Lecture 11 – Digital signatures, UF-CMA, RSA, Schnorr, PKI

TEK4500

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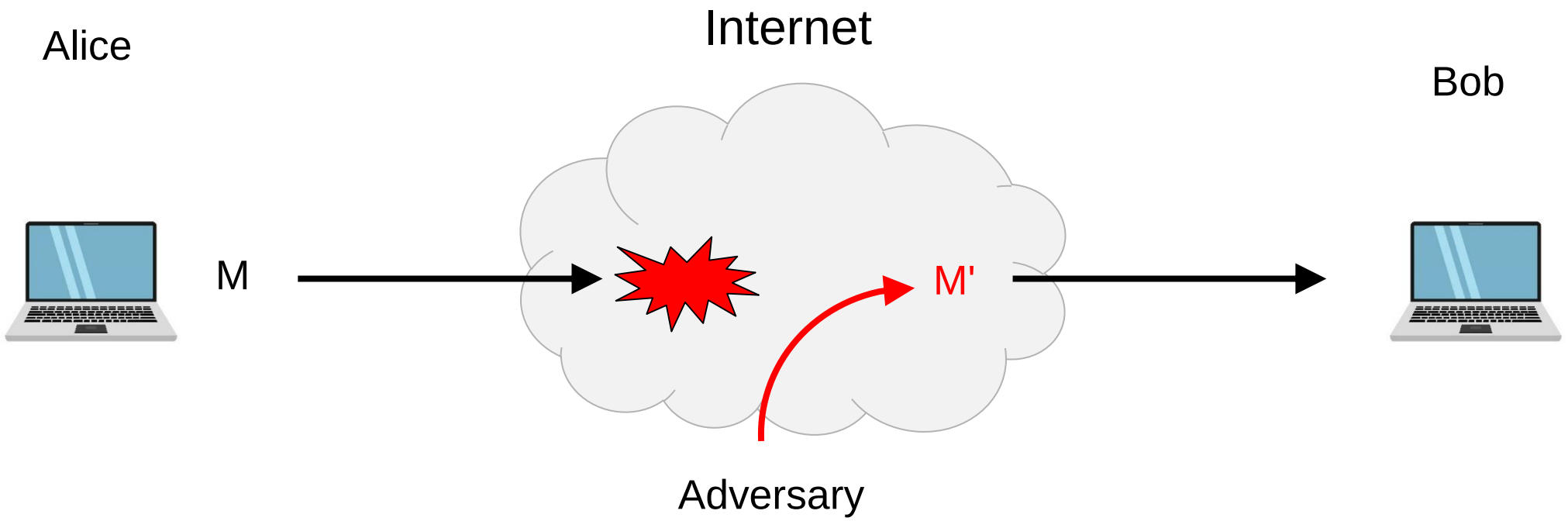
Basic goals of cryptography

	Message privacy	**Message integrity / authentication**
Symmetric keys	Symmetric encryption	Message authentication codes (MAC)
Asymmetric keys	Asymmetric encryption (a.k.a. public-key encryption)	Digital signatures

(Key exchange)

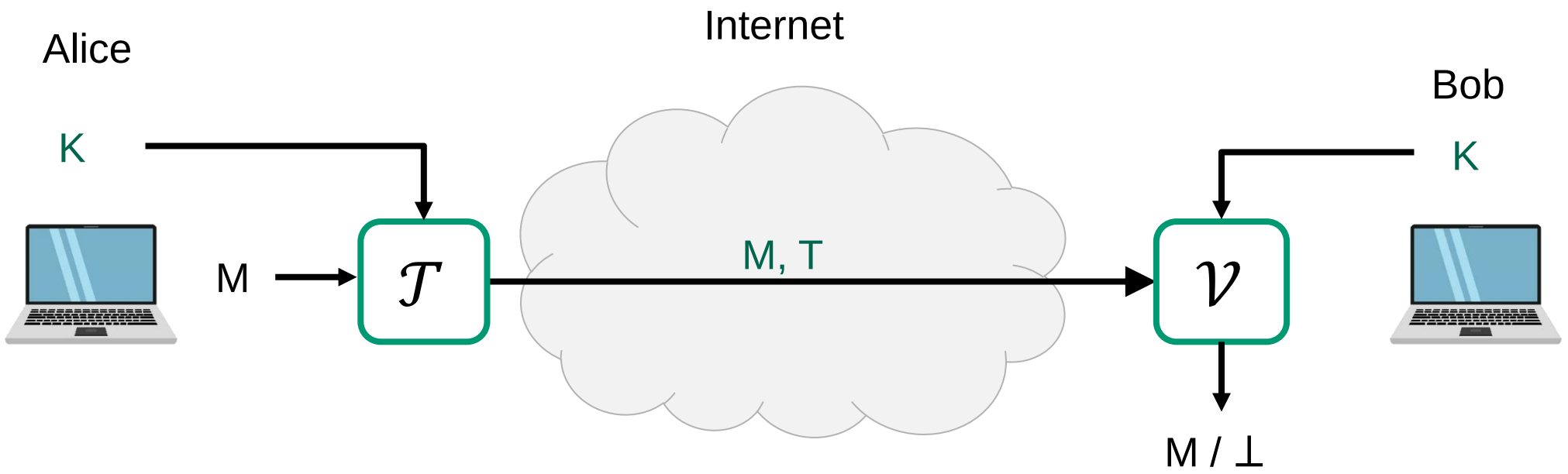
What is cryptography?



Security goals:

- **Data privacy:** adversary should not be able to read message M
- **Data integrity:** adversary should not be able to modify message M
- **Data authenticity:** message M really originated from Alice

MACs

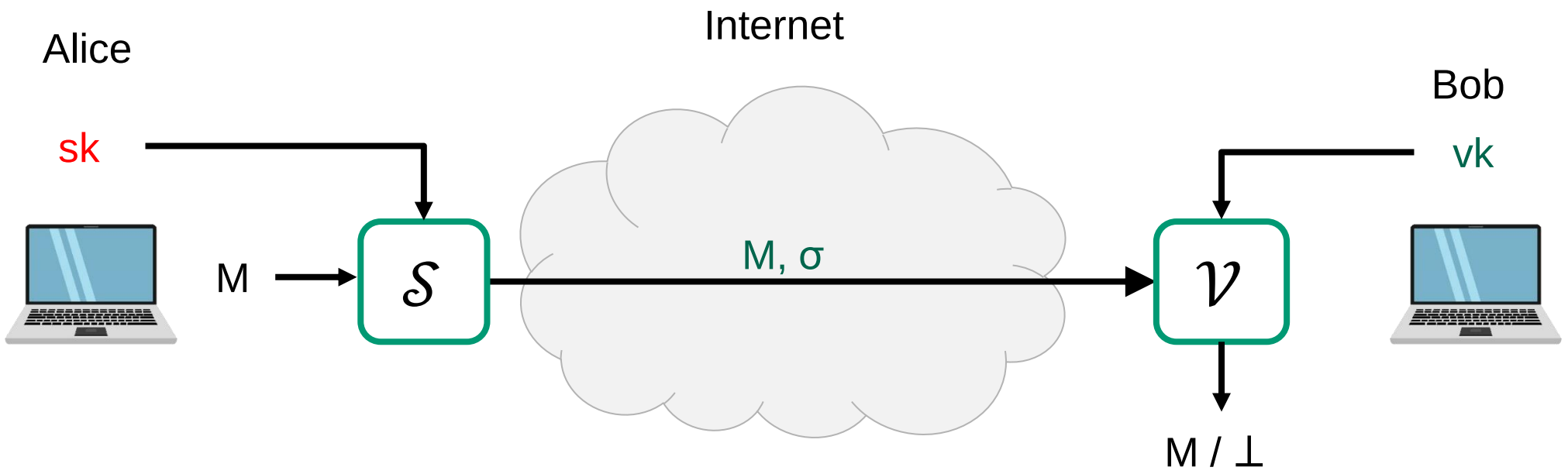


$\mathcal{T}$: tagging algorithm (public)

K : tagging / verification key (secret)

$\mathcal{V}$: verification algorithm (public)

Digital signatures



$\mathcal{T}$: tagging algorithm (public)

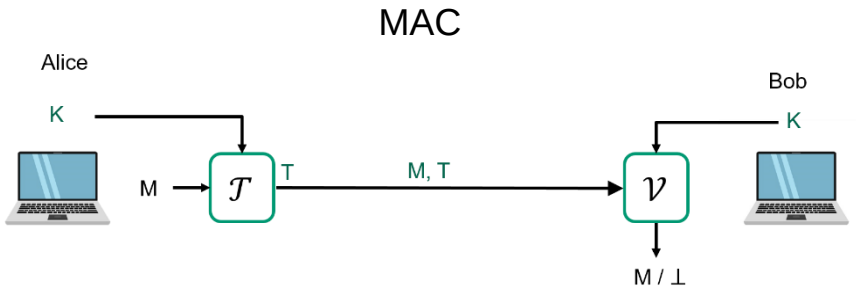
$\mathcal{V}$: verification algorithm (public)

sk : signing key (secret)

