

INF5490 RF MEMS

LN03: Modeling, design and analysis

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Today's lecture

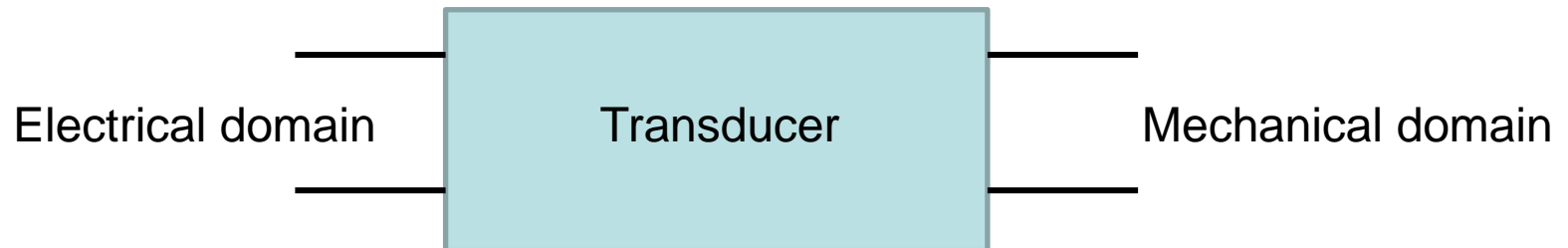
- Operating principles for MEMS components
 - Transducer principles
 - Sensor principles
- Methods for RF MEMS modeling
 - **1. Simple mathematical modeling**
 - **2. Converting to electrical equivalents**
 - **(3. Analyzing using Finite Element Methods)**
 - → LN04

References

- In addition to the course book (Varadan et al):
- For additional reading ("cursory"):
- Senturia, Microsystem design, Kluwer 2002
- Course EE 5349/4328 (University of Texas at Arlington)
 - *"Fundamental MEMS, an Introduction to Microelectromechanical Systems (MEMS)"*
 - <http://arri.uta.edu/acs/jmireles/MEMSclass/lecture12.pdf>

(RF) MEMS transducers

- **Electromechanical transducers**
 - Transforming
electrical energy \leftrightarrow mechanical energy



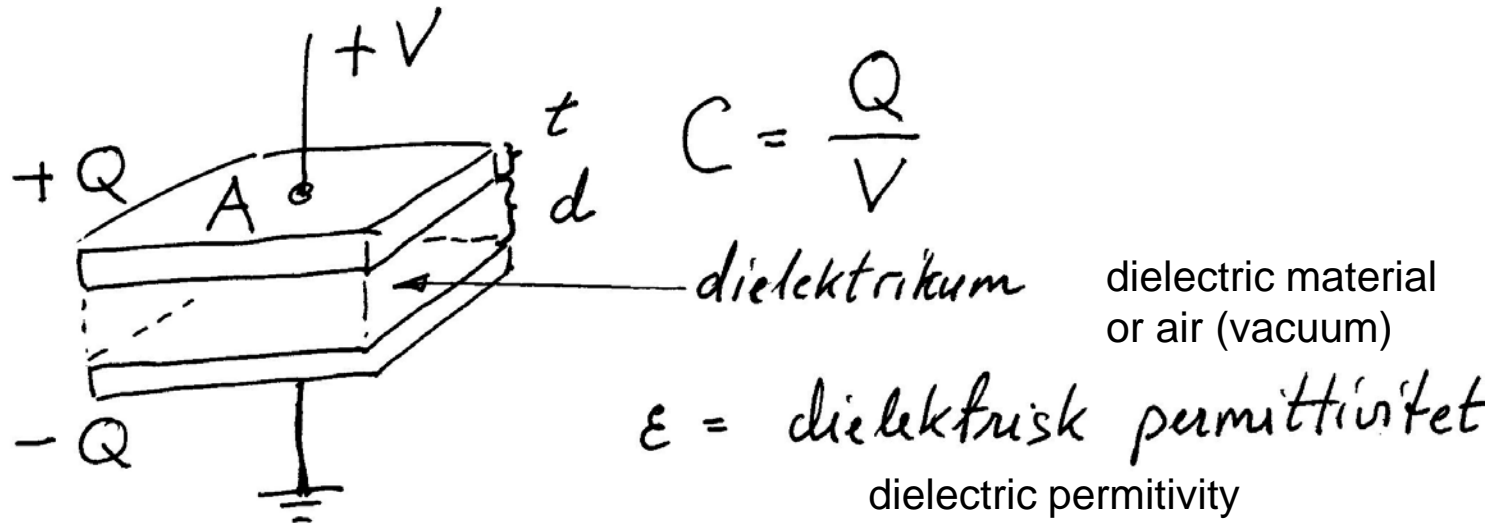
(RF) MEMS transducers

- **Electromechanical transducers**
 - Transforming
electrical energy \leftrightarrow mechanical energy
- Basic transducer principles
 - Electrostatic \rightarrow
 - Electromagnetic
 - Electro thermal
 - Piezoelectric

Transducer principles

- **Electrostatic** transducers
 - Principle: **force** exists between **electric charges**
 - "Coulombs law"
 - Implemented by using a **capacitor with movable "plates"**
 - **Vertical** movement: parallel plates
 - **Horizontal** movement: comb structures

Parallel plate capacitor



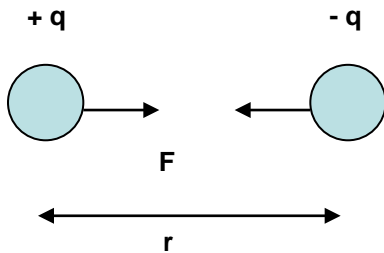
- Stored energy when mechanical work is performed on the unit can be **converted** to electrical energy
- Stored energy when electrical work is performed on the unit can be **converted** to mechanical energy

Simple mathematical modeling

- Important equations for many RF MEMS components:
 - → **Parallel plate capacitor!**
 - Study **electrostatic** actuation of the capacitor with one plate suspended by a spring
 - Calculate **”pull-in”**
 - Formulas and figures →

Electrostatics

Electric force between charges: **Coulombs law**



$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

Electric field = force pr. unit charge

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Work done by a force = change in potential energy

$$W_{a \rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l} = U_a - U_b$$

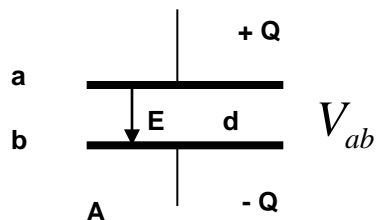
Potential, V = potential energy pr. unit charge

$$V = \frac{U}{q_0}$$

Voltage = potential difference

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

Capacitance



Definition of capacitance

$$C = \frac{Q}{V_{ab}}$$

Surface charge density = σ

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A} \cdot \frac{1}{\epsilon_0}$$

