
Lecture 9 – Diffie-Hellman key exchange II, elliptic curves, conspiracies

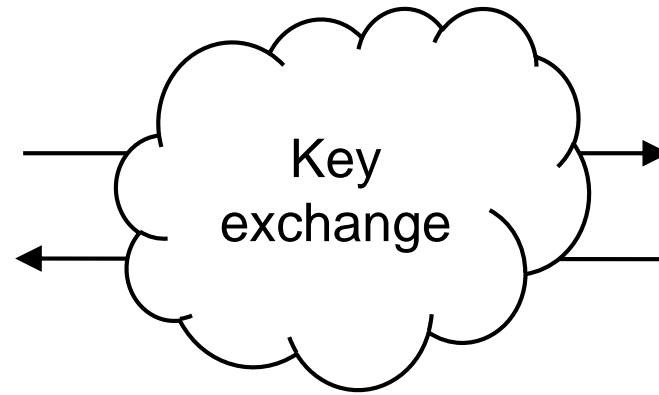
TEK4500

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Diffie-Hellman



Recap: groups

Definition: A **group** (G, \circ) is a set G together with a binary operation \circ satisfying the axioms

- $\mathcal{Q}1:$ $(a \circ b) \circ c = a \circ (b \circ c)$ (associativity)
- $\mathcal{Q}2:$ $\exists e \in G : e \circ a = a \circ e$ (identity)
- $\mathcal{Q}3:$ $\exists a^{-1} \in G : a \circ a^{-1} = a^{-1} \circ a = e$ (inverse)

Definition: A group is **cyclic** if there exists $g \in G$ such that $G = \langle g \rangle = \{\dots, g^{-2}, g^{-1}, g^0, g^1, g^2, g^3, \dots\}$

Examples:

$$(\mathbf{Z}, +) = \langle 1 \rangle = \langle -1 \rangle$$

$$(\mathbf{Z}_n, +_n) = \langle 1 \rangle$$

$$(\mathbf{Z}_p^*, \cdot) \quad (\mathbf{Z}_7^*, \cdot) = \langle 3 \rangle = \langle 5 \rangle = \{1, 3, 2, 6, 4, 5\}$$

Not cyclic groups:

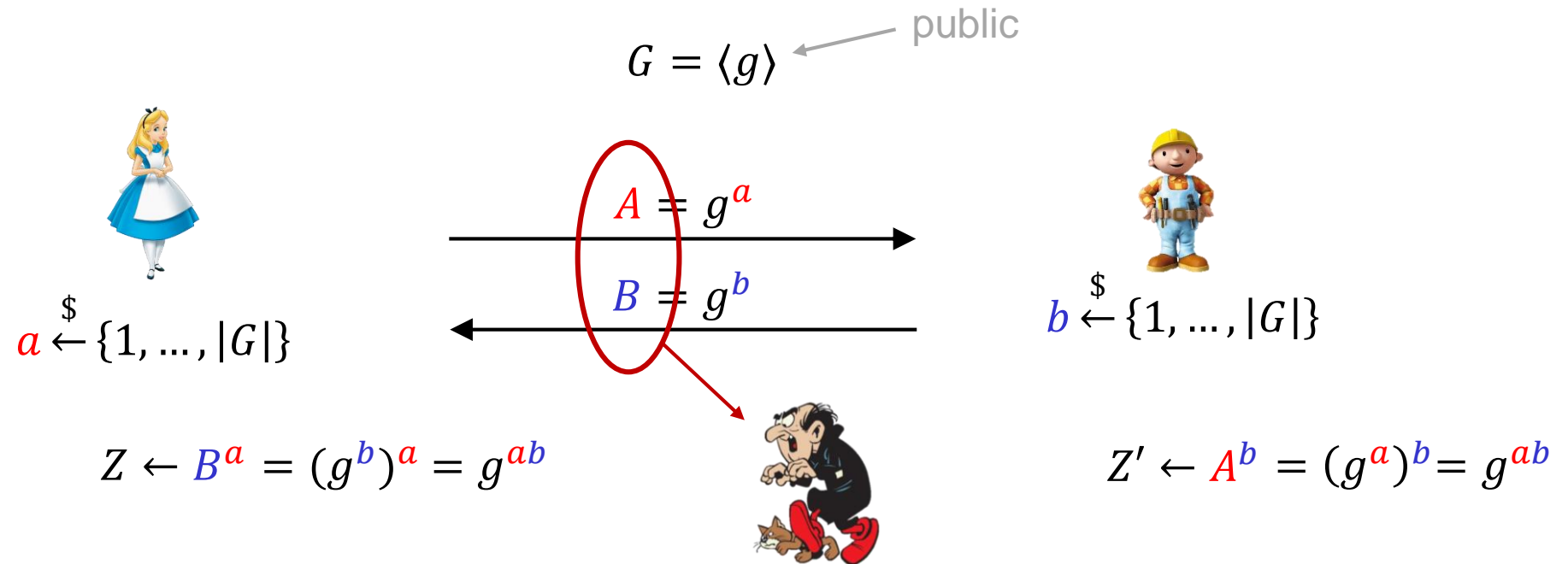
$$(\mathbf{R}, +)$$

$$(\mathbf{R}^*, \cdot)$$

(G, \star)

\star	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Diffie-Hellman



Security:

- Must be hard to compute $Z \leftarrow g^{ab}$ given g, A, B (DH assumption)
- Must be hard to find a (or b) given g, A, B (DLOG assumption)

Solving DLOG: the baby-step giant-step algorithm

Given: $X \leftarrow g^x$

$$n \leftarrow \lceil \sqrt{|G|} \rceil$$

$Y \leftarrow g^n$

Find: x

Look for i, j such that:

$$Xg^i = Y^j$$

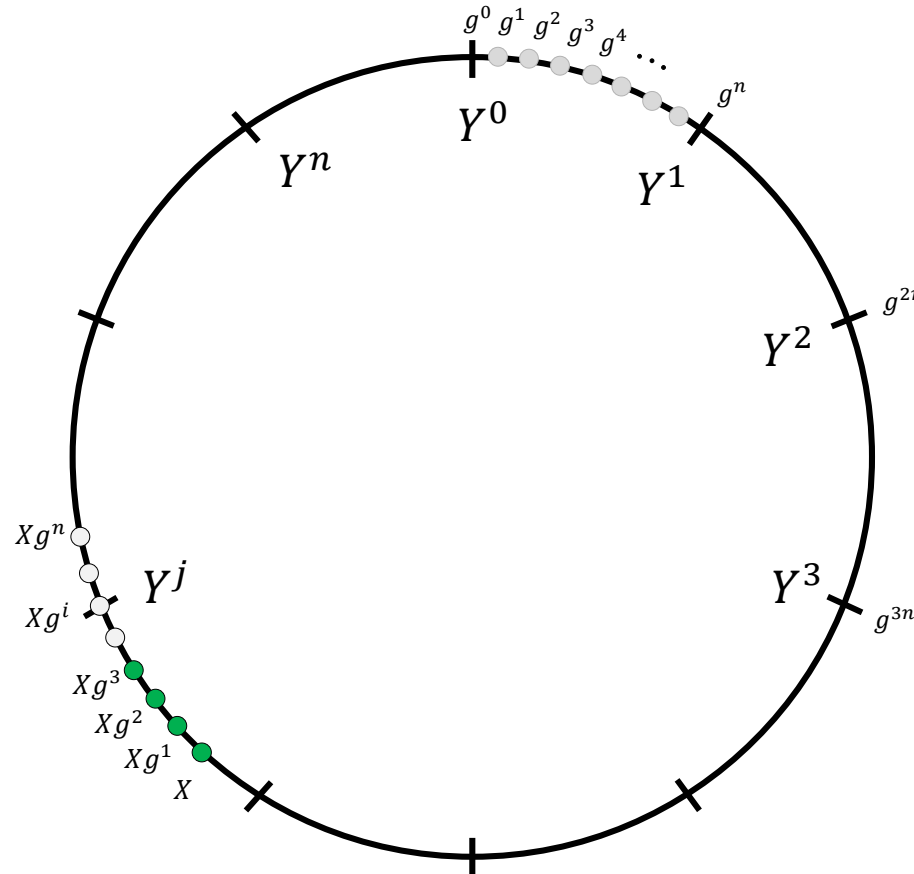
$$X = Y^j g^{-i}$$

$$X = g^{nj} g^{-i}$$

$$g^x = g^{nj} g^{-i}$$

$$g^x = g^{nj-i}$$

$x = nj - i$



$$\underbrace{O(\sqrt{|G|})}_{Y^0, Y^1, \dots, Y^n}$$

$$\underbrace{Xg^0, Xg^1, \dots, Xg^n}_{\tilde{O}(\sqrt{|G|})}$$

Time + memory: $\tilde{O}(\sqrt{|G|})$

Generic algorithms for solving DLOG

- Baby-step, giant-step: time $\mathcal{O}(\sqrt{|G|})$ memory $\mathcal{O}(\sqrt{|G|})$
- Pollard's rho: time $\mathcal{O}(\sqrt{|G|})$ memory $\mathcal{O}(1)$
- Pohlig-Hellman: time $\max_p \mathcal{O}(\sqrt{p})$ memory $\mathcal{O}(1)$ $(|G| = p_1^{e_1} p_2^{e_2} \cdots p_t^{e_t})$

- **Consequence:** for DLOG to be hard $\sqrt{|G|}$ must be large enough!
 - $|G| \approx 2^{128}$ only gives $\sqrt{2^{128}} = 2^{64}$ security
 - $|G| \approx 2^{256}$ only gives $\sqrt{2^{256}} = 2^{128}$ security
 - $|G| \approx 2^{512}$ only gives $\sqrt{2^{512}} = 2^{256}$ security
 - etc...

Non-generic algorithms for DLOG

- Unfortunately, (\mathbf{Z}_p^*, \cdot) is *not* a generic group!
- *Much* faster specific algorithms exist for solving DLOG in \mathbf{Z}_p^*
 - Index-calculus
 - Elliptic-curve method
 - Special number-field sieve (SNFS)
 - General number-field sieve (GNFS)

} *exceptionally* complicated algorithms, requiring very advanced mathematics!
- Current DLOG-solving record: $|\mathbf{Z}_p^*| \approx 2^{795}$ using GNFS (Heninger et al. '19)
 - Previous records: https://en.wikipedia.org/wiki/Discrete_logarithm_records
- $|\mathbf{Z}_p^*| \geq 2^{2048}$ typically required as a minimum today

Diffie-Hellman in (\mathbb{Z}_p^*, \cdot) – what is large enough?

