with zero spacing
    - slicing the bulk material into sub-layers **in contact**
    - make etchings and re-fillings





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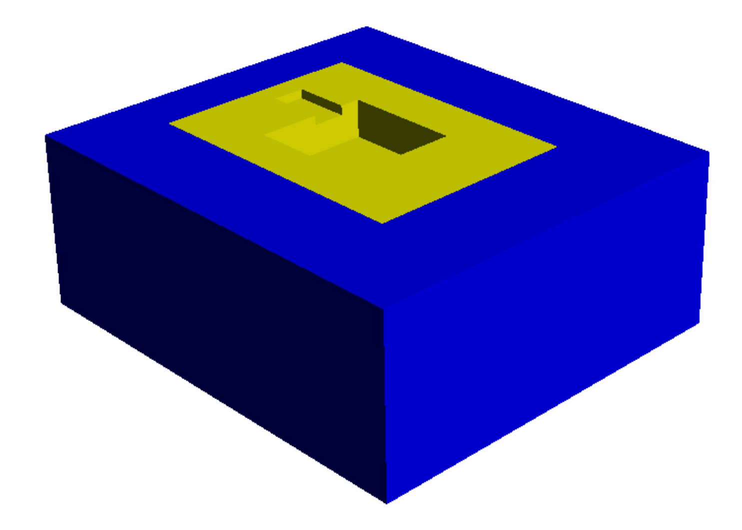
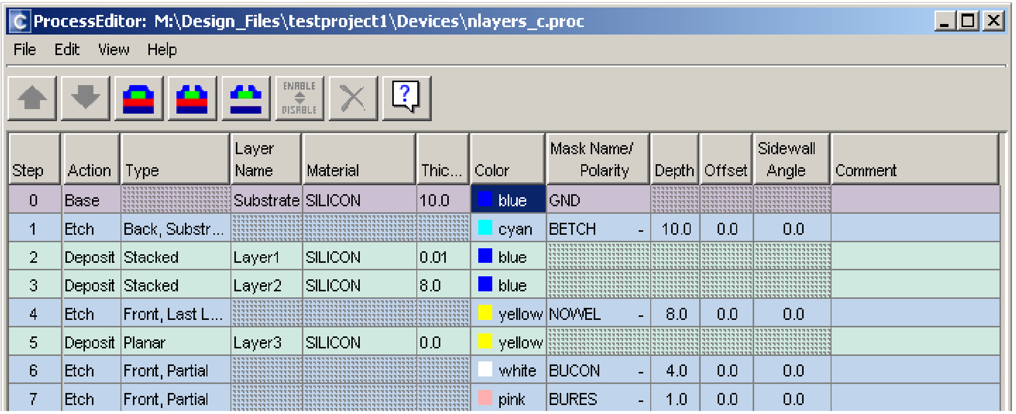
Step	Action	Type	Layer Name	Material	Thic...	Color	Mask Name/ Polarity	Depth	Offset	Sidewall Angle	Comment
0	Base		Substrate	SILICON	10.0	🟦 blue	GND				
1	Etch	Back, Substr...				🟦 cyan	BETCH -	10.0	0.0	0.0	
2	Deposit	Stacked	Layer1	SILICON	0.01	🟦 blue					
3	Deposit	Stacked	Layer2	SILICON	8.0	🟦 blue					
4	Etch	Front, Last L...				🟨 yellow	NOWEL -	8.0	0.0	0.0	

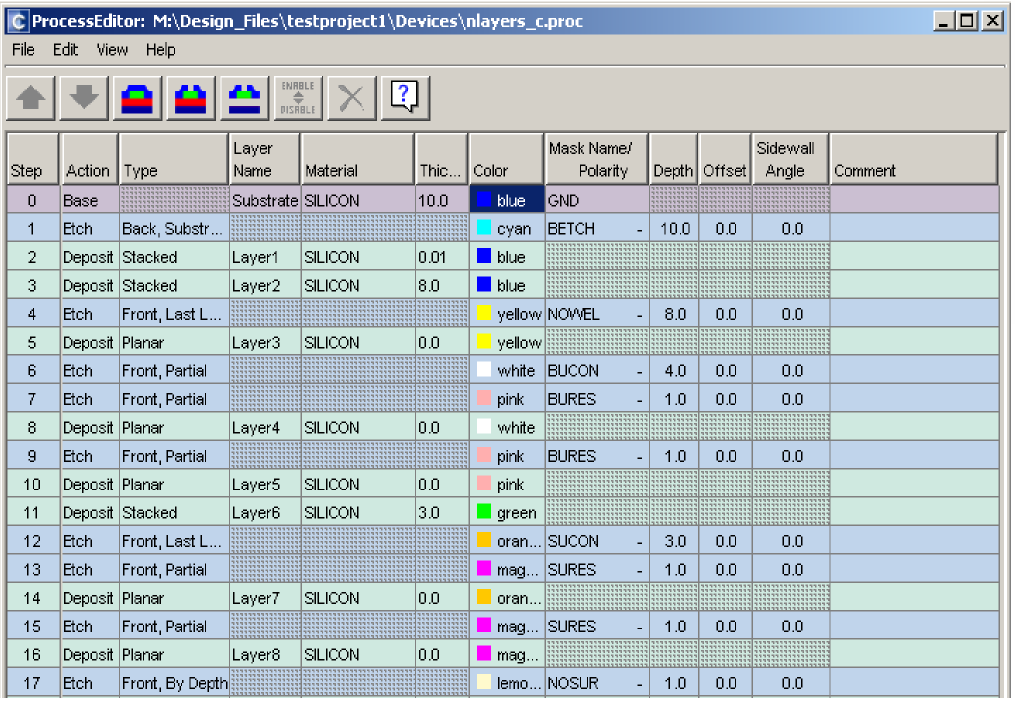
Two slices of the base material stacked. **N-well** opening





Step	Action	Type	Layer Name	Material	Thic...	Color	Mask Name/ Polarity	Depth	Offset	Sidewall Angle	Comment
0	Base		Substrate	SILICON	10.0	blue	GND				
1	Etch	Back, Substr...				cyan	BETCH -	10.0	0.0	0.0	
2	Deposit	Stacked	Layer1	SILICON	0.01	blue					
3	Deposit	Stacked	Layer2	SILICON	8.0	blue					
4	Etch	Front, Last L...				yellow	NOWEL -	8.0	0.0	0.0	
5	Deposit	Planar	Layer3	SILICON	0.0	yellow					
6	Etch	Front, Partial				white	BUCON -	4.0	0.0	0.0	
7	Etch	Front, Partial				pink	BURES -	1.0	0.0	0.0	

N-well in-filling. Etching holes for **buried conductor** implant and **buried resistor** implant



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8	Deposit	Planar	Layer4	SILICON	0.0	white					
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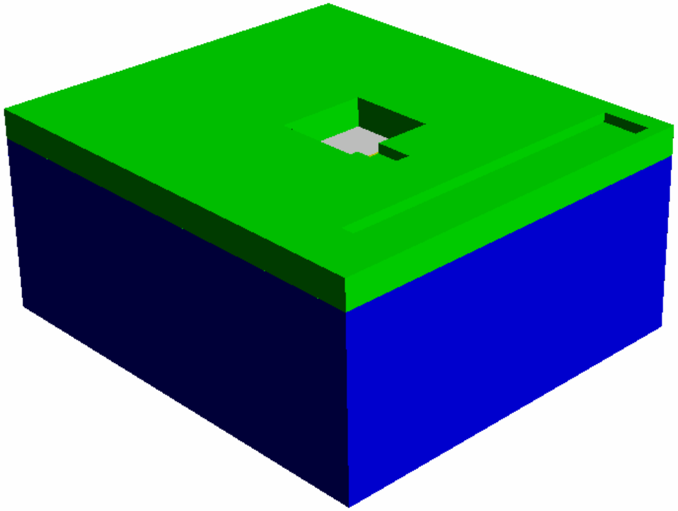
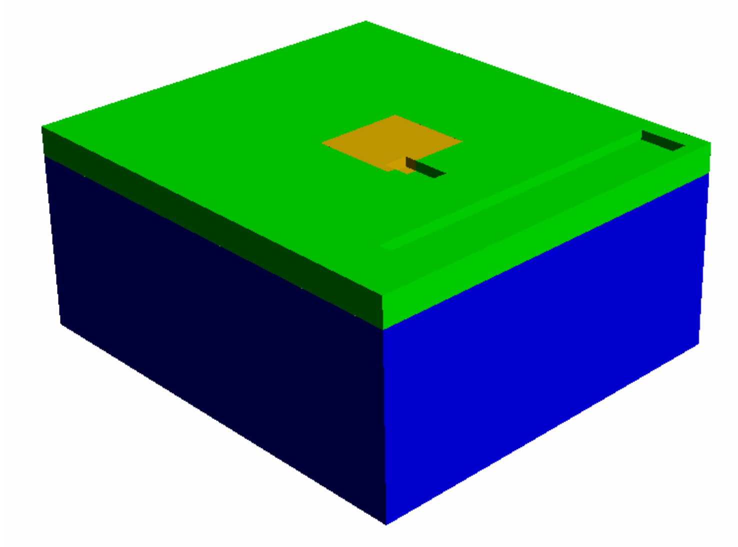
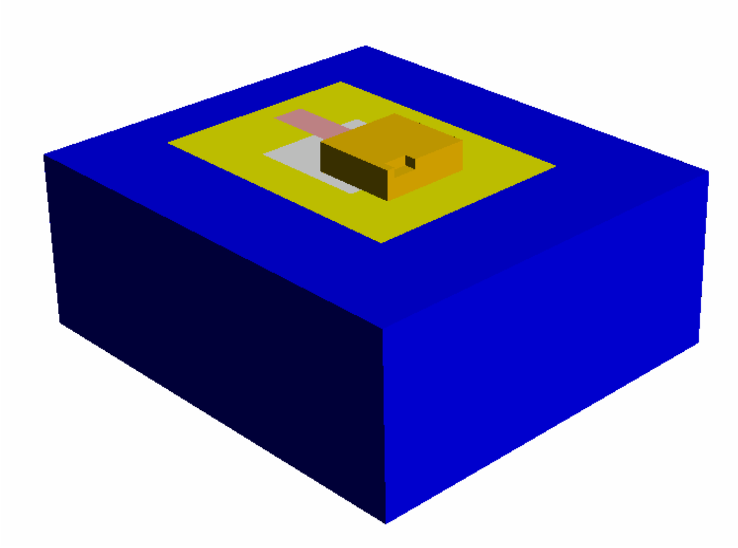
16	Deposit	Planar	Layer8	SILICON	0.0	mag...					
17	Etch	Front, By Depth				lemo...	NOSUR -	1.0	0.0	0.0	



Add **epi-layer**. Etch holes for **surface conductor** and **surface resistor**, -fill in. Etch hole for n+ implant. (Implants are invisible)

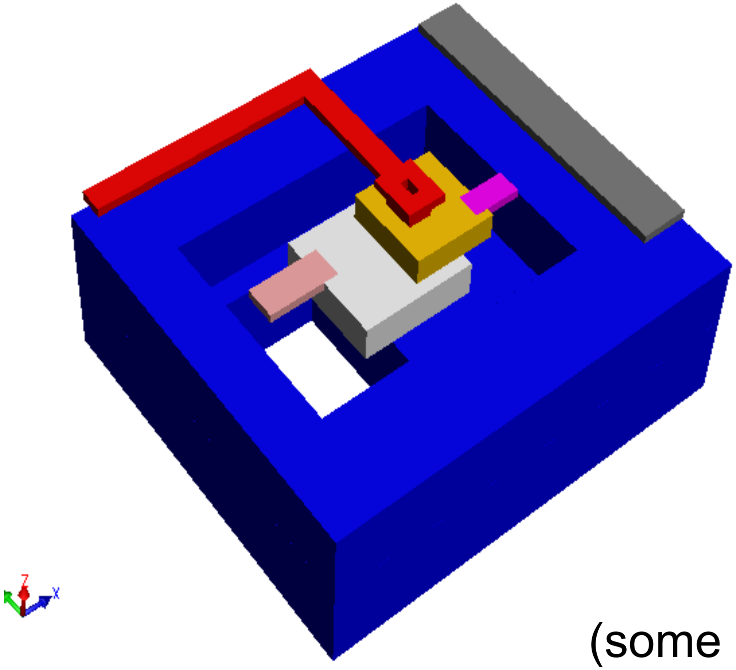


Surface conductor is made visible

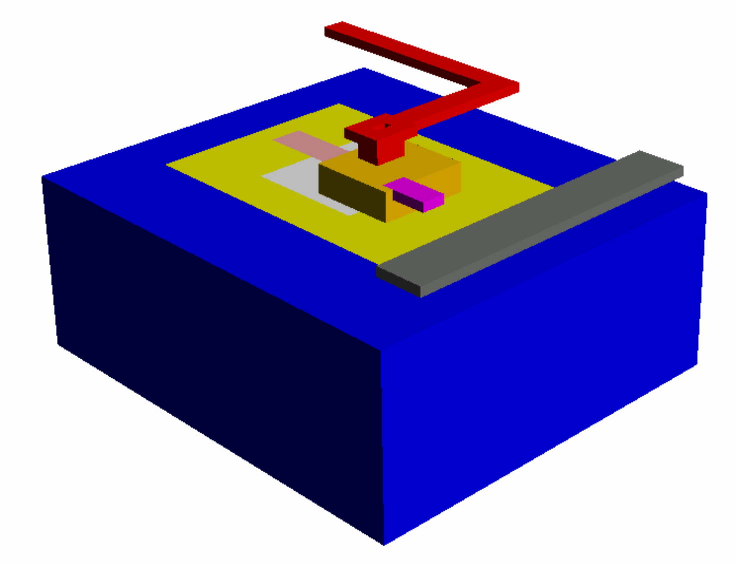


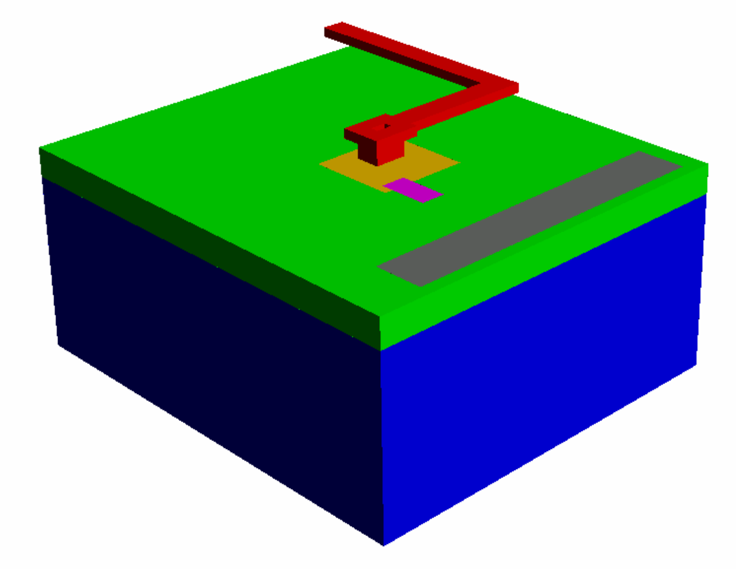
Epi-layer is invisible

3D model building: expansion



(some layers invisible)