Voltage

$$V_{ab} = E \cdot d = \frac{Q}{A\epsilon_0} \cdot d$$

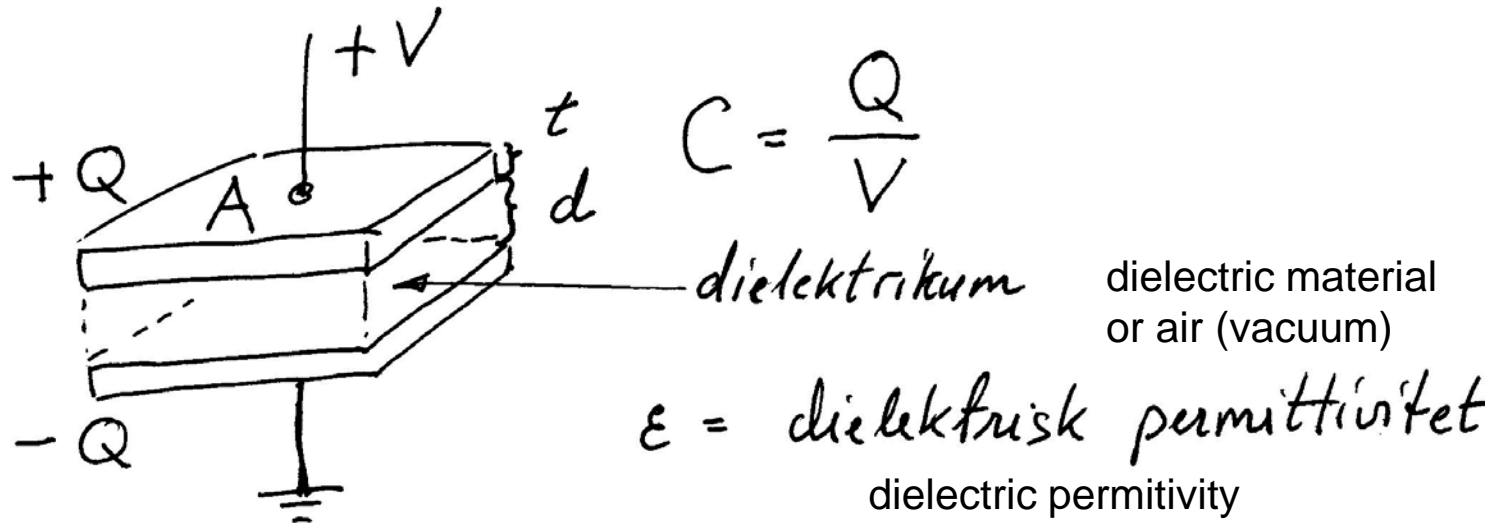
$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$

Energy stored in a capacitor, C ,
that is charged to a voltage V_0 at a current

$$i = \dot{Q} = C \frac{dV}{dt}$$

$$U = \int v \cdot i \cdot dt = \int v \cdot C \frac{dv}{dt} \cdot dt = C \int_0^{V_0} v \cdot dv = \frac{1}{2} C V_0^2 = \frac{\epsilon_0 A}{2d} V_0^2$$

Parallel plate capacitor



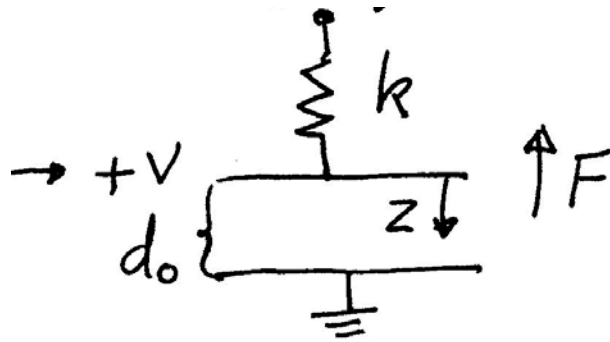
Attractive force between plates

$$F = -\frac{\partial U}{\partial d} = -\frac{\partial}{\partial d} \left(\frac{\epsilon A}{2d} V^2 \right) = \frac{\epsilon A V^2}{2d^2}$$

Movable capacitor plate

- Assumptions for calculations:
 - Suppose air between plates
 - Spring attached to upper plate
 - Spring constant: k
 - Spring force!
 - Voltage is turned on
 - Electrostatic attraction
 - Electrostatic force!
 - At equilibrium
 - Forces up and forces down are in balance →

Force balance



k = spring constant

$$F_{\text{spring}} = k \cdot x$$

deflection from start position

d_0 = gap at 0V and zero spring strain

$$d = d_0 - z$$

$$z = d_0 - d$$

Force on upper plate with voltage V and distance d :

$$F_{\text{net}} = - \frac{\epsilon A V^2}{2 d^2} + k (d_0 - d) = 0 \text{ at equilibrium}$$

Two equilibrium positions

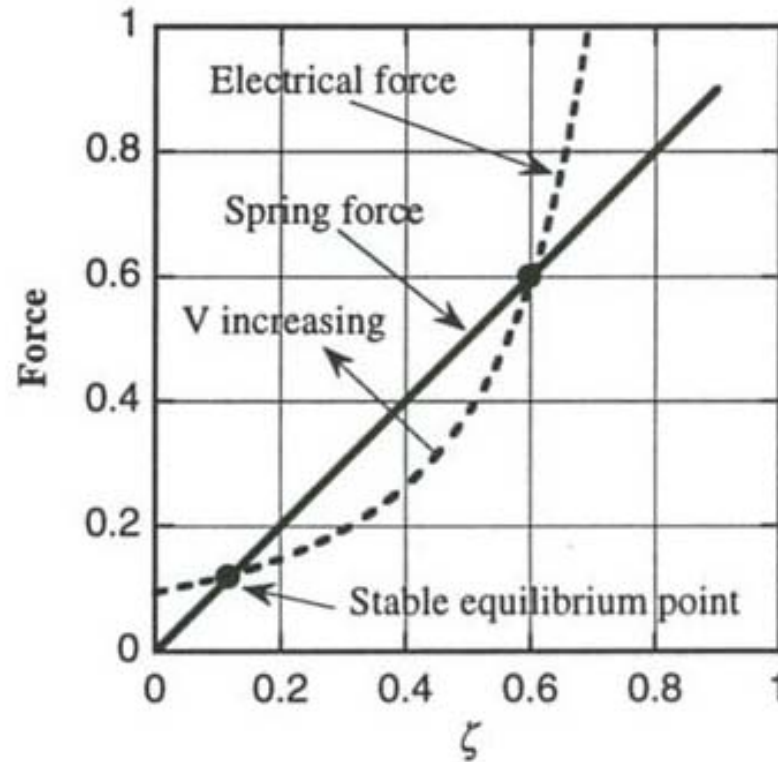


Figure 6.7. Electrical and spring forces for the voltage-controlled parallel-plate electrostatic actuator, plotted for $V/V_{PI} = 0.8$.

$$\zeta = 1 - d/d_0$$

Senturia

Stability

- How the forces develop when d decreases
 - Suppose a small perturbation in the gap at constant voltage

$$\delta F_{\text{net}} = \left. \frac{\partial F_{\text{net}}}{\partial d} \right|_V \cdot \delta d$$

$$\delta F_{\text{net}} = \left(\frac{\epsilon A V^2}{d^3} - k \right) \delta d$$

Suppose the gap decreases $\delta d < 0$

If the upward force also decreases,
the system is **UNSTABLE!**

$$\delta F_{\text{net}} < 0,$$

Stability, contd.