$p =$ 171254583176141379301960419792575778264088323240375085733932929816426671397476217788024387752387285929683446135893799323484756135034769321631669738132186983438164632891441853629126025225404949830905314972329658295365245072698488256583114202993359222957097432675083225259667739503949192575768420387716327420441424710535098501236058838158571626669177751934961573726561955583052720098912760065140004093658772181713883199238963093777917625906143118496429613802248519404604217104493688927252974870395873936387909672274883295377481008150475878590270591798350563488168080923804611822387520198054002990623911454389104774092183 $\approx 2^{2048}$



$A = 2$

$\text{mod } p$



323170060713110073003389139264238282488179412411402391
128420097514007417066343542226196894173635693471179017
379097041917546058732091950288537589861856221532121754
125149017745202702357960782362488842461894775876411059
286460994117232454266225221932305409190376805242355191
256797158701170010580558776510388618472802579760549035
697325615261670813393617995413364765591603683178967290
731783845896806396719009772021941686472258710314113364
293195361934716365332097170774482279885885653692086452
96636077250268955059283627511211740969729980684105543
595848665832916421362182310789909994486524682624169720
35911852507045361090559

$\$ \leftarrow \{1 \dots p\}$

665680077810100734070476416898417408020670352441378967
997224101900193957211854827569676933273935982997835638
067380762809024311842715496660113976613131478051784487
322446809608196031803691263486170912693625523249742383
264476433978409434004770395167011614217279552198029936
108023398643999746539201834933168220864789880837484536
563067997761270826642223539437541791058416731683176995
365899867720283590420932417377927106884773199080517477
615608675490982255098980545292032908358093913531433702
049429047413071525556307299541445514389721222256380833
743627991333762777307118317040885305789910094293548481
5501309561942770687524

$B = 2$

$\text{mod } p$

665680077810100734070476416898417408020670352441378967
997224101900193957211854827569676933273935982997835638
067380762809024311842715496660113976613131478051784487
322446809608196031803691263486170912693625523249742383
264476433978409434004770395167011614217279552198029936
108023398643999746539201834933168220864789880837484536
563067997761270826642223539437541791058416731683176995
365899867720283590420932417377927106884773199080517477
615608675490982255098980545292032908358093913531433702
049429047413071525556307299541445514389721222256380833
743627991333762777307118317040885305789910094293548481
5501309561942770687524

$\$ \leftarrow \{1 \dots p\}$

323170060713110073003389139264238282488179412411402391
128420097514007417066343542226196894173635693471179017
379097041917546058732091950288537589861856221532121754
125149017745202702357960782362488842461894775876411059
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665680077810100734070476416898417408020670352441378967
997224101900193957211854827569676933273935982997835638
067380762809024311842715496660113976613131478051784487
322446809608196031803691263486170912693625523249742383
264476433978409434004770395167011614217279552198029936
108023398643999746539201834933168220864789880837484536
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365899867720283590420932417377927106884773199080517477
615608675490982255098980545292032908358093913531433702
049429047413071525556307299541445514389721222256380833
743627991333762777307118317040885305789910094293548481
5501309561942770687524

$Z \leftarrow 2$

$\text{mod } p$



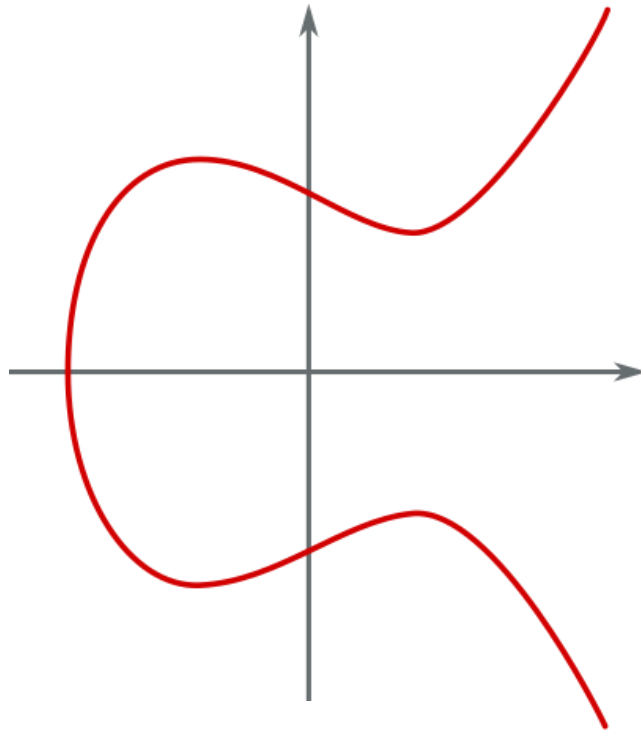
Z_p^*



Group
where GNFS
doesn't work

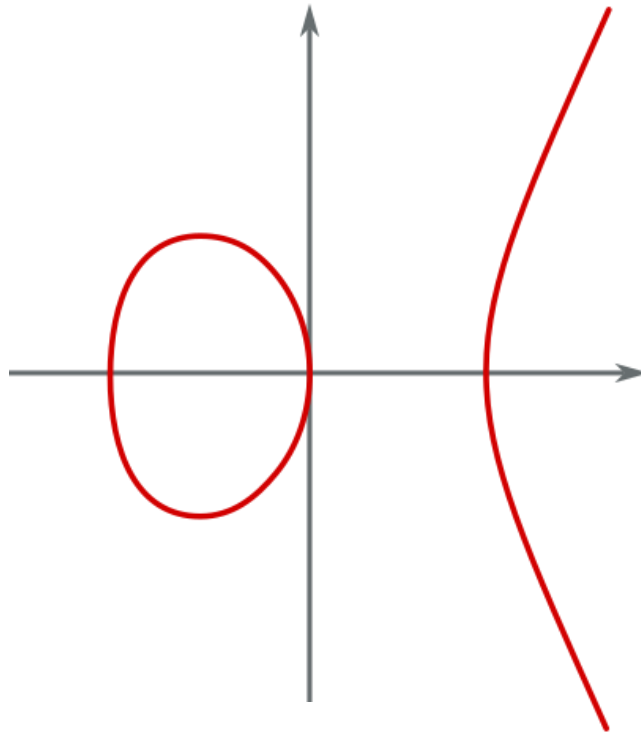
Better alternatives to Z_p^* ?

Elliptic curves



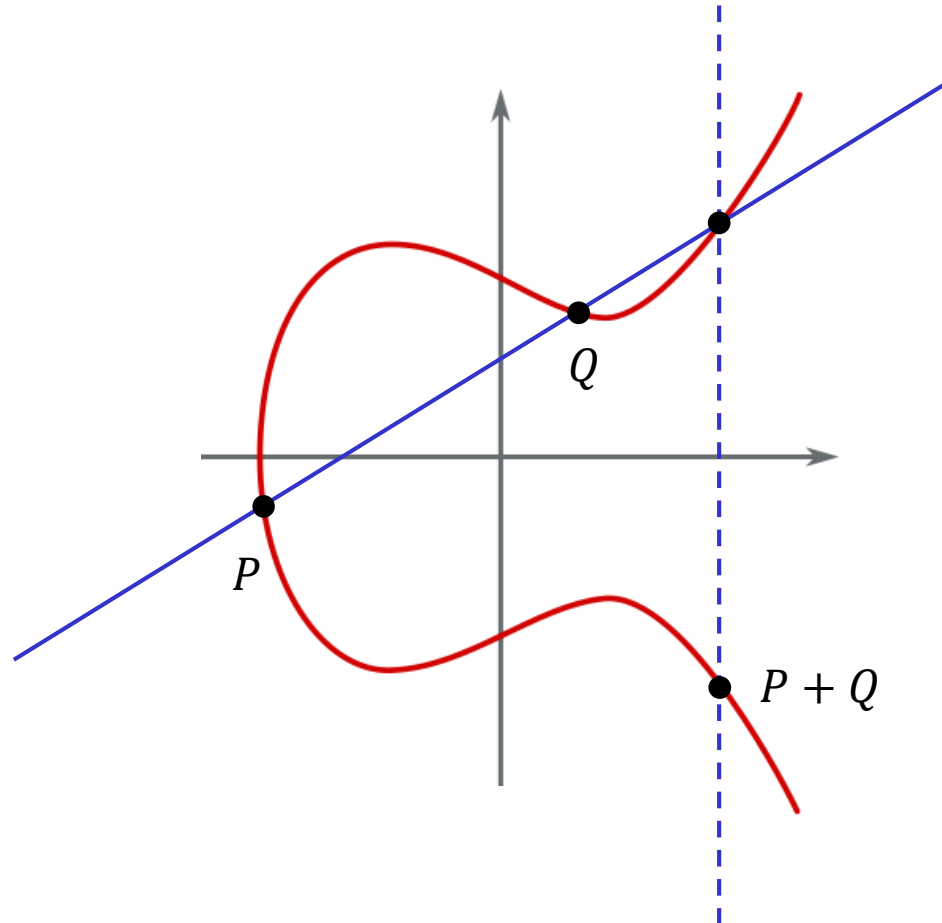
$$y^2 = x^3 - 2x + 3 \quad x, y \in \mathbf{R}$$

Elliptic curves



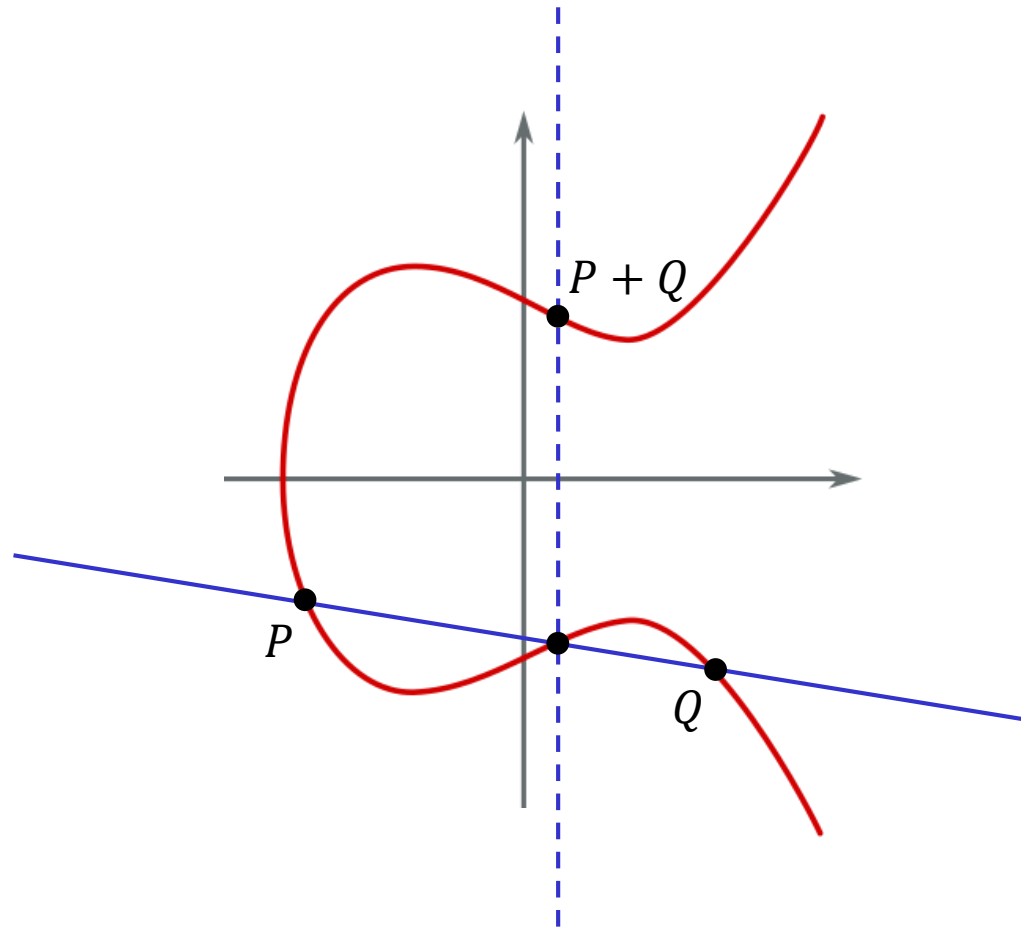
$$y^2 = x^3 - 9x - 1 \quad x, y \in \mathbf{R}$$