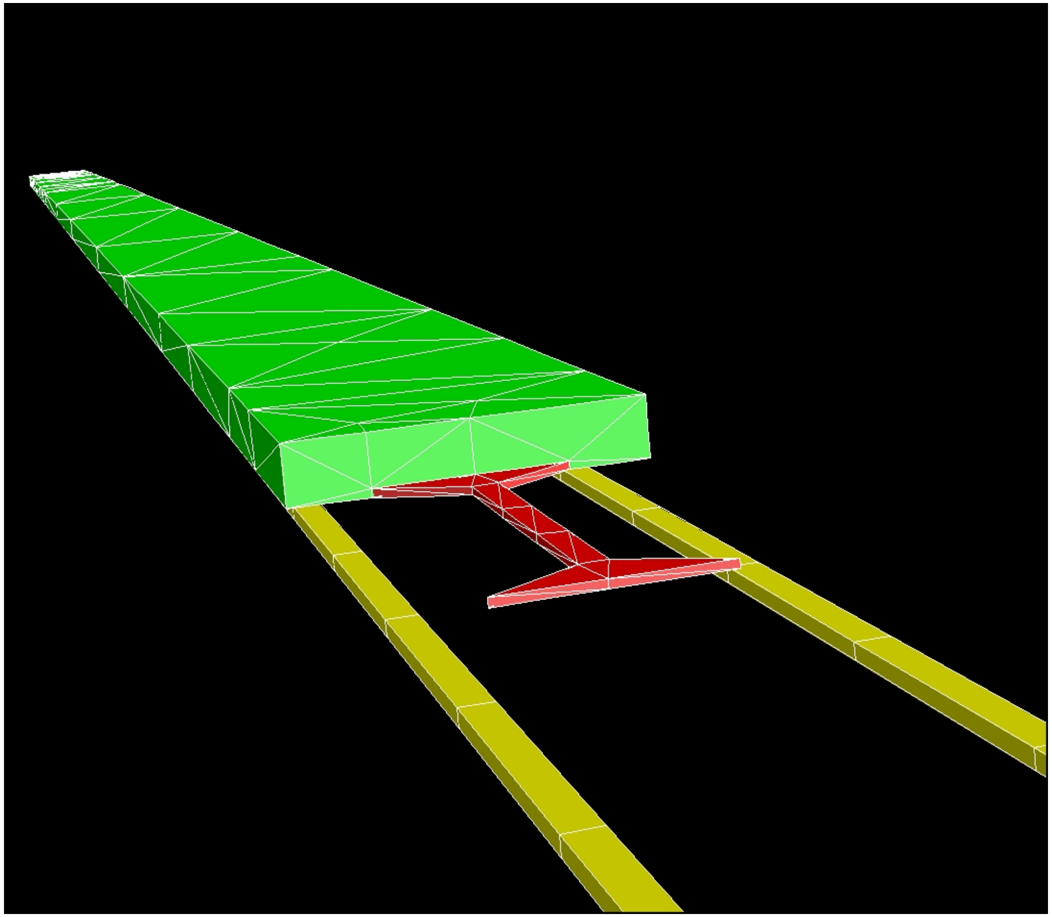


Complete structure with some layers made invisible

3D modeling procedure

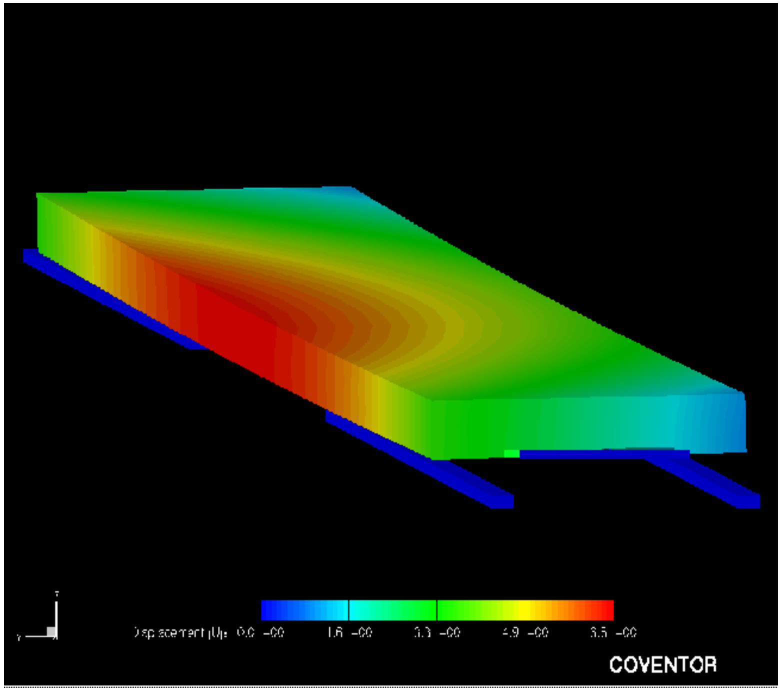
- To introduce one diffusion:
  - Etch base material
  - Fill in implanted material
    - use **“deposit planar” with thickness = 0**
- To introduce multiple overlapping diffusions:
  - Etch base material with all (overlapping) diffusion masks (the deepest first)
  - Fill in the deepest implanted material
  - Re-etch the remaining diffusion openings
  - Fill in the next deepest implant etc.

Meshed model

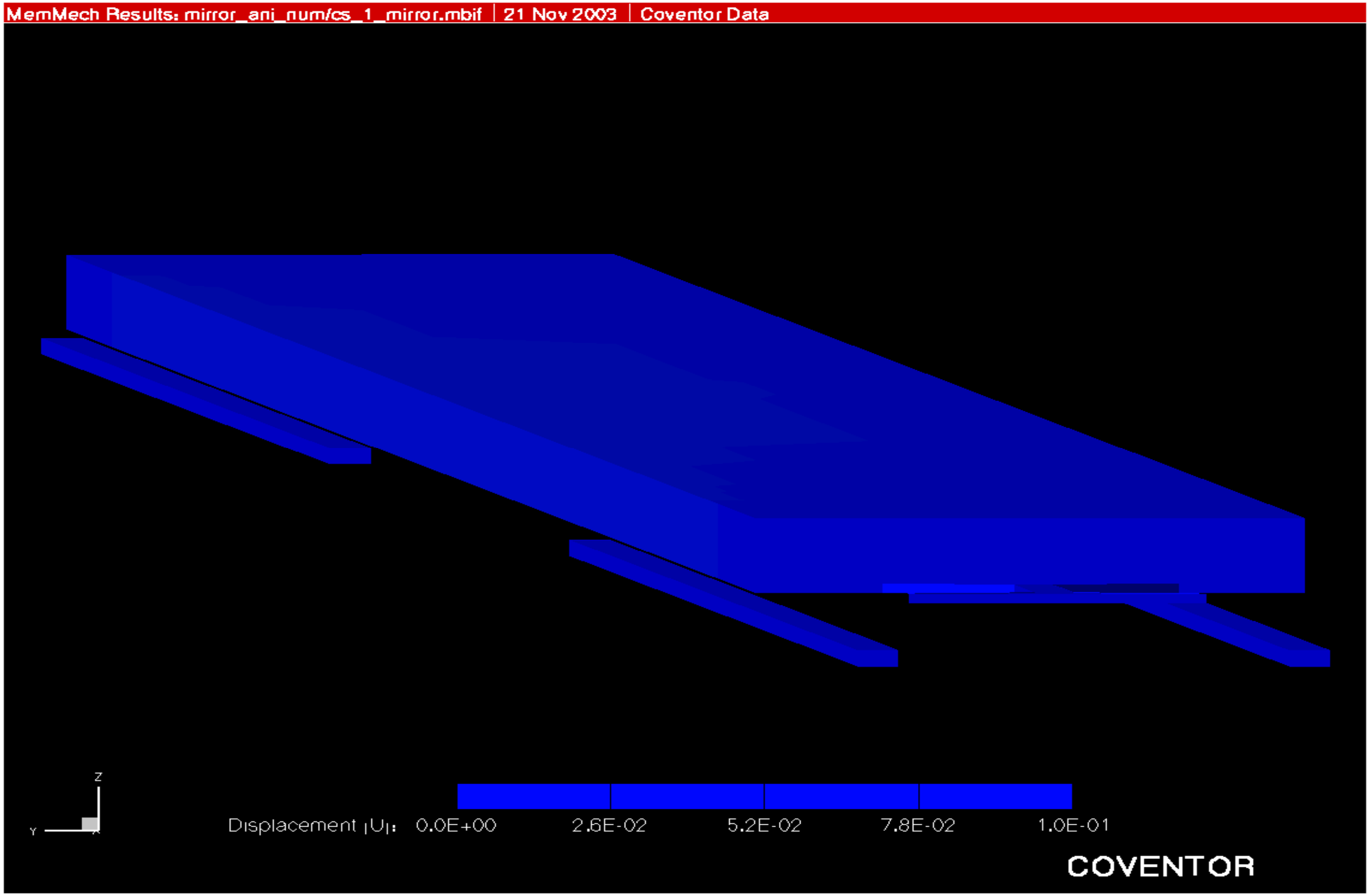


- Mirror meshed by tetrahedrons
  - 23 µm, 3 µm
- Electrodes meshed by Manhatten bricks
  - 5 µm
- Rather coarse dim due to **pull-in** analysis

Mirror deflection, snapshot



Simulation: pull-in



Today’s lecture

- Modeling: 3. Finite Element Method analysis

- RF circuit design
  - → ”Multi disciplinary”
- Electromagnetic waves
- Skin depth
- Passive components at high frequencies
- Transmission line theory
- Two-port networks
  - S-parameters
- Filters
- Q-factor

RF- and microwave design is multi disciplinary

- **Theoretical** fundament
  - Electromagnetism, electromagnetic waves
  - Signal processing

- **Technology**, practical aspects
  - Circuit theory
  - Kirchhoff's laws for current and voltage

- Some topics of today's lecture is also covered in INF5480
  - ”RF-circuits, theory and design” (Tor Fjeldly, fall semester)

- INF5490: → **Critical issues covered in *one* lecture!**

RF circuit design

- Some important questions
  - How do circuits behave at high frequencies?
  - Why do component functionality change?
  - At what frequencies is standard circuit analysis not valid?
  - What “new” circuit theory is needed?
  - How can this theory come into practical use?
    - → *Figures and equations from R. Ludwig et al: ”RF Circuit Design”*

Electromagnetic waves

- Electric and magnetic fields

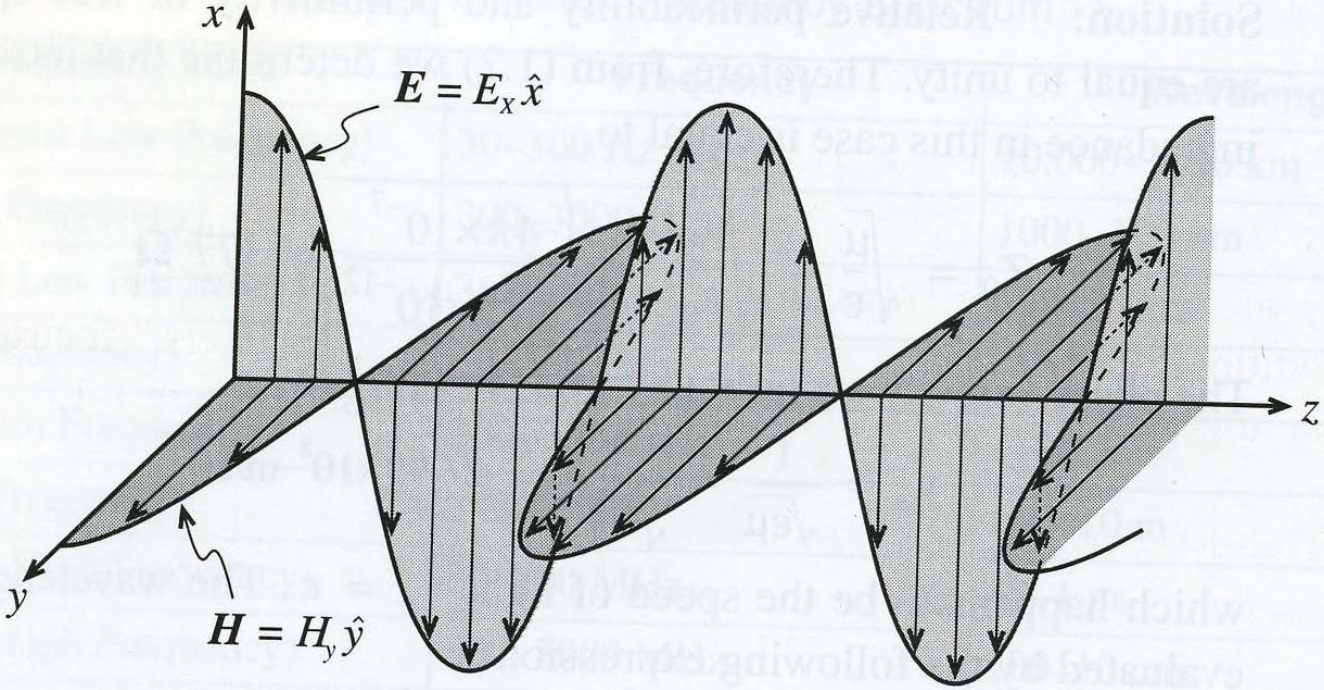


Figure 1-3 Electromagnetic wave propagation in free space. The electric and magnetic fields are recorded at a fixed instance in time as a function of space ($\hat{x}$, $\hat{y}$ are unit vectors in x- and y-direction).