Stability condition:

$$\left. \frac{\partial F_{net}}{\partial d} \right|_V < 0$$

$$k > \frac{\epsilon A V^2}{d^3}$$

Pull-in when:

$$k = \frac{\epsilon A V_{PI}^2}{d_{PI}^3}$$

Pull-in

$$F_{net} = 0$$

$$\frac{\epsilon A V_{PI}^2}{2 d_{PI}^2} = k (d_0 - d_{PI})$$

$\uparrow = \frac{\epsilon A V_{PI}^2}{d_{PI}^3}$

Pull-in when:

$$d_{PI} = \frac{2}{3} d_0$$

$$V_{PI} = \sqrt{\frac{8 k d_0^3}{27 \epsilon A}}$$

Pull-in

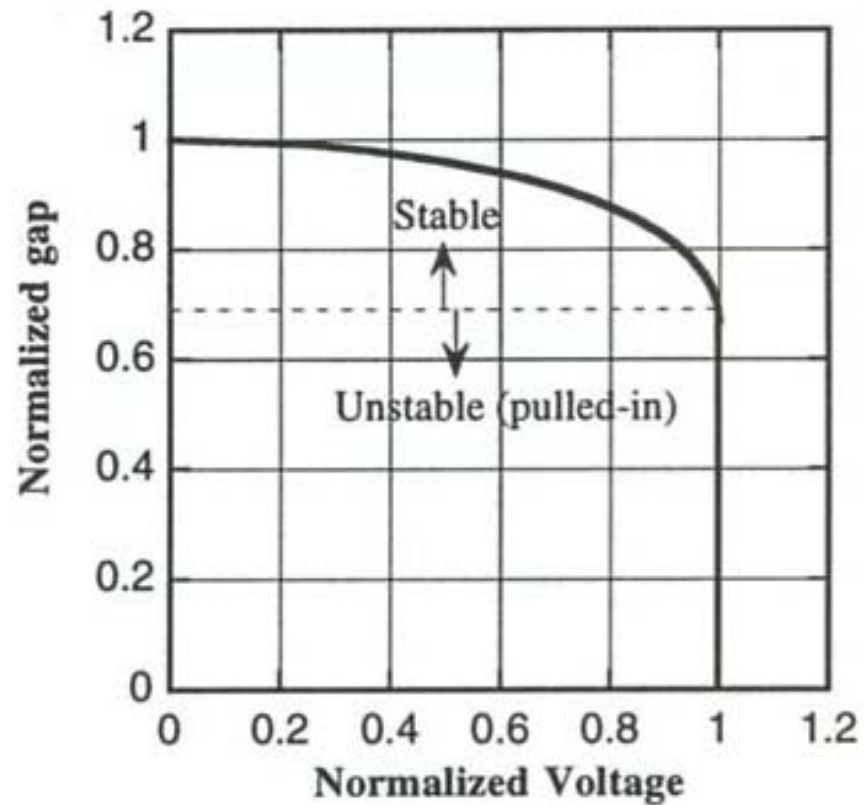


Figure 6.8. Normalized gap as a function of normalized voltage for the electrostatic actuator.

Senturia

Electrostatic transducers: summary

- + Simple principle and fabrication
- + Actuation (movement) controlled by voltage
 - voltage \rightarrow charges \rightarrow attractive force \rightarrow movement
- + Movement gives current
 - movement \rightarrow variable capacitor \rightarrow current when voltage is constant:
 $Q = V C$ and $i = dQ/dt = V dC/dt$
- \div Need environmental protection (dust)
 - Packaging required (vacuum)
- \div Transduction mechanism is non-linear
 - ... for variation of distance between plates ...
 - Force is not proportional to voltage
 - Solution: small signal variations around a DC voltage
- The most used form of electromechanical energy conversion

(RF) MEMS transducers

- **Electromechanical transducers**
 - Transforming
electrical energy \leftrightarrow mechanical energy
- Basic transducer principles
 - Electrostatic
 - **Electromagnetic**
 - **Electro thermal**
 - **Piezoelectric**

Transducer principles, contd.

- **Electromagnetic** transducers
 - Magnetic windings pull the element
 - ÷ More complicated processes
- **Electro thermal** actuators
 - Different thermal expansion due to temperature gradients
 - Different materials
 - Each with its: TCE – Thermal Coefficient of Expansion
 - Different locations
 - **Large deflections** can be obtained
 - **Slow!**

Transducer principles, contd.

- **Piezoelectric** transducers
 - In some **anisotropic** crystalline materials the charges will be displaced when **stressed** → electric field
 - stress = "mechanical stress" (Norw: "mekanisk spenning")
 - Similarly, **strain** results when an electric field is applied (relative shrinking or prolongation of unit)
 - strain = "mechanical strain" (Norw: "mekanisk tøyning")
 - Ex. PZT (lead zirconate titanates) – ceramic material
- (Electrostrictive transducers
 - Mechanical deformation by electric field
- (Magnetostrictive transducers
 - Deformation by magnetic field)

Comparing different transducer principles

Table 1.4 Comparison of electromechanical transducers

| Actuator | Fractional stroke (%) | Maximum energy density (J cm^{-3}) | Efficiency | Speed |
|------------------|-----------------------|---|------------|-------|
| Electrostatic | 32 | 0.004 | High | Fast |
| Electromagnetic | 50 | 0.025 | Low | Fast |
| Piezoelectric | 0.2 | 0.035 | High | Fast |
| Magnetostrictive | 0.2 | 0.07 | Low | Fast |
| Electrostrictive | 4 | 0.032 | High | Fast |
| Thermal | 50 | 25.5 | Low | Slow |

Source: Wood, Burdess and Hariss, 1996.

