INF-5610, Mathematical models in medicine

Forelesere:

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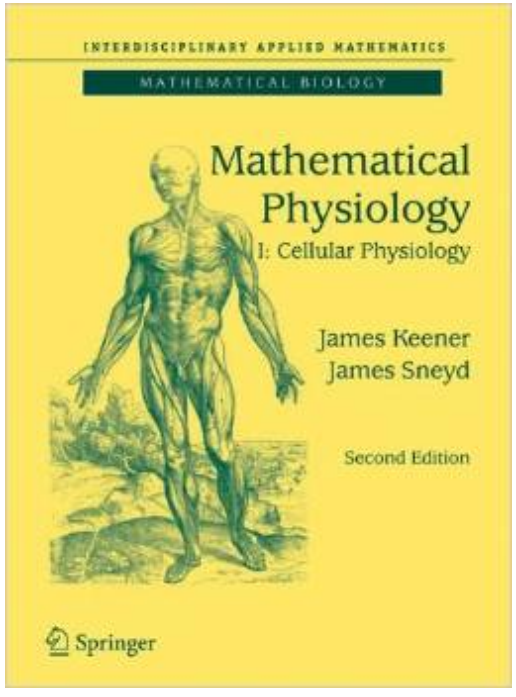
Topics:

- Properties of cells (heart cells in particular)
- Electrical currents in the heart and the body (ECG)
- Models for these phenomena
- (Numerical methods for the models)

One mandatory assignment (two for PhD students)

Literature

J. Keener and J. Sneyd, Mathematical Physiology, second edition (two volumes)



Oral exam

- Six topics are given, no later than two weeks before the exam
- You prepare a 20 min lecture on each topic
- In the exam you draw one topic, and give a lecture on this
- Questions will be asked from the other topics

Lecture plan, part I

- Anatomy, about cells and the heart. Keener & Sneyd (KS)
- Fundamental biophysical processes. KS chap 2
- Ion channels KS. chap 3
- Excitability and signal propagation. KS chap 4
- Neurons and cell to cell coupling. KS chap 7& 8
- Calcium dynamics. KS chap 5

Lecture plan, part II

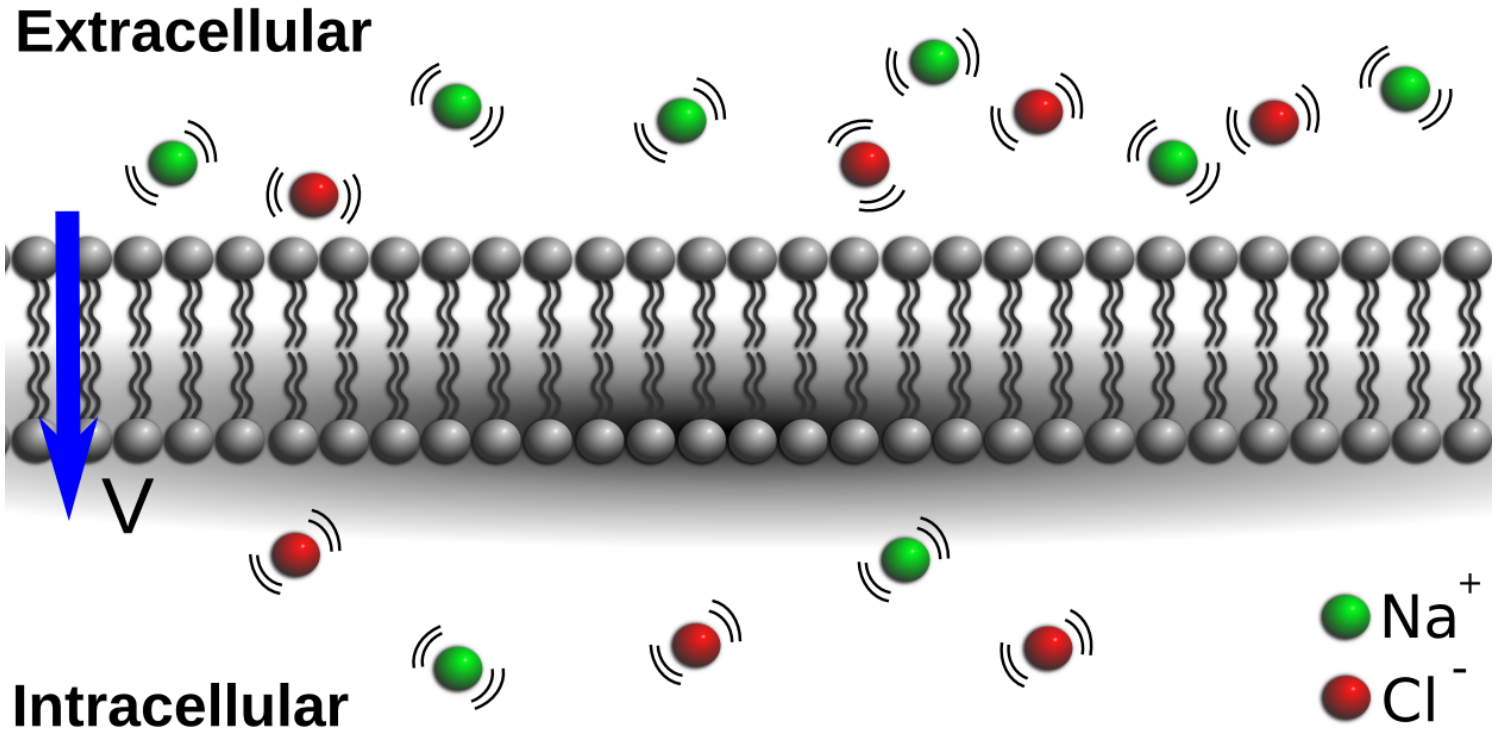
- The electrocardiogram. KS chap 12.
- Bidomain model. KS chap 11.
- Muscle contraction. KS chap 15.
- (Circulation models. KS chap 11.)
- (Numerical methods)

