

INF5490 RF MEMS

LN08: RF MEMS resonators II

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Department of Informatics, UoO

Today's lecture

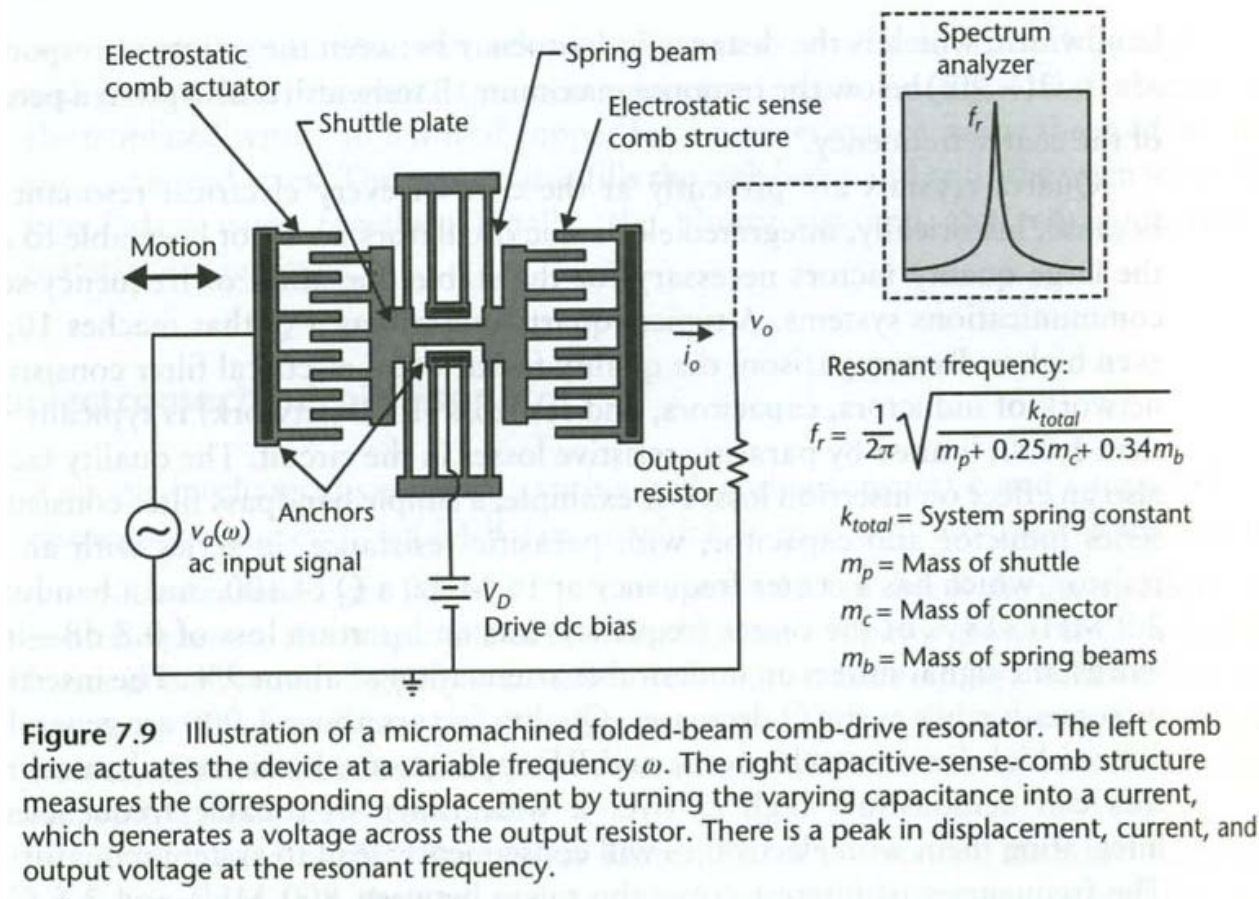
- Lateral vibrating resonator:
Comb resonator
 - **Working principle**
 - Detailed **modeling**
 - A) "phasor"-modeling
 - B) modeling by converting between mechanical and electrical energy domains

Lateral and vertical movement

- Lateral movement in the resonator
 - Parallel to substrate
 - Folded beam comb structure
- Vertical movement (next lecture)
 - Vertical to substrate
 - clamped-clamped beam (c-c beam)
 - free-free beam (f-f beam)

Comb resonator

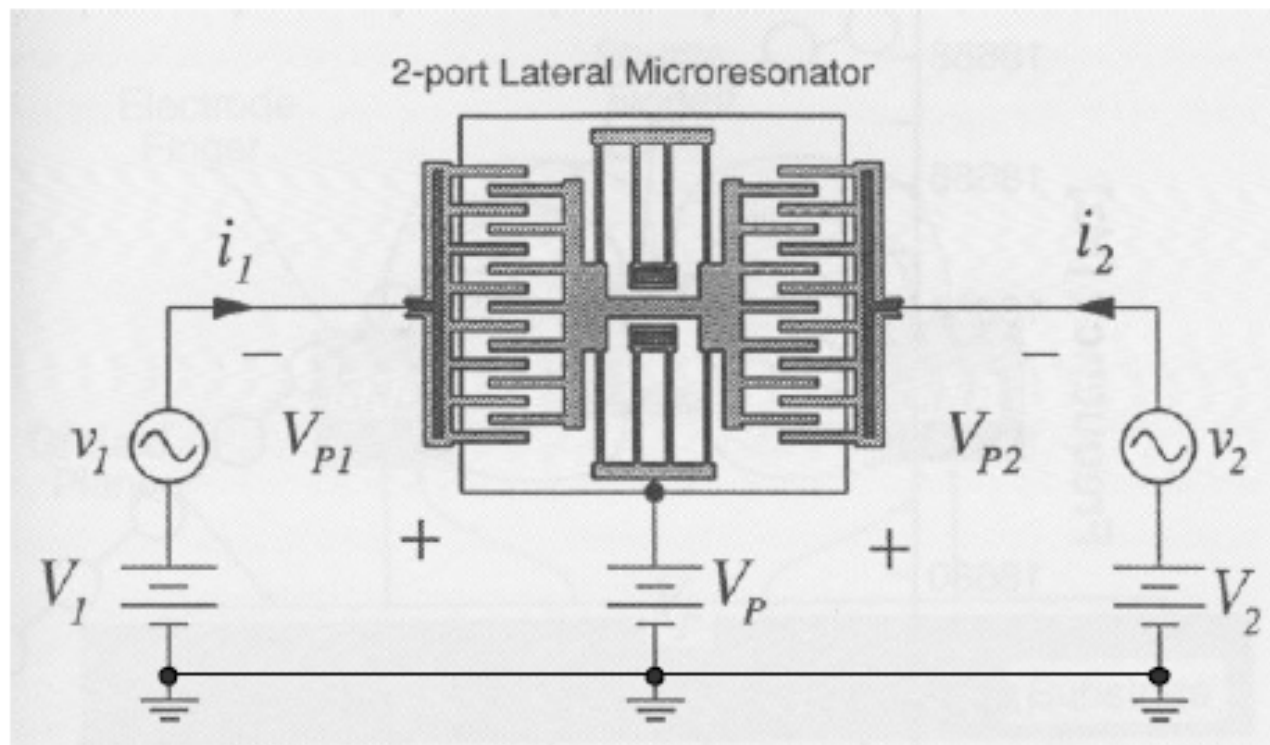
- Fixed comb + movable, suspended comb
- Suspended by folded springs, compact layout
- Total capacitance between the combs can be varied
- Applied bias (+ or -) generates an electrostatic force between left anchor-comb and "shuttle"-comb. Shuttle pulled to the left in the plane



Detailed modeling

- Modeling of **lateral comb structure**
 - A) "Phasor"-modeling ala [UoC, Berkeley](#)
 - Detailed calculations included
 - B) Conversion between energy domains
 - Material from [UCLA](#)
- In next lecture, LN09, the **c-c beam** will be modeled with reference to the book
 - T. Itoh et al: "RF Technologies for Low Power Wireless Communications", chap. 12: "Transceiver Front-End Architectures Using Vibrating Micromechanical Signal Processors", by Clark T.-C. Nguyen

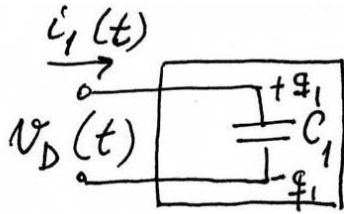
The Lateral Resonator as a “Two-Port”



Calculation procedure

- **A.** Model the comb as a two-port. Analyze first the input port
- **B.** When the comb moves the input capacitance will have a static and a variable component
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- **F.** Compare the expression to Y for a L-C-R-circuit and find equivalent elements
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- **J.** Set up a two port equivalent circuit
- **K.** Set up a complete two-port-model