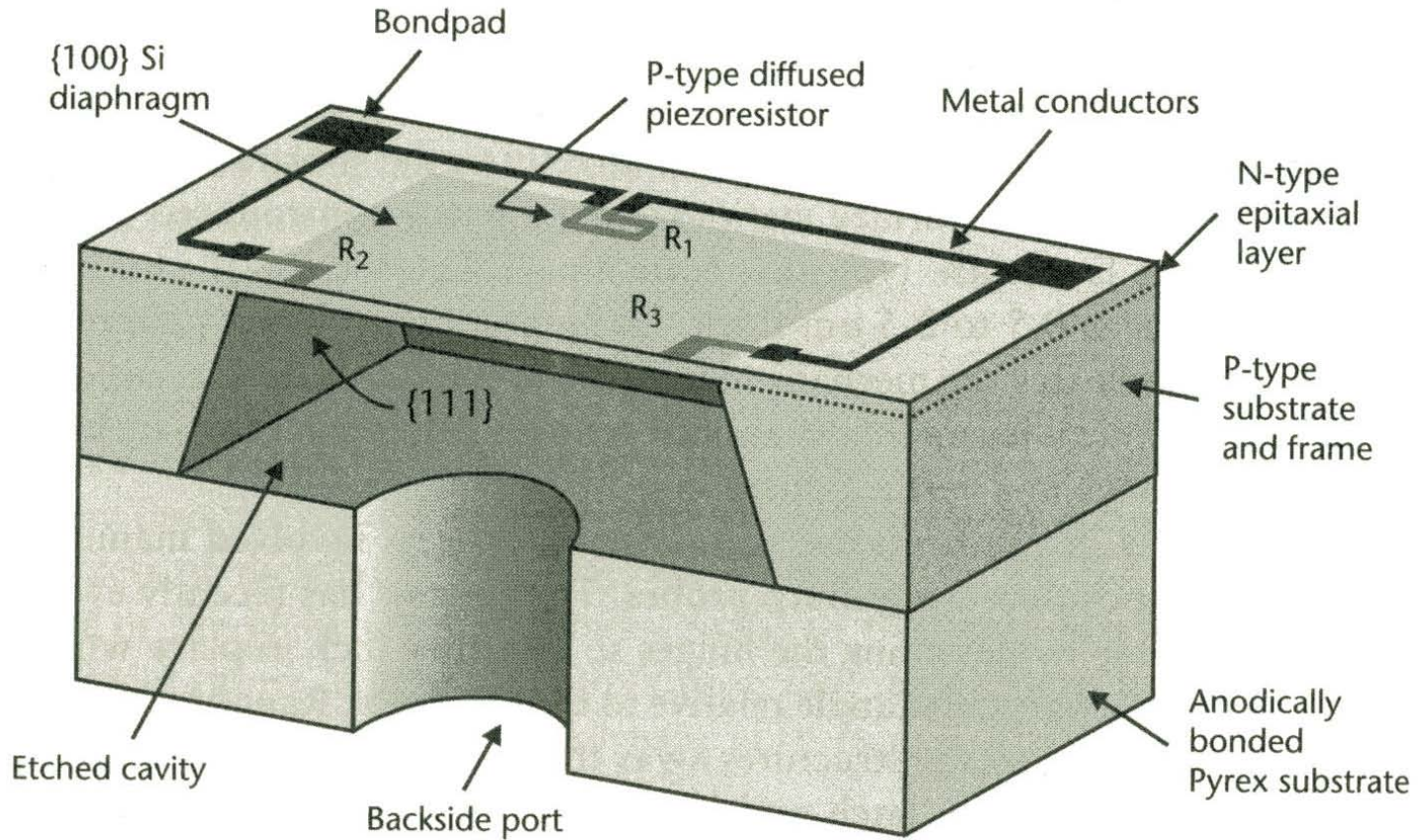
Sensor principles

- Transducers can be used as sensors
 - Piezoresistive detection
 - Capacitive detection
 - Piezoelectric detection
 - Resonance detection

Sensor principles

- **Piezoresistive** detection
 - Resistance varies due to external pressure/stress
 - Resistor value is proportional to strain (“tøyning”)
 - Piezoresistors placed on membrane where **strain is maximum**
 - At the peripheries
 - Used in pressure sensors
 - Deflection of membrane influences R values
 - + **Simple** principle
 - ÷ Performance of piezoresistive micro sensors is **temperature dependent**

Pressure sensor



Sensor principles, contd.

- **Capacitive** detection
 - Exploiting capacitance variations
 - Pressure \rightarrow change in $C \rightarrow$ electric signal
 - Can influence oscillation frequency, charge or voltage
 - Potentially higher performance than piezoresistive detection
 - + Better sensitivity
 - + Can detect small pressure variations
 - + **High stability**

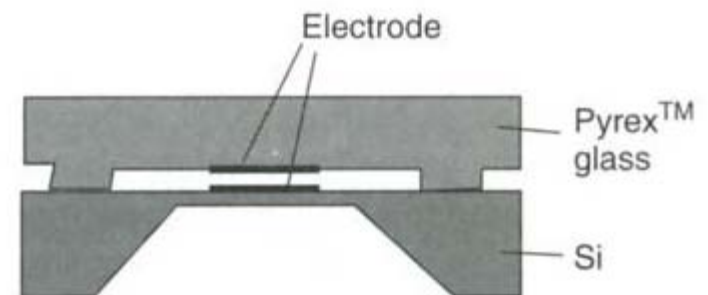


Figure 1.19 Capacitive sensing structure

Sensor principles, contd.

- **Piezoelectric** detection
 - Electric charge distribution changed due to external force → electric field → current
- **Resonance** detection
 - Using resonating structures
 - Analogy: stress variation on a string gives strain and is changing the “**natural**” **resonance frequency**

Methods for modeling RF MEMS

- **1. Simple mathematical modeling**
 - Ex. As explained for the parallel plate capacitor
- Use equations, formulas describing the physical phenomena
 - Simplification, approximations necessary
 - a) Explicit solutions for simple problems
 - Linearization around a bias point
 - b) Numerical solution of a set of equations
 - Typical: differential equations
- **+ Gives the designer insight/ understanding**
 - How the performance changes by parameter variations
 - May be used for initial estimates

Methods for modeling RF MEMS

- 1. Simple mathematical modeling
- **2. Converting to electrical equivalents**
- **3. Analysis using Finite Element Methods**

A mechanical system can be modeled as an electrical circuit

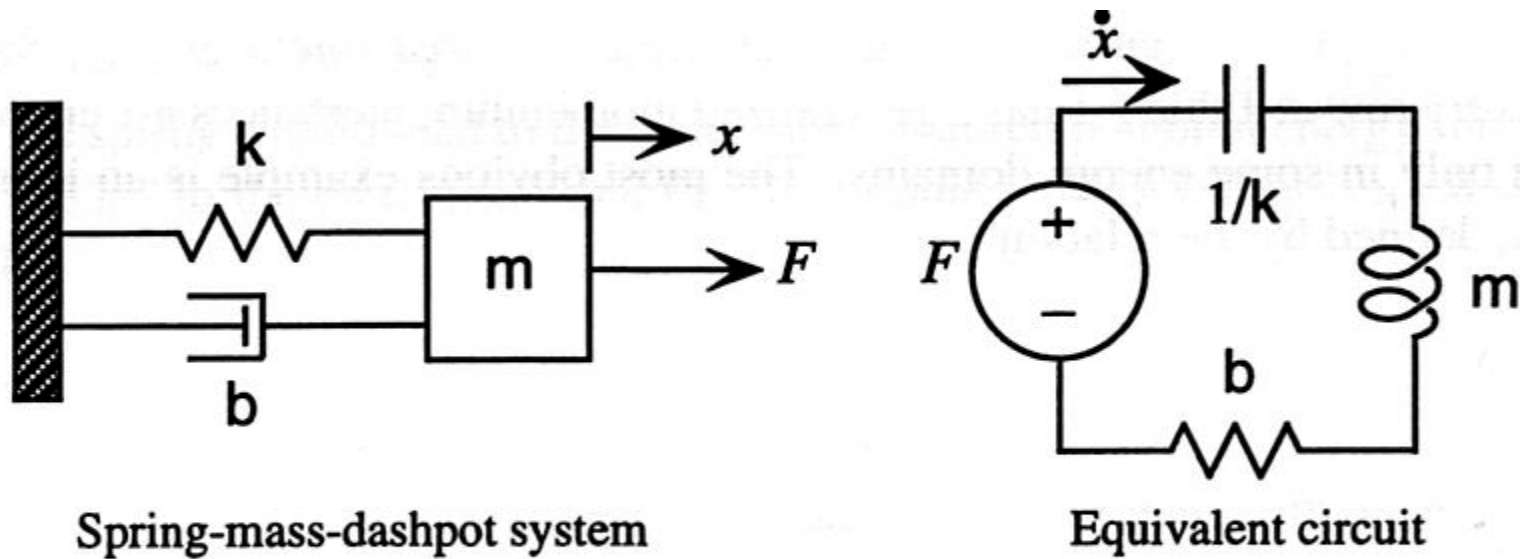


Figure 5.9. Translating mechanical to electrical representations.

$$C=1/k \quad L=m \quad R=b$$

2. Converting to electrical equivalents

- Mechanical behavior can be modeled using **electrical circuit elements**
 - Mechanical structure → simplifications → equivalent electrical circuit
 - ex. spring/mass/damper system → R, C, L -equivalent
 - Then: Possible to “interconnect” electrical and mechanical **energy domains**
 - Simplified modeling and co-simulation of electronic and mechanical parts of the system
 - Proper **analysis-tools** can be used
 - Ex. SPICE

Converting to electrical equivalents, contd.