Elliptic curves



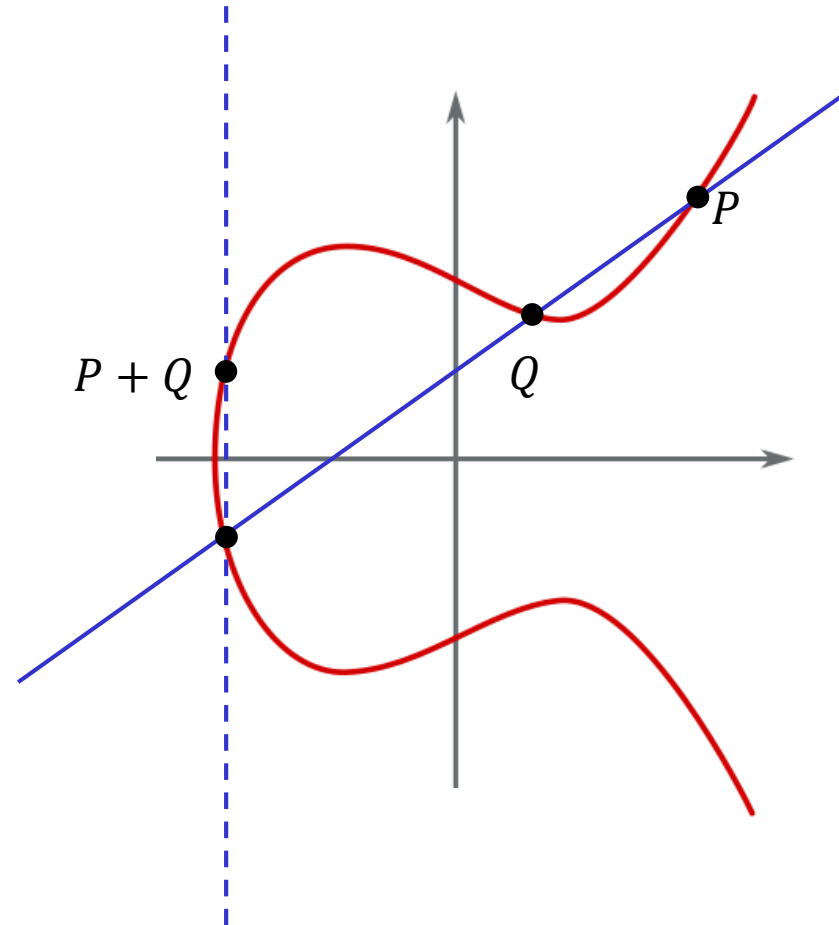
$$y^2 = x^3 - 2x + 3 \quad x, y \in \mathbf{R}$$

Elliptic curves



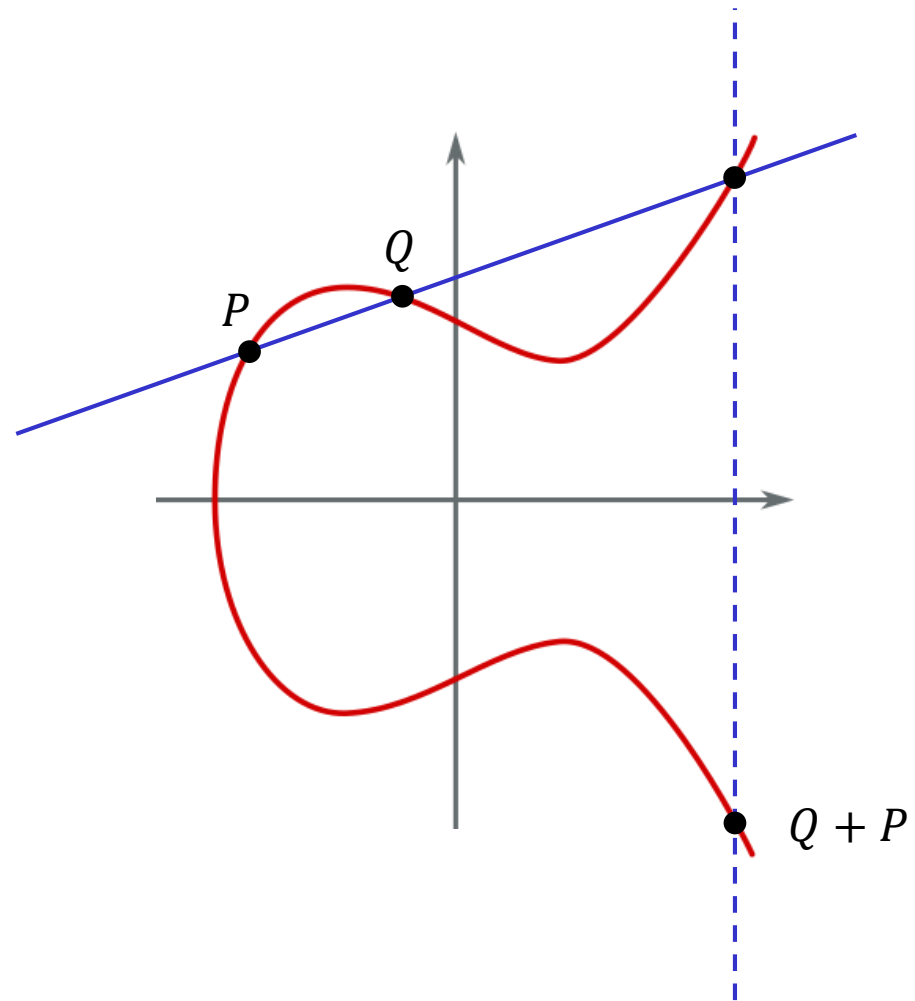
$$y^2 = x^3 - 2x + 3 \quad x, y \in \mathbf{R}$$

Elliptic curves



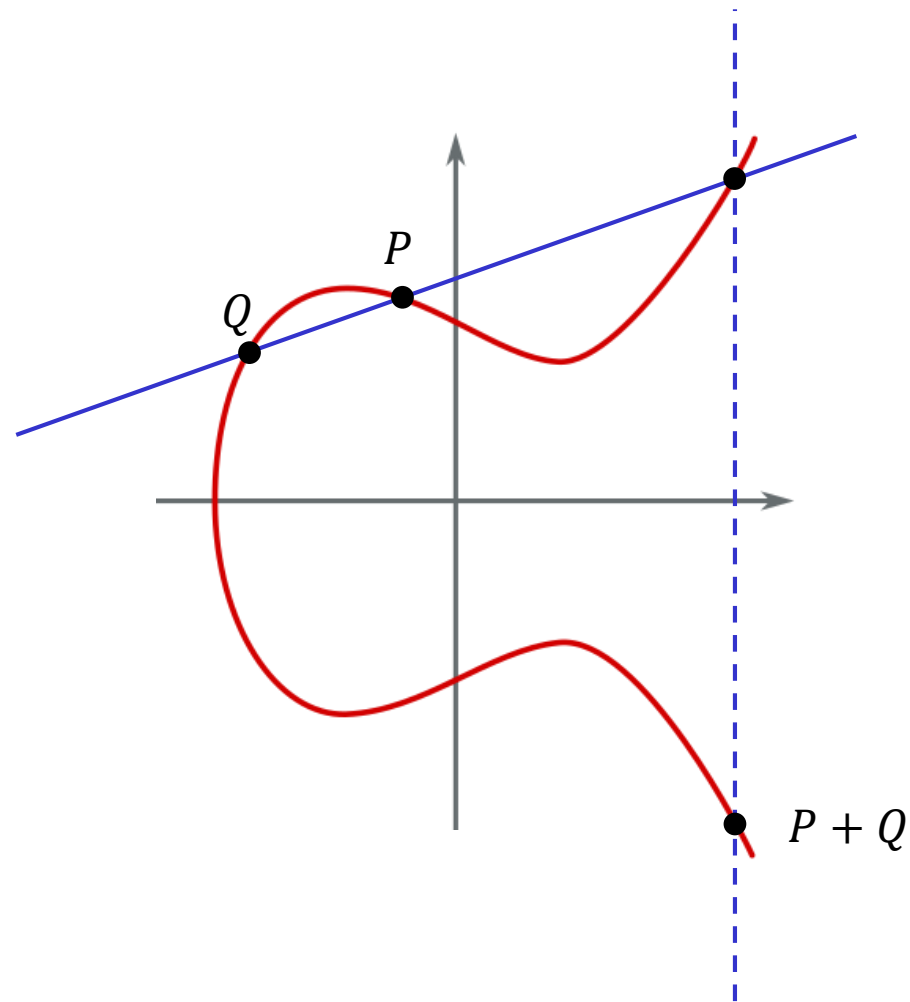
$$y^2 = x^3 - 2x + 3 \quad x, y \in \mathbf{R}$$