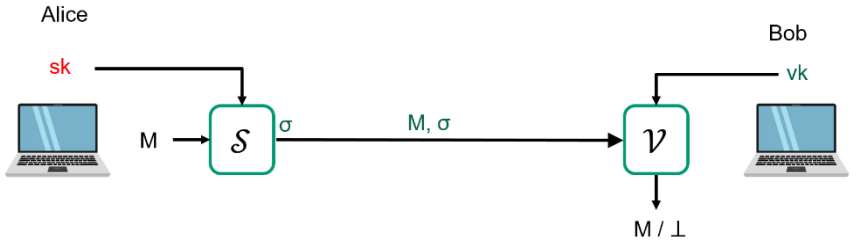
vk : verification key (public)

MACs vs. digital signatures

- MACs can only be verified by party sharing the same key



- Digital signatures can be verified by *anyone*

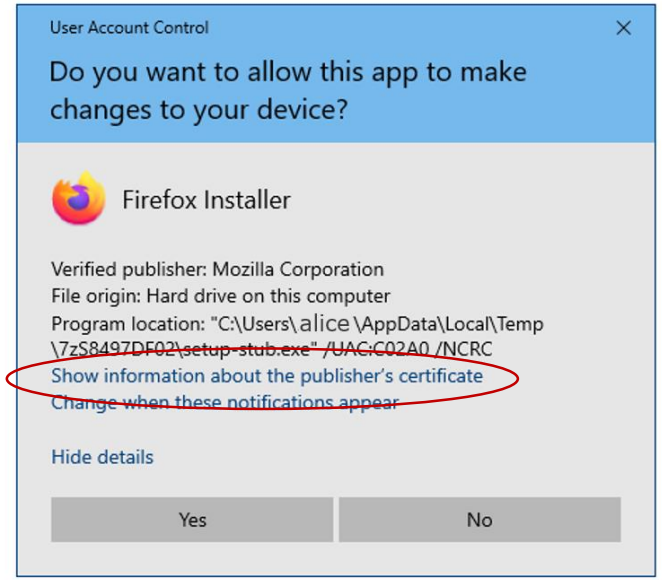


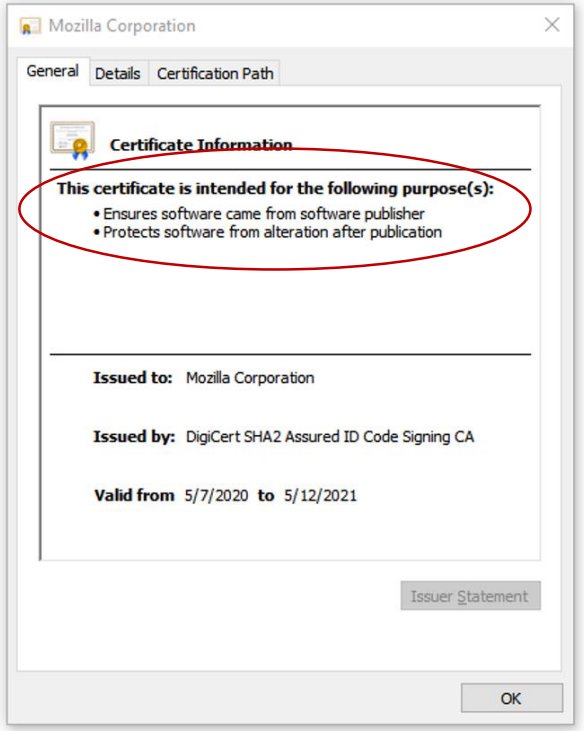
Digital signature

- **Non-repudiation:** Alice cannot deny having created σ
  - But she can deny having created T (since Bob could have done it)

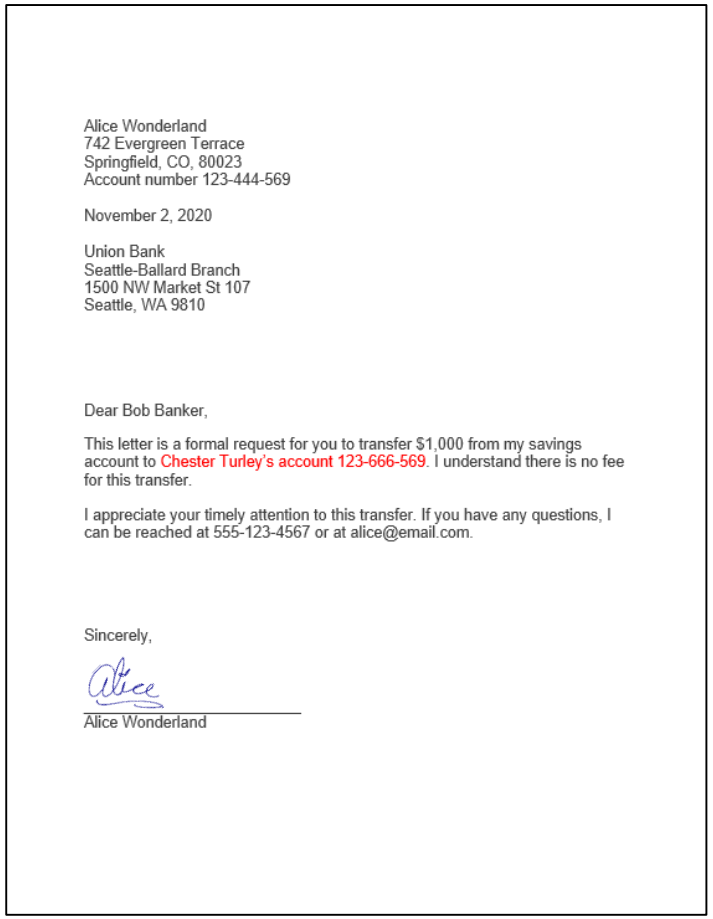
Applications of digital signatures

- Electronic document signing
- HTTPS / TLS certificates
- Software installation
- Email sender authentication
- Bitcoin





Signing electronically



Alice Wonderland
742 Evergreen Terrace
Springfield, CO, 80023
Account number 123-444-569

November 2, 2020

Union Bank
Seattle-Ballard Branch
1500 NW Market St 107
Seattle, WA 9810

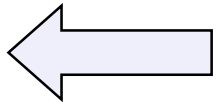
Dear Bob Banker,

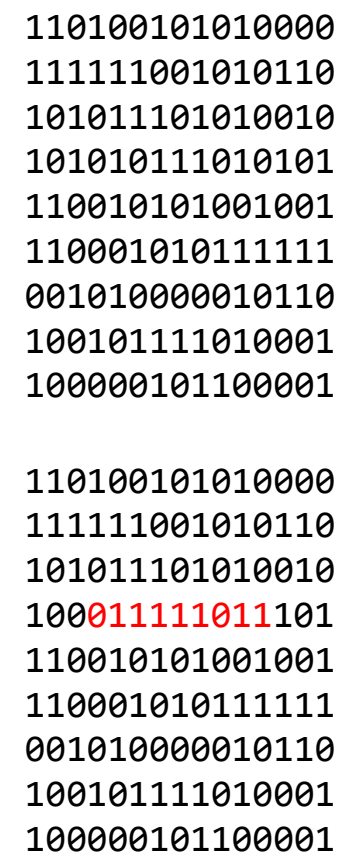
This letter is a formal request for you to transfer $1,000 from my savings account to Chester Turley's account 123-666-569. I understand there is no fee for this transfer.

I appreciate your timely attention to this transfer. If you have any questions, I can be reached at 555-123-4567 or at alice@email.com.

Sincerely,

Alice Wonderland





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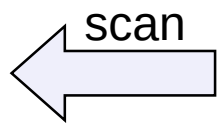

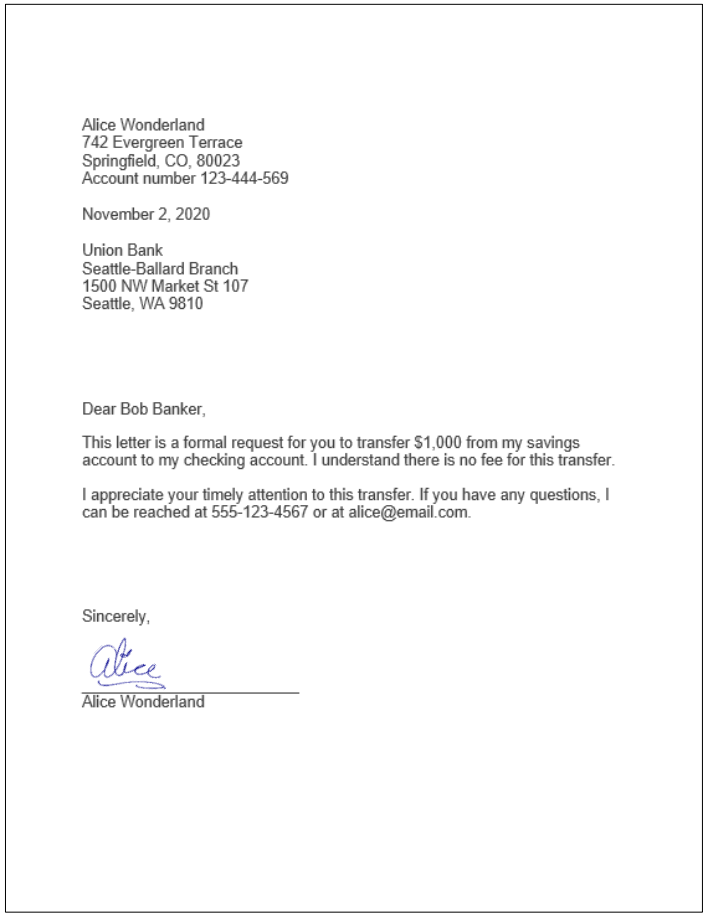

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Alice Wonderland

Digital signatures – syntax

A **digital signature** scheme is a tuple of algorithms $\Sigma = (\text{KeyGen}, \text{Sign}, \text{Vrfy})$