A. Model the comb as a two-port. Analyze first the input port

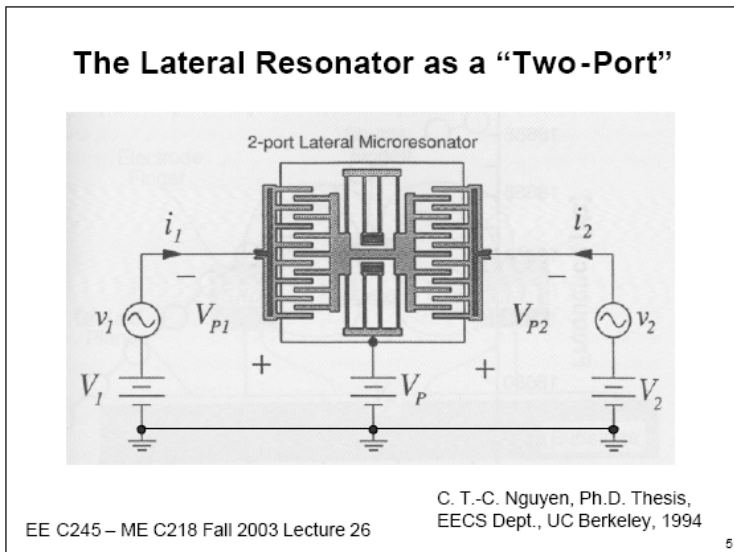


$$q_1 = C_1 v_D$$

$$\dot{q}_1(t) = i_1(t) = C_1 \frac{dv_D}{dt} + v_D \frac{dC_1}{dt}$$

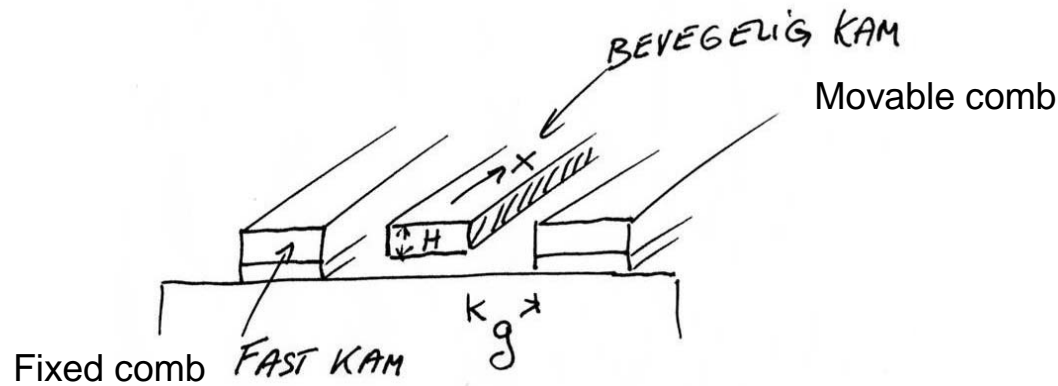
$$v_D(t) = V_1 + v_1(t) - V_P = -V_{P1} + v_1 \cos \omega t$$

$$V_{P1} = V_P - V_1$$



V_{P1} = positive when $V_P > V_1$

B. When the comb moves the input capacitance will have a static and variable component



$$C_1(t) = C_{01} + C_{m1}(t)$$

$$C_1 = \frac{\epsilon_0 A}{g} = \frac{\epsilon_0 \cdot x \cdot 2H \cdot n}{g}$$

$$C_1(t) = C_{01}(\text{fixed}) + C_{m1}(\text{prop. with } x(t))$$

$$C_1(t) = C_{01} + \frac{\partial C_1}{\partial x} \cdot x(t)$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \dots$$

← general formula

C. Find the input current versus displacement, X

$$i_1(t) = C_1 \frac{dv_D}{dt} + v_D \frac{dC_1}{dt}$$

$$= C_1 \frac{dv_1(t)}{dt} + (-V_{P1} + v_1(t)) \frac{\partial C_1}{\partial x} \frac{\partial x}{\partial t} = \left[C_{01} + \frac{\partial C_1}{\partial x} \cdot x(t) \right] \frac{dv_1(t)}{dt} + \dots$$

$$= C_{01} \frac{dv_1(t)}{dt} + \frac{\partial C_1}{\partial x} \cdot x(t) \cdot \frac{\partial v_1(t)}{\partial t} - V_{P1} \frac{\partial C_1}{\partial x} \frac{\partial x}{\partial t} + v_1(t) \frac{\partial C_1}{\partial x} \frac{\partial x}{\partial t}$$

$$= C_{01} \frac{dv_1(t)}{dt} + \frac{\partial C_1}{\partial x} \underbrace{\left(x \cdot \frac{\partial v_1}{\partial t} + v_1 \frac{\partial x}{\partial t} \right)}_{\frac{\partial}{\partial t}(x \cdot v_1)} - V_{P1} \frac{\partial C_1}{\partial x} \frac{\partial x}{\partial t}$$

$$\frac{\partial}{\partial t}(x \cdot v_1), \text{ where } v_1 = v_0 \cos \omega t, \text{ } x = x_0 \cos \omega t$$

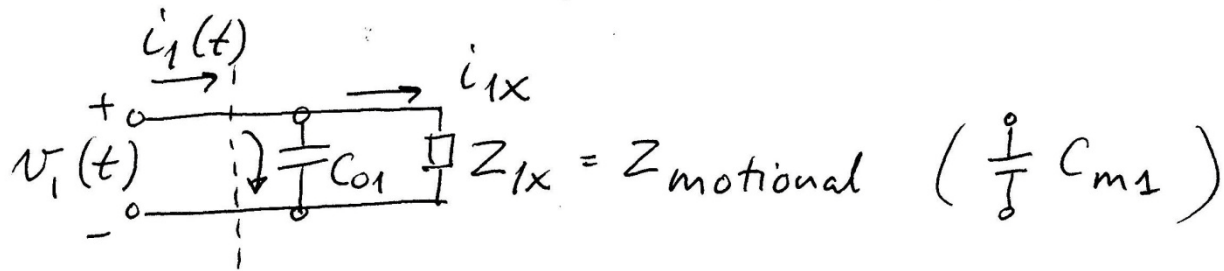
$$(x \cdot v_1) \cong \cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$$

double frequency, small contribution

$$i_1(t) \approx C_{01} \frac{\partial v_1(t)}{\partial t} - V_{P1} \frac{\partial C_1}{\partial x} \frac{\partial x(t)}{\partial t}$$

Current into the DC-capacitance

"motional current"



$$i_{1x}(t) = -V_{P1} \frac{\partial C_1}{\partial x} \frac{\partial x(t)}{\partial t} = \left(-V_{P1} \frac{\partial C_1}{\partial t} \right)$$

"motional current"

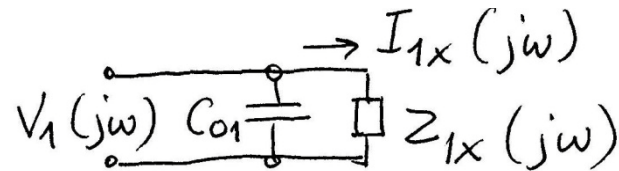
$$I_{1x}(j\omega) = -V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot X(j\omega)$$

phasor-form of "motional current"

= current as function of movement ("displacement")

D. Calculate the input admittance, Y ("motional admittance")

- D1. Find Y versus X



$$Y_{1x}(j\omega) = \frac{I_{1x}(j\omega)}{V_1(j\omega)} = -V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot \frac{X(j\omega)}{V_1(j\omega)}$$

← displacement
← voltage

- D2. X depends on the electrostatic force, F, and m, b and k

$$Y_{1x}(j\omega) = -V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot \frac{X(j\omega)}{F_d(j\omega)} \cdot \frac{F_d(j\omega)}{V_1(j\omega)}$$

F_d depends of m, b og k

voltage V_1 creates an electrostatic force F_d

D3. F depends on the applied bias, V

Relationship between force and voltage can be found from:

$$U = \frac{1}{2} C_1 v_D^2(t) \quad \text{Potential energy, } V_D \text{ is independent of } x$$

$$F = \frac{\partial U}{\partial x} = \frac{1}{2} v_D^2(t) \cdot \frac{\partial C_1}{\partial x} \quad \text{non-linear relation}$$

$$F = F_0 + f \cos \omega t, \quad v_D = -V_{P1} + v_1 \cos \omega t \quad \text{Linearizing around a DC-point}$$

$$F_0 + f \cos \omega t = \frac{1}{2} (-V_{P1} + v_1 \cos \omega t)^2 \cdot \frac{\partial C_1}{\partial x} \quad \text{Substitute}$$

$$= \frac{1}{2} (V_{P1}^2 - 2 \cdot V_{P1} \cdot v_1 \cos \omega t + v_1^2 \cos^2 \omega t) \cdot \frac{\partial C_1}{\partial x}$$