Important wave parameters:

Electric field $E_x = E_{0x} \cos(\omega t - \beta z)$

Magnetic field $H_y = H_{0y} \cos(\omega t - \beta z)$

Angular frequency: ω Propagation constant: β

Wave is periodic, repeating when: $\beta \cdot z = 2\pi$

Wavelength: $z = \lambda = \frac{2\pi}{\beta}$

The wave propagates a distance λ during the time T = period

Propagation velocity:
(in vacuum: c)

$$v_p \cdot T = \lambda$$

$$v_p = \lambda \cdot \frac{1}{T} = \lambda \cdot f = \frac{2\pi}{\beta} \cdot \frac{\omega}{2\pi} = \frac{\omega}{\beta}$$

Important wave parameters, contd.

For a position z = constant, the wave repeats after a period T:

$\omega T = 2\pi$ and $\omega = 2\pi / T = 2\pi f$

in which f = frequency

Frequency and wavelength

- In vacuum: $\lambda * f = c$
  - Increasing frequency → decreasing wavelength

- At high frequencies (RF) is the wavelength comparable to the circuit dimensions
  - →

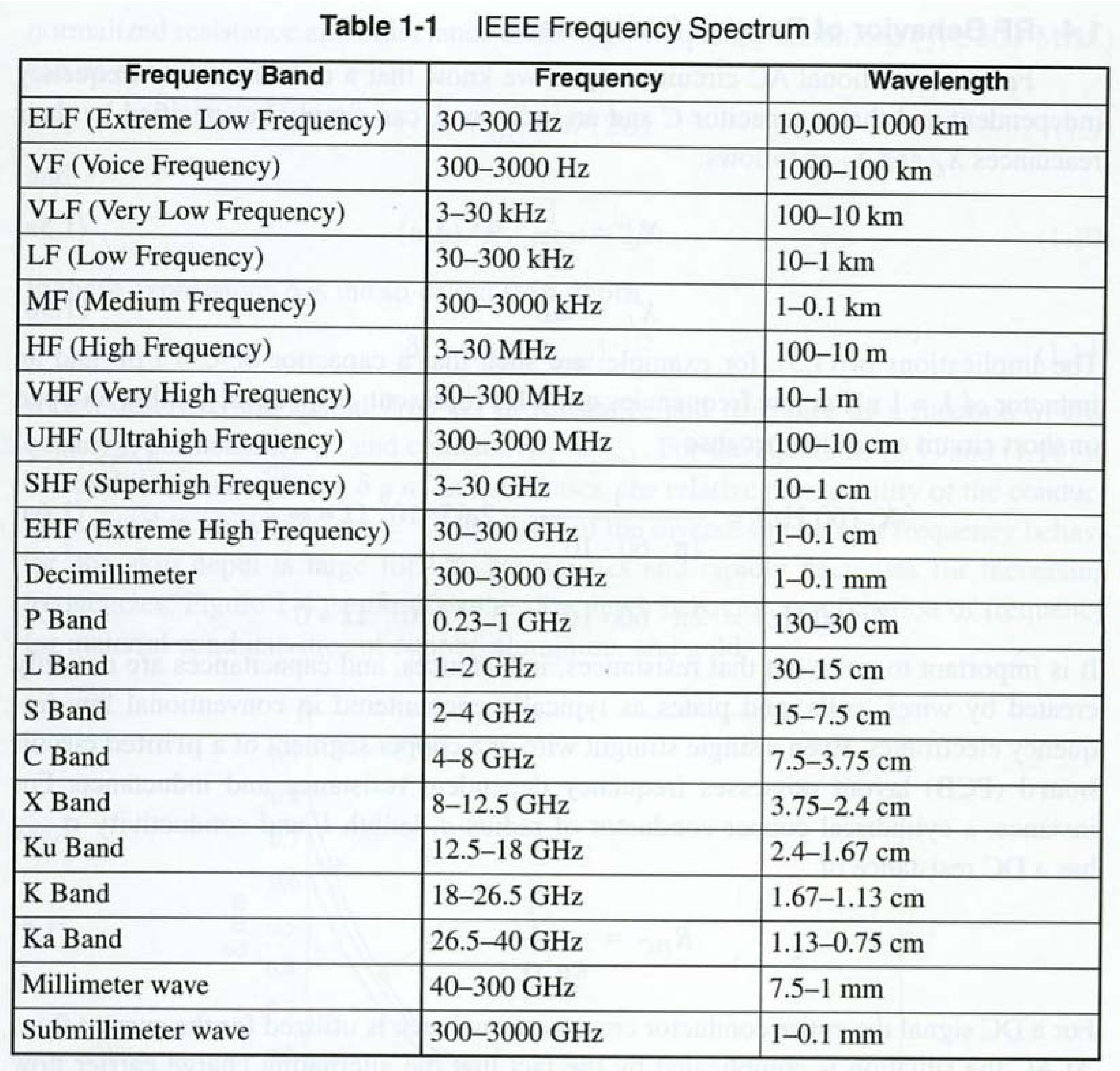


Table 1-1 IEEE Frequency Spectrum

Frequency Band	Frequency	Wavelength
ELF (Extreme Low Frequency)	30–300 Hz	10,000–1000 km
VF (Voice Frequency)	300–3000 Hz	1000–100 km
VLF (Very Low Frequency)	3–30 kHz	100–10 km
LF (Low Frequency)	30–300 kHz	10–1 km
MF (Medium Frequency)	300–3000 kHz	1–0.1 km
HF (High Frequency)	3–30 MHz	100–10 m
VHF (Very High Frequency)	30–300 MHz	10–1 m
UHF (Ultrahigh Frequency)	300–3000 MHz	100–10 cm
SHF (Superhigh Frequency)	3–30 GHz	10–1 cm
EHF (Extreme High Frequency)	30–300 GHz	1–0.1 cm
Decimillimeter	300–3000 GHz	1–0.1 mm
P Band	0.23–1 GHz	130–30 cm
L Band	1–2 GHz	30–15 cm
S Band	2–4 GHz	15–7.5 cm
C Band	4–8 GHz	7.5–3.75 cm
X Band	8–12.5 GHz	3.75–2.4 cm
Ku Band	12.5–18 GHz	2.4–1.67 cm
K Band	18–26.5 GHz	1.67–1.13 cm
Ka Band	26.5–40 GHz	1.13–0.75 cm
Millimeter wave	40–300 GHz	7.5–1 mm
Submillimeter wave	300–3000 GHz	1–0.1 mm

Two important laws

- **Faradays law**
  - Varying **magnetic field** induces **current**

- **Amperes law**
  - **Current** is setting up a **magnetic field**

Faradays law

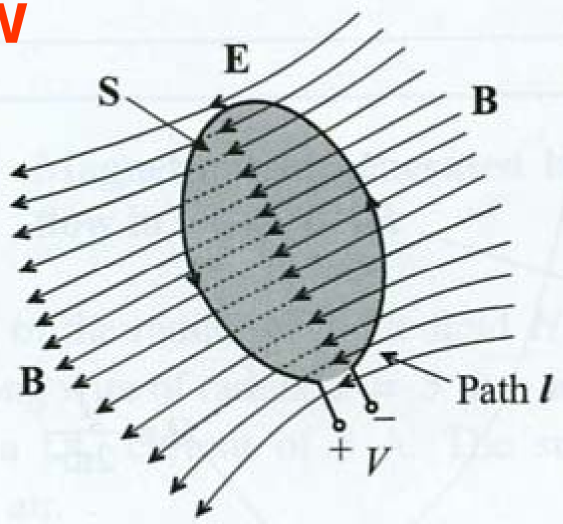


Figure 2-15 The time rate of change of the magnetic flux density induces a voltage.

$$\oint \overline{E} \cdot d\overline{l} = -\frac{d}{dt}\iint \overline{B} \cdot d\overline{S}$$

$$\overline{B} = magnetic_{.} \quad flux - density$$

$$\overline{B} = \mu \cdot \overline{H}$$

$$\mu = permeability = \mu_0 \cdot \mu_r$$

$$\overline{H} = magnetic\_ field$$

Amperes law

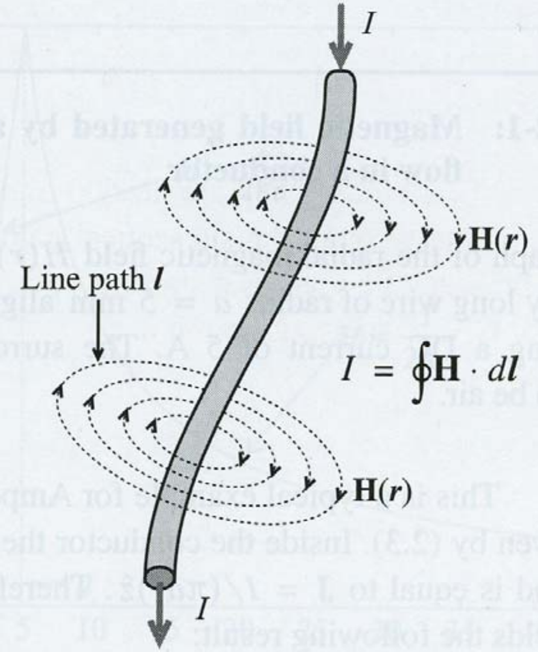


Figure 2-13 Ampère's law linking the current flow to the magnetic field.

$$I = \oint \overline{H} \cdot d\overline{l} = \iint \overline{J} \cdot d\overline{S}$$

”Skin depth”

- Signal transmission at increasing frequency
  - **DC** signal:
    - Current is flowing in whole cross section
  - **AC** signal (sequence of arguments for the operation):
    - Varying current induces an alternating magnetic field (Amperes law)
    - Magnetic field strength higher for small radius
    - Increased time variation of magnetic field in centre
    - Varying magnetic field induces an electric field (Faradays law)
    - Induced electric field (opposing the original one) increases in strength towards the centre of the conductor

Current density for various frequencies

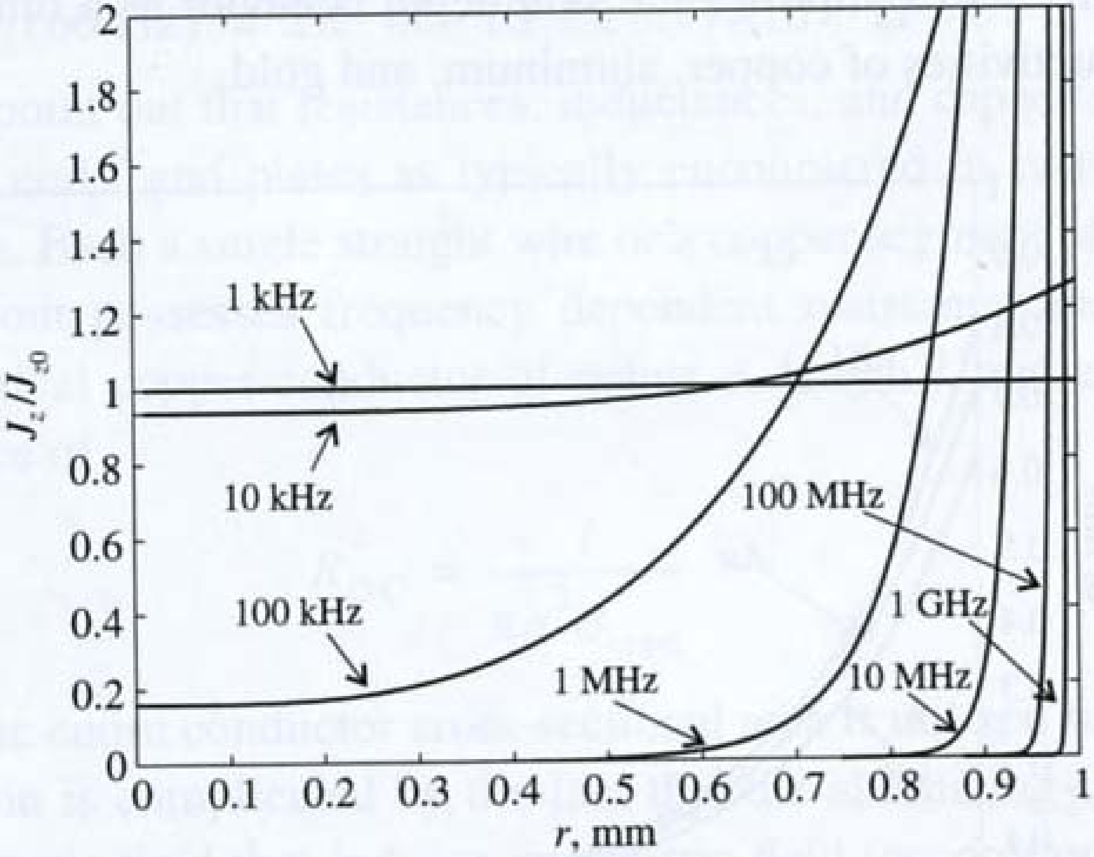


Figure 1-5(b) Frequency behavior of normalized AC current density for a copper wire of radius $a = 1$ mm.

Skin depth, contd.

- Resistance R increases towards centre of conductor
  - Current close to **surface** at increasing frequency
  - Formula: ”skin-depth” →
    - Current density reduced by a factor 1/e
- What does this mean for practical designs? →

$$\delta = (\pi f \mu \sigma_{\mathrm{cond}})^{-1/2}$$

magnetic permeability

conductivity

”Skin-depth”

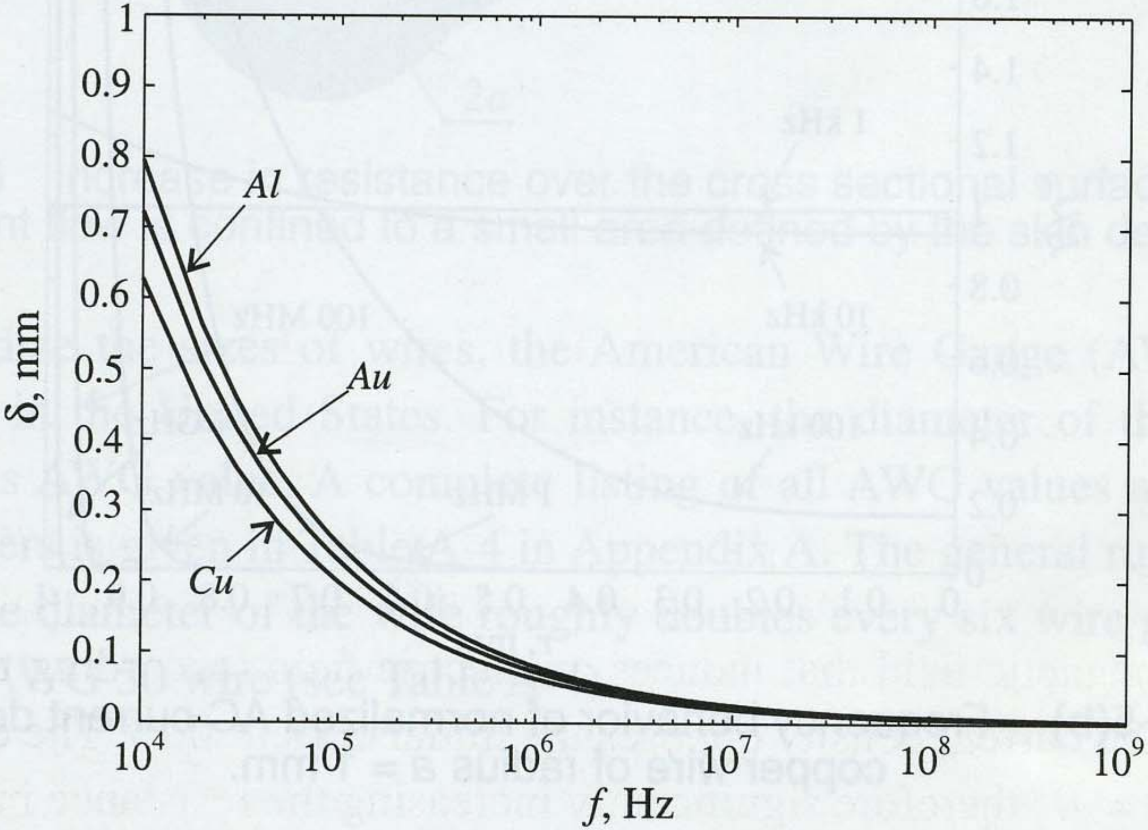


Figure 1-4 Skin depth behavior of copper $\sigma_{Cu} = 64.516 \times 10^6$ S/m, aluminum $\sigma_{Al} = 40.0 \times 10^6$ S/m, and gold $\sigma_{Au} = 48.544 \times 10^6$ S/m.