- We will discuss:
 - Relevant circuit theory
 - Conversion principles
 - **effort - flow**
 - Example of conversion
 - Mechanical resonator
 - In a future lecture:
 - Co-existence and coupling between various energy domains

Circuit theory

- Basic circuit elements: R, C, L
- Current and voltage equations for basic elements (low frequency)
 - Ohms law, C and L-equations
 - $V = RI$, $I = C \, dV/dt$, $V = L \, dI/dt$
 - Laplace transformation
 - From differential equations to algebraic (s-polynomial)
 - → Complex impedances: R , $1/sC$, sL
- Kirchhoffs equations
 - Σ currents into nodes = 0, Σ voltages in loops = 0

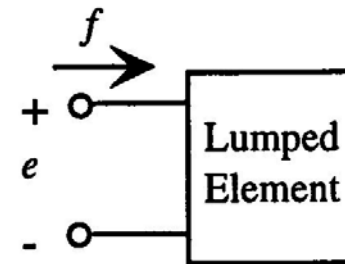
Effort - flow

- Electrical circuits are described by a **set of variables: *conjugate power variables***
 - Voltage V : **across** or **effort** variable
 - Current I : **through** or **flow** variable
 - *An effort variable drives a flow variable through an impedance, Z*

- Circuit element is modeled as a **1-port** with terminals

- Same current ($f = \text{flow}$) in and out and through the element

- ***Positive flow*** into a terminal defining a ***positive effort***



Energy-domains, analogies

- Various energy domains exist
 - Electric, mechanical/elastic, thermal, for liquids etc.
- ***For every energy domain it is possible to define a set of **conjugate power variables** that may be used as basis for lumped component modeling using equivalent circuits elements***
- Table 5.1 Senturia ->

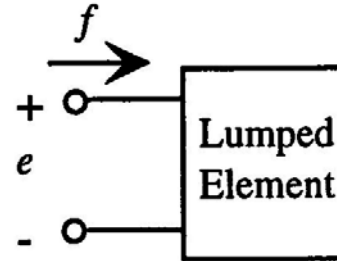
Ex. of conjugate power variables

| Energy Domain | Effort | Flow | Momentum | Displacement |
|---------------------------|----------------------------|--------------------------------|-------------------------------|-------------------|
| Mechanical translation | Force F | Velocity \dot{x}, v | Momentum p | Position x |
| Fixed-axis rotation | Torque τ | Angular velocity ω | Angular momentum J | Angle θ |
| Electric circuits | Voltage V, v | Current I, i | ... | Charge Q |
| Magnetic circuits | Magnetomotive force MMF | Flux rate $\dot{\phi}$ | ... | Flux ϕ |
| Incompressible fluid flow | Pressure P | Volumetric flow Q | Pressure momentum Γ | Volume V |
| Thermal | Temperature T | Entropy flow rate \dot{S} | ... | Entropy S |

Conjugate power variables: e, f

- Properties

- $e * f = \text{power}$
- $e / f = \text{impedance}$



- Generalized **displacement** represents the state, f. ex. position or charge

$$f(t) = \dot{q}(t)$$

$$q(t) = \int_{t_0}^t f(t) dt + q(t_0)$$

- $e * q = \text{energy}$

Generalized momentum

$$p(t) = \int_{t_0}^t e(t) dt + p(t_0)$$

– Mechanics: “impulse”

- $F \cdot dt = mv - mv_0$

– General: **p * f = energy**

Ex.: Mechanical energy domain

$$e = F \quad (\text{kraft})$$

force

$$f = v, \dot{x} \quad (\text{hastighet})$$

velocity

$$g = x \quad (\text{posisjon}) = \int \dot{x} dt$$

position

$$p = p \quad (\text{momentum}) = \int F dt$$

momentum

$$(\text{kraft} \times \text{tid})$$

force x time

$$e \cdot f \rightarrow F \cdot \dot{x} = \frac{F \Delta x}{\Delta t} = \frac{\text{arbeid}}{\text{tid}} = \text{effekt}$$

work/time = power

$$e \cdot g \rightarrow F \cdot x = \text{kraft} \times \text{vei} = \text{arbeid} = \text{energi}$$

force*distance = work = energy

$$p \cdot f \rightarrow p \cdot \dot{x} = m v \cdot v = m v^2 = \text{energi}$$

energy

Ex.: Electrical energy domain

$$e = V \quad (\text{spenning}) \quad \text{voltage}$$

$$f = I \quad (\text{strøm}) \quad \text{current}$$

$$q = \int I dt = Q \quad (\text{ladning}) \quad \text{charge}$$

$$p = \text{n.a.}$$

$$e \cdot f \rightarrow V \cdot I = \text{effekt} \quad \text{power}$$

$$e \cdot q \rightarrow V \cdot Q = V \int I dt = \text{energi} \quad \text{energy}$$

$e \rightarrow V$ - convention

- **Senturia** and **Tilmans** use the **$e \rightarrow V$ -convention**
- Ex. electrical and mechanical circuits
 - $e \rightarrow V$ (voltage) equivalent to F (force)
 - $f \rightarrow I$ (current) equivalent to v (velocity)
 - $q \rightarrow Q$ (charge) equivalent to x (position)
 - $e * f =$ "power" injected into the element

H. Tilmans, Equivalent circuit representation of electromagnetical transducers:
I. Lumped-parameter systems, J. Micromech. Microeng., Vol. 6, pp 157-176, 1996

Other conventions

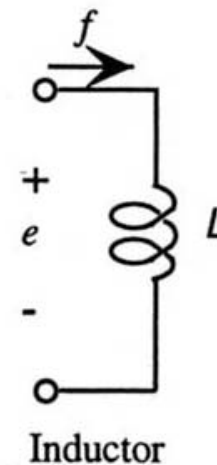
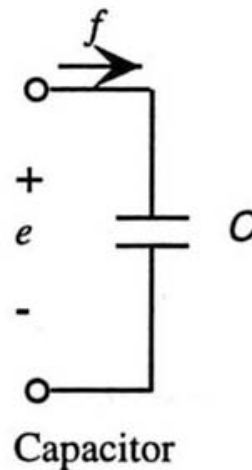
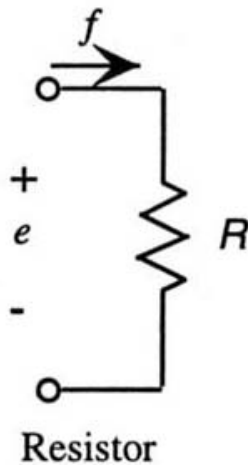
- Different conventions exist for defining **through-** or **across-variables**

Table 5.2. Different conventions for assigning circuit variables.

| Convention | Across Variable | Through Variable | Product | Principal Use |
|-------------------------------|-----------------|------------------|-------------|---|
| $e \rightarrow V$ * | e | f | power | electric circuit elements |
| $f \rightarrow V$ alternative | f | e | power | mechanical circuit elements |
| Thermal | T | \dot{Q} | Watt-Kelvin | thermal circuits |
| HDL | q | e | energy | HDL circuit representation of mechanical elements |

Generalized circuit elements

- **One-port** circuit elements
 - R, dissipating element
 - C, L, energy-storing elements
 - Elements can have a **general function!**
 - Can be used in **various energy domains**



Generalized capacitance

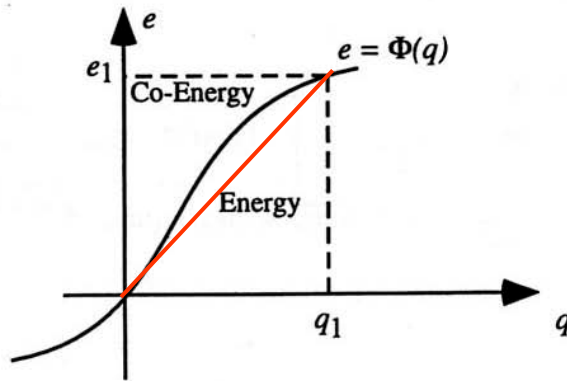


Figure 5.5. Illustrating energy and co-energy for a generalized capacitor.