Levels of modeling

- Body
- Organ
- Tissue
- Cell
- Organelles
- Proteins

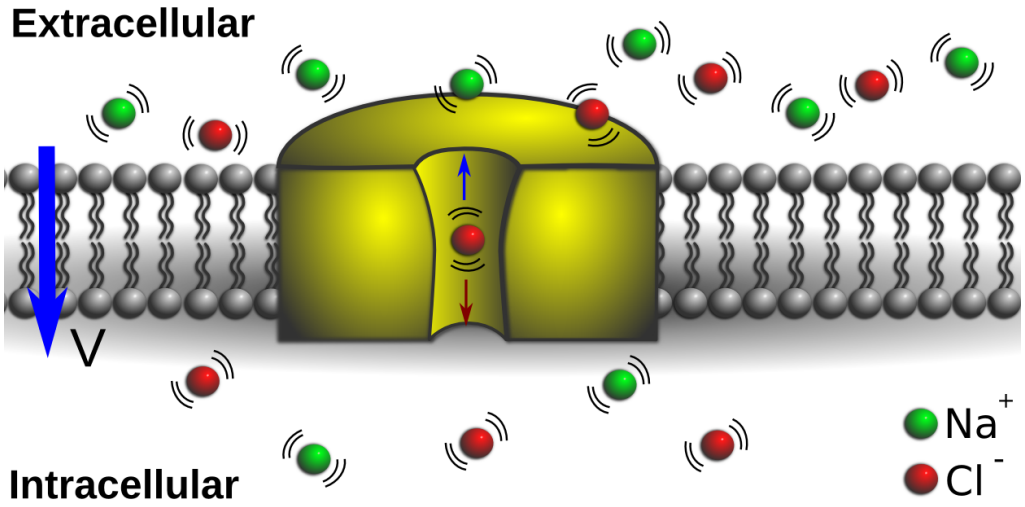
We will in this course focus mainly on the levels of cells and tissue.

The Cell Membrane



- Consist of a bilipid layer
- Embedded proteins for transport control
- Selectively permeable
- Maintains concentration gradients
- Has a transmembrane potential

The cell membrane (II)



Two types of transmembrane flow

Passive: Diffusion along the concentration gradient

- Through the membrane (H_2O, O_2, CO_2)
- Through specialized channels (Na^+, K^+, Cl^-)
- Carrier mediated transport

Active: Energy driven flow against the gradients

- ATP driven pumps ($Na^+ - K^+$, Ca^{2+})
- Exchangers driven by concentration gradients ($Na^+ - Ca^{2+}$)