Elliptic curves



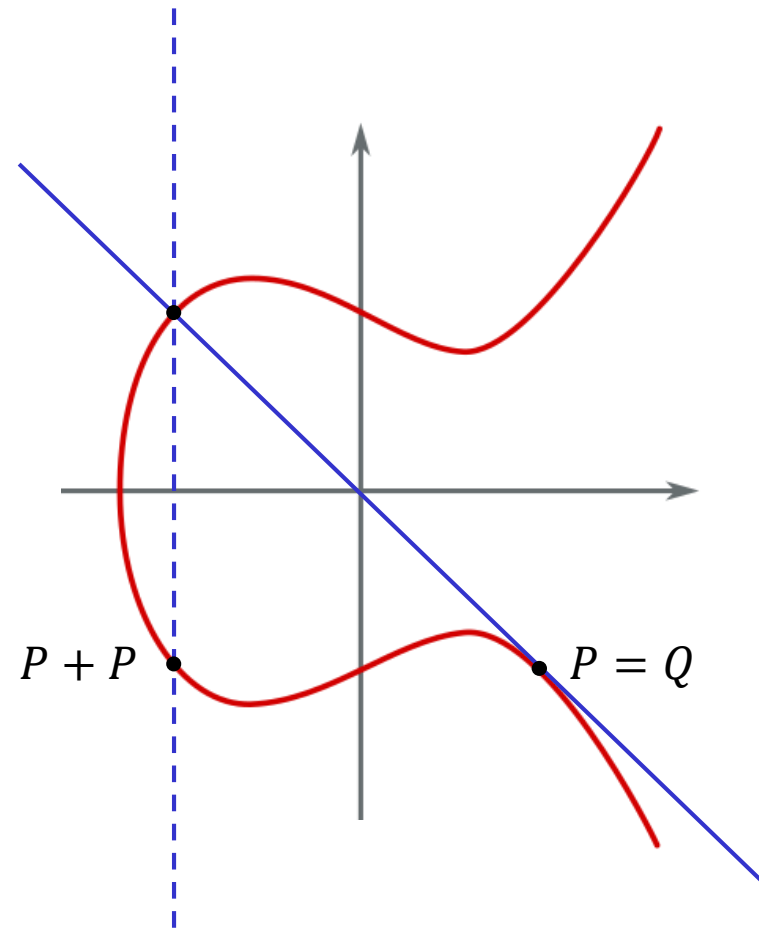
$$y^2 = x^3 - 2x + 3 \quad x, y \in \mathbf{R}$$

Elliptic curves



$$y^2 = x^3 - 2x + 3 \quad x, y \in \mathbf{R}$$

Elliptic curves

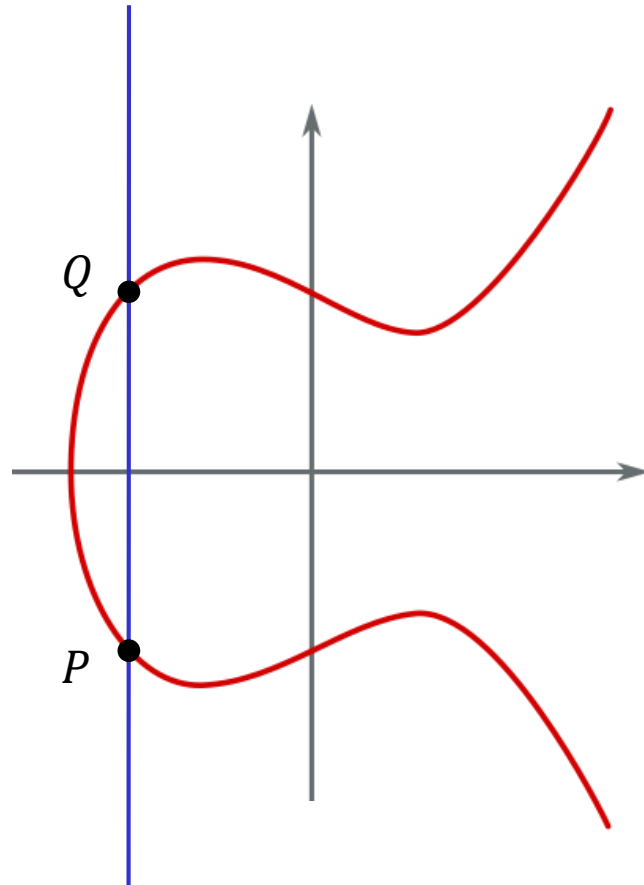


$$y^2 = x^3 - 2x + 3 \quad x, y \in \mathbf{R}$$

Elliptic curves

$$P + Q \stackrel{\text{def}}{=} \mathcal{O}$$

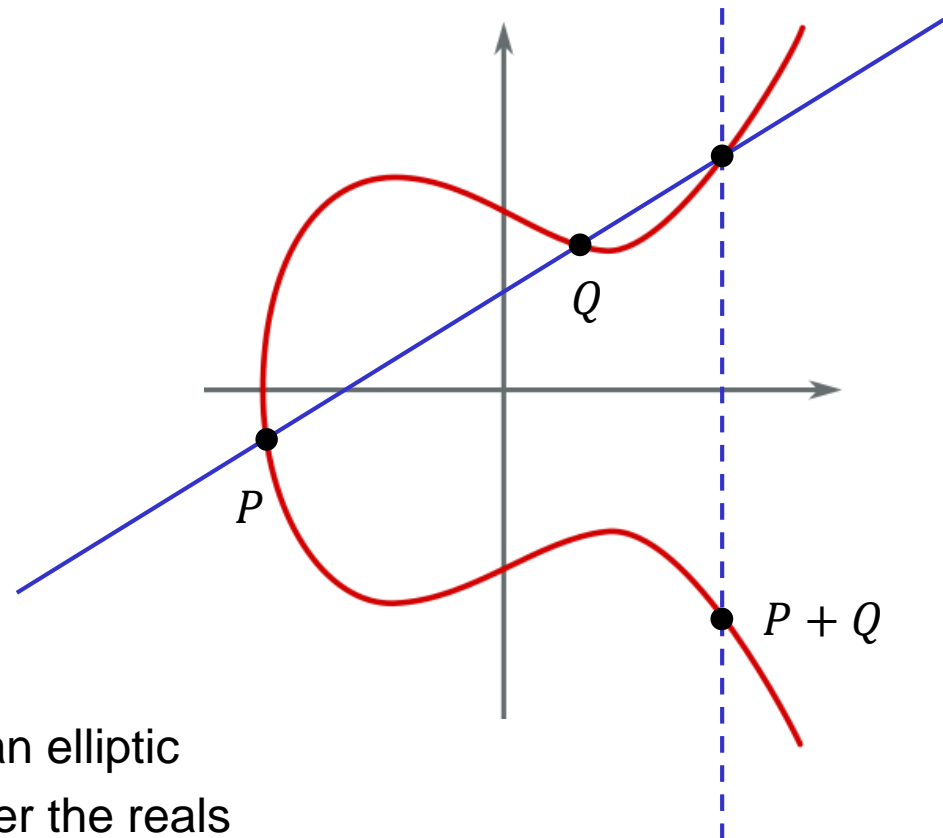
Identity element



$$y^2 = x^3 - 2x + 3 \quad x, y \in \mathbf{R}$$

Elliptic curves

Theorem: the points on an elliptic curve, together with \mathcal{O} , is an abelian group under "geometric point addition"



Notation: $(E(\mathbf{R}), +)$ – an elliptic curve group defined over the reals

$$y^2 = x^3 + ax + b$$

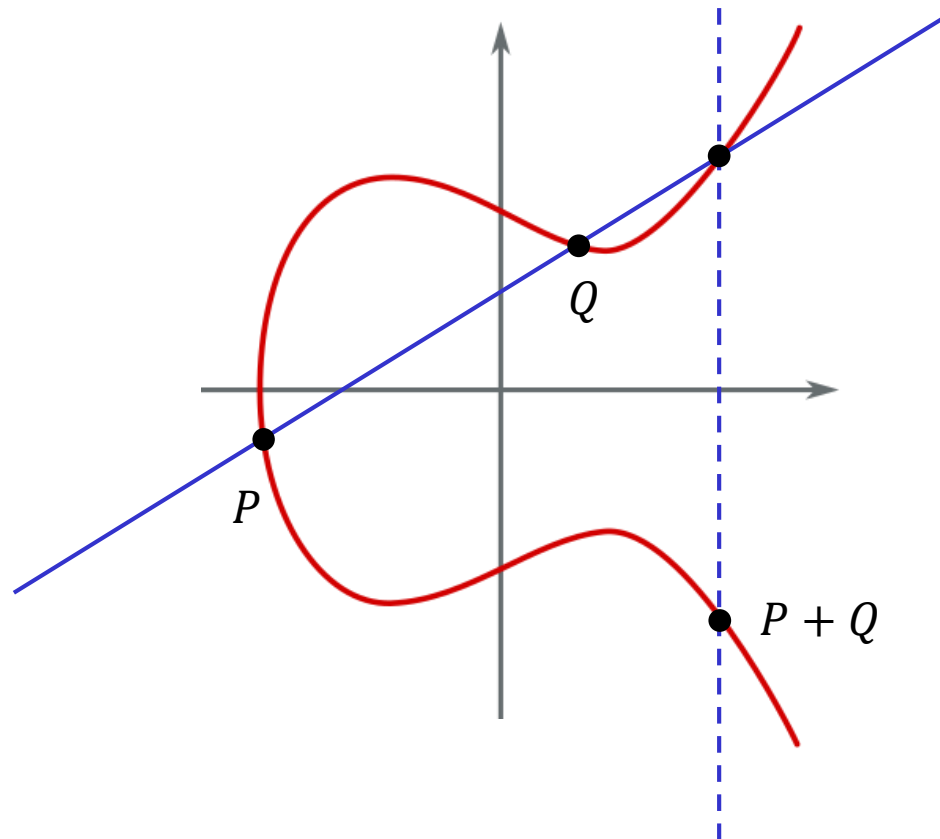
$$a, b, x, y \in \mathbf{R}$$

$$P + Q = (x_1, y_1) + (x_2, y_2) = (x_3, y_3)$$

$$x_3 = \frac{(x_1x_2 - 2a)x_1x_2 - 4b(x_1 + x_2) + a^2}{(x_1x_2 + a)(x_1 + x_2) + 2y_1y_2 + 2b}$$

$$y_3 = \frac{x_1x_2(x_1 + x_2) - x_3((x_1 + x_2)^2 - x_1x_2 + a) - y_1y_2 - b}{y_1 + y_2}$$

Elliptic curves



$$y^2 = x^3 + ax + b$$

$$a, b, x, y \in \mathbf{Z}_p$$

$$P + Q = (x_1, y_1) + (x_2, y_2) = (x_3, y_3)$$

$$x_3 = \frac{(x_1x_2 - 2a)x_1x_2 - 4b(x_1 + x_2) + a^2}{(x_1x_2 + a)(x_1 + x_2) + 2y_1y_2 + 2b}$$