Passive components at high frequencies

- Equivalent circuit diagram for resistor

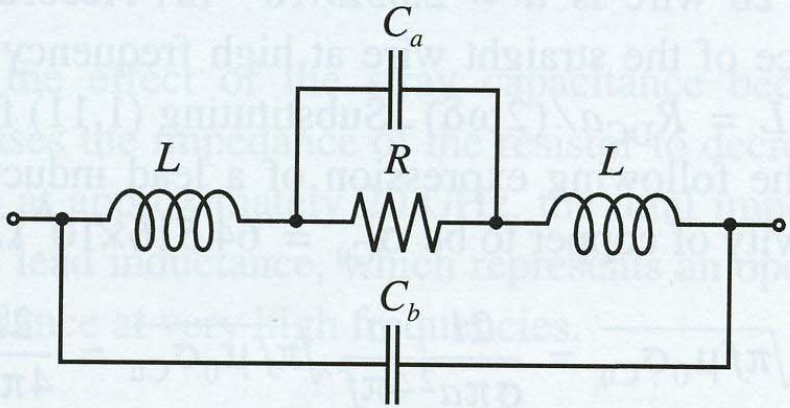
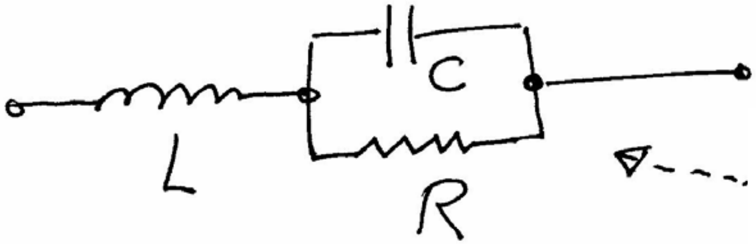


Figure 1-8 Electric equivalent circuit representation of the resistor.

Calculating resistor-impedance

Simplified model:



$$G = \frac{1}{R} + sC$$

$$z = sL + \frac{1}{\frac{1}{R} + sC} = sL + \frac{R}{1 + sRC}$$

$$z(j\omega) = j\omega L + \frac{R}{1 + j\omega RC}$$

Impedance versus frequency

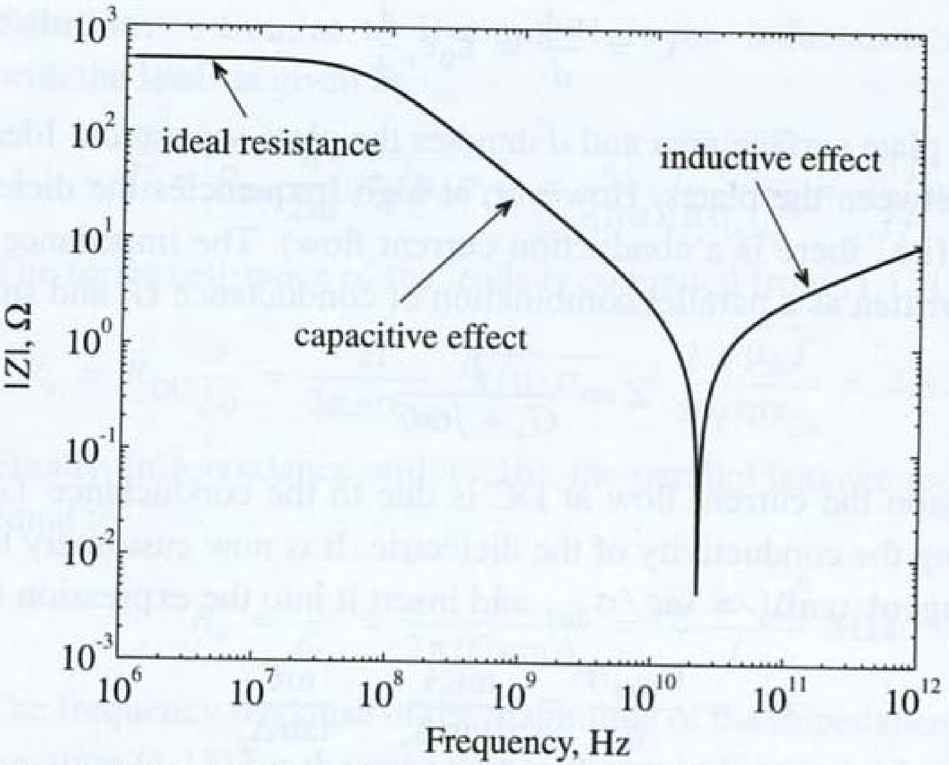


Figure 1-10 Absolute impedance value of a 500-Ω thin-film resistor as a function of frequency.

$Limits:$

$$z(j\omega) \rightarrow R, \quad når \quad \omega \rightarrow 0$$

$$z(j\omega) \rightarrow j\omega L, \quad når \quad \omega \rightarrow \infty$$

Resonance when terms cancel

$$sL = -\frac{R}{1 + sRC}$$

$$LRCs^2 + Ls + R = 0$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s = -\frac{1}{2RC} \pm j\sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$$

High frequency capacitor

- Equivalent circuit

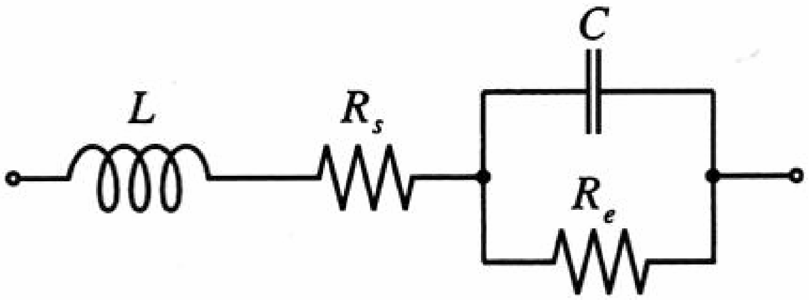


Figure 1-11 Electric equivalent circuit for a high-frequency capacitor.

Impedance versus frequency

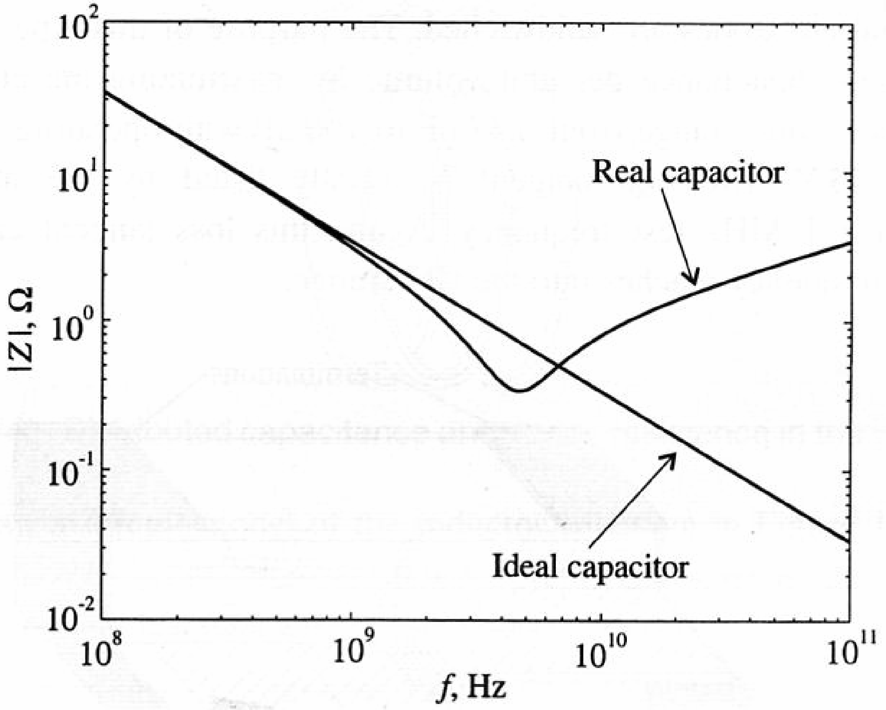


Figure 1-12 Absolute value of the capacitor impedance as a function of frequency.

High frequency inductor

- Equivalent circuit

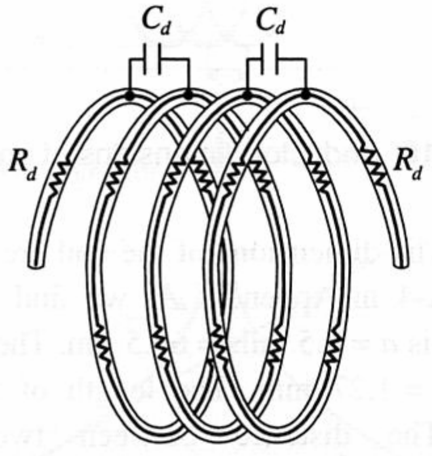


Figure 1-14 Distributed capacitance and series resistance in the inductor coil.

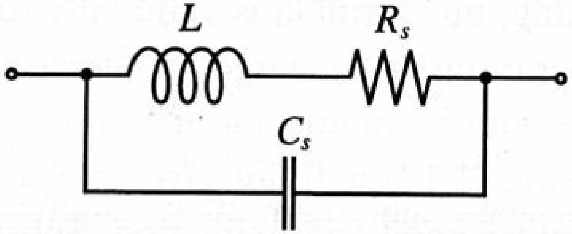


Figure 1-15 Equivalent circuit of the high-frequency inductor.

Impedance versus frequency

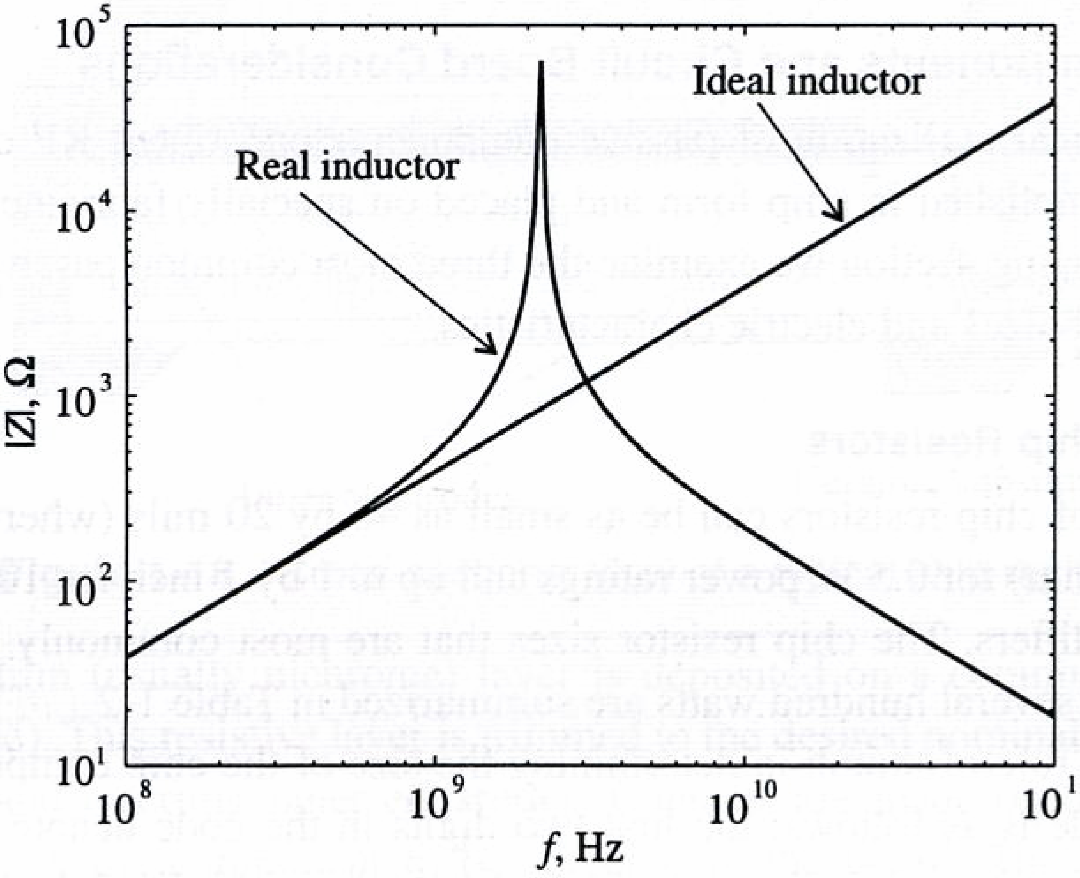


Figure 1-17 Frequency response of the impedance of an RFC.