Cardiac propagation

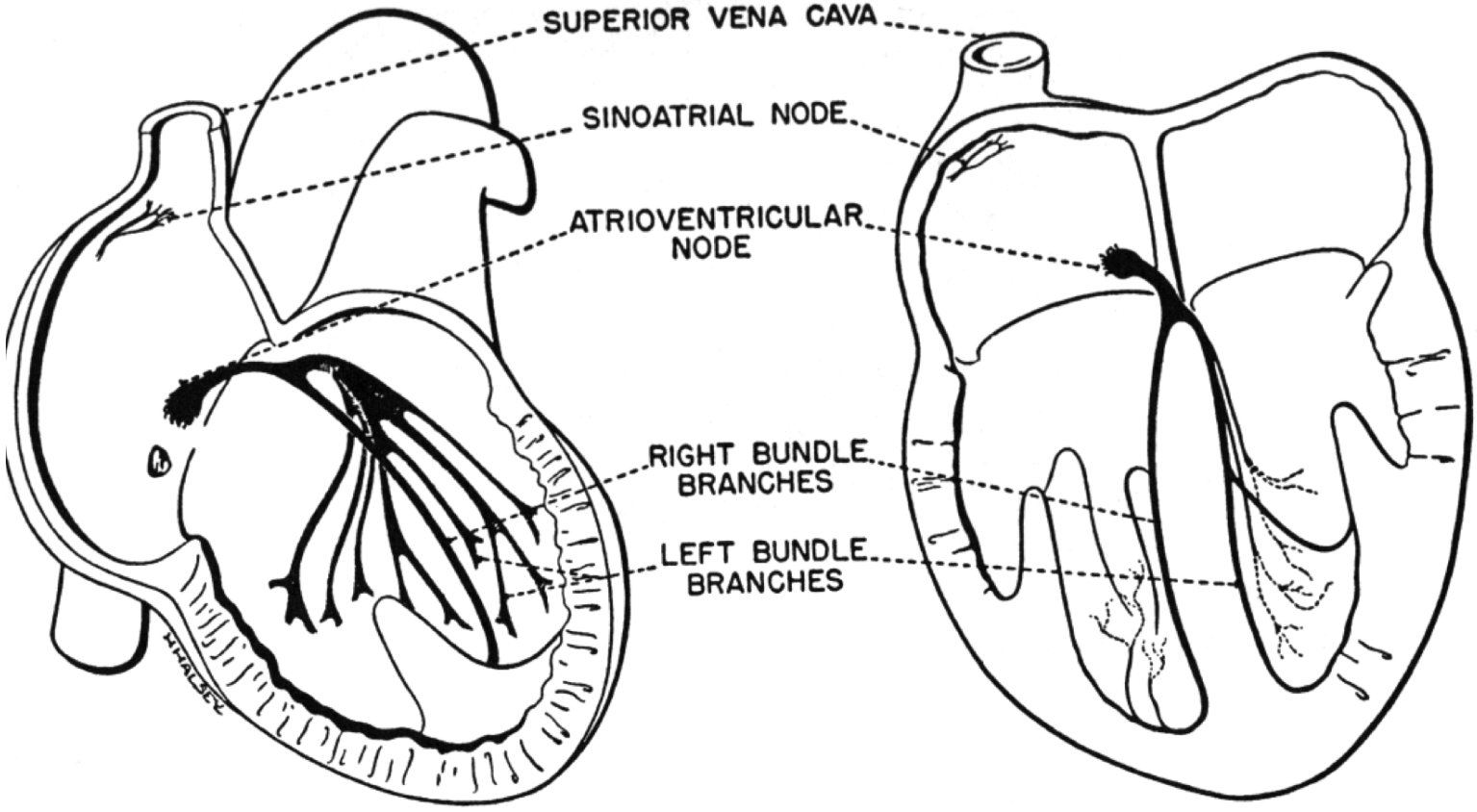
Cardiac cells has two properties and corresponding function

- Excitable $\rightarrow$ Propagates the AP
- Contractive $\rightarrow$ Pumps blood

Furthermore, the arrival of an AP triggers contraction. Cell to cell coupling. Two types:

- Tight junctions: Transfer mechanical energy
- Gap junctions: Inter cellular channels where ions can flow

The conduction system



The path of electrical signal in the heart

- Originates in the sinoatrial node (sinusknuten)
- APs spreads throughout the atria
- The atria and ventricles are separated by an insulating membrane
- Only path of conduction through the AtrioVentricular node
- Slow propagation through the AV node,
- From the AV node the signal propagate through Purkinje fibers, which have a high conductivity
- These fibers end at the endocardial surface of the ventricles
- The arrival of AP at these endings depolarize the tissue and the wavefront spreads out from these locations.

Propagation in both 1D and 3D.