$\text{KeyGen} : (\,) \to \mathcal{SK} \times \mathcal{VK}$

$(sk, vk) \xleftarrow{\$} \text{KeyGen}$

$\text{Sign} : \mathcal{SK} \times \mathcal{M} \to \mathcal{S}$

$\text{Sign}(sk, M) = \text{Sign}_{sk}(M) = \sigma$

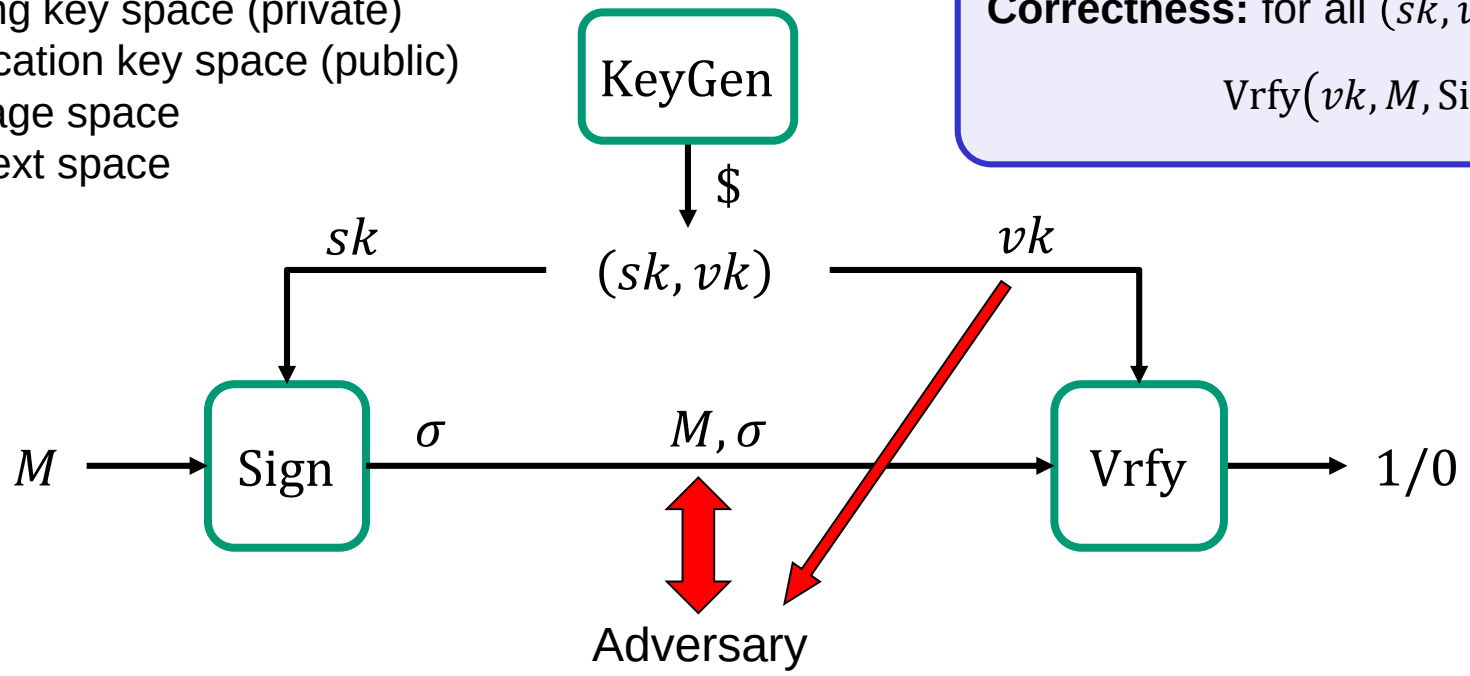
$\text{Vrfy} : \mathcal{VK} \times \mathcal{M} \times \mathcal{S} \to \{0,1\}$

$\text{Vrfy}(vk, M, \sigma) = \text{Vrfy}_{vk}(M, \sigma) = 1/0$

- $\mathcal{SK}$ – signing key space (private)
- $\mathcal{VK}$ – verification key space (public)
- $\mathcal{M}$ – message space
- $\mathcal{C}$ – ciphertext space

Correctness: for all $(sk, vk) \leftarrow \text{KeyGen}$:

$$\text{Vrfy}\big(vk, M, \text{Sign}(sk, M)\big) = 1$$



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Digital signatures – security: UF-CMA

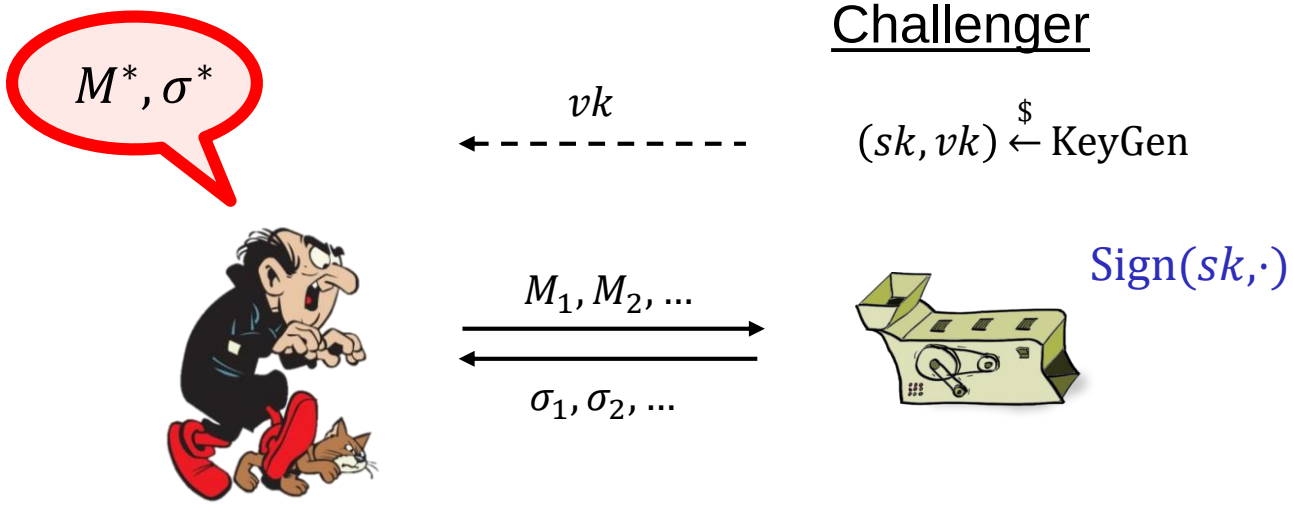
$\mathbf{Exp}_{\Sigma}^{\text{uf-cma}}(A)$

1. $(sk, vk) \xleftarrow{\$} \Sigma.\text{KeyGen}$
2. $\text{Msgs} \leftarrow [\,]$ // bookkeeping
3. $(M^*, \sigma^*) \leftarrow A^{\mathcal{S}(\cdot)}(vk)$
4. **if** $\Sigma.\text{Vrfy}(vk, M^*, \sigma^*) = 1$ and $M^* \notin \text{Msgs}$ **:**
5. **return** 1
6. **else**
7. **return** 0

$\mathcal{S}(M)$

---

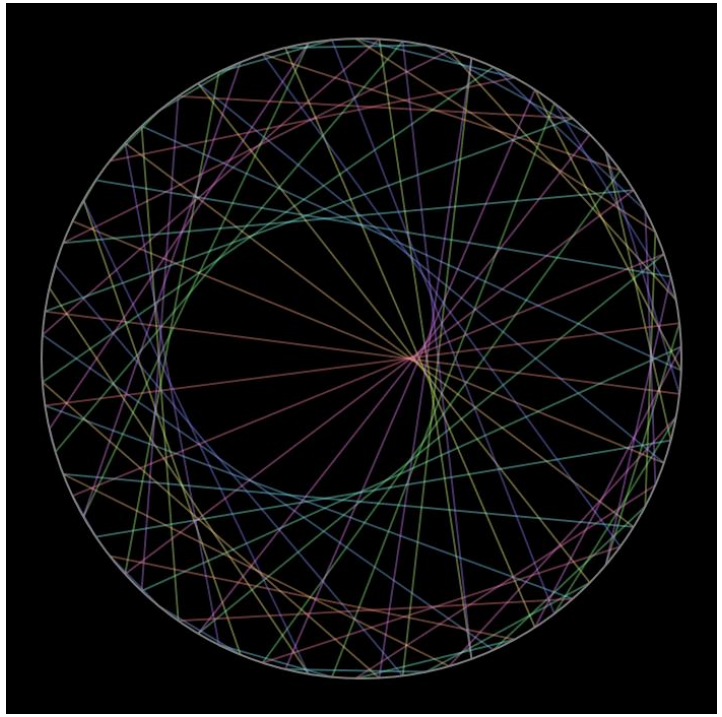
1. $\sigma \leftarrow \Sigma.\text{Sign}(sk, M)$
2. $\text{Msgs.add}(M)$
3. **return** σ