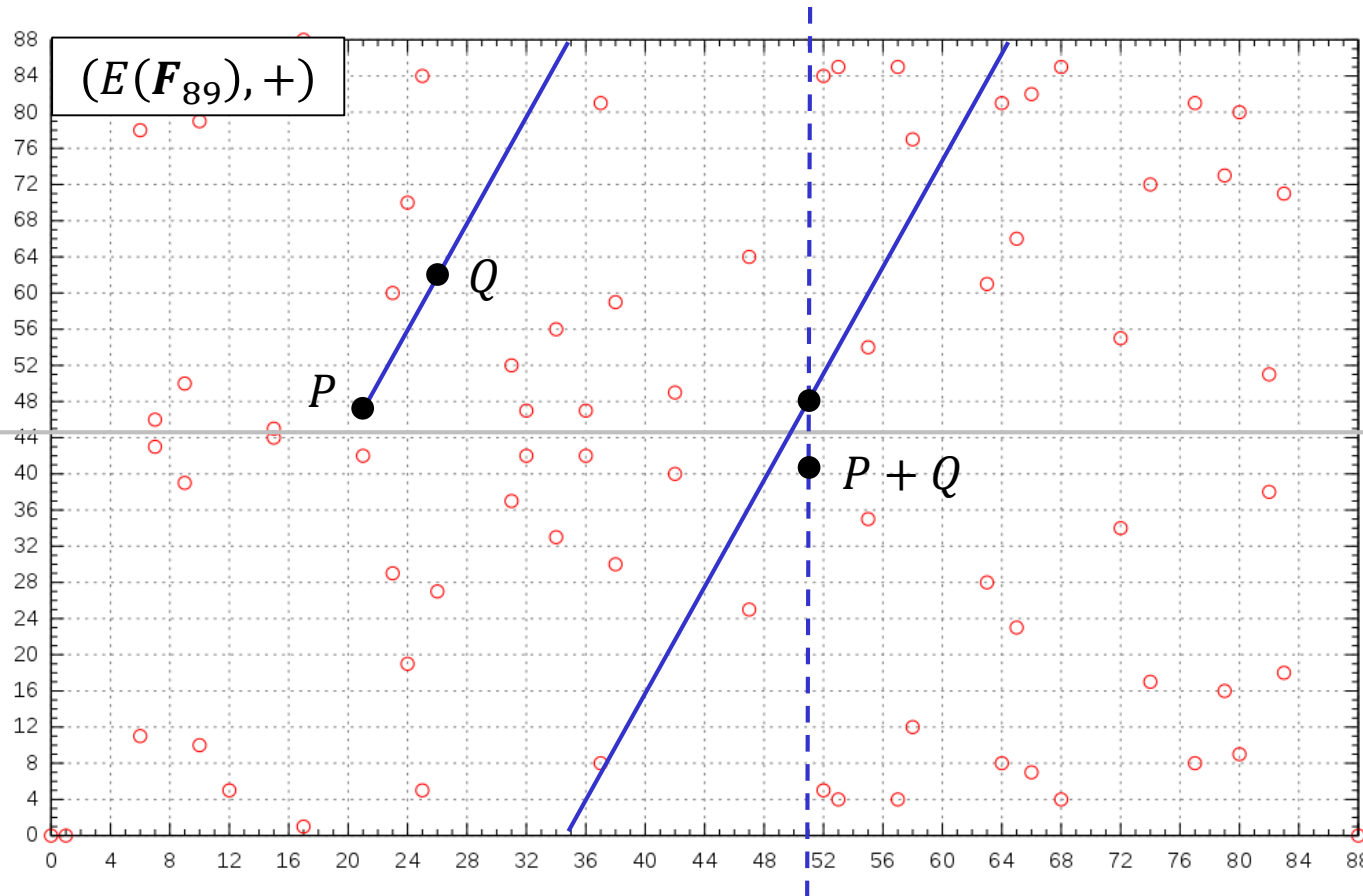
$$y_3 = \frac{x_1x_2(x_1 + x_2) - x_3((x_1 + x_2)^2 - x_1x_2 + a) - y_1y_2 - b}{y_1 + y_2}$$

Still valid!

Elliptic curves over \mathbb{Z}_p

Theorem: the points on an elliptic curve, together with \mathcal{O} , is an abelian group under "geometric point addition"

Notation: $(E(\mathbb{F}_p), +)$ = an elliptic curve group defined over \mathbb{F}_p



$$y^2 = x^3 + ax + b$$

$$a, b, x, y \in \mathbb{Z}_{89}$$

$$P + Q = (x_1, y_1) + (x_2, y_2) = (x_3, y_3)$$

$$x_3 = \frac{(x_1x_2 - 2a)x_1x_2 - 4b(x_1 + x_2) + a^2}{(x_1x_2 + a)(x_1 + x_2) + 2y_1y_2 + 2b}$$

$$y_3 = \frac{x_1x_2(x_1 + x_2) - x_3((x_1 + x_2)^2 - x_1x_2 + a) - y_1y_2 - b}{y_1 + y_2}$$

$(\mathbb{Z}_p, +)$ and (\mathbb{Z}_p^*, \cdot)

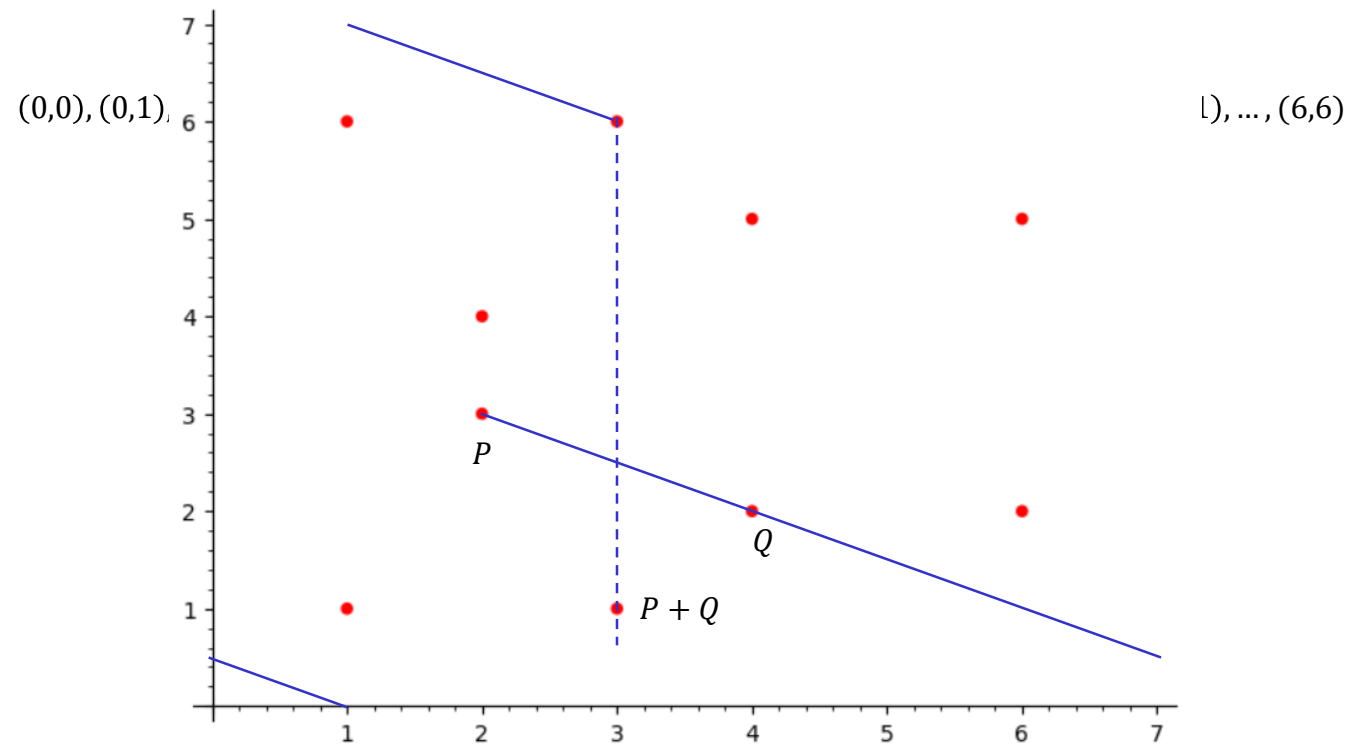
Finite field $(\mathbb{F}_p, +, \cdot)$

Still valid!

Elliptic curves – example

$$E : y^2 = x^3 + x - 1$$

$$x, y \in \mathbf{Z}_7$$



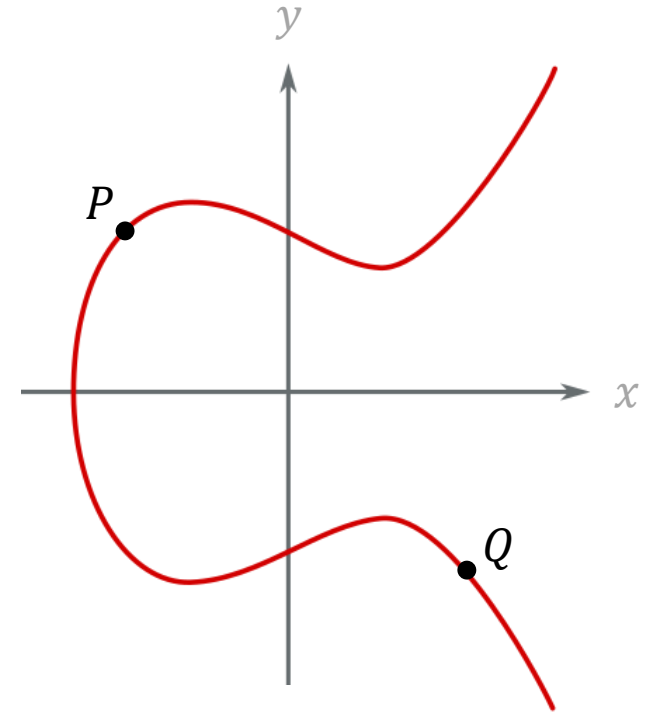
$$(E(\mathbf{Z}_7), +) = \{\mathcal{O}, (1,1), (1,6), (2,3), (2,4), (3,1), (3,6), (4,2), (4,5), (6,2), (6,5)\}$$

Elliptic curves summary

- $P = (x, y)$ point on elliptic curve
- x and y are elements in \mathbf{Z}_p
- Points P and Q can be added to get another point $P + Q$
- Special case: add P to itself n times

$$nP = P + P + \dots + P$$

- DLOG problem: given P and nP for **secret** n , find n



$E(\mathbf{Z}_p)$ – properties

- (\mathbf{Z}_p^*, \cdot) is *not* a generic group
 - Specialized attacks (GNFS) exploit algebraic structure \Rightarrow parameters must be bigger to compensate
 - $|\mathbf{Z}_p^*| \geq 2^{2048}$ required for security today
 - Bigger parameters \Rightarrow slower systems
- Currently no attacks manage to exploit the algebraic structure of $(E(\mathbf{Z}_p), +)$
 - Best-know attacks are *generic attacks*:
 - **Nechaev '94 & Shoup '97**: *Generic* algorithms for solving DLOG require time $\Omega(\sqrt{|G|})$
 - Consequently: elliptic curve crypto can use *much* smaller parameters
 - $|E(\mathbf{Z}_p)| = 2^{256}, 2^{384}, 2^{512}$ common in practice \Rightarrow much faster than \mathbf{Z}_p^* -based crypto

Diffie-Hellman in (\mathbb{Z}_p^*, \cdot)