Compare with a **simplified case**:

- a **linear** capacitor

$$Q = V \cdot C$$

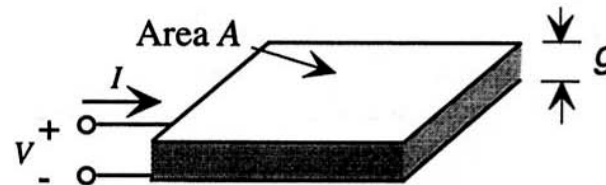
$$V = \frac{1}{C} \cdot Q$$

\Downarrow

$$e = \frac{1}{C} \cdot q$$

$$C = \frac{\epsilon A}{g}$$

definition of C



Generalized capacitance, contd.

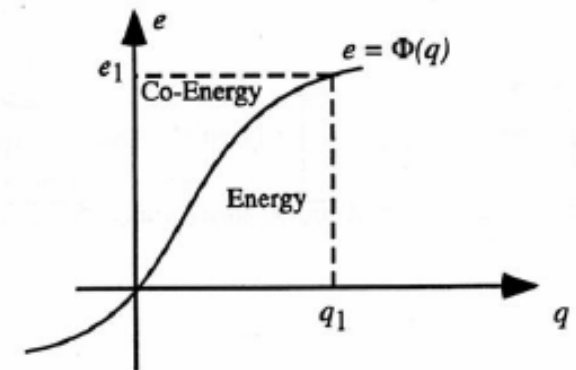
Capacitance is associated with stored **potential energy**

$$\mathcal{W}(q_1) = \int_0^{q_1} e \, dq = \int_0^{q_1} \Phi(q) \, dq \quad (5.10)$$

Co-energy:

$$\mathcal{W}^*(e) = eq - \mathcal{W}(q) \quad (5.11)$$

$$\mathcal{W}^*(e_1) = \int_0^{e_1} q \, de = \int_0^{e_1} \Phi^{-1}(e) \, de \quad (5.12)$$



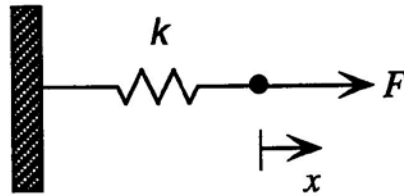
Energy stored in parallel plate capacitor

Energy:
$$W(Q) = \int_0^Q e \cdot dq = \int_0^Q \frac{q}{C} \cdot dq = \frac{Q^2}{2C}$$

Co-energy:
$$W^*(V) = \int_0^V q \cdot de = \int_0^V C \cdot v \cdot dv = \frac{CV^2}{2}$$

$$W^*(V) = W(Q) \quad \text{for linear capacitance}$$

Mechanical spring



Hook's law: $F = k \cdot x$

Stored energy $W(x_1) = \int_0^{x_1} F(x)dx = \frac{1}{2}kx_1^2$ (5.18)

Compare with capacitor $W(Q) = \frac{1}{2} \cdot \frac{1}{C} \cdot Q^2$

Q displacement

x_1 displacement

→ 1/C equivalent to k

"Compliance"

- "Compliance" = "inverse stiffness"

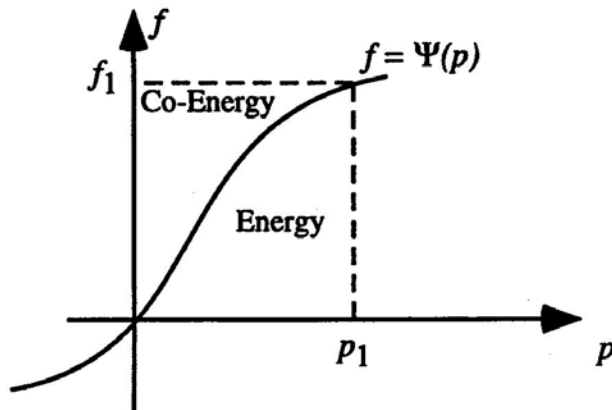
$$C_{spring} = \frac{1}{k}$$

- Stiff spring → small capacitor
- Soft spring → large capacitor

Generalized inductance

Energy also defined as:

$$\int e dt = \int v dt = \int m \cdot v dt = \text{flow} \times \text{momentum}$$




Energy = stored kinetic energy

$$W(p_1) = \int_0^{p_1} f(p) dp$$

Ex.: Electrical inductor

Co-energy: $W^*(f) = \int_0^{f_1} p(f) df$

 $V = L \frac{dI}{dt}$

$$p = \int e dt = \int V dt = \int L \frac{dI}{dt} dt = \int L dI$$

$$p(f) = p(I) = LI$$

$$W^*(f_1) = W^*(I_1) = \int_0^{I_1} L \cdot I \cdot dI = \frac{1}{2} L I_1^2$$

Analogy between mass (mechanical inertance) and inductance L

A mechanical system has **linear momentum**: $p = mv$

Flow: $\phi = v = \frac{p}{m}$

$$W(p_1) = \int_0^{p_1} f(p) dp = \int_0^{p_1} \frac{p}{m} dp = \frac{p_1^2}{2m}$$

Co-energy:

$$W^*(v_1) = \int_0^{v_1} p(v) dv = \int_0^{v_1} (mv) dv = \frac{1}{2} m v_1^2$$

Analogy between m and L

$$W^*(f_1) = W^*(I_1) = \int_0^{I_1} L \cdot I \cdot dI = \frac{1}{2} L I_1^2$$

Compare with: $W^*(v_1) = \frac{1}{2} m v_1^2$

$$I_1 = \text{flow}$$

$$v_1 = \text{---}$$

L is equivalent to m

m = L inertance

Mechanical inertance = mass m
is analog to inductance L

Interconnecting elements

- $e \rightarrow V$ follows two basic principles
 - Elements that share a **common flow**, and hence a common variation of displacement, are connected in **series**
 - Elements that share a **common effort** are connected in **parallel**

Ex. of interconnection:

”Direct transformation”: **series connection**

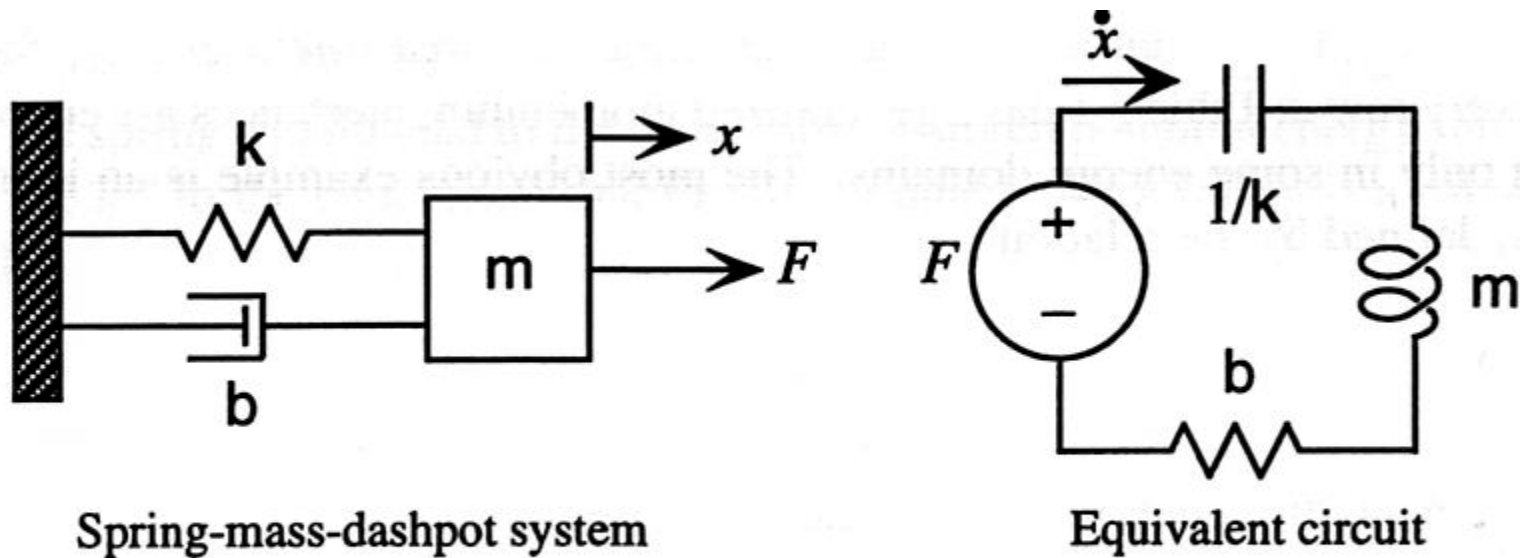
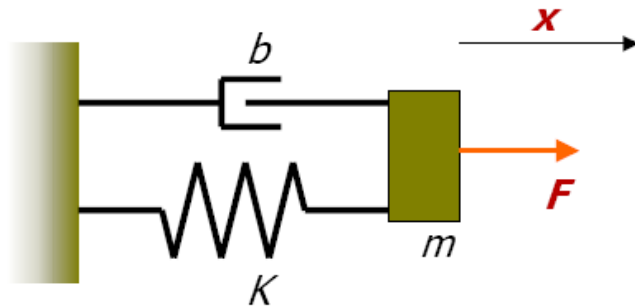


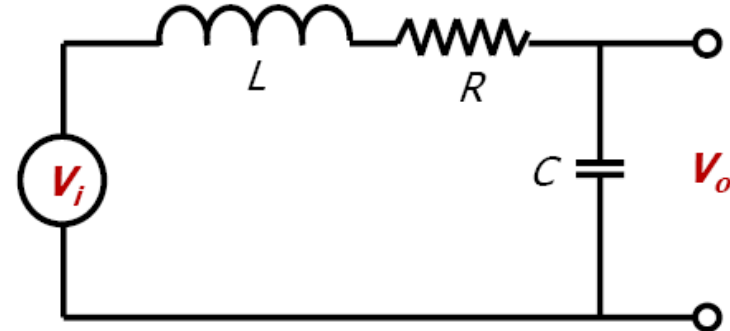
Figure 5.9. Translating mechanical to electrical representations.

Mechanical / Electrical Systems



Input : external force F
 Output : displacement x
 $m\ddot{x}(t) + b\dot{x}(t) + Kx(t) = F$
 m mass, b damping, K stiffness
 Transfer function :

$$H(s) = \frac{x}{F} = \frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{K}{m}}$$

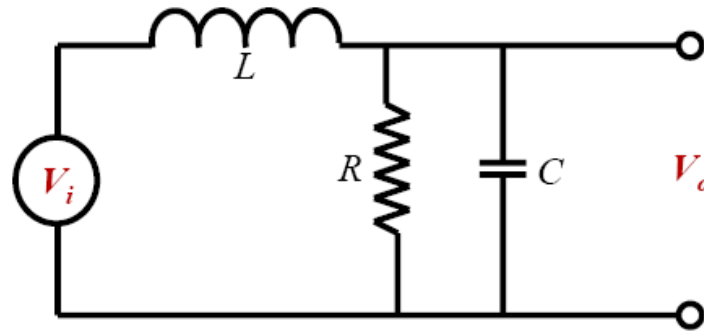


Input : voltage V_i
 Output : voltage V_o
 $L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = V_i$
 L induct., R resist., C capacit.
 Transfer function :

$$H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Mechanical / Electrical Systems

Alternative circuit:



Input : voltage V_i

Output : voltage V_o

$$L\ddot{q}(t) + \frac{L}{RC}\dot{q}(t) + \frac{1}{C}q(t) = V_i$$

L inductance, R resistance, C capacitance

Transfer function :

$$H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Resonators

- Analogy between mechanical and electrical system:
 - Mass m - inductivity L
 - Spring K - capacitance C
 - Damping b - resistance R (depending where R is placed in circuit)
- Solution to 2nd order differential equation:

$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

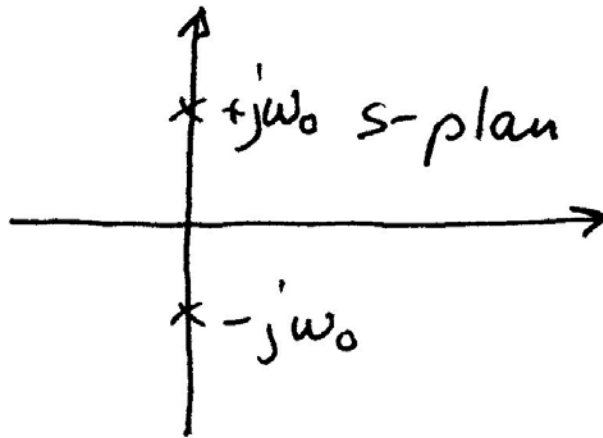
$$\omega_0 = 2\pi f_0 \text{ natural frequency}$$

$$\omega_0 = \sqrt{\frac{K}{m}} \text{ mechanical system, } \omega_0 = \sqrt{\frac{1}{LC}} \text{ electrical system}$$

$$Q \text{ quality factor}$$

System without damping ($b=0$, $R=0$)

$$H(s) = \frac{\omega_0^2}{s^2 + \omega_0^2} = \frac{\omega_0^2}{(s + j\omega_0)(s - j\omega_0)}$$



$$|H(j\omega_0)| = \infty$$

$$H(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

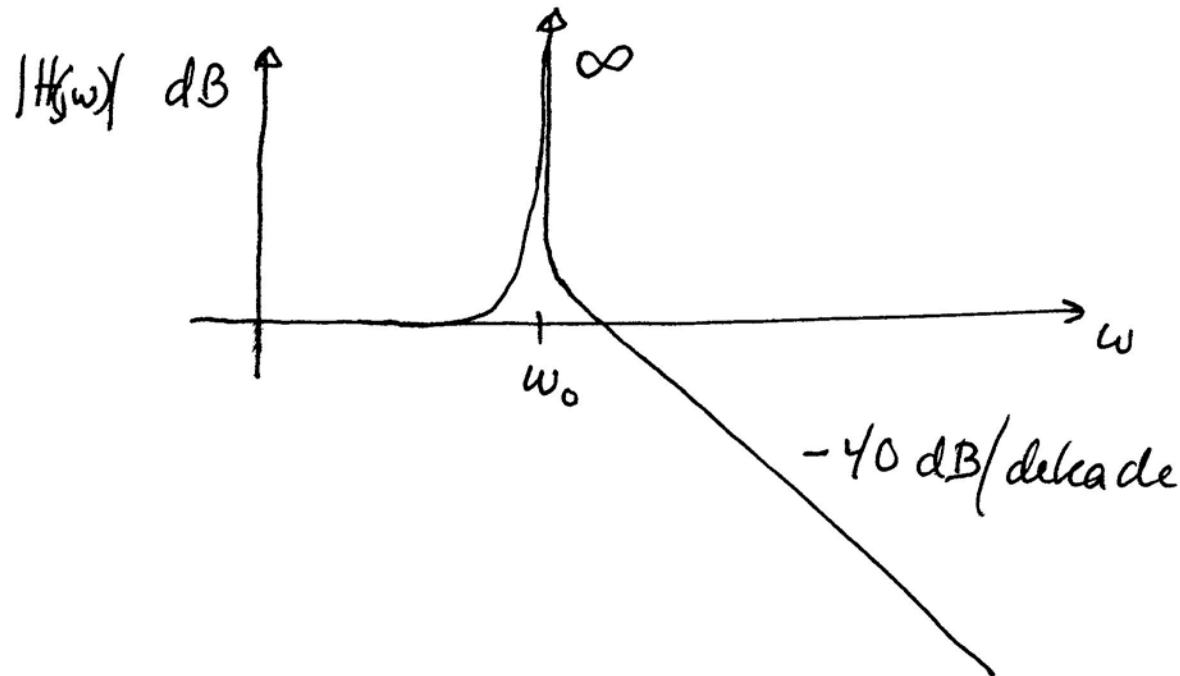
$$\omega_0 = \sqrt{\frac{1}{LC}}, \omega_0 = \sqrt{\frac{k}{m}}$$