$\underbrace{\hspace{10em}}_{\cos 2\omega t \text{ - term}}$

$$f \cos \omega t = -V_{P1} \cdot v_1 \cos \omega t \cdot \frac{\partial C_1}{\partial x} \quad \text{Comparing AC-terms}$$

$$f_{d,\omega} = -V_{P1} \frac{\partial C_1}{\partial x} v_1(t) \quad \leftarrow \text{LINEAR RELATION!}$$

$$F_d(j\omega) = -V_{P1} \frac{\partial C_1}{\partial x} \cdot V_1(j\omega) \quad \text{In phasor-form}$$

$$\frac{F_d(j\omega)}{V_1(j\omega)} = -V_{P1} \frac{\partial C_1}{\partial x}$$

Relation between displacement and force:

$$\frac{X(s)}{F_d(s)} = \frac{1}{ms^2 + bs + k} = \frac{1}{k} \frac{k/m}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

D2. X depends on the electrostatic force, F, and m, b and k

$$\omega_0^2 = k/m, \quad b/m = \omega_0/Q$$

Substitute

$$Q = \frac{\sqrt{k/m}}{b/m} = \frac{\sqrt{km}}{b}$$

$$\frac{X(s)}{F_d(s)} = \frac{1}{k} \cdot \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \xrightarrow{s=j\omega} \frac{1}{k} \cdot \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + j\frac{\omega_0\omega}{Q}}$$

$$\frac{X(j\omega)}{F_d(j\omega)} = \frac{1}{k} \cdot \frac{1}{[1 - (\omega/\omega_0)^2] + j\frac{\omega}{Q\omega_0}}$$

E. Find an expression for Y (dynamic behavior)

$$\begin{aligned} Y_{1x}(j\omega) &= -V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot \frac{X(j\omega)}{F_d(j\omega)} \cdot \frac{F_d(j\omega)}{V_1(j\omega)} \\ &= -V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot \frac{1/k}{\left[1 - (\omega/\omega_0)^2\right] + j \frac{\omega}{\omega_0 Q}} \cdot \left(-V_{P1} \frac{\partial C_1}{\partial x}\right) \end{aligned}$$

$$\eta = V_{P1} \frac{\partial C_1}{\partial x}$$

← η defined

$$Y_{1x}(j\omega) = \eta^2 \cdot j\omega \cdot \frac{1/k}{\left[1 - (\omega/\omega_0)^2\right] + j \frac{\omega}{\omega_0 Q}}$$

$$I_{1x}(j\omega) = [\dots] \cdot V_1(j\omega)$$

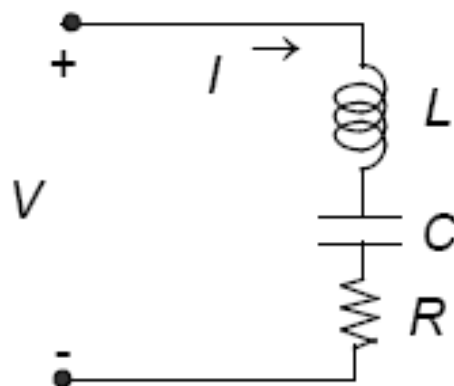
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F.

Series L - C - R Admittance

The current through an L - C - R branch is:

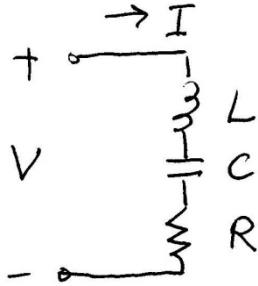


$$\frac{I(j\omega)}{V(j\omega)} = \frac{j\omega C}{1 - (\omega / \omega_o)^2 + j(\omega RC)}$$

$$\omega_o^{-2} = LC$$

Match terms in motional admittance \rightarrow find equivalent elements

Current through the L-C-R-circuit



$$V = I(sL + 1/sC + R)$$

$$\frac{I(s)}{V(s)} = \frac{sC}{s^2LC + sRC + 1}$$

$$Y(j\omega) = \frac{I(j\omega)}{V(j\omega)} = \frac{j\omega C}{-\omega^2 LC + j\omega RC + 1}$$

Introduce

$$\omega_0^2 = \frac{1}{LC}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Y(j\omega) = \frac{j\omega C}{[1 - (\omega/\omega_0)^2] + j\omega RC} = \frac{j\omega C}{[\dots] + j\frac{\omega}{\omega_0 Q}}$$

$$RC = \frac{1}{\omega_0 Q}, \quad Q = \frac{1}{\omega_0 RC} = \frac{\sqrt{LC}}{RC} = \sqrt{\frac{L}{C}} \cdot \frac{1}{R}$$

Which gives:

$$Y(j\omega) = \frac{j\omega C}{\left[1 - (\omega/\omega_0)^2\right] + j\frac{\omega}{\omega_0 Q}}$$

Compare to

$$Y_{1x}(j\omega) = \eta^2 \cdot \frac{j\omega \cdot 1/k}{\left[1 - (\omega/\omega_0)^2\right] + j\frac{\omega}{\omega_0 Q}}$$

This results in:

$$C_{x1} = \eta^2 / k$$

$$\omega_0^2 = k/m = 1/LC \Rightarrow L_{x1} = \frac{1}{C} \cdot \frac{m}{k} = \frac{k}{\eta^2} \cdot \frac{m}{k} = \frac{m}{\eta^2}$$

$$RC = \frac{1}{Q\omega_0} = \frac{1}{Q\sqrt{k/m}} \Rightarrow R_{x1} = \frac{1}{C} \cdot \frac{1}{Q\sqrt{k/m}} = \frac{k}{\eta^2} \frac{\sqrt{m}}{Q\sqrt{k}} = \frac{\sqrt{km}}{Q\eta^2}$$

η = Electromagnetic coupling coefficient

$$I_{x1}(\omega_0) = \frac{V_1(\omega_0)}{R_{x1}} \quad \text{At resonance the impedances from L and C cancel}$$

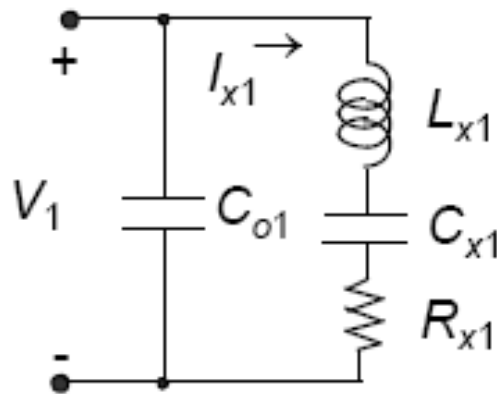
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G. Equivalent Circuit for Input Port

A series L-C-R circuit results in the identical expression →
find equivalent values L_{x1} , C_{x1} , and R_{x1}

$$L_{x1} = \frac{m}{\eta^2} \quad C_{x1} = \frac{\eta^2}{k} \quad R_{x1} = \frac{\sqrt{km}}{Q\eta^2} \quad \eta = V_{p1} \frac{\partial C_1}{\partial x} = \text{electromechanical coupling coefficient}$$



At resonance, the impedances of the inductance and the capacitance *cancel out* →

$$I_{x1} = \frac{V_1}{R_{x1}}$$

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H. Find the output current for a given input

$$i_{1x}(t) = -V_{P1} \frac{\partial C_1}{\partial t}$$

This displacement causes the output capacitance C2 also to change.
Output current due to displacement ($v_2 = 0V$, short-circuited):

$$i_2(t) = -V_{P2} \frac{\partial C_2}{\partial t} = -V_{P2} \frac{\partial C_2}{\partial x} \frac{\partial x}{\partial t}$$

$$I_2(j\omega) = -V_{P2} \frac{\partial C_2}{\partial x} \cdot j\omega \cdot X(j\omega)$$

In phasor-form

$$X(j\omega) = \frac{1/k}{[1 - (\omega/\omega_0)^2] + j \frac{\omega}{\omega_0 Q}} \cdot F_d(j\omega)$$

$$F_d(j\omega) = -V_{P1} \frac{\partial C_1}{\partial x} \cdot V_1(j\omega)$$

voltage \rightarrow force \rightarrow displacement \rightarrow current

$$\Rightarrow I_2(j\omega) = \frac{V_{P1} V_{P2} \frac{\partial C_1}{\partial x} \frac{\partial C_2}{\partial x}}{[1 - (\omega/\omega_0)^2] + j \frac{\omega}{\omega_0 Q}} \cdot j\omega \cdot (1/k) \cdot V_1(j\omega)$$