$p =$ 171254583176141379301960419792575778264088323240375085733932929816426671397476217788024387752387285929683446135893799323484756135034769321631669738132186983438164632891441853629126025225404949830905314972329658295365245072698488256583114202993359222957097432675083225259667739503949192575768420387716327420441424710535098501236058838158571626669177751934961573726561955583052720098912760065140004093658772181713883199238963093777917625906143118496429613802248519404604217104493688927252974870395873936387909672274883295377481008150475878590270591798350563488168080923804611822387520198054002990623911454389104774092183 $\approx 2^{2048}$



$A = 2$

$\text{mod } p$



323170060713110073003389139264238282488179412411402391
128420097514007417066343542226196894173635693471179017
379097041917546058732091950288537589861856221532121754
125149017745202702357960782362488842461894775876411059
286460994117232454266225221932305409190376805242355191
256797158701170010580558776510388618472802579760549035
697325615261670813393617995413364765591603683178967290
731783845896806396719009772021941686472258710314113364
293195361934716365332097170774482279885885653692086452
966360772502689555059283627511211740969729980684105543
595848665832916421362182310789909994486524682624169720
35911852507045361090559

$\$ \leftarrow \{1 \dots p\}$

665680077810100734070476416898417408020670352441378967
997224101900193957211854827569676933273935982997835638
067380762809024311842715496660113976613131478051784487
322446809608196031803691263486170912693625523249742383
264476433978409434004770395167011614217279552198029936
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365899867720283590420932417377927106884773199080517477
615608675490982255098980545292032908358093913531433702
049429047413071525556307299541445514389721222256380833
743627991333762777307118317040885305789910094293548481
5501309561942770687524

$B = 2$

$\text{mod } p$

665680077810100734070476416898417408020670352441378967
997224101900193957211854827569676933273935982997835638
067380762809024311842715496660113976613131478051784487
322446809608196031803691263486170912693625523249742383
264476433978409434004770395167011614217279552198029936
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563067997761270826642223539437541791058416731683176995
365899867720283590420932417377927106884773199080517477
615608675490982255098980545292032908358093913531433702
049429047413071525556307299541445514389721222256380833
743627991333762777307118317040885305789910094293548481
5501309561942770687524

$\$ \leftarrow \{1 \dots p\}$

323170060713110073003389139264238282488179412411402391
128420097514007417066343542226196894173635693471179017
379097041917546058732091950288537589861856221532121754
125149017745202702357960782362488842461894775876411059
286460994117232454266225221932305409190376805242355191
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665680077810100734070476416898417408020670352441378967
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365899867720283590420932417377927106884773199080517477
615608675490982255098980545292032908358093913531433702
049429047413071525556307299541445514389721222256380833
743627991333762777307118317040885305789910094293548481
5501309561942770687524

$Z \leftarrow 2$

$\text{mod } p$

Diffie-Hellman in $(E(\mathbb{Z}_p), +)$

$$p = 115792089210356248762697446949407573530086143415290314195533631308867097853951 \approx 2^{256}$$



$$A = \begin{matrix} 890802873957740768864044652762626807642 \\ 97347818898641719959470631357066012729 \end{matrix} \cdot Q$$



890802873957740768864044652762626807642
97347818898641719959470631357066012729

$\$ \leftarrow \{1 \dots p\}$

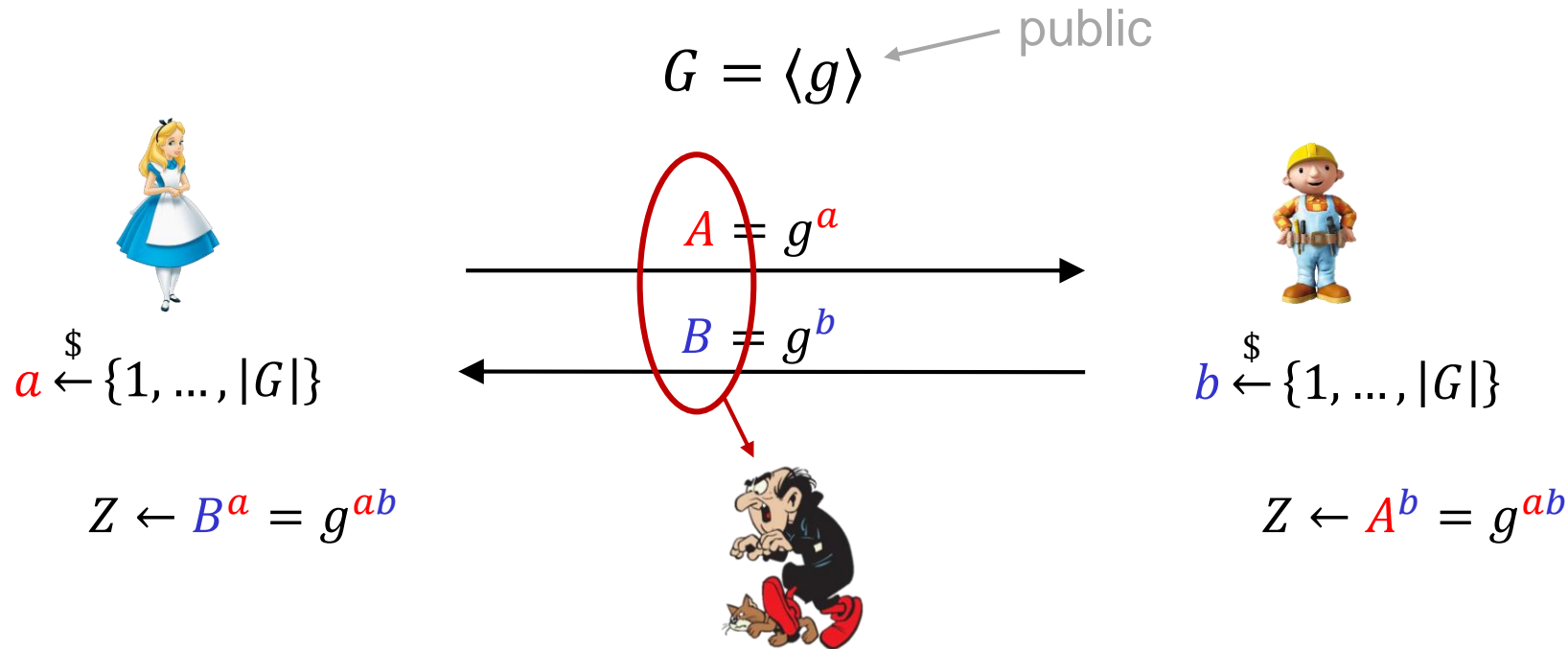
47777099444069171196101629885784192859
74852857762532427086507801748071058272

$\$ \leftarrow \{1 \dots p\}$

$$B = \begin{matrix} 47777099444069171196101629885784192859 \\ 74852857762532427086507801748071058272 \end{matrix} \cdot Q$$

$$Z \leftarrow \begin{matrix} 890802873957740768864044652762626807642 \\ 97347818898641719959470631357066012729 \end{matrix} \times \begin{matrix} 47777099444069171196101629885784192859 \\ 74852857762532427086507801748071058272 \end{matrix} \cdot Q$$

Diffie-Hellman – security



Security (given G, g, A, B):

- Must be hard to compute $Z \leftarrow g^{ab}$
- Must be hard to find a (or b)
- $G = (\mathbf{Z}_p, +)$
- $G = (\mathbf{Z}_p^*, \cdot)$
- $G = (E(\mathbf{F}_p), +)$

(DH assumption)

(DLOG assumption)

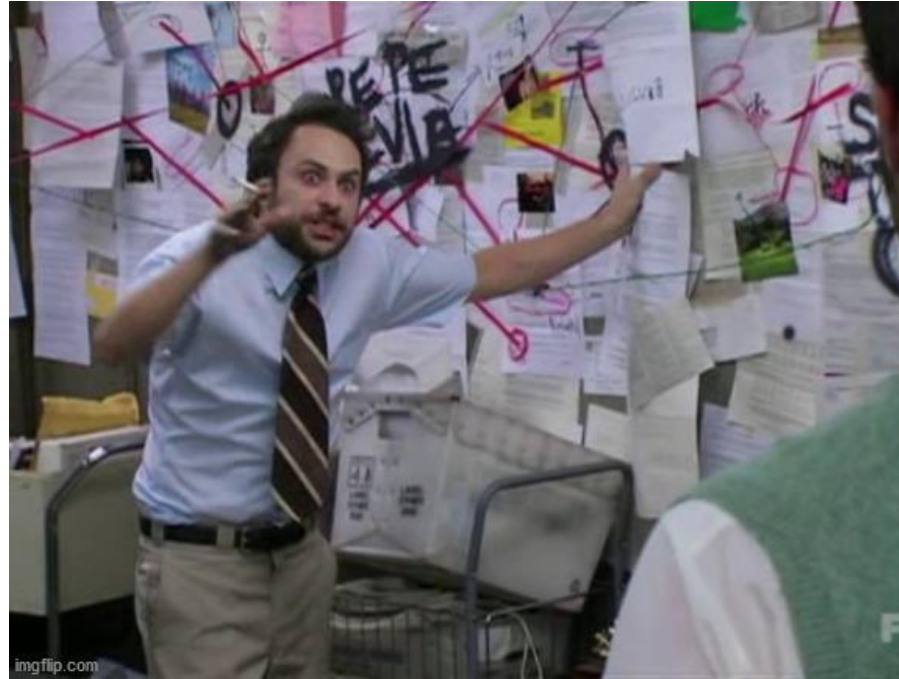
not secure

secure for $|\mathbf{Z}_p^*|$ large enough (conjectured)

secure for $|E(\mathbf{F}_p)|$ large enough (conjectured)

Cryptographic groups in practice

- (\mathbf{Z}_p^*, \cdot) groups:
 - **TLS 1.3**: five specific groups allowed
 - size $\approx 2^{2048}, 2^{3072}, 2^{4096}, 2^{6144}, 2^{8192}$ (RFC 7919)
 - **IKEv2** (IPsec key exchange protocol): MODP groups
 - size $\approx 2^{768}, 2^{1024}, 2^{1536}, 2^{2048}, 2^{3072}, 2^{4096}, 2^{6144}, 2^{8192}$ (RFC 7296 and RFC 3526)
 - all p 's are **safe primes** (i.e., of the form $p = 2q + 1$ where q is prime)
- $(E(\mathbf{Z}_p^*), +)$ groups
 - NIST curves: P-224, P-256, P-384, P-521
 - Curve25519 ($E : y^2 = x^3 + 486662x^2 + x$ and $p = 2^{255} - 19$) (Daniel J. Bernstein)
 - Curve448 ($E : y^2 + x^2 = 1 - 39081x^2y^2$ and $p = 2^{448} - 2^{224} - 1$) (Mike Hamburg)



After the break...