System without damping, contd.

$$H(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

$$|H(j\omega)| = 1 \quad \text{near } \omega \ll \omega_0 \quad 0 \text{ dB}$$

$$|H(j\omega)| = -\left(\frac{\omega_0}{\omega}\right)^2 \quad \text{near } \omega \gg \omega_0 \quad -40 \text{ dB/decade}$$

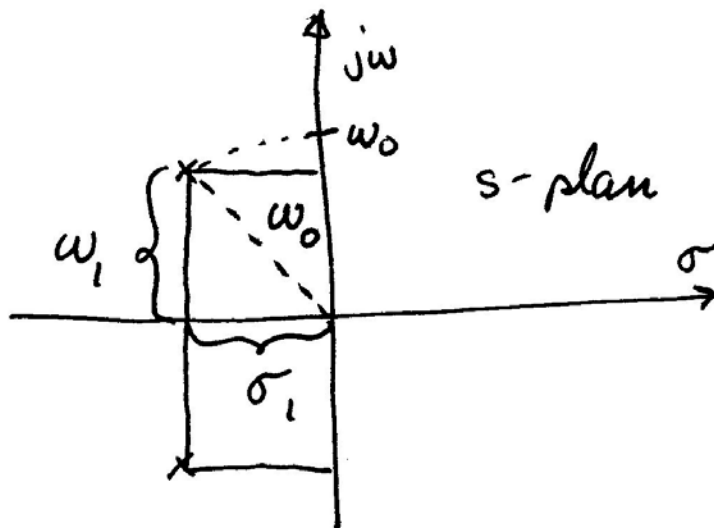


With damping

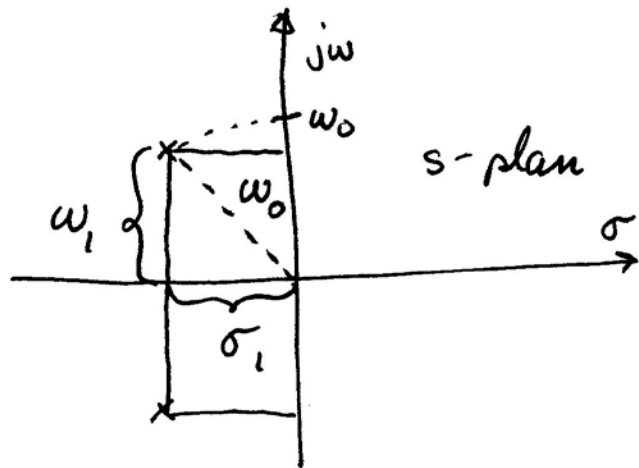
$$s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0$$

$$s = -\frac{\omega_0}{2Q} \pm j \omega_0 \sqrt{1 - \frac{1}{4Q^2}}$$

$$= -\sigma_1 \pm j \omega_1$$



Damped system, contd.



$$\frac{\omega_0}{Q} = \frac{b}{m} = \frac{1}{\tau}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\sigma_1 = \frac{1}{2\tau} = \frac{b}{2m}$$

$$\omega_1 = \omega_0 \sqrt{1 - \frac{1}{4\omega_0^2 \tau^2}} = \omega_0 \sqrt{1 - \frac{b^2}{4km}}$$

$$\omega_1^2 + \sigma_1^2 = \omega_0^2$$

Mechanical Resonator

- Frequency and phase shift under damping:

$$x(t) = Ae^{-t/2\tau} \cos(\omega_1 t + \varphi)$$

$$\tau = m/b \text{ damping time}$$

$$\omega_1 = \omega_0 \sqrt{1 - \frac{1}{4\omega_0^2 \tau^2}} = \omega_0 \sqrt{1 - \frac{b^2}{4Km}}$$

φ phase shift

- Energy dissipation:

$$E(t) = E_0 e^{-t/\tau}$$

What is the meaning of "damping time"?

τ = damping time

$$e^{-t/2\tau} \Big|_{t=\tau} = e^{-1/2} = \frac{1}{\sqrt{e}}$$

Power

Effektum

$$|x(t)|^2 \Big|_{t=\tau} = \frac{1}{e}$$

$$x(t) = A e^{-t/2\tau} \cos(\omega_1 t + \varphi)$$

$$x(0) = A \cdot \cos \varphi \quad \begin{array}{l} \text{initialbedingungen} \\ \text{initial conditions} \end{array}$$

Q-factor and damping time

Generell ligning

General equation

$$s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0$$

$$\Rightarrow s^2 + \frac{1}{\tau} s + \omega_0^2 = 0$$

$$Q = \omega_0 \tau$$

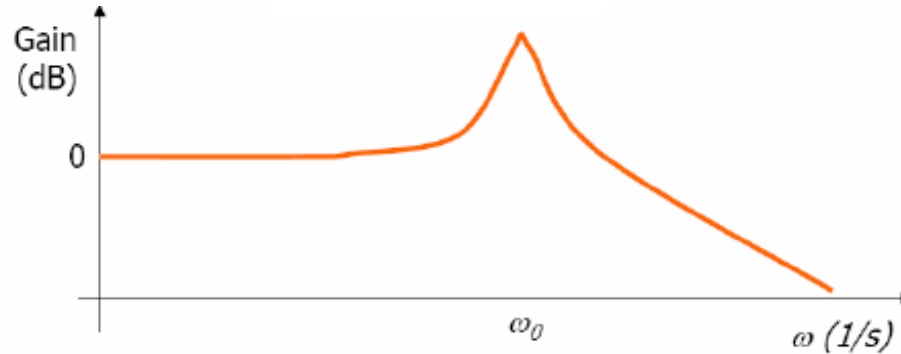
$$\tau = \frac{m}{b} \quad \begin{array}{l} \text{mekanisk} \\ \text{mechanical} \end{array}$$

$$\tau = \frac{L}{R} \quad \begin{array}{l} \text{elektrisk} \\ \text{electrical} \end{array}$$

$$Q_{\text{mek}} = \frac{\omega_0 m}{b}$$

$$Q_{\text{el}} = \frac{\omega_0 L}{R}$$

Amplitude at resonance for forced vibrations



$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H(j\omega) = \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + j \frac{\omega \omega_0}{Q}}$$

$$|H(j\omega_0)| = \left| \frac{\omega_0^2}{0 + j \frac{\omega_0^2}{Q}} \right| = Q$$