I. Calculate the ratio between the output and input currents ("forward current gain")

"Forward current gain"

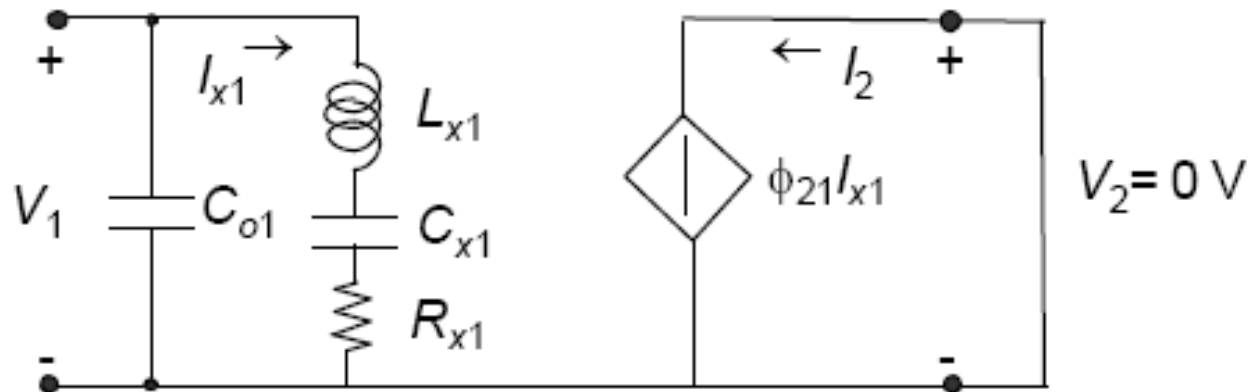
$$\Phi_{21} = \frac{I_2(j\omega)}{I_{x1}(j\omega)} = \frac{-V_{P2} \frac{\partial C_2}{\partial x} \cdot j\omega \cdot X(j\omega)}{-V_{P1} \frac{\partial C_1}{\partial x} \cdot j\omega \cdot X(j\omega)} = \frac{V_{P2} \frac{\partial C_2}{\partial x}}{V_{P1} \frac{\partial C_1}{\partial x}}$$

$$I_2(j\omega) = \Phi_{21} \cdot I_{x1}(j\omega), \quad V_2 = 0$$

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J. Two-Port Equivalent Circuit ($v_2 = 0$)

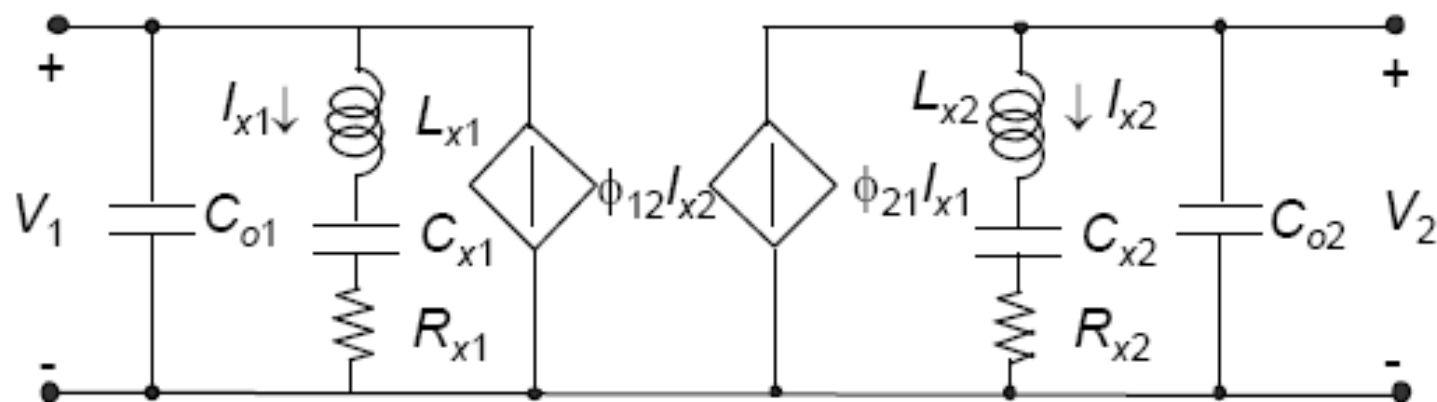


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K.

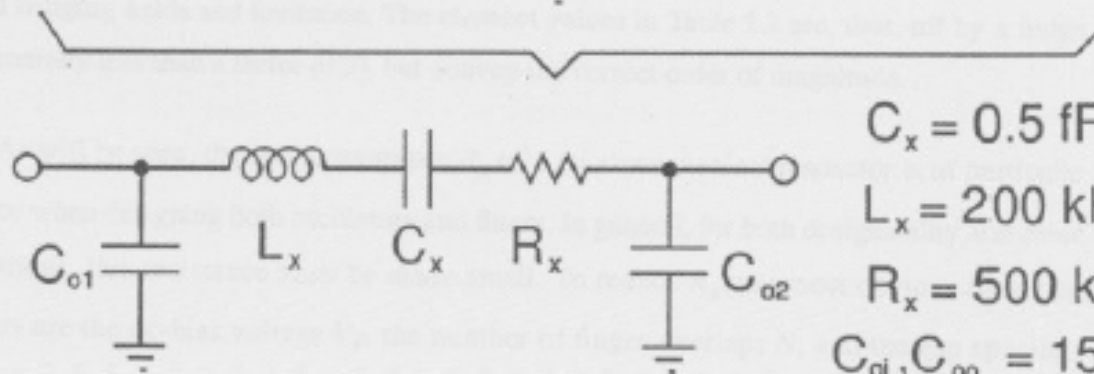
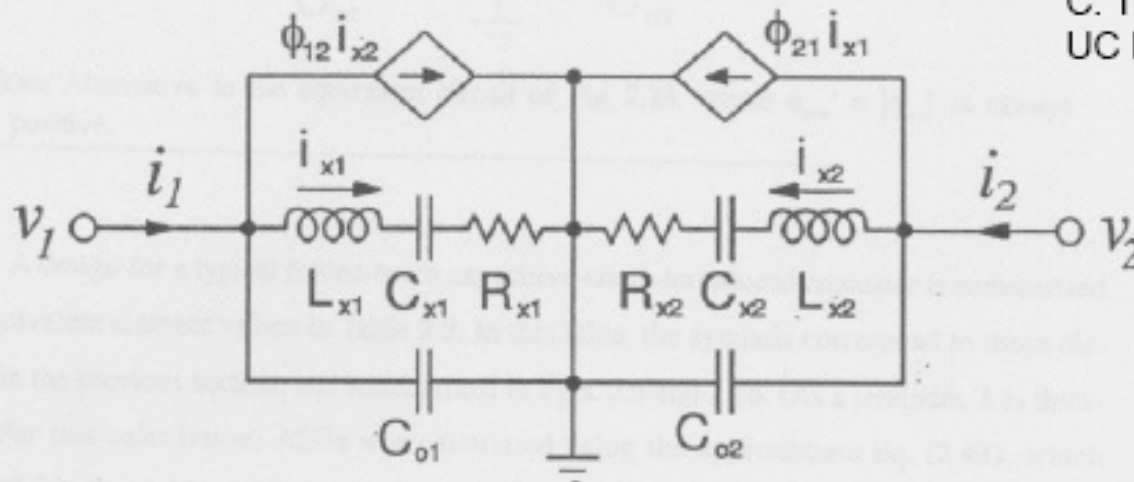
Complete Two-Port Model



Symmetry implies that modeling can be done from port 2, with port 1 shorted \rightarrow superimpose the two models

Equivalent Circuit for Symmetrical Resonator ($\phi_{21} = \phi_{12} = 1$)