A has **forged** a signature if σ^* is valid for M^*

Definition: The **UF-CMA-advantage** of an adversary A is

$$\mathbf{Adv}_{\Sigma}^{\text{uf-cma}}(A) = \Pr\left[\mathbf{Exp}_{\Sigma}^{\text{uf-cma}}(A) \Rightarrow 1\right]$$



RSA signatures

Textbook RSA signatures

$n = p \cdot q$

$$C^e = M^e \bmod n$$

Textbook RSA signatures

$n = p \cdot q$

$$\sigma = \sigma^{ed} = M^{d} \bmod n$$

Textbook RSA signatures

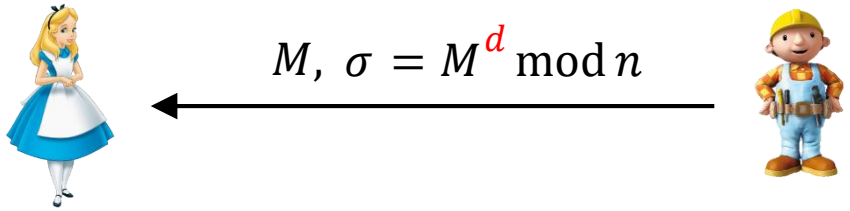
$n = p \cdot q$

RSA.KeyGen

1. $p, q \xleftarrow{\$}$ two random prime numbers
2. $n \leftarrow p \cdot q$
3. $\phi(n) = (p-1)(q-1)$
4. **choose** e such that $\gcd(e, \phi(n)) = 1$
5. $d \leftarrow e^{-1} \bmod \phi(n)$
6. $sk \leftarrow (n, d)$ $\quad pk \leftarrow (n, e)$
7. **return** (sk, pk)

RSA.Sign$\big((n, d), M\big)$

1. $\sigma \leftarrow M^d \bmod N$
2. **return** σ



RSA.Vrfy$\big((n, e), M, \sigma\big)$

1. **if** $\sigma^e = M \bmod N$ **then**
2. **return** 1
3. **else**
4. **return** 0

Textbook RSA signatures – (in)security

$A_1(n, e)$

1. $\sigma \xleftarrow{\$} \boldsymbol{Z}_n^*$

$\mathbf{Adv}_{\mathrm{RSA}}^{\text{uf-cma}}(A_1) = 1$

$\sigma^e = M \bmod n$

$A_2(n, e)$

1. Pick arbitrary $M_1 \in \boldsymbol{Z}_n^*$ (different from M)

$\mathbf{Adv}_{\mathrm{RSA}}^{\text{uf-cma}}(A_2) = 1$

$(\sigma_1 \cdot \sigma_2)^e \stackrel{?}{=} M \bmod n$

$\left(M_1^d \cdot M_2^d\right)^e = M_1^{ed} \cdot M_2^{ed} = M_1 \cdot M_2 = M_1 \cdot M \cdot M_1^{-1} = M \bmod n$

RSA.KeyGen

1. $p, q \xleftarrow{\$}$ two random prime numbers
2. $n \leftarrow p \cdot q$
3. $\phi(n) = (p-1)(q-1)$
4. **choose** e such that $\gcd(e, \phi(n)) = 1$
5. $d \leftarrow e^{-1} \bmod \phi(n)$
6. $sk \leftarrow (n, d)$ $\quad pk \leftarrow (n, e)$
7. **return** (sk, pk)

RSA.Sign$((n, d), M)$

1. $\sigma \leftarrow M^d \bmod N$
2. **return** σ

RSA.Vrfy$((n, e), M, \sigma)$

1. **if** $\sigma^e = M \bmod N$ **then**
2. $\quad$ **return** 1
3. **else**
4. $\quad$ **return** 0

Textbook RSA signatures

$H : \{0,1\}^* \to \boldsymbol{Z}_n^*$

RSA.KeyGen

1. $p, q \xleftarrow{\$}$ two random prime numbers
2. $n \leftarrow p \cdot q$
3. $\phi(n) = (p-1)(q-1)$
4. **choose** e such that $\gcd(e, \phi(n)) = 1$
5. $d \leftarrow e^{-1} \bmod \phi(n)$
6. $sk \leftarrow (n, d)$ $\quad pk \leftarrow (n, e)$
7. **return** (sk, pk)

RSA.Sign$\big((n, d), M\big)$

1. $\sigma \leftarrow M^d \bmod N$
2. **return** σ

$M,\ \sigma = M^d \bmod n$

RSA.Vrfy$\big((n, e), M, \sigma\big)$

1. **if** $\sigma^e = M \bmod N$ **then**
2. $\quad$ **return** 1
3. **else**
4. $\quad$ **return** 0

RSA message space:

$\mathcal{M} = \boldsymbol{Z}_n^*$

$\mathcal{M} = \{0,1\}^*$

Actually want

Hashed-RSA

$H : \{0,1\}^* \to \boldsymbol{Z}_n^*$

RSA.KeyGen

1. $p, q \xleftarrow{\$}$ two random prime numbers
2. $n \leftarrow p \cdot q$
3. $\phi(n) = (p-1)(q-1)$
4. **choose** e such that $\gcd(e, \phi(n)) = 1$
5. $d \leftarrow e^{-1} \bmod \phi(n)$
6. $sk \leftarrow (n, d)$ $\quad pk \leftarrow (n, e)$
7. **return** (sk, pk)

RSA.Sign$\big((n, d), M\big)$

1. $\sigma \leftarrow H(M)^d \bmod N$
2. **return** σ

$M,\ \sigma = H(M)^d \bmod n$

RSA.Vrfy$\big((n, e), M, \sigma\big)$

1. **if** $\sigma^e = H(M) \bmod N$ **then**
2. $\quad$ **return** 1
3. **else**
4. $\quad$ **return** 0

Hashed-RSA – security

$A_1(n, e)$

1. $\sigma \xleftarrow{\$} \boldsymbol{Z}_n^*$
2. $M \leftarrow \sigma^e \bmod n$
3. ~~Output (M, σ)~~ Output (X, σ) // $H(X) = M$

$\sigma^e = H(M) \bmod n$

Hard to find!

$A_2(n, e)$

1. Pick arbitrary $M_1 \in \boldsymbol{Z}_n^*$ (different from M)
2. $M_2 \leftarrow$ ~~$M \cdot M_1^{-1} \bmod n$~~ // $H(M_1) \cdot H(M_2) = H(M)$
3. Query $\sigma_1 \leftarrow \mathcal{S}(M_1)$ and $\sigma_2 \leftarrow \mathcal{S}(M_2)$
4. Output $(M, \sigma_1 \cdot \sigma_2 \bmod n)$

$(\sigma_1 \cdot \sigma_2)^e \stackrel{?}{=} H(M) \bmod n$

Hard to find!

$\left(H(M_1)^d \cdot H(M_2)^d\right)^e = H(M_1) \cdot H(M_2) \neq H(M) \bmod n$

RSA.KeyGen

1. $p, q \xleftarrow{\$}$ two random prime numbers
2. $n \leftarrow p \cdot q$
3. $\phi(n) = (p-1)(q-1)$
4. **choose** e such that $\gcd\left(e, \phi(n)\right) = 1$
5. $d \leftarrow e^{-1} \bmod \phi(n)$
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RSA.Sign$\left((n, d), M\right)$

1. $\sigma \leftarrow H(M)^d \bmod N$
2. **return** σ

RSA.Vrfy$\left((n, e), M, \sigma\right)$

1. **if** $\sigma^e = H(M) \bmod N$ **then**
2. $\quad$ **return** 1
3. **else**
4. $\quad$ **return** 0

Hashed-RSA – security

- Factoring + RSA-problem must be hard
- What are the requirements of H?
  - Must be collision-resistant:

$$H(X) = H(Y) \implies H(X)^d = H(Y)^d = \sigma$$

  - Is this enough?
    - Unknown
    - However, if we assume that H is *perfect** then

Theorem: For *any* UF-CMA adversy A against Hashed-RSA making q sign queries, there is an algorithm B solving the RSA-problem:

$$\mathbf{Adv}^{\text{uf-cma}}_{\text{H-RSA}}(A) \leq q \cdot \mathbf{Adv}^{\text{RSA}}_{n,e}(B)$$

where H is assumed perfect*

*random oracle

RSA.KeyGen

1. $p, q \xleftarrow{\$}$ two random prime numbers
2. $n \leftarrow p \cdot q$
3. $\phi(n) = (p-1)(q-1)$
4. **choose** e such that $\gcd(e, \phi(n)) = 1$
5. $d \leftarrow e^{-1} \bmod \phi(n)$
6. $sk \leftarrow (n, d)$ $\quad$ $pk \leftarrow (n, e)$
7. **return** (sk, pk)

RSA.Sign$((n, d), M)$

1. $\sigma \leftarrow H(M)^d \bmod N$
2. **return** σ

RSA.Vrfy$((n, e), M, \sigma)$

1. **if** $\sigma^e = H(M) \bmod N$ **then**
2. $\quad$ **return** 1
3. **else**
4. $\quad$ **return** 0

$$Y = g^{x}$$

Signatures from discrete logarithms

Schnorr signatures

$(G,*) = \langle g \rangle$

$H : G \times \mathcal{M} \to \boldsymbol{Z}_p^*$

KeyGen

1. $b \xleftarrow{\$} \{1 \dots |G|\}$
2. $B \leftarrow g^b$
3. **return** $(sk = b, vk = B)$