Transmission line theory

- Frequency increases → wavelength decreases (λ)
- When λ is comparable with component dimensions, there will be a **voltage drop over the component!!**
  - → Current and voltage are not constant
- Voltage and current are **waves** that propagate along conductors and components
  - Position dependent value →
  - Signal should propagate along **transmission lines**
  - **Reflections, characteristic impedances** must be controlled

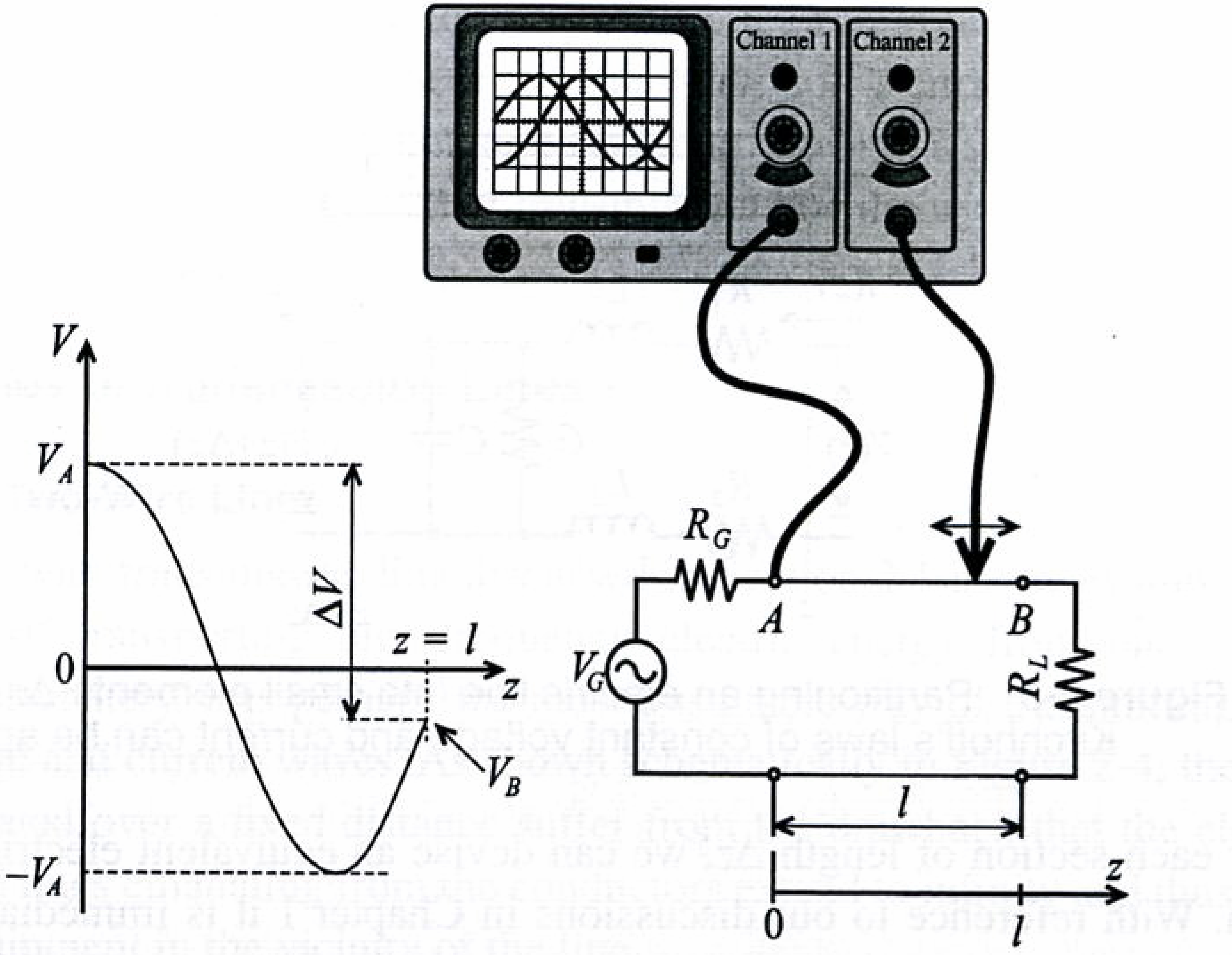


Figure 2-2 Amplitude measurements of 10 GHz voltage signal at the beginning (location *A*) and somewhere in between a wire connecting load to source.

Transmission line

- A conductor has to be modeled as a transmission line

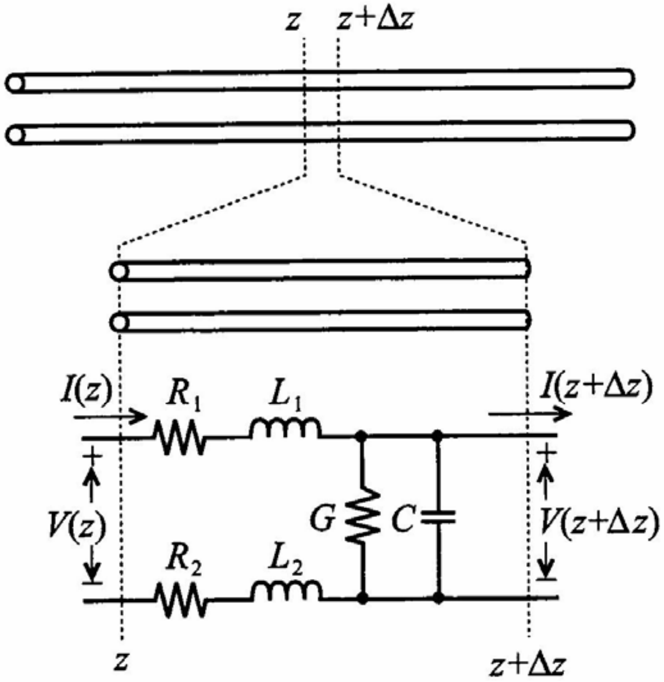


Figure 2-3 Partitioning an electric line into small elements Δz over which Kirchhoff's laws of constant voltage and current can be applied.

The line is divided into infinitesimal sub-units

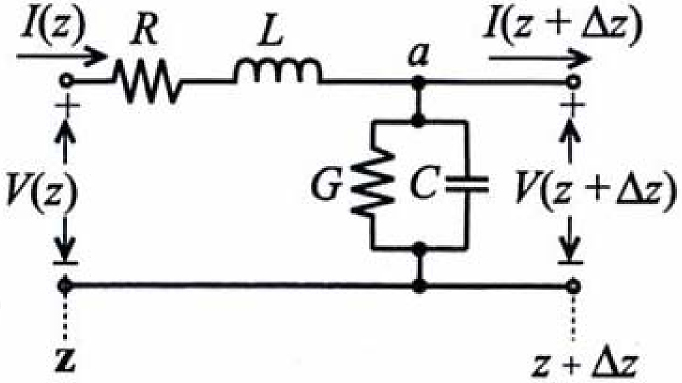
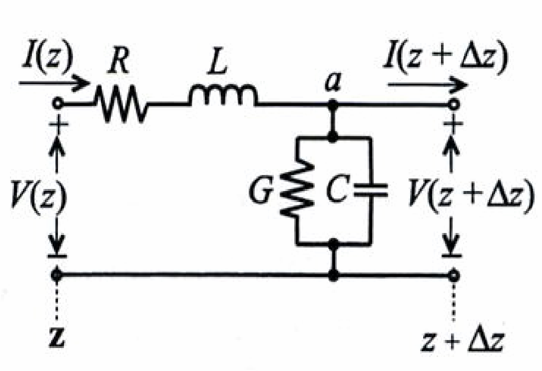


Figure 2-17 Segment of a transmission line with voltage loop and current node.

Use Kirchhoff's laws

- Will give 2 coupled 1.order diff-equations



$$(R + j\omega L)I(z)\Delta z + V(z + \Delta z) \;=\; V(z) \qquad (2.26)$$

$$\lim_{\Delta z \to 0}\left(-\frac{V(z+\Delta z) - V(z)}{\Delta z}\right) \;=\; -\frac{dV(z)}{dz} \;=\; (R + j\omega L)I(z) \qquad (2.27)$$

$$\boxed{-\frac{dV(z)}{dz} \;=\; (R + j\omega L)I(z)} \qquad (2.28)$$

$$I(z) - V(z+\Delta z)(G + j\omega C)\Delta z = I(z+\Delta z) \tag{2.29}$$

$$\lim_{\Delta z \to 0} \frac{I(z+\Delta z) - I(z)}{\Delta z} = \boxed{\frac{dI(z)}{dz} = -(G + j\omega C)V(z)} \tag{2.30}$$

$$\boxed{\frac{d^2V(z)}{dz^2} - k^2 V(z) = 0} \tag{2.31}$$

$$k = k_r + jk_i = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{2.32}$$

$$\boxed{\frac{d^2I(z)}{dz^2} - k^2 I(z) = 0} \tag{2.33}$$

Solution: 2 waves

- The solution is waves in a **positive** and **negative** direction

$$V(z) = V^+e^{-kz} + V^-e^{+kz} \tag{2.34}$$

$$I(z) = I^+e^{-kz} + I^-e^{+kz} \tag{2.35}$$

$$I(z) = \frac{k}{(R+j\omega L)}(V^+e^{-kz} - V^-e^{+kz}) \tag{2.36}$$

(Jmfr.2.27)

Characteristic line-impedance: $Z_0 = \frac{V^+}{I^+} = -\frac{V^-}{I^-}$

$$\boxed{Z_0 = \frac{(R+j\omega L)}{k} = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}}} \tag{2.37}$$

Impedance for lossless transmission line

$$Z_0 = \sqrt{L/C}$$

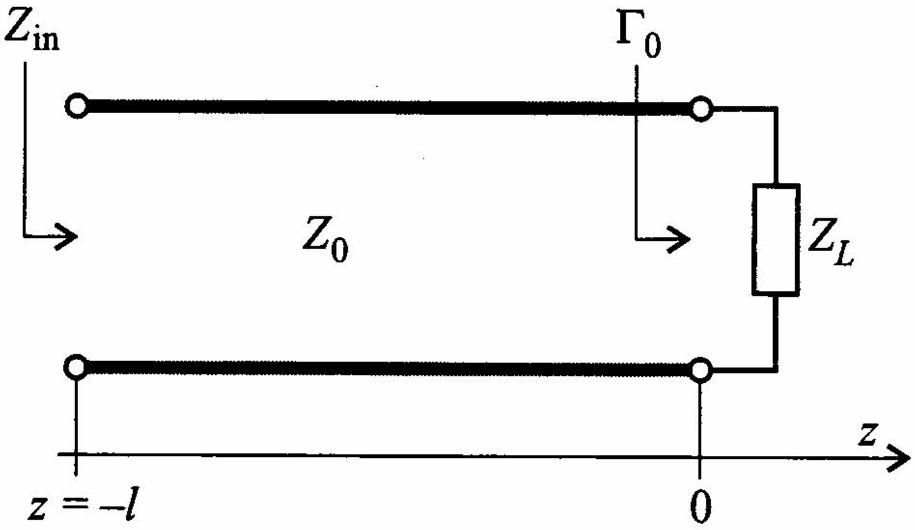


Figure 2-23 Terminated transmission line at location $z = 0$.

Reflection

- How to avoid reflections and have good signal propagation?
- Definition of **reflection coefficient →**

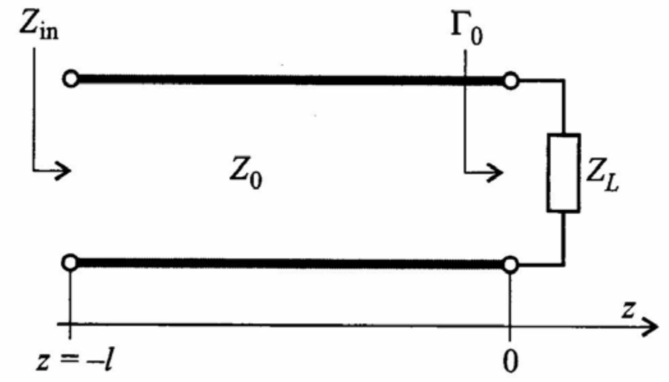


Figure 2-23 Terminated transmission line at location $z = 0$.

Reflection coefficient

$$\Gamma_0 = \frac{V^-}{V^+}$$

← definition of reflection coefficient for z = 0

$$V(z) = V^+(e^{-kz} + \Gamma_0 \cdot e^{+kz})$$

$$I(z) = \frac{V^+}{Z_0}(e^{-kz} - \Gamma_0 \cdot e^{+kz})$$

Impedance for z = 0:

$$Z(0) = \frac{V(0)}{I(0)} = Z_0 \frac{1+\Gamma_0}{1-\Gamma_0} = Z_L$$

= load impedance

$$\boxed{\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Various terminations

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Open line
→ reflection with equal polarity

$$Z_L = \infty \Rightarrow \Gamma_0 = 1$$

Short circuit
→ Reflection with inverse polarity

$$Z_L = 0 \Rightarrow \Gamma_0 = -1$$

No reflection when:

$$Z_0 = Z_L \Rightarrow \Gamma_0 = 0$$

→ **”MATCHING”**

Standing waves

- Short circuiting gives standing waves $(Z_L = 0)$

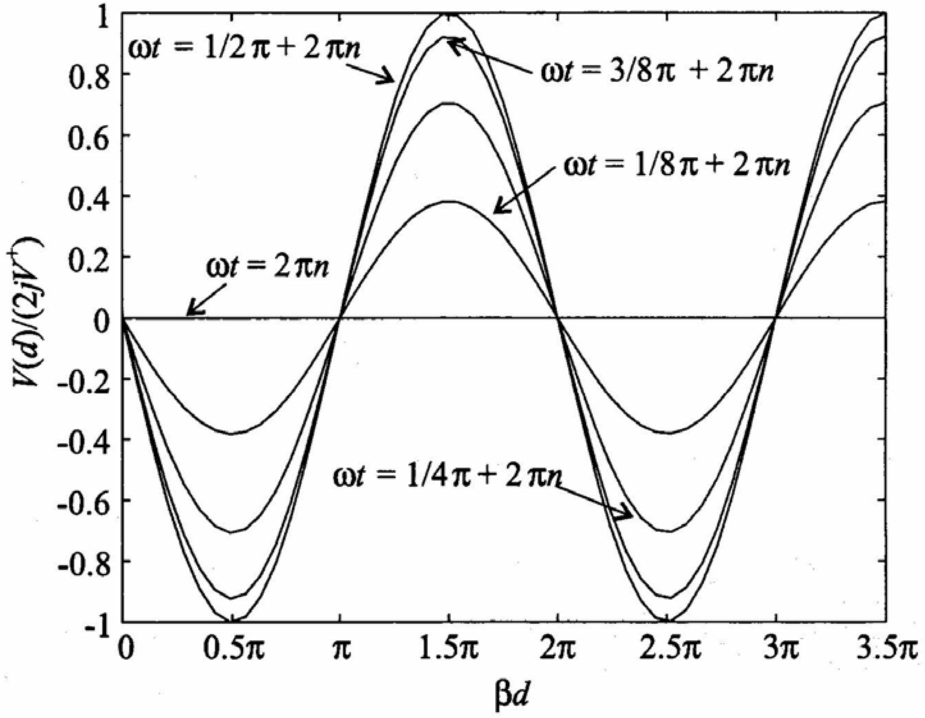


Figure 2-25 Standing wave pattern for various instances of time.