C. T.-C. Nguyen, Ph.D.,
UC Berkeley, 1994



$$C_x = 0.5 \text{ fF}$$

$$L_x = 200 \text{ nH}$$

$$R_x = 500 \text{ k}\Omega$$

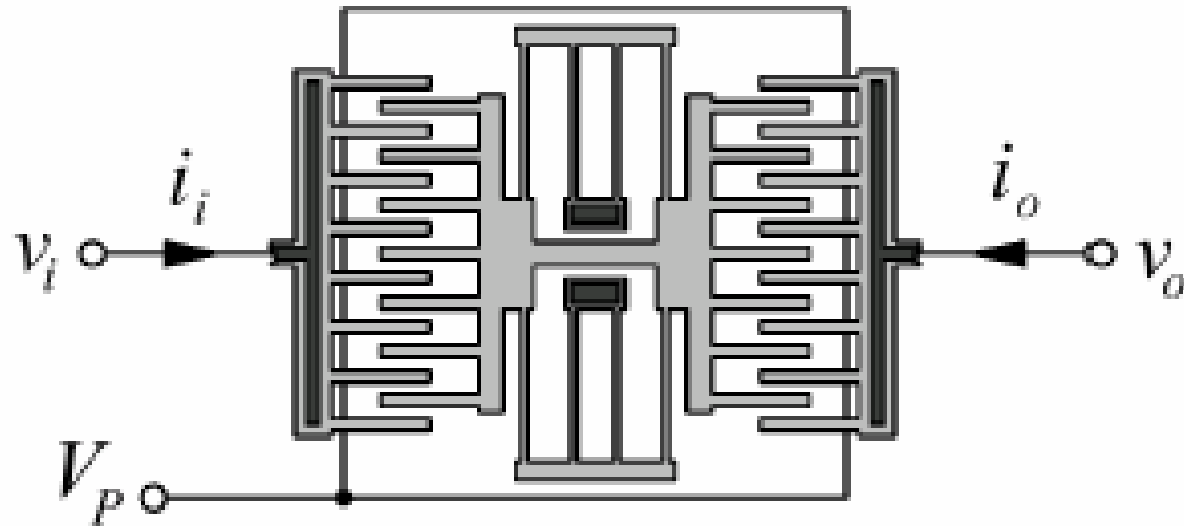
$$C_{o1}, C_{o2} = 15 \text{ fF}$$

Alternative modeling

- B) Exploit **conversion** between mechanical and electrical energy domains
 - Slides from UCLA
- Supported by lecture notes →

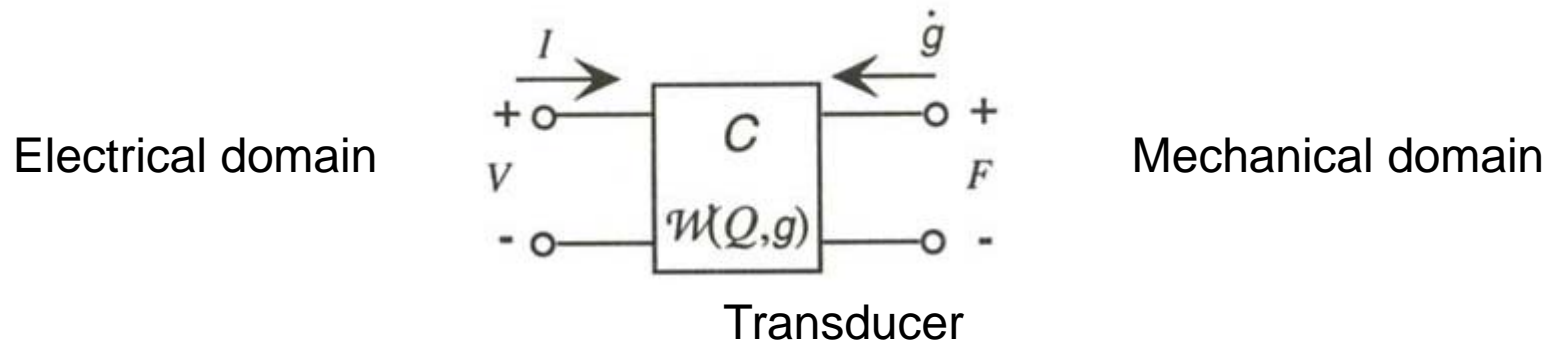
Two-Port Micromechanical Resonator Using Comb-Drive Actuator

2-port Lateral Microresonator



Conversion between energy domains

- Both vertical and lateral resonator structures may be described by a **generalized non-linear capacitance, C , interconnecting** energy-domains

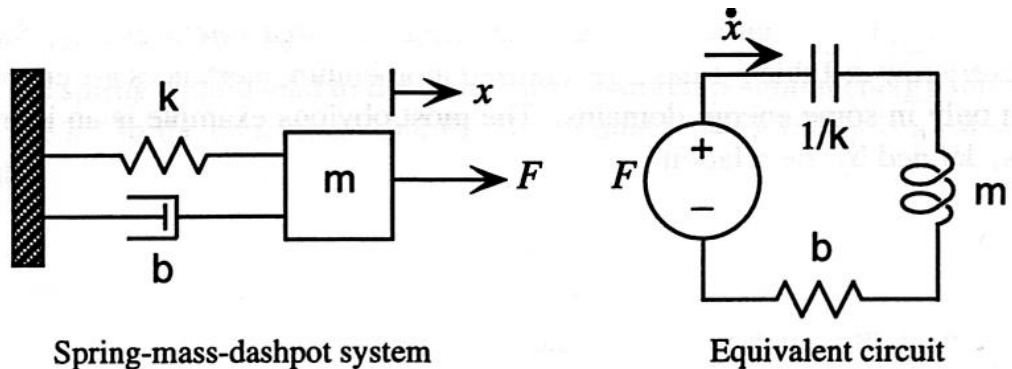


Interconnecting where there is **no energy loss**

Procedure

- First, transform the mechanical domain impedances to an **electrical representation**
 - The mechanical components are modeled as lumped electrical components
- NB! You are still in the mechanical domain!

- $C = 1/k$
- $L = m$
- $R = b$



- Power-variables
 - Effort = force \rightarrow voltage
 - Flow = velocity \rightarrow current

Interconnecting different energy domains

- 1. Each energy domain is transformed to its electrical equivalent
- 2. Domains are interconnected by a generalized non-linear capacitance, C
- 3. Transformer and gyrator may be used for **interconnecting** if a **linear relationship** exists between the power-variables!
 - Problem: Transducer C is generally **NOT** a linear 2-port
- 4. Then, must **linearize** the 2-port transducer to be able to substitute it with a **transformer**
- 5. The transformer can "be removed" by recalculating the component **values** to **new** ones
 - → **Electromechanical coupling coefficient used!** = turn ratio
 - → Results in a common circuit diagram

Interaction between energy domains

- Suppose **linear** relation between power variables
 - → A linear 2-port element can be used:
 - Use a transformer or gyrator

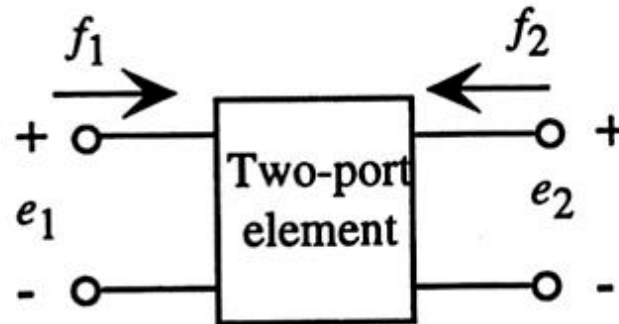


Figure 5.11. General two-port element.

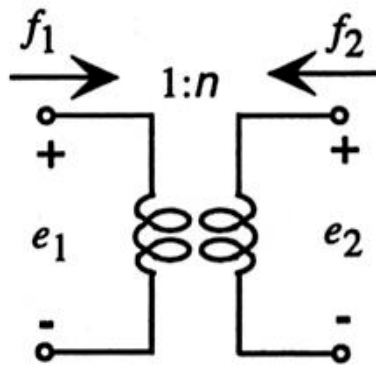
power in = power out NO POWER LOSS

$$e_1 f_1 + e_2 f_2 = 0 \quad (5.41)$$

Transformer

TRANSFORMER:

$$\begin{pmatrix} e_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \begin{pmatrix} e_1 \\ f_1 \end{pmatrix} \quad (5.42)$$



Transformer

$$e_2 = n \cdot e_1$$
$$f_2 = -\frac{1}{n} f_1$$

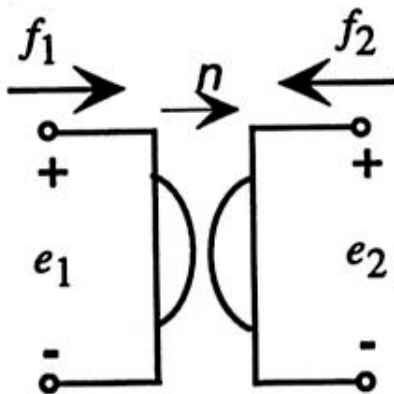
n = "turns ratio"

Ex. V and F can be interconnected

Gyrator

GYRATOR:

$$\begin{pmatrix} e_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} 0 & n \\ -\frac{1}{n} & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ f_1 \end{pmatrix} \quad (5.43)$$