Sign$(sk = b, M)$

1. $r \xleftarrow{\$} \{1 \dots |G|\}$
2. $R \leftarrow g^r$
3. $h \leftarrow H(R, M)$
4. $s \leftarrow r - bh \bmod p$
5. **return** $\sigma = (h, s)$

M, σ

Vrfy$\big(vk = B, M, \sigma = (h, s)\big)$

1. $R' \leftarrow g^s * B^h$
2. $h' \leftarrow H(R', M)$
3. **if** $h' = h$ **then**
4. **return** 1
5. **else**
6. **return** 0

Correctness: $\mathrm{Vrfy}\big(vk, M, \mathrm{Sign}(sk, M)\big) = 1$

$$h' = H(R', M) = H\big(g^s B^h, M\big) = H\big(g^{r-bh} g^{bh}, M\big) = H\big(g^{r-bh+bh}, M\big) = H(g^r, M) = H(R, M) = h$$

Schnorr signatures

$(G,*) = \langle g \rangle$

$H : G \times \mathcal{M} \to \boldsymbol{Z}_p^*$

KeyGen

1. $b \xleftarrow{\$} \{1 \dots |G|\}$
2. $B \leftarrow g^b$
3. **return** $(sk = b, vk = B)$

Vrfy$\big(vk = B, M, \sigma = (h,s)\big)$

1. $R' \leftarrow g^s * B^h$
2. $h' \leftarrow H(R', M)$
3. **if** $h' = h$ **then**
4. **return** 1
5. **else**
6. **return** 0

M, σ

Sign$(sk = b, M)$

1. $r \xleftarrow{\$} \{1 \dots |G|\}$
2. $R \leftarrow g^r$
3. $h \leftarrow H(R, M)$
4. $s \leftarrow r - bh \bmod p$
5. **return** $\sigma = (h,s)$

Security:

- DLOG must be hard in G
- H must be collision-resistant, one-way, etc.
- Attacker must essentially solve $g^r = g^s * g^{bh} \iff r = s + bh \iff s = r - bh$
- r must be picked new *every time*!

$\sigma = (h,s)$

$\sigma' = (h',s')$

$\overset{\text{same } r}{\Longrightarrow} \quad s - s' = (r - bh) - (r - bh') = b \cdot (h' - h) \implies b = (s - s') \cdot (h' - h)^{-1} \bmod p$

Learned private long-term key!

Discrete-log-based signatures

- Schnorr
  - Elegant design
  - Has formal security proof (based on DLOG problem and H assumed perfect)
  - Was patented

- (EC)DSA
  - Non-patented alternative
  - More complicated design than Schnorr
  - No security proof
  - Standardized by NIST (designed by NSA)
  - Very widely used
  - Same sharp edge as Schnorr
    - Reuse of r leaks long-term signing key
    - Broke all PlayStation 3's produced by Sony


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VIDEO GAMES

Sony: PlayStation Breach Involves 70 Million Subscribers

Chris Morris | @MorrisatLarge

Published 4:30 PM ET Tue, 26 April 2011 | Updated 5:26 PM ET Tue, 26 April 2011

Public-key infrastructure (PKI)

What are identities?

"Alice", g^a

"Bob", g^b

$K \leftarrow g^{ab}$

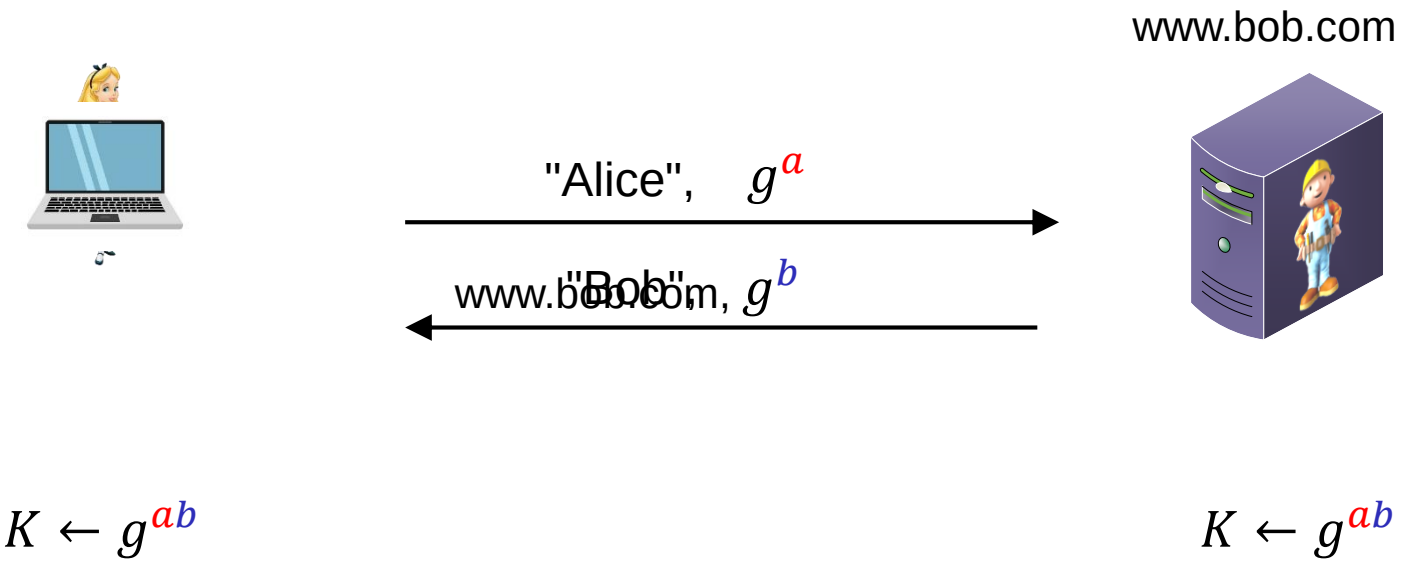
$K \leftarrow g^{ab}$

There are many Alice's and many Bob's

How do we know that g^a belongs to *this* particular Alice, and g^b to this particular Bob?

Need to **bind** public keys to entities

Identities on the internet



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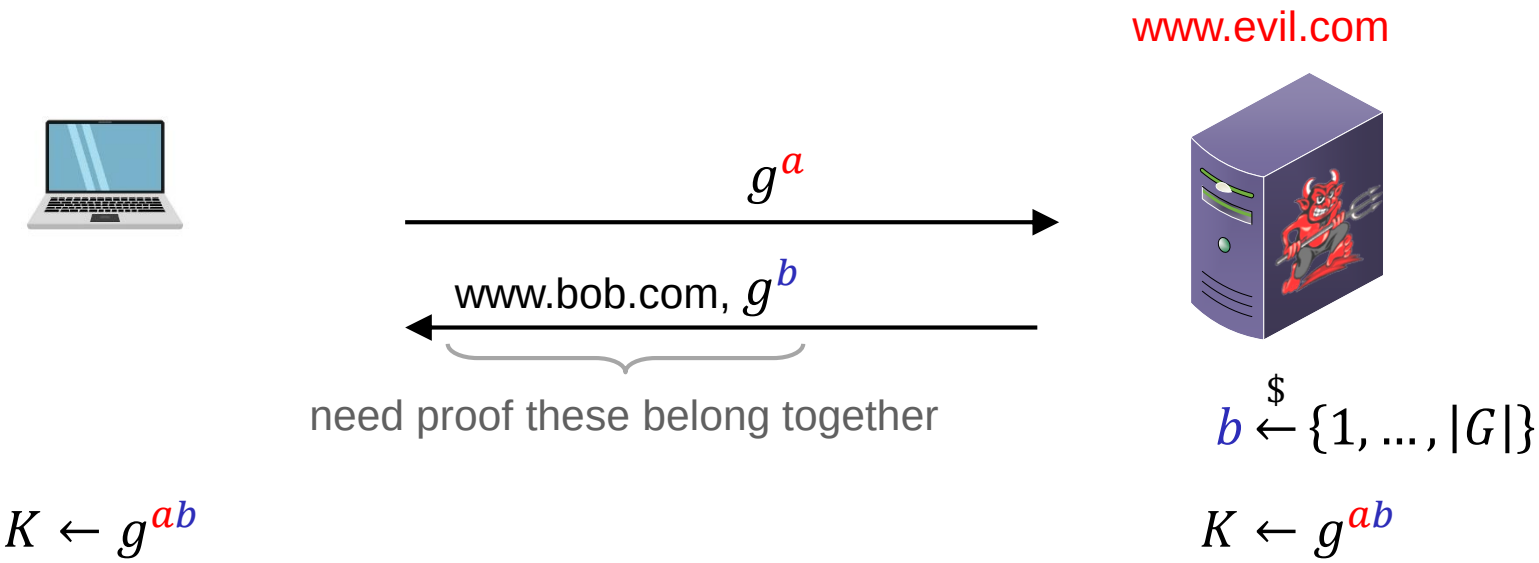
How do we know that g^a belongs to *this* particular Alice, and g^b to this particular Bob?