Juniper Networks

- Juniper Networks: big manufacturer of network equipment (routers, VPNs, firewalls, etc.)
 - Major customers: telcos, banks, US DoD

- **2015:**

IMPORTANT JUNIPER SECURITY ANNOUNCEMENT

During a recent internal code review, Juniper discovered **unauthorized code in ScreenOS** that could allow a knowledgeable attacker to gain administrative access to NetScreen® devices and to decrypt VPN connections.

- Hackers obtained access to source code repository
- Only made one change:

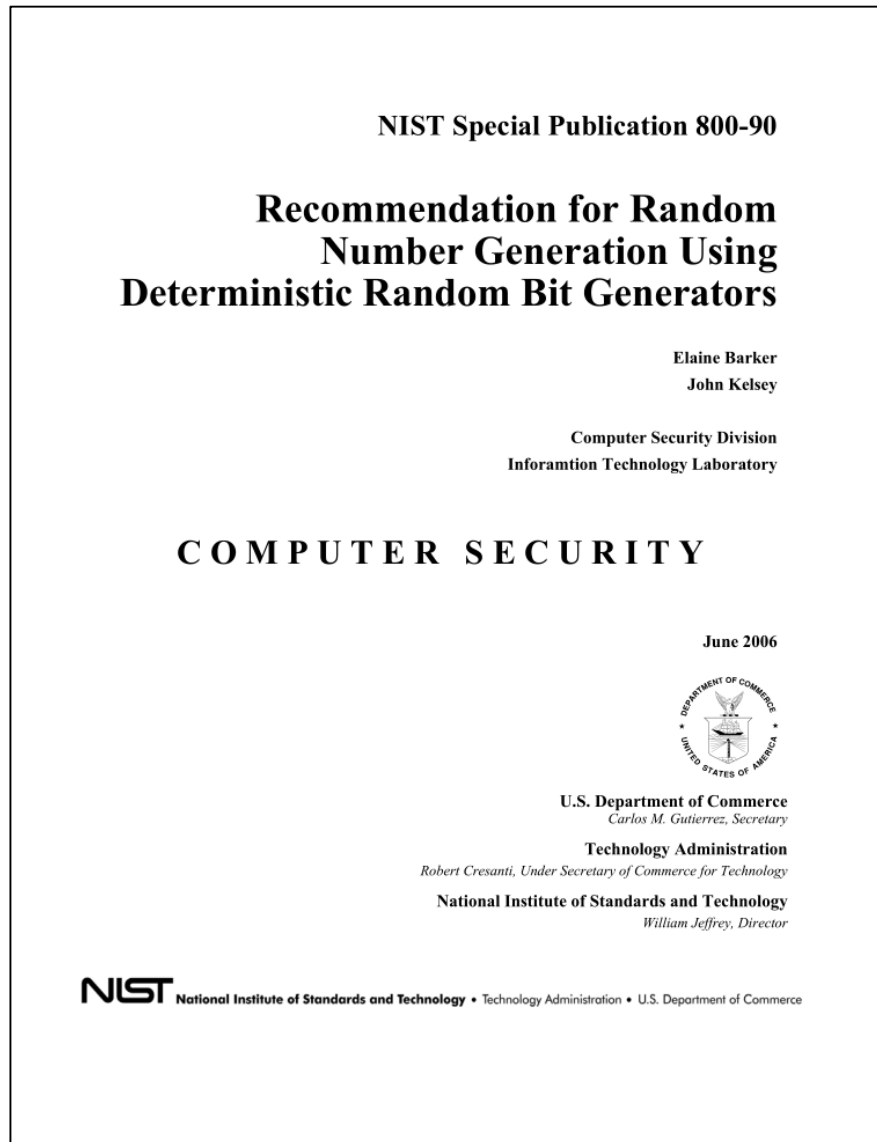
--- Qx = 2c55e5e45edf713dc43475effe8813a60326a64d9ba3d2e39cb639b0f3b0ad10

+++ Qx = 9585320eeaf81044f20d55030a035b11bece81c785e6c933e4a8a131f6578107

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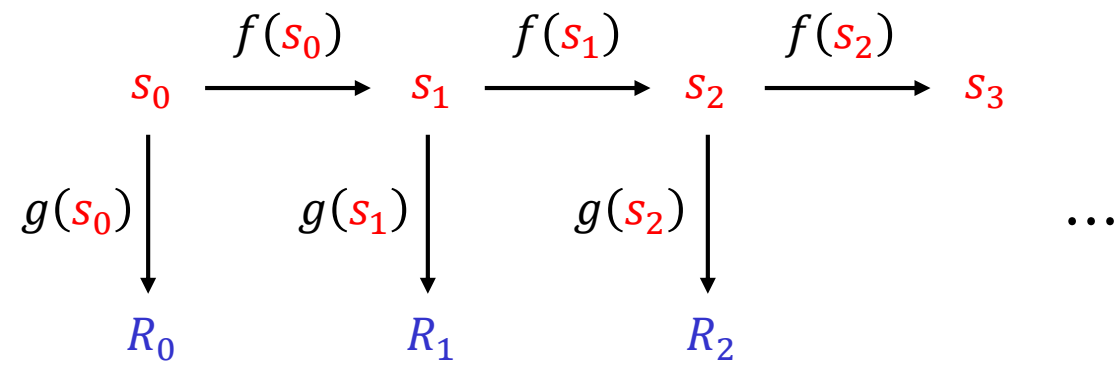


PRNG standardization



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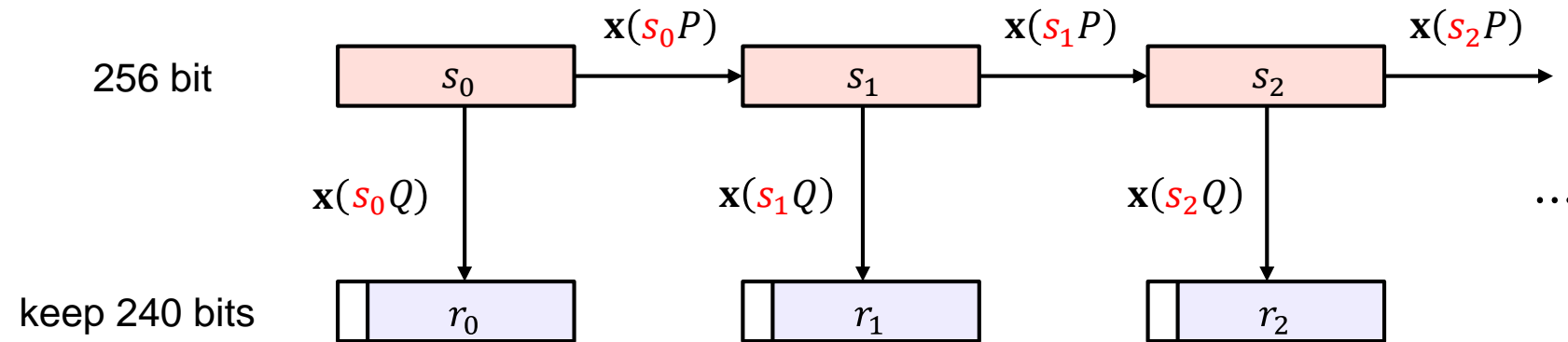
PRNG



Dual EC DRBG

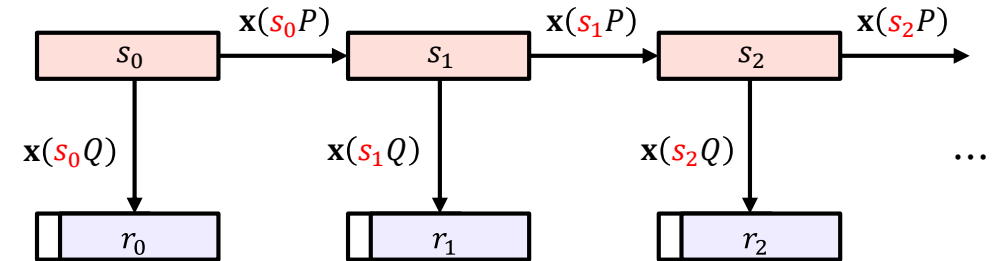
P, Q : **public** points on an elliptic curve

$\mathbf{x}(\cdot)$: x-coordinate (256 bits)



Dual EC DBRG – something's fishy

- Dual EC is *slow*
 - Orders of magnitude slower than HMAC/AES-CTR based alternatives
- **2006** – Kristian Gjøsteen: Dual EC is *not* a good PRNG
 - Can distinguish output from random with $\text{Adv}_{\text{DualEC}}^{\text{prg}}(KG) \approx 0.0011$
 - Slightly improved by Schoenmakers and Sidorenko
- **2007** – Shumow and Ferguson: Dual EC can be *backdoored*
 - What if $P = dQ$ for a *secret* d only you know?
 - If you know full s_1Q , compute $d(s_1Q) = s_1(dQ) = s_1P = s_2$
 - Because of truncation need to guess top 16 bits ($\approx 2^{16}$ additional work)
- **2007** – NIST adds appendix to standard on how to create P and Q yourself
 - Continues to recommend existing P and Q
- Most cryptographers: who cares? No one is going to use Dual EC anyway ...
- **2013** – Edward Snowden leak: a project called Bullrun exists within the NSA
 - Purpose: "Insert vulnerabilities into commercial encryption systems, IT systems, networks, and endpoint communications devices used by targets."
 - Turns out Juniper Networks made Dual EC their PRNG in ScreenOS from 2008



Where does Q (and P) come from?

From: John Kelsey [mailto:john.kelsey@nist.gov]
Sent: Wednesday, October 27, 2004 11:17 AM
To: Don Johnson
Subject: Minding our Ps and Qs in Dual_EC

Do you know where Q comes from in Dual_EC_DRBG?

Thanks,
-Joh

Subject: RE: Minding our Ps and Qs in Dual_EC
From: "Don Johnson" <DJohnson@cygnacom.com>
Date: Wed, October 27, 2004 11:42 am
To: "John Kelsey" <john.kelsey@nist.gov>
John,

$P=G$.

Q is (in essence) the public key for some random private key.

It could also be generated like a (nother) canonical G , but NSA kyboshed this idea, and I was not allowed to publicly discuss it, just in case you may think of going there.

Don B. Johnson

NIST SP 800-90

June 2006

Appendix A: (Normative) Application-Specific Constants

A.1 Constants for the Dual_EC_DRBG

The Dual_EC_DRBG requires the specifications of an elliptic curve and two points on the elliptic curve. One of the following NIST approved curves with associated points **shall** be used in applications requiring certification under FIPS 140-2. More details about these curves may be found in FIPS PUB 186-3, the Digital Signature Standard.

Each of following curves is given by the equation:

$$y^2 = x^3 - 3x + b \pmod{p}$$

Notation:

p - Order of the field F_p , given in decimal

r - order of the Elliptic Curve Group, in decimal. Note that r is used here for consistency with FIPS 186-3 but is referred to as n in the description of the Dual_EC_DRBG.

a - (-3) in the above equation

b - coefficient above

The x and y coordinates of the base point, i.e., generator G , are the same as for the point P .