RF-circuits

- A high frequency circuit may be viewed as
  - a finite number of **transmission line sections** interconnected with **discrete active** and **passive** components

Two-port network

- Circuits can be made up of simple parts:
  - **Two-ports**
- Two-port-description can be used to simplify analysis of complex networks
- Different types of two-ports
  - **Z, Y, h-matrix**
    - Each one is used in different situations and has **different properties when interconnected**
    - Z → series, Y → parallel, hybrid
  - Figure →

Two-ports at low frequencies

- **Open** and **shorts** are used for two-ports to determine **Z** (impedance) or **Y** (admittance) at low frequencies

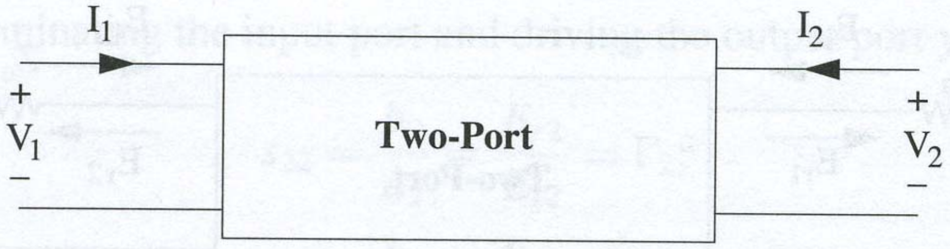


FIGURE 3.6. Port variable definitions

Multiport-network

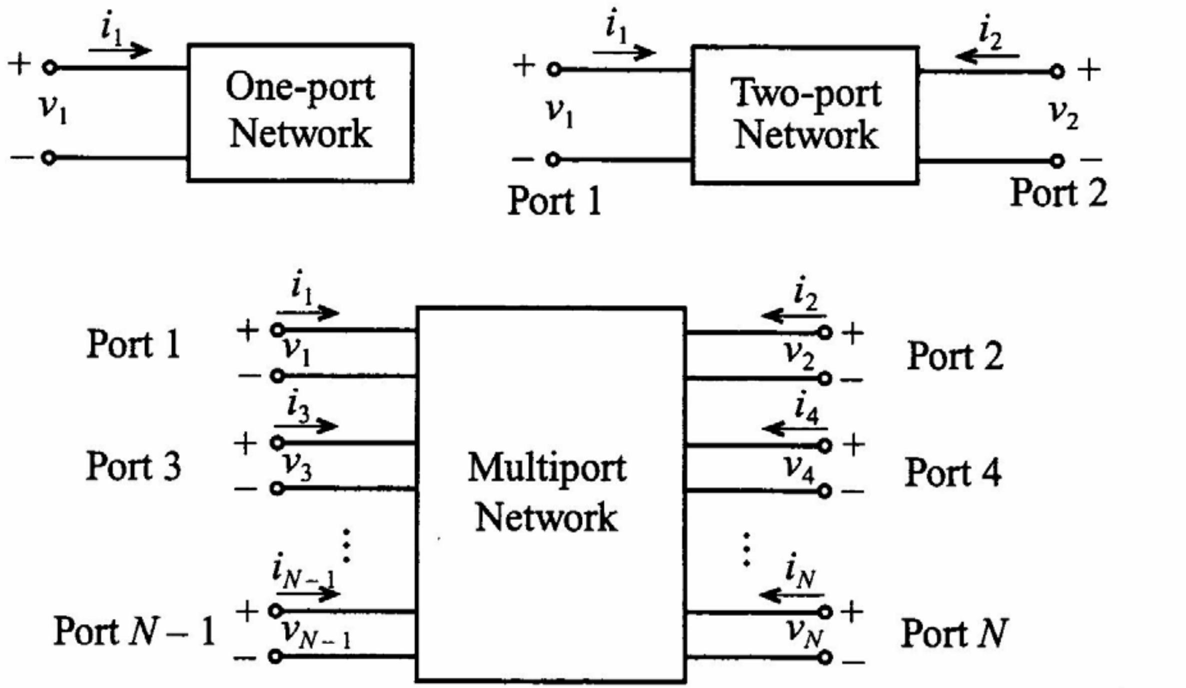
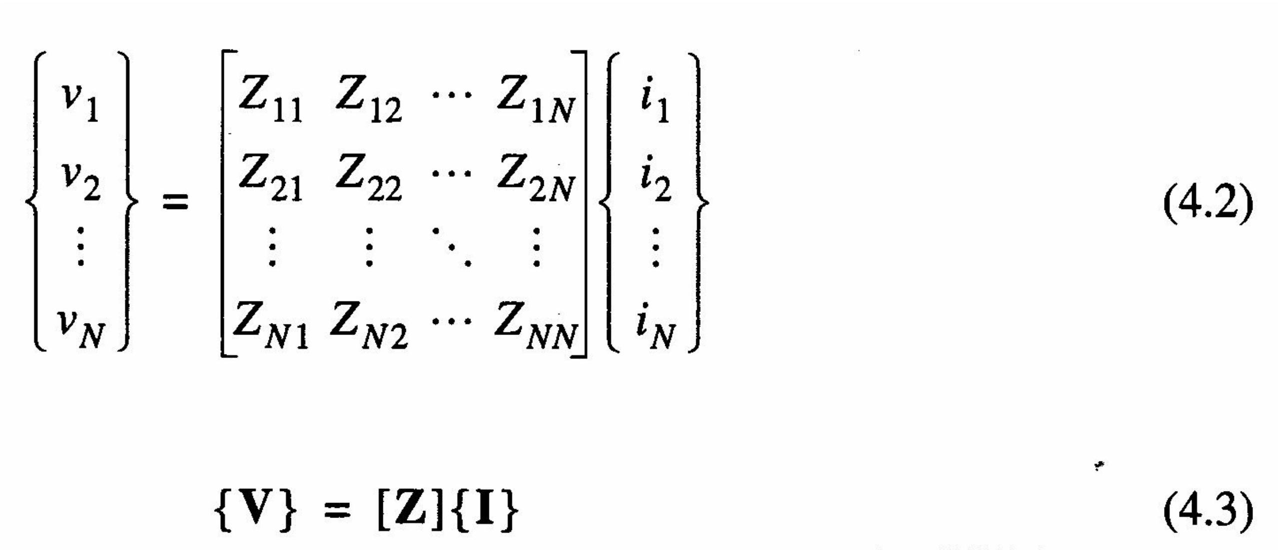


Figure 4-1 Basic voltage and current definitions for single- and multiport network.

Ex. Z-matrix



$$\begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{Bmatrix} \tag{4.2}$$

$$\{\mathbf{V}\} = [\mathbf{Z}]\{\mathbf{I}\} \tag{4.3}$$

ABCD network

$$\begin{Bmatrix} v_1 \\ i_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} v_2 \\ -i_2 \end{Bmatrix} \tag{4.10}$$

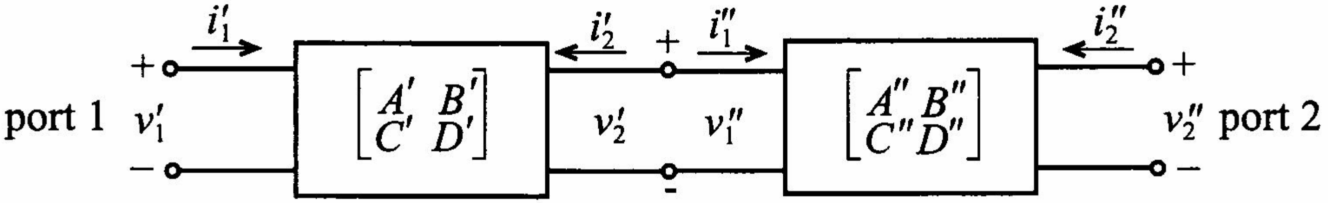


Figure 4-9 Cascading two networks.

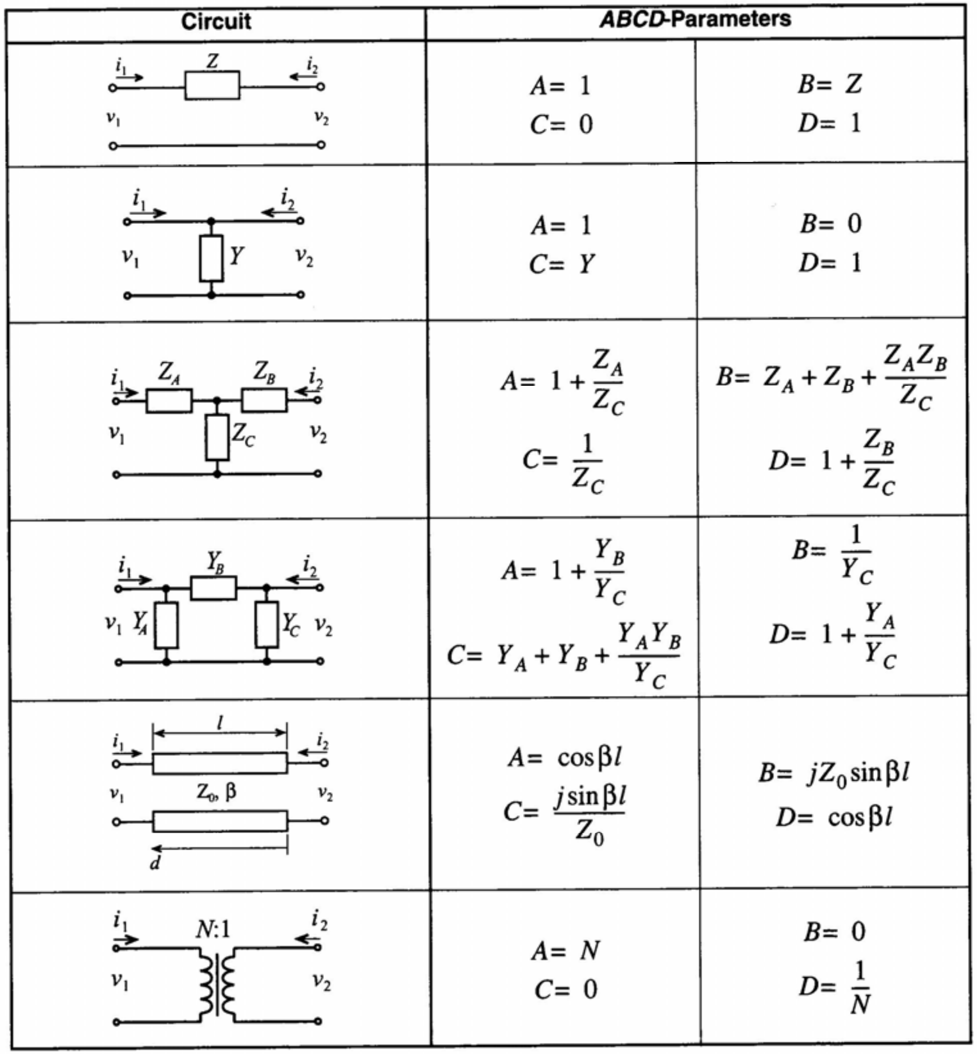
$$\begin{Bmatrix} v_1 \\ i_1 \end{Bmatrix} = \begin{Bmatrix} v_1{}' \\ i_1{}' \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} v_2{}' \\ -i_2{}' \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} v_1{}'' \\ i_1{}'' \end{Bmatrix}$$

$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{Bmatrix} v_2{}'' \\ -i_2{}'' \end{Bmatrix} \tag{4.21}$$

Cascade coupling made easy

ABCD-parameters for ”useful” 2-ports

Table 4-1 *ABCD*-Parameters of Some Useful Two-Port Circuits.



Circuit	***ABCD*-Parameters**	
i_1 Z i_2 v_1 v_2	$A= 1$ $C= 0$	$B= Z$ $D= 1$
i_1 i_2 v_1 Y v_2	$A= 1$ $C= Y$	$B= 0$ $D= 1$
i_1 Z_A Z_B i_2 v_1 Z_C v_2	$A= 1+\frac{Z_A}{Z_C}$ $C= \frac{1}{Z_C}$	$B= Z_A+Z_B+\frac{Z_A Z_B}{Z_C}$ $D= 1+\frac{Z_B}{Z_C}$
i_1 Y_B i_2 v_1 Y_A Y_C v_2	$A= 1+\frac{Y_B}{Y_C}$ $C= Y_A+Y_B+\frac{Y_A Y_B}{Y_C}$	$B= \frac{1}{Y_C}$ $D= 1+\frac{Y_A}{Y_C}$
l i_1 i_2 v_1 Z_0, β v_2 d	$A= \cos\beta l$ $C= \frac{j\sin\beta l}{Z_0}$	$B= jZ_0\sin\beta l$ $D= \cos\beta l$
i_1 $N{:}1$ i_2 v_1 v_2	$A= N$ $C= 0$	$B= 0$ $D= \frac{1}{N}$