Need to **bind** public keys to entities – internet: bind public keys to **domain names**

Identities on the internet

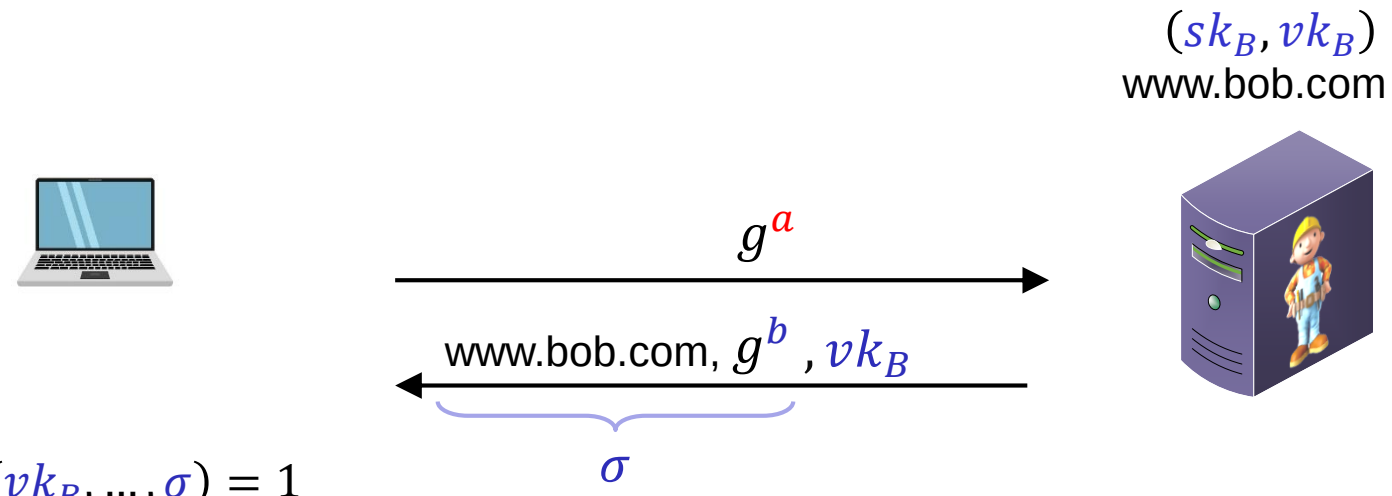


There are many Alice's and many Bob's

How do we know that g^a belongs to *this* particular Alice, and g^b to this particular Bob?

Need to **bind** public keys to entities – internet: bind public keys to **domain names**

Authenticated key exchange



if $\mathrm{Vrfy}(vk_B, \ldots, \sigma) = 1$

$K \leftarrow g^{ab}$

else

abort

$K \leftarrow g^{ab}$

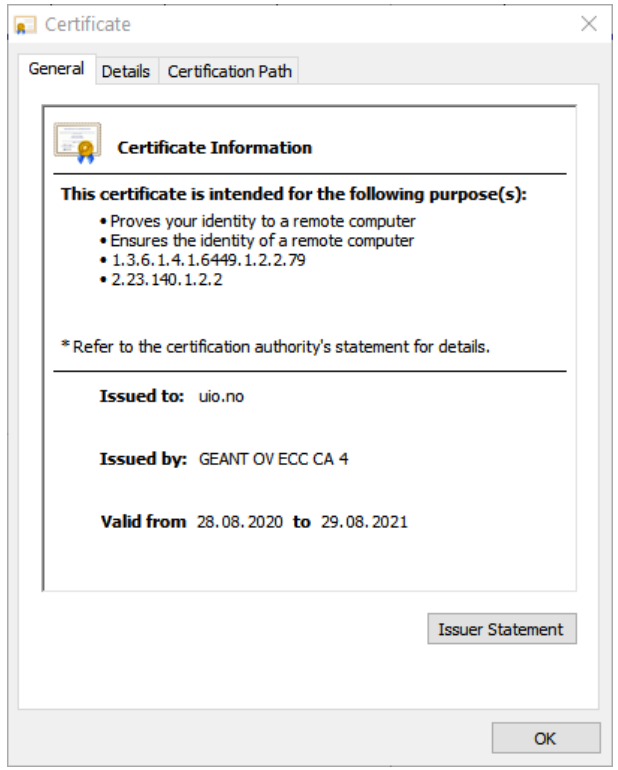
But why should we trust this vk_B? Could have been created by the adversary itself

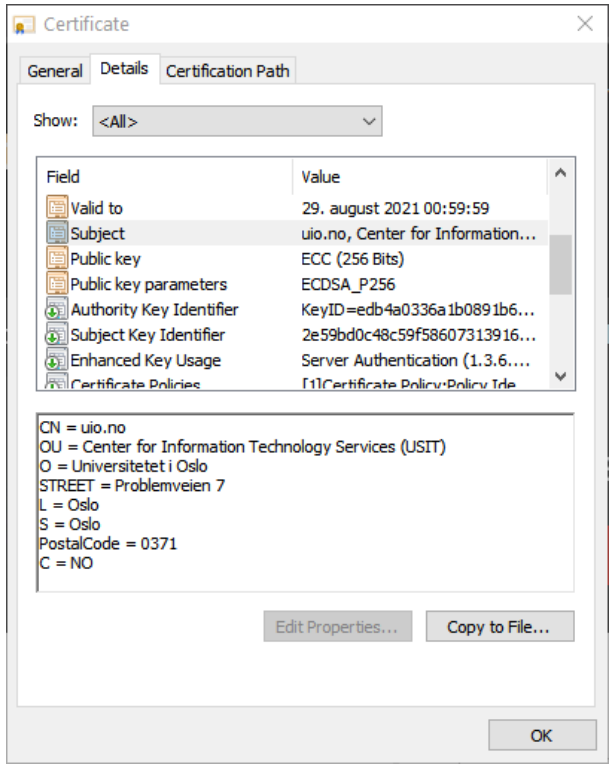
Digital certificates

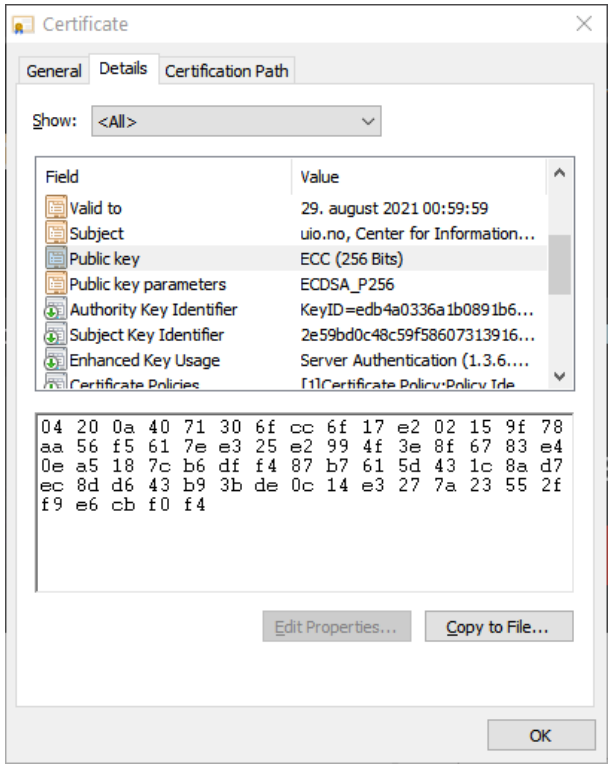
- **Digital certificate:** a way of binding a public key to an entity

- A certificate consists of:
  - The public key of the entity
  - A bunch of information identifying the entity
    - Name
    - Address
    - Occupation
    - URL
    - Email-address
    - Phone number
    - …
  - A *digital signature* on all the above by a **certificate authority (CA)**

Digital certificates







Certificate authorities (CA)

- **CA:** an issuer of digital certificates

- Acts as a trusted third-party, certifying (i.e., signing) the public keys of other entities
  - Verifies the identity of a claimed public-key owner