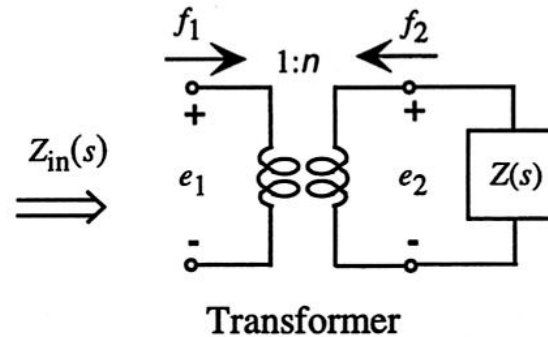


Gyrator

$$e_2 = n \cdot f_1$$
$$f_2 = -\frac{1}{n} e_1$$

The impedances can be transformed

$$Z_{in}(s) = \frac{Z(s)}{n^2}$$

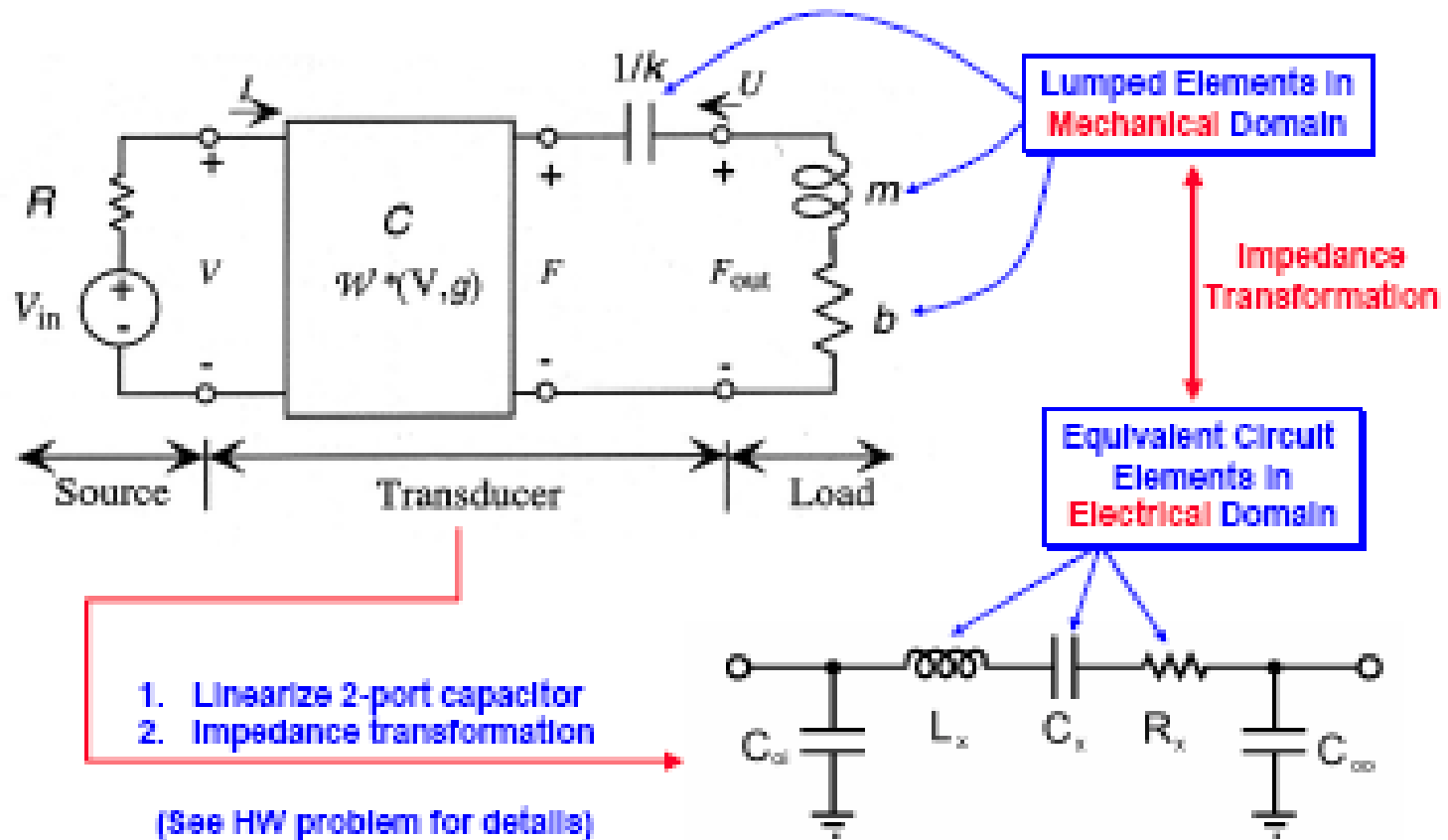


n = coupling coefficient between energy domains

$$Z_{in}(s) = \frac{e_1}{f_1}$$

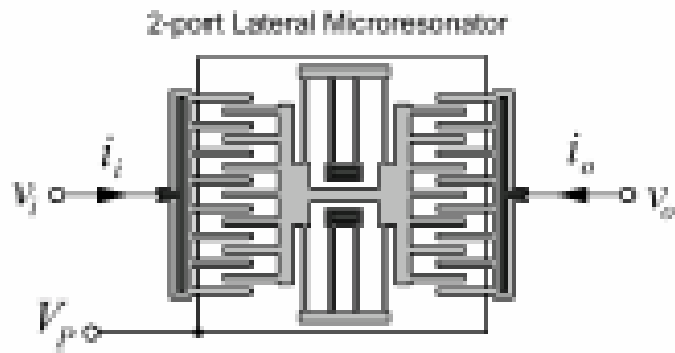
$$Z(s) = \frac{e_2}{-f_2} = \frac{n \cdot e_1}{\frac{1}{n} \cdot f_1} = n^2 \cdot \frac{e_1}{f_1} = n^2 \cdot Z_{in}(s)$$

Lumped Element Model (Senturia's Book)

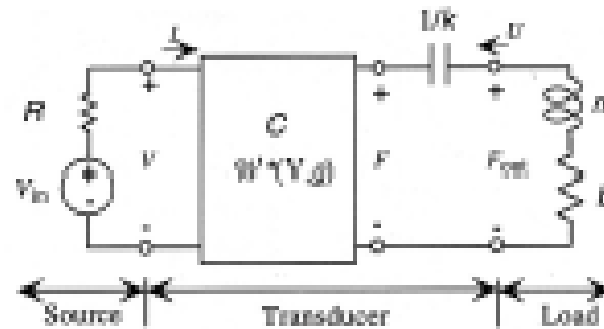


Linearized Transducers

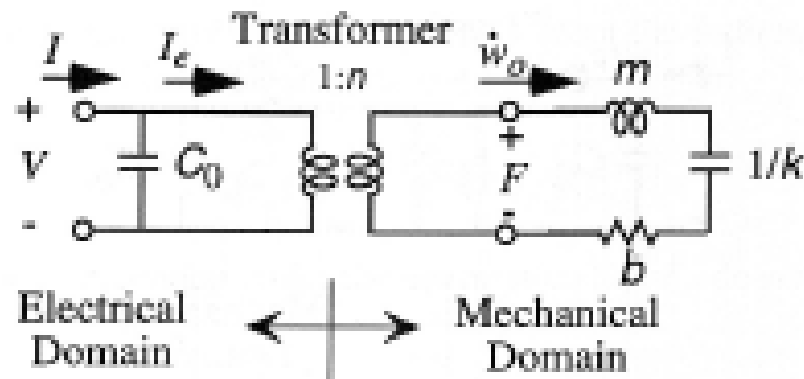
Physical Circuit



Equivalent Circuit (Nonlinear)



Linearized Equivalent Circuit



Procedure

- Investigate relation between "efforts" and "flows" in the 2 domains
- **Efforts:** calculation procedure
 - 1. Start with an expression for potential energy
 - 2. Calculate force
 - 3. Look at perturbations around the DC-bias
 - 4. Find the relationship between AC-terms
 - → A linear relationship is obtained!