Conversion between different 2-port types

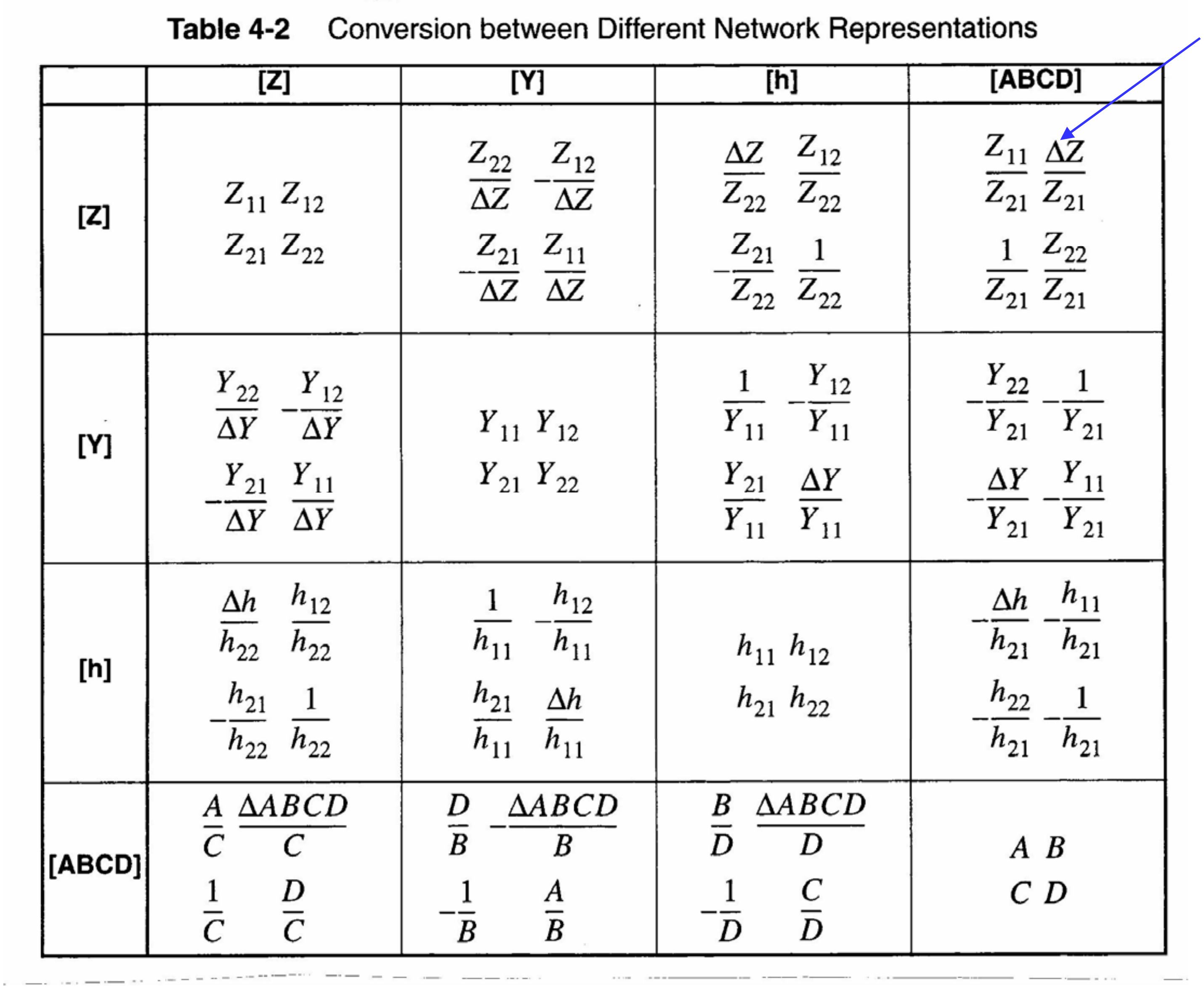


Table 4-2 Conversion between Different Network Representations

	[Z]	[Y]	[h]	[ABCD]
[Z]	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{matrix}$	$\begin{matrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$
[Y]	$\begin{matrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{matrix}$	$\begin{matrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{matrix}$
[h]	$\begin{matrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$
[ABCD]	$\begin{matrix} \frac{A}{C} & \frac{\Delta ABCD}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{D}{B} & -\frac{\Delta ABCD}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta ABCD}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$

determinant

Two-ports at high frequencies

For high frequencies: Difficult to provide adequate shorts and opens due to **reflections**

Introduce: ”scattering” parameters (**S-parameters**)

Then: line terminated in its characteristic impedance → gives no reflections!

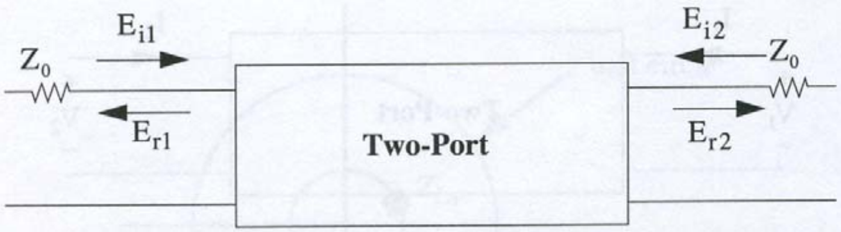


FIGURE 3.7. S-parameter port variable definitions

S-parameters

- 2-port used for definition of S-parameters
- **”Power waves”** defined as

$$a_n = \frac{1}{2\sqrt{Z_0}}(V_n + Z_0 I_n) \tag{4.36a}$$

$$b_n = \frac{1}{2\sqrt{Z_0}}(V_n - Z_0 I_n) \tag{4.36b}$$

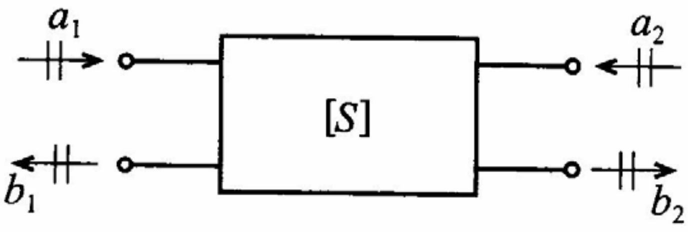


Figure 4-14 Convention used to define *S*-parameters for a two-port network.

Use incident and reflected voltage waves

- The solution is waves in a **positive** and **negative** direction

$$V(z) = V^+ e^{-kz} + V^- e^{+kz} \qquad (2.34)$$

$$I(z) = I^+ e^{-kz} + I^- e^{+kz} \qquad (2.35)$$

$$I(z) = \frac{k}{(R + j\omega L)}(V^+ e^{-kz} - V^- e^{+kz}) \qquad (2.36)$$

(Jmfr.2.27)

Characteristic line-impedance: $Z_0 = \frac{V^+}{I^+} = -\frac{V^-}{I^-}$

$$\boxed{Z_0 = \frac{(R + j\omega L)}{k} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}} \qquad (2.37)$$

I. $V(z) = V^+ e^{-kz} + V^- e^{+kz}$

II. $I(z) = \frac{1}{Z_0}\left(V^+ e^{-kz} - V^- e^{+kz}\right)$

I. $V^+ e^{-kz} = V(z) - V^- e^{+kz}$

II. $V^- e^{+kz} = V^+ e^{-kz} - Z_0 \cdot I(z)$

I. $V^+ e^{-kz} = \frac{1}{2}\left(V(z) + Z_0 I(z)\right)$

II. $V^- e^{+kz} = \frac{1}{2}\left(V(z) - Z_0 I(z)\right)$

Define:

Incident power wave: $a_n = \frac{1}{\sqrt{Z_0}} \cdot V_n^+ e^{-kz}$ Port $n = 1, 2$

Reflected —"—"—: $b_n = \frac{1}{\sqrt{Z_0}} \cdot V_n^- e^{+kz}$

$\frac{1}{\sqrt{Z_0}}$ = scaling factor

I. $a_n = \frac{1}{2\sqrt{Z_0}} \cdot (V_n(z) + Z_0 I_n(z))$

II $b_n = \frac{1}{2\sqrt{Z_0}} \cdot (V_n(z) - Z_0 I_n(z))$

I + II. $a_n + b_n = \frac{1}{\sqrt{Z_0}} \cdot V_n(z)$

$a_n - b_n = \sqrt{Z_0} \cdot I_n(z)$

Calculate the power of the wave :

$$P_n = \frac{1}{2} \mathrm{Re} \left\{ V_n \cdot I_n^* \right\}$$

I. $V_n = \sqrt{Z_0} \, (a_n + b_n)$

II. $I_n = \frac{1}{\sqrt{Z_0}} \, (a_n - b_n)$

I. $V_n = \sqrt{Z_0} \left[(a_{nR} + j a_{ni}) + (b_{nR} + j b_{ni}) \right]$

$= \sqrt{Z_0} \left[(a_{nR} + b_{nR}) + j \, (a_{ni} + b_{ni}) \right]$

II. $I_n = \frac{1}{\sqrt{Z_0}} \left[(a_{nR} - b_{nR}) + j \, (a_{ni} - b_{ni}) \right]$

$I_n^* =$ - - - - - ÷ - - - -

$$V_n \cdot I_n^* = (a_{nR}+b_{nR})\cdot(a_{nR}-b_{nR}) + j(\ ) + j(\ )$$
$$+ (a_{ni}+b_{ni})\cdot(a_{ni}-b_{ni})$$

$$P_n = \frac{1}{2} R_e(V_n \cdot I_n^*) = \frac{1}{2}\left[(a_{nR}^2 - b_{nR}^2) + (a_{ni}^2 - b_{ni}^2)\right]$$
$$= \frac{1}{2}\left[(a_{nR}^2 + a_{ni}^2) - (b_{nR}^2 + b_{ni}^2)\right]$$

$$P_n = \frac{1}{2}\left(|a_n|^2 - |b_n|^2\right)$$

$n=1,2$ (ports)

Power of incident wave

Power of reflected wave

↑ = square of magnitude

Normalizing by $\sqrt{Z_0}$: convenient!

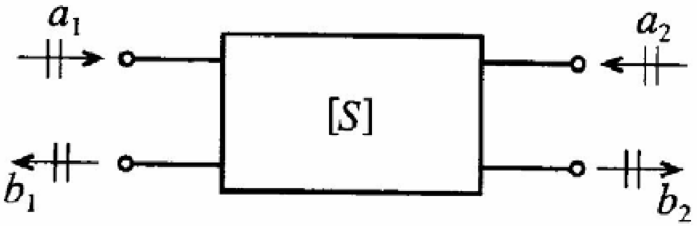
Definition of S-parameters

- The power is:

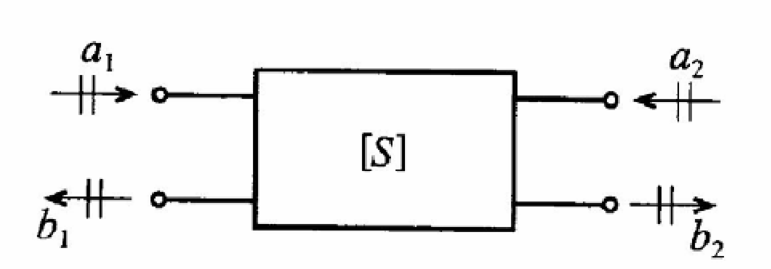
$$P_n = \tfrac{1}{2}\mathrm{Re}\{V_n I_n^*\} = \tfrac{1}{2}(|a_n|^2 - |b_n|^2)$$

S-parameters

$$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$



Interpretation of S-parameters



$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2 = 0} \equiv \frac{\text{reflected power wave at port 1}}{\text{incident power wave at port 1}} \qquad (4.42a)$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0} \equiv \frac{\text{transmitted power wave at port 2}}{\text{incident power wave at port 1}} \qquad (4.42b)$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1 = 0} \equiv \frac{\text{reflected power wave at port 2}}{\text{incident power wave at port 2}} \qquad (4.42c)$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1 = 0} \equiv \frac{\text{transmitted power wave at port 1}}{\text{incident power wave at port 2}} \qquad (4.42d)$$

Measuring S-parameters

- S-parameters are measured when lines are **terminated** with their **characteristic impedances**

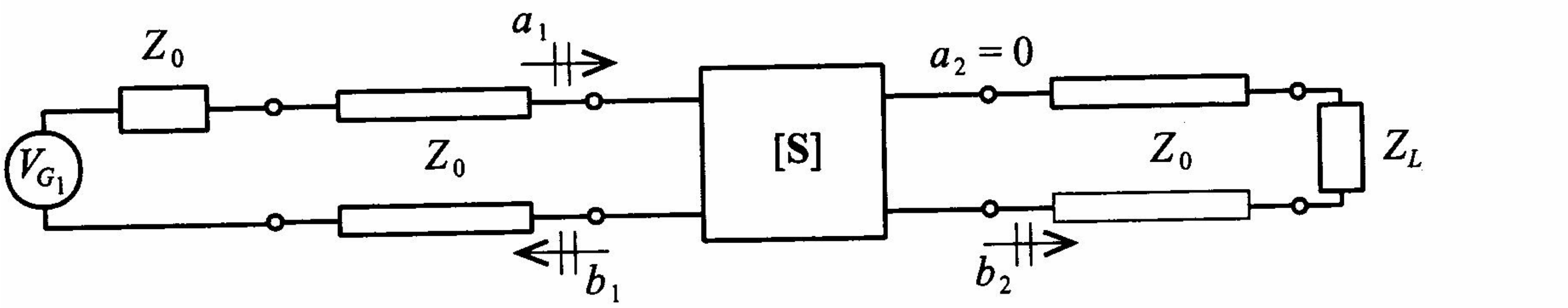
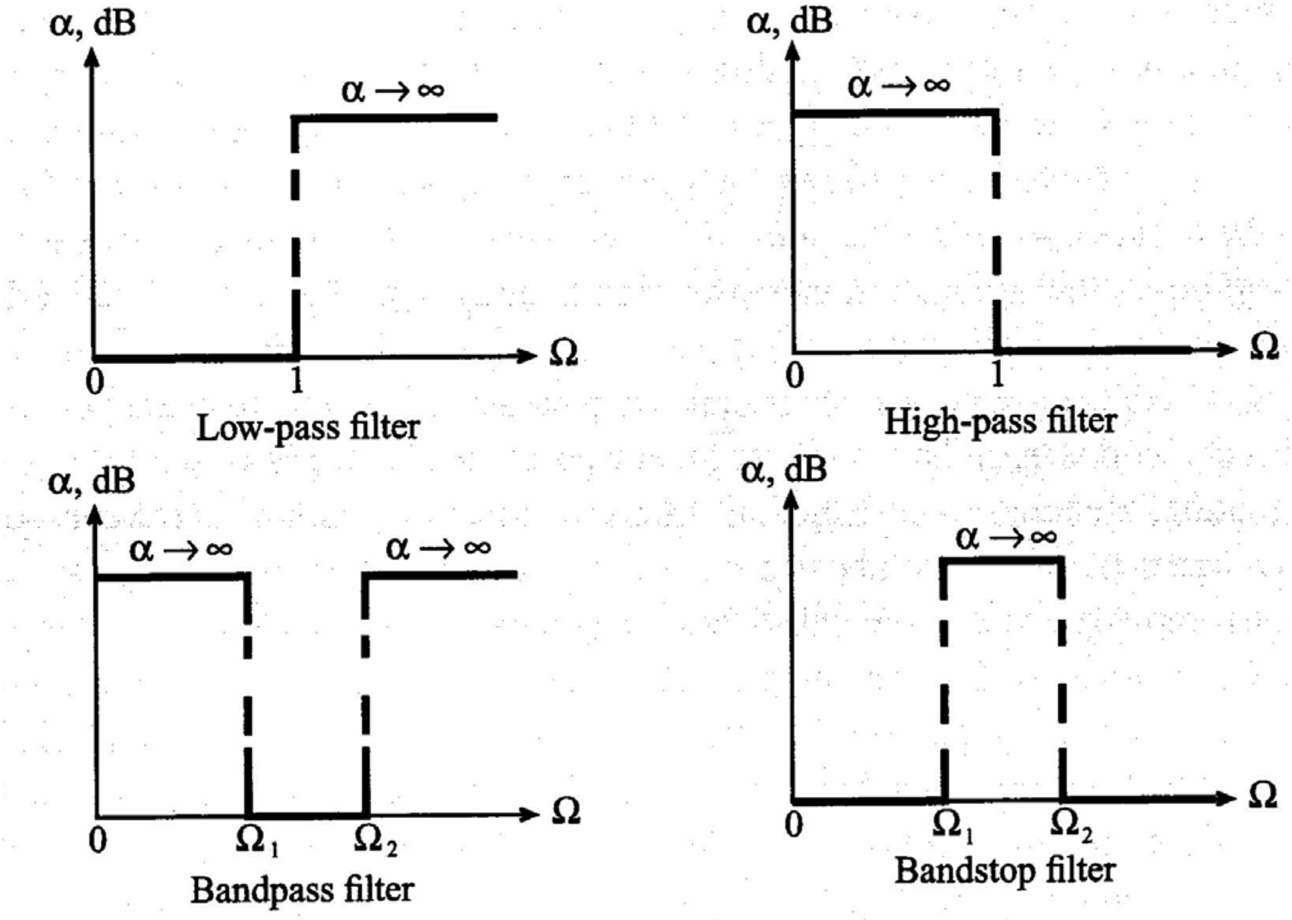


Figure 4-15 Measurement of S_{11} and $S_{21}$by matching the line impedance Z_0 at port 2 through a corresponding load impedance $Z_L = Z_0$.