- The basis of a **public-key infrastructure (PKI)**

Authenticated key exchange + PKI

(sk_B, vk_B)
www.bob.com

g^a

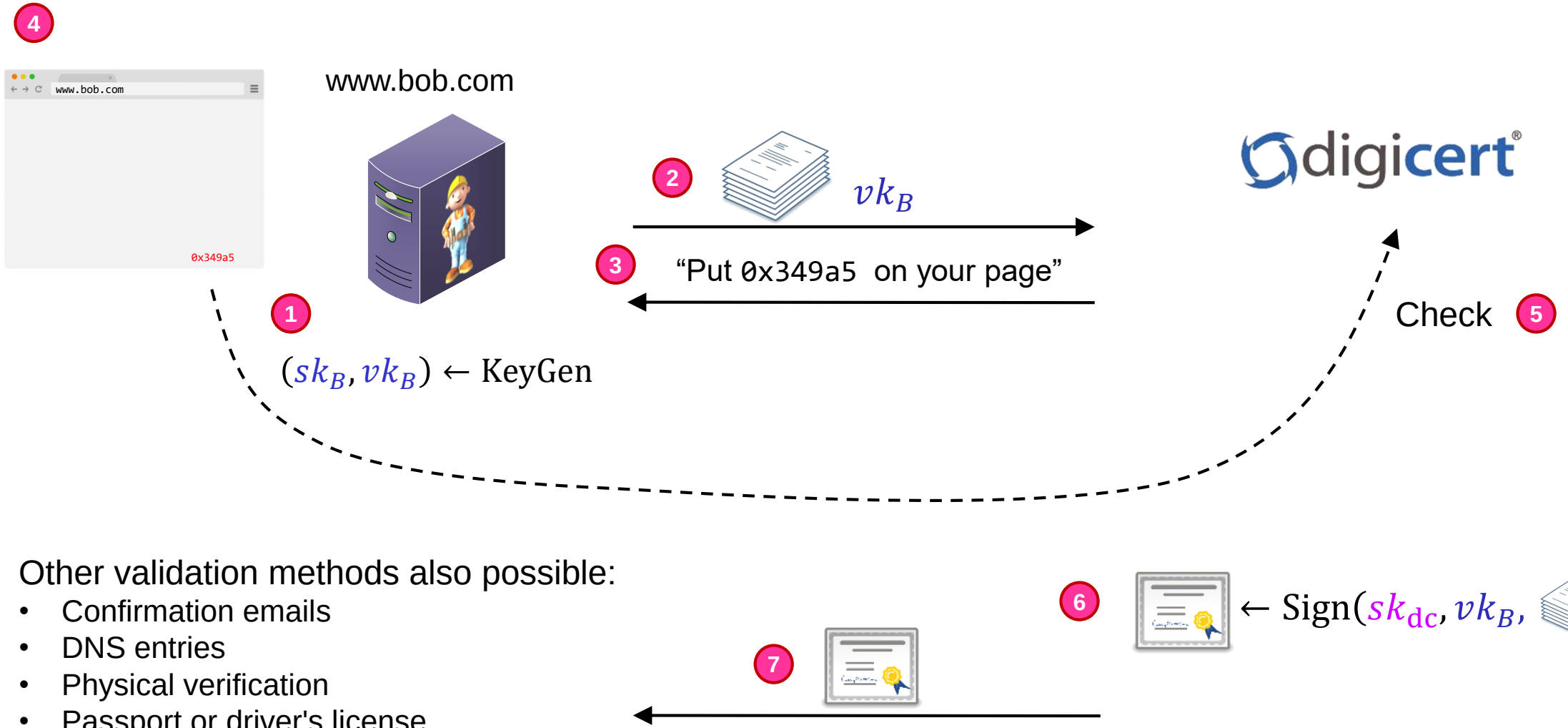
www.bob.com, g^b, vk_B

σ σ vk_{CA}

if $\text{Vrfy}(vk_B, \ldots, \sigma) = 1$
and $\text{Vrfy}(vk_{\text{CA}}, \ldots, \sigma) = 1$
$K \leftarrow g^{ab}$
else
abort

$K \leftarrow g^{ab}$

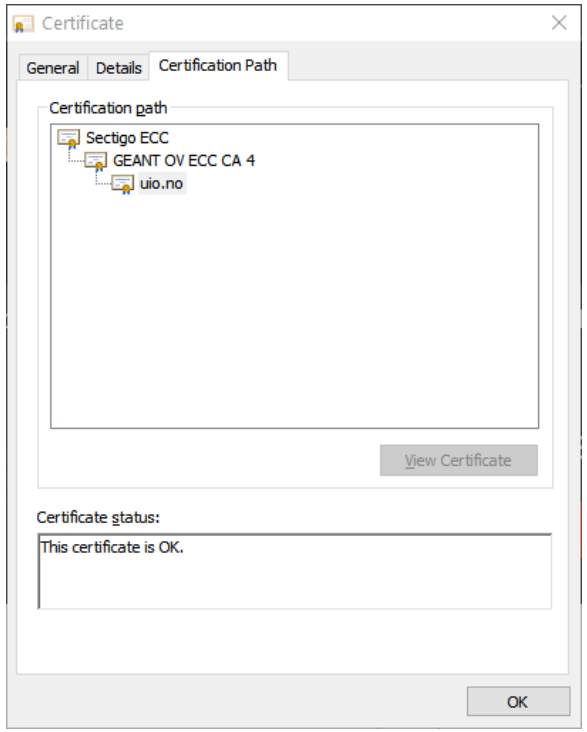
How to get a signed certificate?



Other validation methods also possible:

- Confirmation emails
- DNS entries
- Physical verification
- Passport or driver's license

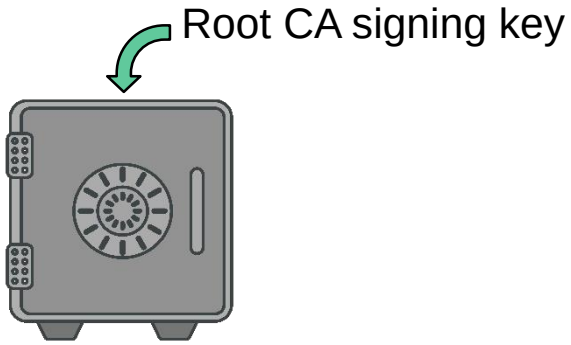
Certificate chains



Root CAs

- **Root CAs:** CAs that sign other CAs' public keys
  - \+ only a few root CAs need to be trusted by end-users
  - \+ root CAs can distribute the signing + verification load to smaller CAs
  - \- single point of failure; private key must be *very heavily* guarded

  - Root CAs for the internet: a few large multinational corporations



COMODO

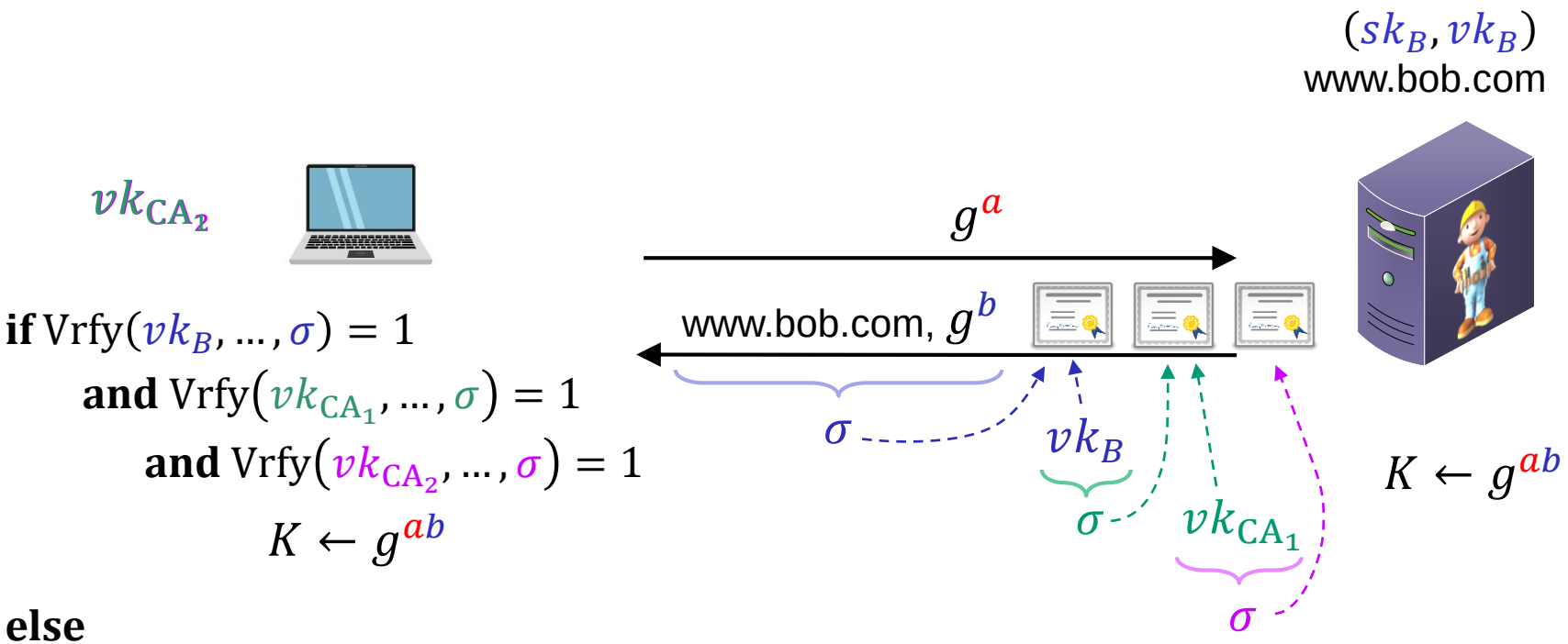
IdenTrust part of HID Global

digicert

GoDaddy

Let's Encrypt

HTTPS / TLS + PKI



How to become an internet root CA?