Relation between "efforts"

$$F = \frac{\partial \mathcal{W}^*}{\partial x} = \frac{1}{2} V^2 \frac{\partial C}{\partial x}$$

$$F = F_{dc} + f \cdot \sin(\omega t)$$

$$V = V_{dc} + v \cdot \sin(\omega t)$$

$$F_{dc} + f \cdot \sin(\omega t) = \frac{1}{2} (V_{dc} + v \cdot \sin(\omega t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2} \left((V_{dc})^2 + 2 \cdot V_{dc} \cdot v \cdot \sin(\omega t) \right) \frac{\partial C}{\partial x}$$

$$f = V_{dc} \cdot \frac{\partial C}{\partial x} \cdot v \quad \leftarrow \text{AC terms}$$

effort (mechanical domain) = const. * effort (electrical domain)

Similarly for relationship between FLOWS:

Linearization – Small Signal Analysis

Relations between "Efforts"

$$F = \frac{\partial W^*}{\partial x} = \frac{1}{2} V^2 \frac{\partial C}{\partial x}$$

$$F = F_{dc} + f \cdot \sin(\omega t)$$

$$V = V_{dc} + v \cdot \sin(\omega t)$$

$$F_{dc} + f \cdot \sin(\omega t) = \frac{1}{2} (V_{dc} + v \cdot \sin(\omega t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2} \left((V_{dc})^2 + 2 \cdot V_{dc} \cdot v \cdot \sin(\omega t) \right) \frac{\partial C}{\partial x}$$

$$f = V_{dc} \cdot \frac{\partial C}{\partial x} \cdot v \quad \leftarrow \text{AC terms}$$

Relations between "Flows"

$$Q = V \cdot C$$

$$I = V \cdot \frac{\partial C}{\partial t} = V \cdot \frac{\partial C}{\partial X} \cdot \frac{\partial X}{\partial t} = V \cdot \frac{\partial C}{\partial X} \cdot \dot{X}$$

$$I = I_{dc} + i \cdot \sin(\omega t)$$

$$X = X_{dc} - x \cdot \sin(\omega t)$$

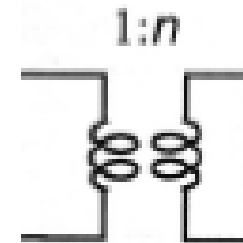
$$i = -V_{dc} \frac{\partial C}{\partial x} \dot{x}$$

Negative sign due to definition of flow direction

Linearized capacitive transducer is a Transformer

$$\begin{pmatrix} f \\ \dot{x} \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \begin{pmatrix} v \\ i \end{pmatrix}$$

Turn Ratio: $n = V_{dc} \frac{\partial C}{\partial x}$



flow (electrical domain) = - const. * flow (mechanical domain)

Current direction, mechanical domain

- **Flow** in the mechanical domain is defined as positive **into** the 2-port transducer
- Choose the current to go **out of** 2-port C. Then we have:
 - **Current goes into** the electrical domain
 - → creates an attractive force on the comb
 - → spring stretches
 - → potential energy is built up
 - → equivalent to charging of an $1/k$ -capacitor

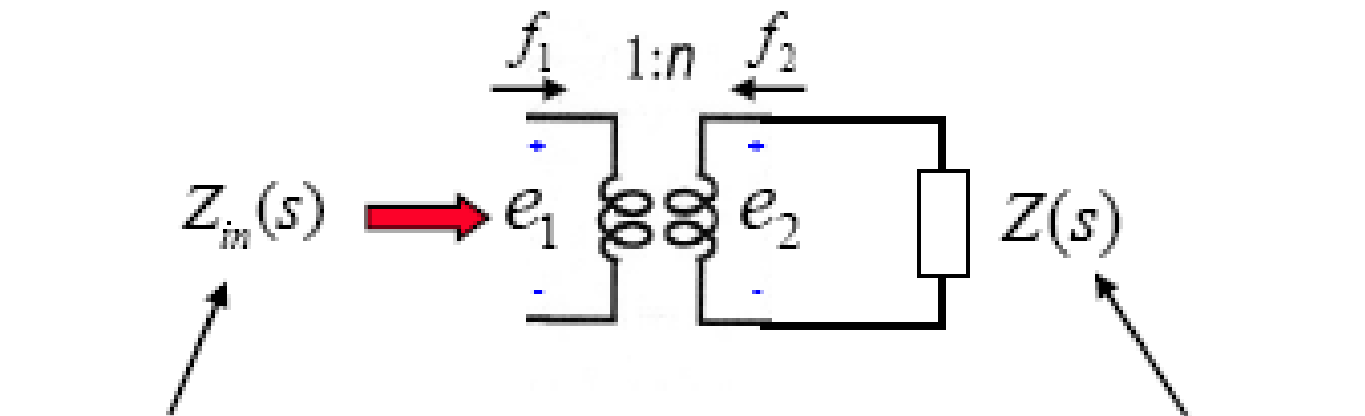
 - → Current increases → charge on the capacitor increases → attractive force increases → **displacement (x) decreases**

Compatible relations both between "efforts" and "flows"

$$f = V_{dc} \cdot \frac{\partial c}{\partial x} \cdot v = n \cdot v \quad \text{du} \quad n = V_{dc} \cdot \frac{\partial c}{\partial x}$$
$$i = -V_{dc} \cdot \frac{\partial c}{\partial x} \cdot \dot{x} = -n \cdot \dot{x} \quad \Rightarrow \quad \dot{x} = -\frac{1}{n} \cdot i$$

- **effort** (mechanical domain) = n * **effort** (electrical domain)
- **flow** (mechanical domain) = $-1/n$ * **flow** (electrical domain)
- A linearized capacitive transducer implemented as a **transformer** can be used!

Impedance Transformation



Equivalent Impedance In Electrical Domain

Impedance In Mechanical Domain

$$\begin{pmatrix} f \\ \dot{x} \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \begin{pmatrix} v \\ i \end{pmatrix}$$

$$Z_{in}(s) = \frac{1}{n^2} Z(s)$$

Transformation of impedances

$$Z_{el} = \frac{1}{n^2} \cdot Z_{mek}$$

Inductor

$$sL_{el} = \frac{1}{n^2} \cdot sL_{mek} = \frac{sm}{n^2} \Rightarrow L_{el} = \frac{m}{n^2}$$

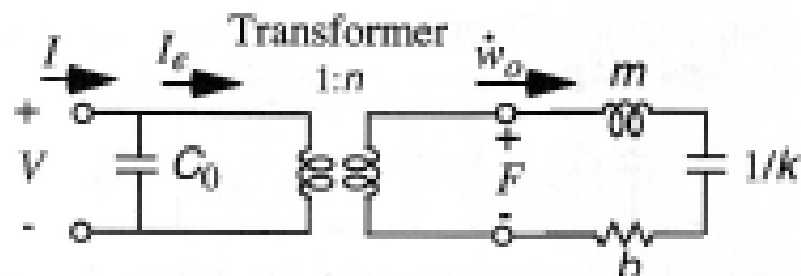
Resistor

$$R_{el} = \frac{1}{n^2} \cdot R_{mek} = \frac{b}{n^2}$$

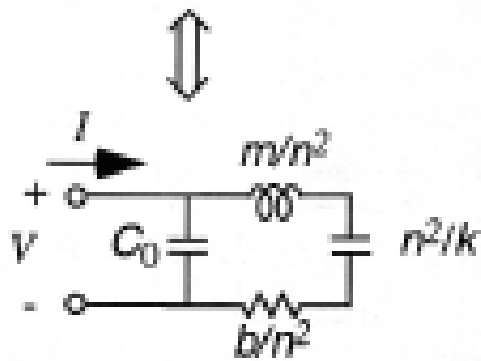
Capacitor

$$\frac{1}{sC_{el}} = \frac{1}{n^2} \cdot \frac{1}{sC_{mek}} = \frac{1}{n^2} \cdot \frac{k}{s} \Rightarrow C_{el} = \frac{n^2}{k}$$

Small Signal Equivalent Circuit of Microresonators



Electrical Domain ↔ Mechanical Domain



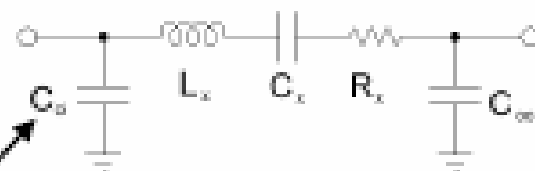
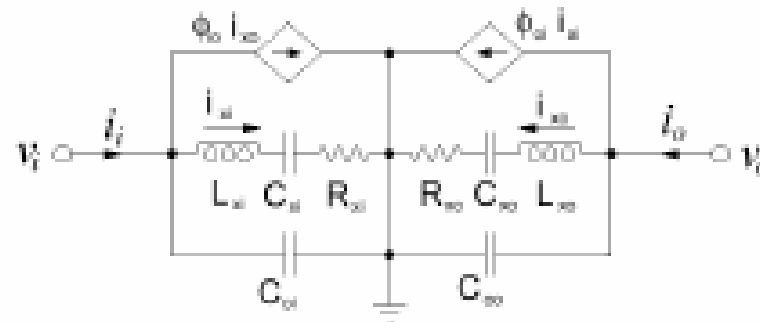
Equivalent Electrical Circuit

Unit of n^2/k is Farad

$$n = V_{dc} \frac{\partial C}{\partial x}$$

Both methods result in the same equivalent circuit:

Equivalent Circuit of 2-Port Resonator (in Electrical Domain)