Filters

- Different filter types



Low-pass filter

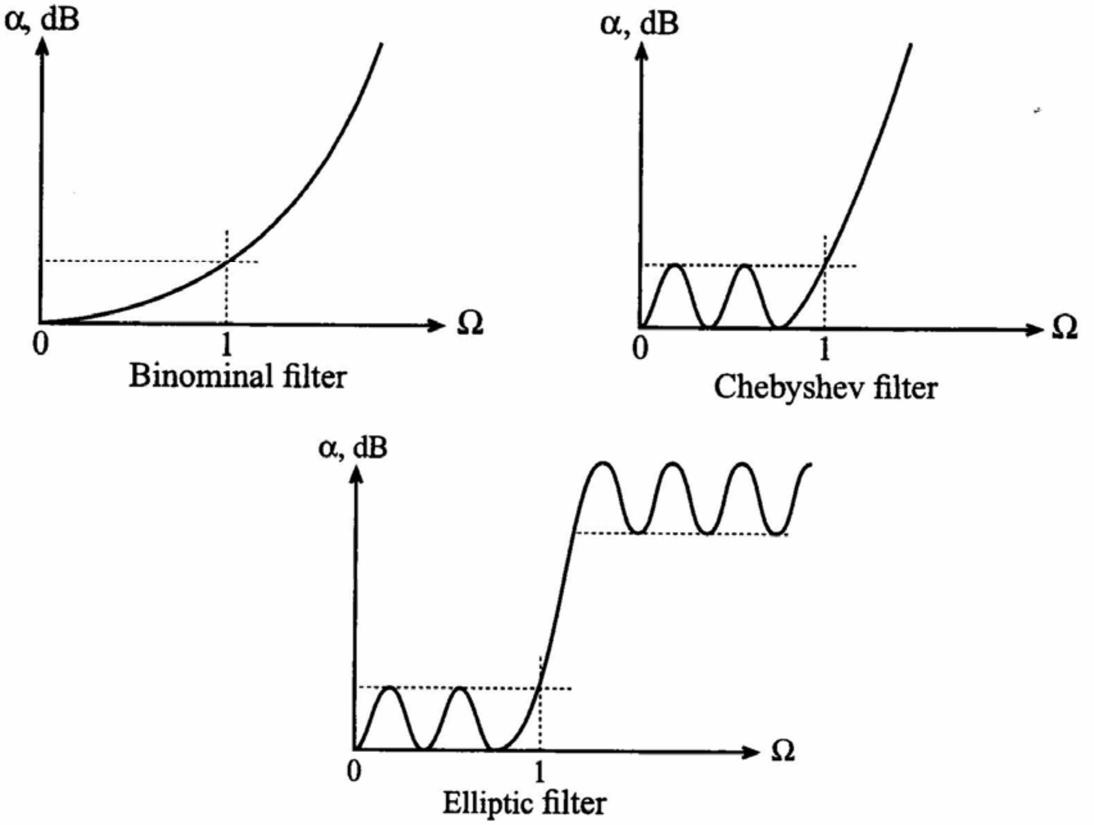
High-pass filter

Bandpass filter

Bandstop filter

Figure 5-1 Four basic filter types.

Ex. of 3 different filter types



Butterworth

Binominal filter

Chebyshev filter

Elliptic filter

Figure 5-2 Actual attenuation profile for three types of low-pass filters.

Filter parameters

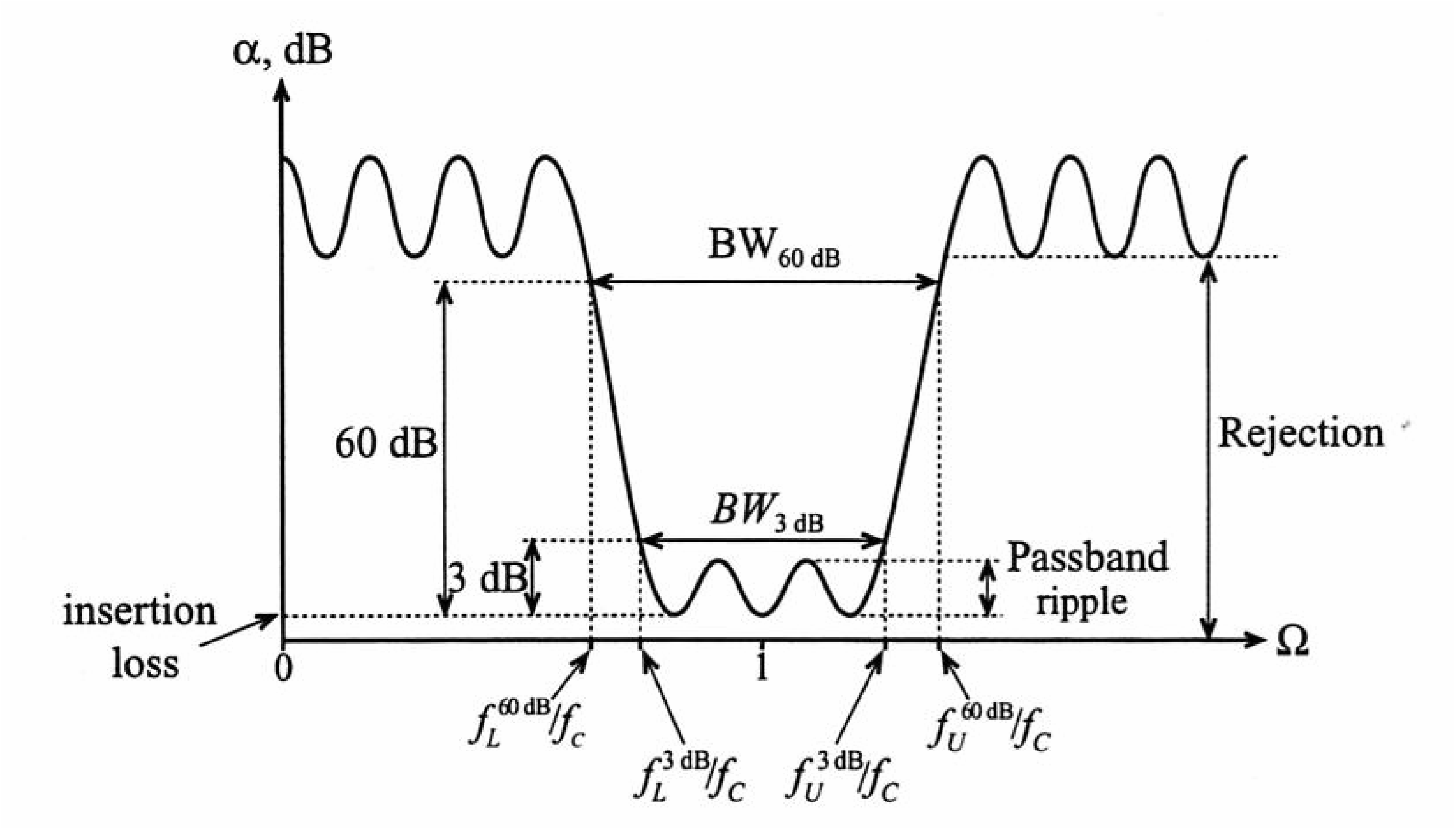


Figure 5-3 Generic attenuation profile for a bandpass filter.

Q-factor

- Definition of **Q-factor**

$$Q = \omega \frac{\text{average stored energy}}{\text{energy loss per cycle}}\bigg|_{\omega = \omega_c} = \omega \frac{\text{average stored energy}}{\text{power loss}}\bigg|_{\omega = \omega_c} = \omega \frac{W_{\text{stored}}}{P_{\text{loss}}}\bigg|_{\omega = \omega_c} \tag{5.4}$$

- Different definitions of the Q-factor exist
  - The definitions are equivalent

$$Q_{LD} = \frac{f_c}{f_U^{3\text{dB}} - f_L^{3\text{dB}}} \equiv \frac{f_c}{BW^{3\text{dB}}}$$

Unloaded – loaded Q

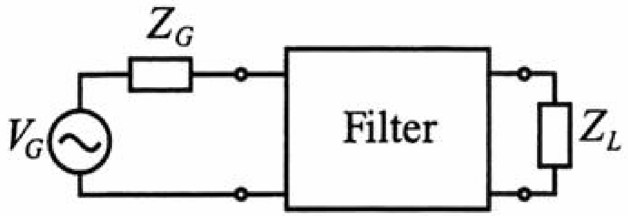
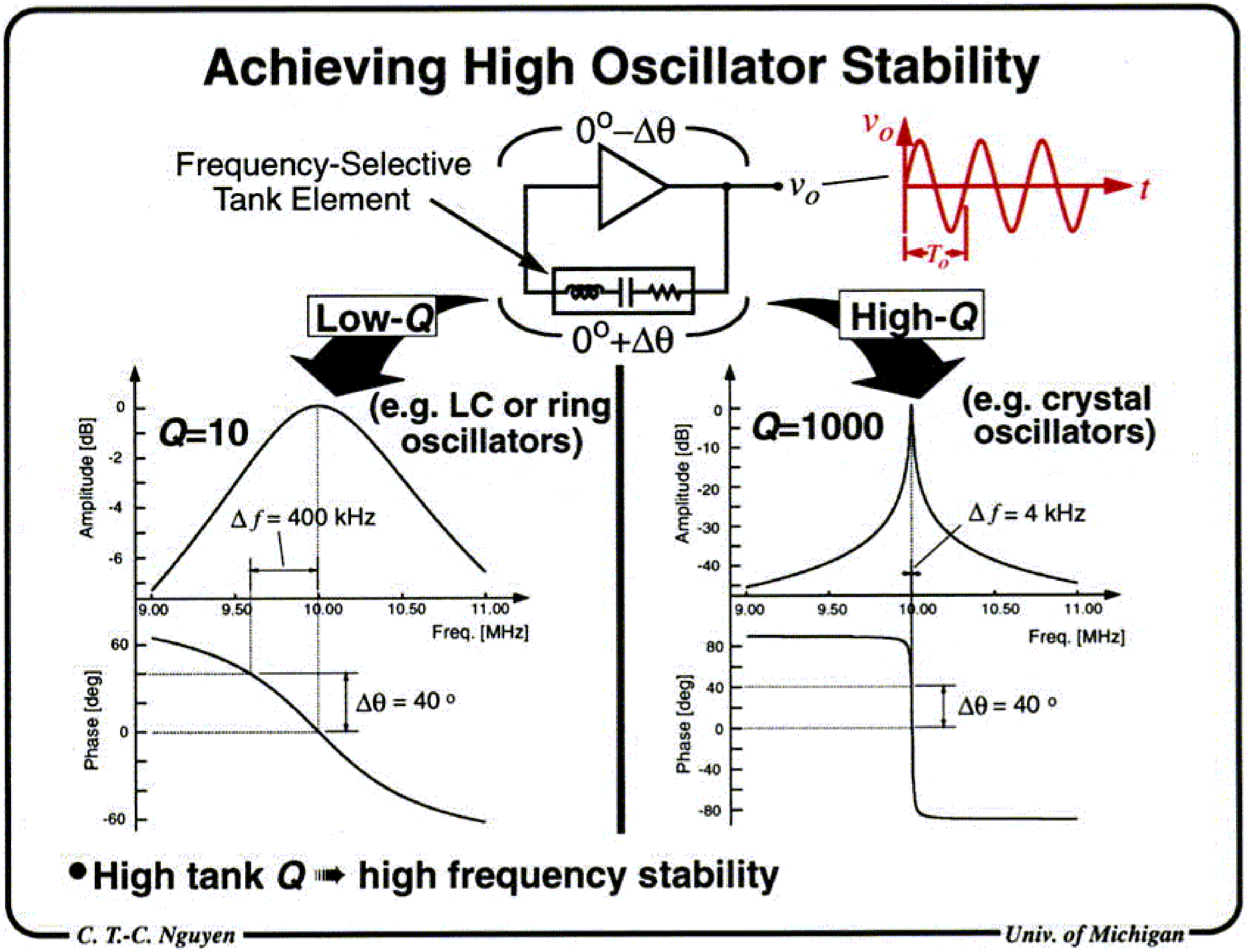


Figure 5-4 Filter as a two-port network connected to an RF source and load.

$$\frac{1}{Q_{LD}} = \frac{1}{\omega}\left(\frac{\text{power loss in filter}}{\text{average stored energy}}\right)\Bigg|_{\omega=\omega_r} + \frac{1}{\omega}\left(\frac{\text{power loss in load}}{\text{average stored energy}}\right)\Bigg|_{\omega=\omega_r} \tag{5.5}$$

$$\boxed{\frac{1}{Q_{LD}} = \frac{1}{Q_F} + \frac{1}{Q_E}}$$

Q-factor is important for frequency stability



Achieving High Oscillator Stability

- **High tank Q ➠ high frequency stability**

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