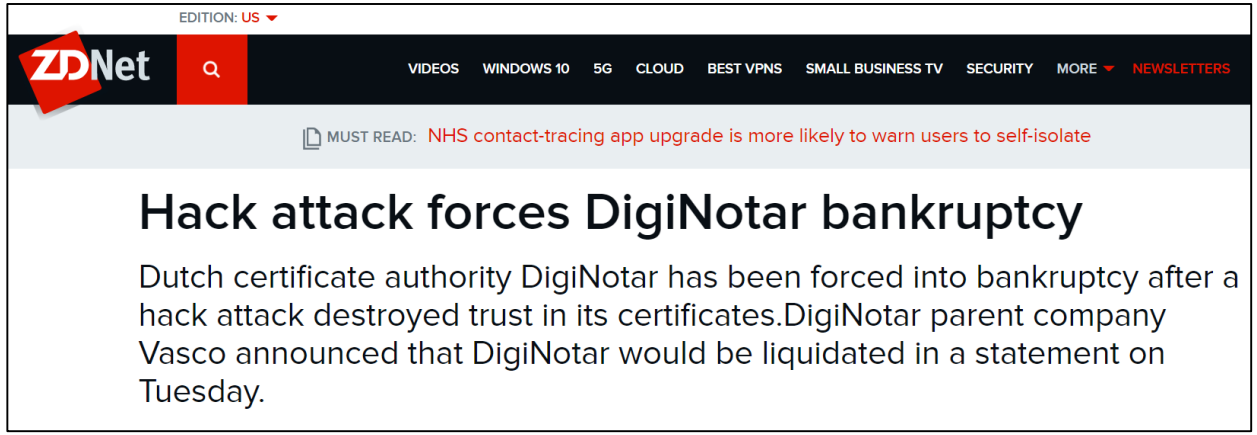
- Need to prove yourself (trust)worthy to browser and OS vendors
  - Microsoft Root Certificate Program
  - Mozilla CA Certificate Program
  - Apple Root Certificate Program
  - Chrome Root CA Program

- Lot's of auditing and paperwork

- Many formal technical and non-technical security requirements
  - CA/Browser forum
  - Baseline Requirements v1.7.3

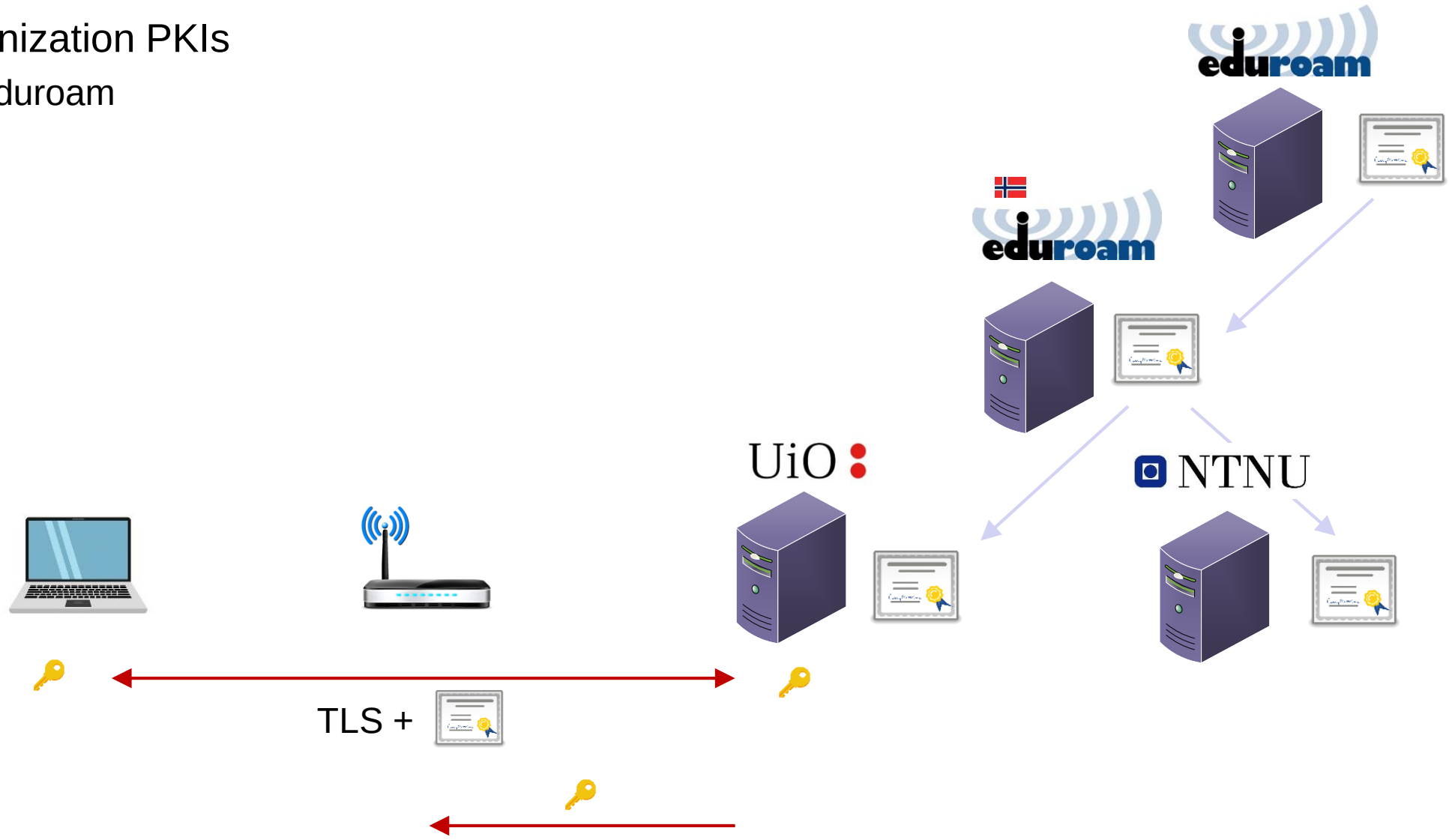
DigiNotar

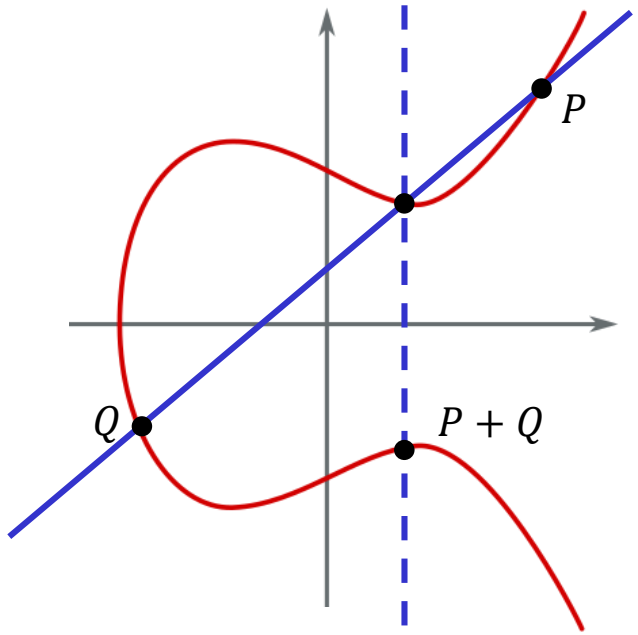
- Dutch root CA
- Lost control of their private signing key in 2011
- Fraudulent certificates issued for Gmail, Yahoo!, Mozilla, WordPress, …
- 30 000 Iranian Gmail users targeted



Other PKIs exist

- Organization PKIs
  - eduroam





$y^2 = x^3 + ax + b$

$a, b, x, y \in \boldsymbol{R}$

End of Part II
(Asymmetric crypto)

Summary of asymmetric cryptography

Primitive	Functionality + syntax	Hardness assumption / security goal	Acronym	Examples
Diffie-Hellman	Derive shared value (key) in a cyclic group $A^b = g^{ab} = B^a$	Discrete logarithm (DLOG) Diffie-Hellman (DH) Decisional Diffie-Hellman (DDH)	PRF	$(\boldsymbol{Z}_p^*,\cdot)$ −DH $(E(\boldsymbol{F}_p),+)$ −DH
RSA function	One-way trapdoor permutation	Factoring problem RSA-problem		Textbook RSA
Public-key encryption	Encrypt variable-length input $\mathrm{Enc} : \mathcal{PK} \times \mathcal{M} \to \mathcal{C}$	Confidentiality: attacker should learn nothing about plaintext (except length) from ciphertexts	IND-CPA IND-CCA	ElGamal Hashed/Padded RSA Fujisaki-Okomoto-transform
Digital signatures	Create signature on variable length input $\mathrm{Sign} : \mathcal{SK} \times \mathcal{M} \to \mathcal{S}$ $\mathrm{Vrfy} : \mathcal{VK} \times \mathcal{M} \times \mathcal{S} \to \{1,0\}$	Integrity: attacker shouldn't be able to forge messages, i.e., create new messages with valid signatures	UF-CMA	Schnorr Hashed-RSA ECDSA

Cryptographic groups	Comment	Computational problem	Best-known attack	Common sizes
$(\boldsymbol{Z}_p^*,\cdot)$	p prime $\lvert\boldsymbol{Z}_p^*\rvert = p-1$	Discrete logarithm	General number field sieve (GNFS)	$\lvert p\rvert \approx 2000-3000$ bits
Subgroups $H < (\boldsymbol{Z}_p^*,\cdot)$	$\lvert H\rvert = q$ (typically prime)	Discrete logarithm	GNFS	$\lvert q\rvert \approx 256$ bits
$(E(\boldsymbol{F}_p),+)$	p prime $\lvert E(\boldsymbol{F}_p)\rvert = q$ (typically) prime $p \neq q$	Discrete logarithm	Generic attacks: Baby-step giant-step, Pollard-rho, Pohlig-Hellman	$\lvert E(\boldsymbol{F}_p)\rvert \approx 256$ bits $\lvert p\rvert \approx 256$ bits
$(\boldsymbol{Z}_n^*,\cdot)$	n not prime $\lvert\boldsymbol{Z}_n^*\rvert = \phi(n)$	Factoring	GNFS	$\lvert n\rvert \approx 2000-4000$ bits

Next week

- Quantum computers