$C_x = 0.5 \text{ fF}$
 $L_x = 200 \text{ nH}$
 $R_x = 500 \text{ k}\Omega$
 $C_{in}, C_{out} = 15 \text{ fF}$

Fixed electrical
 Capacitance
 Between fixed comb
 And ground plane

$$C_{in} = \frac{\eta_n^2}{k} \quad R_x = \frac{\sqrt{k m}}{Q \eta_n^2} \quad \eta_n = V_{ps} \frac{\partial C_n}{\partial x}$$

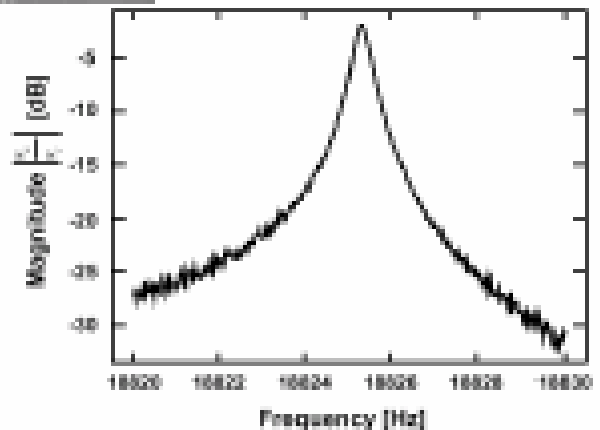
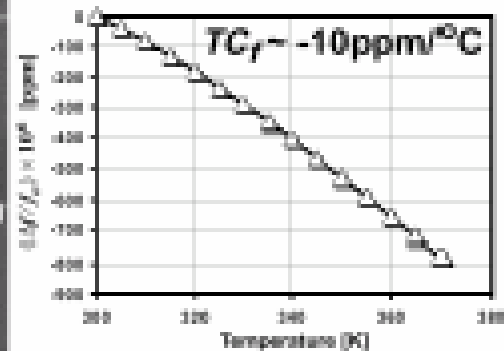
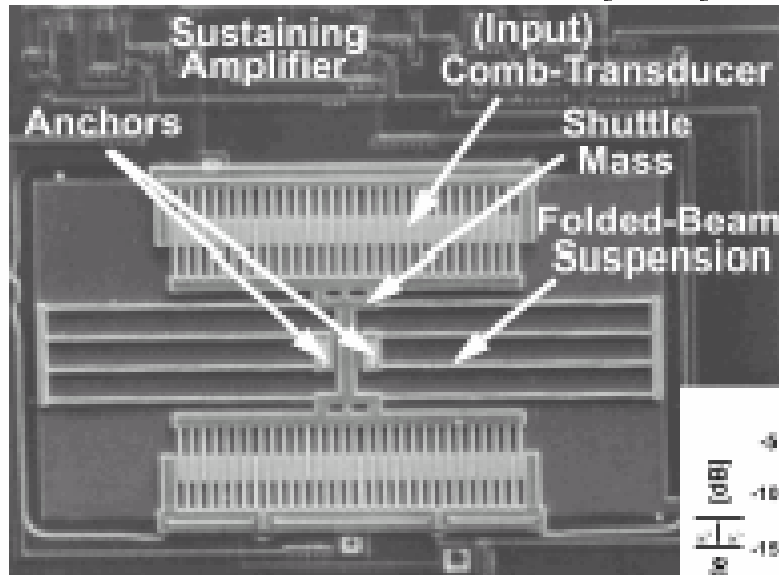
$$L_{in} = \frac{m}{\eta_n^2} \quad \phi_{out} = \frac{\eta_n}{\eta_e}$$

C. T.-C. Nguyen, "Micromechanical resonators for oscillators and filters," Proceedings
 IEEE International Ultrasonics Symposium, Seattle, WA, pp. 489-498, Nov. 7-10, 1986



Comb-Transduced Folded-Beam Microresonator

- Micromachined from *in situ* phosphorous-doped polysilicon



- At right: $Q = 50,000$ measured at 20 mTorr pressure
- ($Q = 27$ at atmospheric pressure)
- Problems: large mass \Rightarrow limited to low frequencies; low coupling

Comb resonator, summary

- Summary of modeling:
- Force: $F_e = \frac{1}{2} \frac{dC}{dx} V^2$ (force is always attractive)
 - Input signal $V_a \cos(\omega t)$
 - $F_e \sim V_a^2 \cdot \frac{1}{2} [1 + \cos(2\omega t)]$
 - Driving force is 2x input-frequency + DC: NOT DESIRABLE
- Add DC bias, V_d
 - $F_e \sim V_d^2 + 2 V_d \cdot V_a \cos \omega t + \text{negligible term } (2\omega t)$
 - Keep linearized AC force-component $\sim V_d \cdot V_a$, which oscillates with the same frequency as V_a : ω
- C increases when finger-overlap increases (comb moves)
 - $\epsilon \cdot A/d$ ($A = \text{comb-thickness} \cdot \text{overlap-length}$)
- $dC/dx = \text{constant}$ for a given design (linear change, C is proportional to length-variation)

Comb-resonator, output current

- A time varying capacitance is established at the output comb
 - Calculate output current when V_d is kept constant and C is varying
 - $I_0 = d/dt (Q) = d/dt (C \cdot V) = V_d \cdot dC/dt = V_d \cdot dC/dx \cdot dx/dt$
 - $I_0 = V_d \cdot dC/dx \cdot \omega \cdot x_{\max}$
 - I_0 plotted versus frequency, shows a BP-characteristic

Comb-resonator, spring constant

- Spring constant for simple beam deflected to the side
 - $k_{\text{beam}} = \text{const} * E * t * (w/L) \text{ exp}^3$
 - $E = \text{Youngs modul, } t = \text{thickness, } w = \text{width, } L = \text{length}$
- Example in figure 7.9:
 - $\text{const} = 1 = 4 * \frac{1}{4}$ (e.g. cantilevers)
 - $k_{\text{total}} = 2 * k_{\text{beam}}$

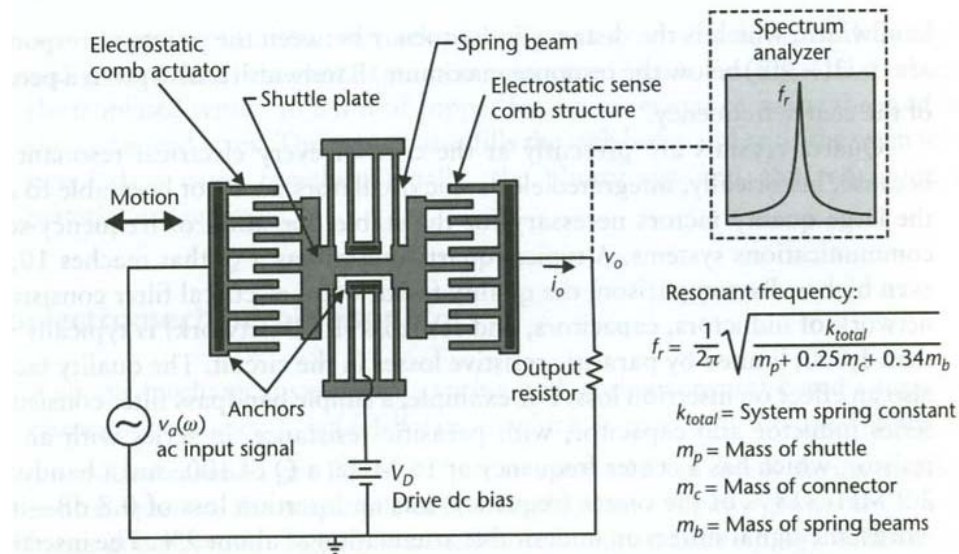


Figure 7.9 Illustration of a micromachined folded-beam comb-drive resonator. The left comb drive actuates the device at a variable frequency ω . The right capacitive-sense-comb structure measures the corresponding displacement by turning the varying capacitance into a current, which generates a voltage across the output resistor. There is a peak in displacement, current, and output voltage at the resonant frequency.

Design parameters

- To obtain a **higher resonance frequency**:
- Total **spring constant** must increase
- **Dynamic mass** must decrease
 - Difficult to achieve because a minimum number of fingers are needed
 - To have good electrostatic coupling (voltage \rightarrow force)
 - Process resolution determines how small the lateral structures can be fabricated (geometrical design rules)
- Frequency can be increased by using **another material** with larger **E/ρ** than Si
 - E/ρ is a measure of the spring constant relative to weight
 - Elastic modulus versus material density
 - Aluminum and titanium has E/ρ lower than Si
 - Si carbide, poly diamond has E/ρ higher than for Si (poly diamond is a research topic)