A.1.1 Curve P-256

$p = 11579208921035624876269744694940757353008614\backslash$
3415290314195533631308867097853951

$r = 11579208921035624876269744694940757352999695\backslash$
522413576034242259061068512044369

$b = 5ac635d8 aa3a93e7 b3ebbd55 769886bc 651d06b0 cc53b0f6 3bce3c3e$
27d2604b

$P_x = 6b17d1f2 e12c4247 f8bce6e5 63a440f2 77037d81 2deb33a0$
f4a13945 d898c296

$P_y = 4fe342e2 fe1a7f9b 8ee7eb4a 7c0f9e16 2bce3357 6b315ece$
cbb64068 37bf51f5

$Q_x = c97445f4 5cde9f0 d3e05e1e 585fc297 235b82b5 be8ff3ef$
ca67c598 52018192

$Q_y = b28ef557 ba31dfcb dd21ac46 e2a91e3c 304f44cb 87058ada$
2cb81515 1e610046

Juniper PRNG

ScreenOS does make use of the Dual_EC_DRBG standard, but is designed to not use Dual_EC_DRBG as its primary random number generator. ScreenOS uses it in a way that should not be vulnerable to the possible issue that has been brought to light. Instead of using the NIST recommended curve points it uses self-generated basis points and then takes the output as an input to FIPS/ANSI X.9.31 PRNG, which is the random number generator used in ScreenOS cryptographic operations.

Juniper Knowledge Base Article KB28205

Juniper PRNG

ScreenOS v.6.1

```
1. char seed[8] // dual_ec output
2. char block[8]; // X9.31 temporary output
3.
4. unsigned int calls_since_reseed;
5.
6. void prng_reseed(void) {
7.
8.     // same as v6.2
9.
10. }
11.     caller-supplied buffer
12. void prng_generate(char *output) {
13.     unsigned int index = 0;
14.     calls_since_reseed++;
15.
16.     if (calls_since_reseed > 10 000) private variable
17.         prng_reseed();
18.
19.     for (; index < 20; index += 8) {
20.         ...
21.         x9_31_generate_block(seed, block);
22.         ...
23.         memcpy(&output[index], block, 8);
24.     }
25. }
```

only 20 bytes

ScreenOS v.6.2

```
1. char seed[32]; // dual_ec output
2. char block[8]; // X9.31 temporary output
3. char output[32]; // prng_generate final output
4. unsigned int index;
5.
6. void prng_reseed(void) {
7.     dual_ec_generate(output, 32);
8.     memcpy(seed, output, 32);
9.     index = 32;
10. }
11.
12. void prng_generate(void) {
13.     index = 0;
14.
15.     prng_reseed();
16.
17.     for (; index < 32; index += 8) {
18.         ...
19.         x9_31_generate_block(seed, block);
20.         ...
21.         memcpy(&output[index], block, 8);
22.     }
23. }
```

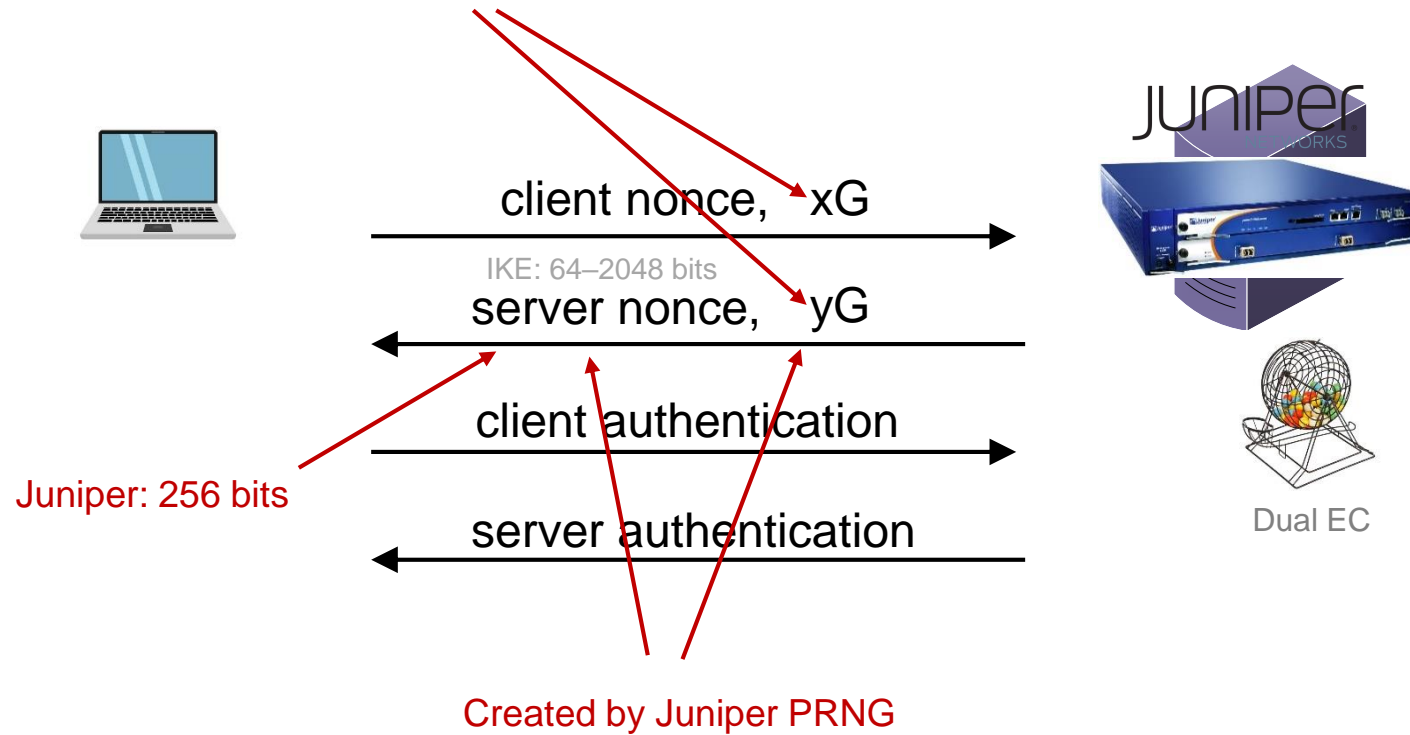
global variable

global shared buffer

Source: *Where did I leave my keys? Lessons from the Juniper Dual EC incident* <https://dl.acm.org/doi/10.1145/3266291>

IKEv2 / IPsec

Knowledge of any of these allows deriving traffic encryption key



Juniper Networks backdoor

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 - Major customers: telcos, banks, US DoD

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```
--- Qx = 2c55e5e45edf713dc43475effe8813a60326a64d9ba3d2e39cb639b0f3b0ad10  
+++ Qx = 9585320eeaf81044f20d55030a035b11bece81c785e6c933e4a8a131f6578107
```

Juniper created Q (yay! No NSA)

Who dis?

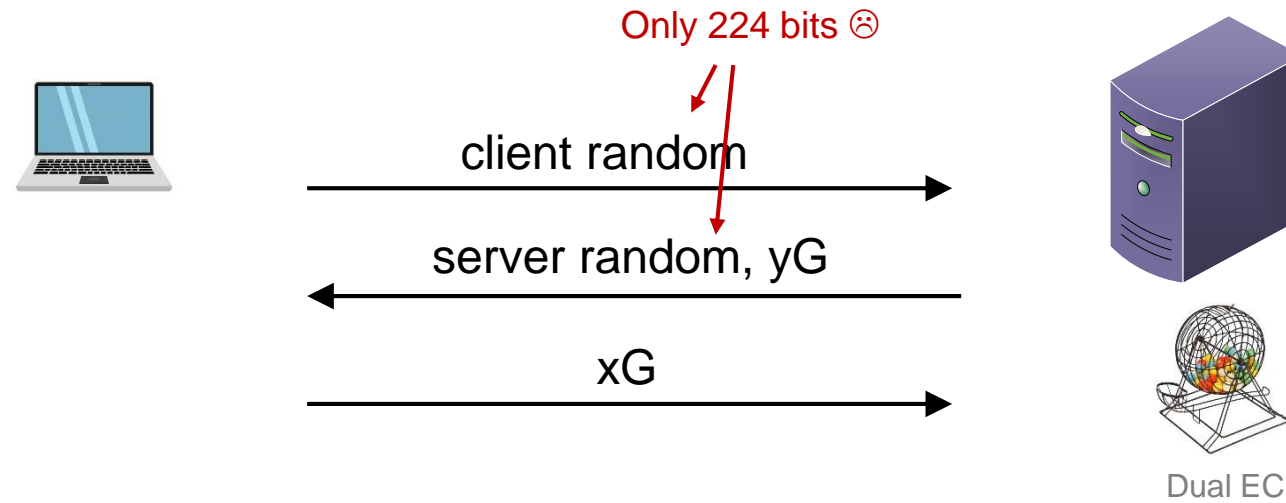
Juniper Networks backdoor

- **2008** – Juniper starts using Dual EC in ScreenOS
- **2012** – Someone hacks into Juniper's code repositories
 - Changes Q point in Dual EC
- **2015** – Juniper discovers intrusion
 - Changes Q back to its original value

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TLS



Extended Random

- NSA: please make the TLS nonces bigger...for reasons ;-)

[Search] [txt|pdf|bibtex] [Tracker] [Email] [Nits]

Versions: [00](#)
Network Working Group
Internet-Draft
Expires: June 16, 2007

E. Rescorla
Network Resonance
M. Salter
National Security Agency
December 13, 2006

Opaque PRF Inputs for TLS
draft-rescorla-tls-opaque-prf-input-00.txt

- Implementing Extended Random makes exploiting Dual EC 10,000 times easier

[Search] [txt|xml|pdf|bibtex] [Tracker] [Email] [Nits]

Versions: [00](#) [01](#) [02](#)
Network Working Group
Internet-Draft
Intended status: Informational
Expires: October 31, 2008

E. Rescorla
RTFM, Inc.
M. Salter
National Security Agency
April 29, 2008

Extended Random Values for TLS
draft-rescorla-tls-extended-random-00.txt

Status of this Memo

By submitting this Internet-Draft, each author represents that any applicable patent or other IPR claims of which he or she is aware will be disclosed, in accordance with Section 6 of BCP 79.

Internet-Drafts are working documents of the Internet Engineering Task Force (IETF), its areas, and its working groups. Note that other groups may also distribute working documents.

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documents of the Internet Engineering Task Force (IETF), its areas, and its working groups. Note that

- No real cryptographic justification exists for making them longer

[Search] [txt|pdf|bibtex] [Tracker] [Email] [Diff1] [Diff2] [Nits]

Versions: [00](#) [01](#)
Network Working Group
Internet-Draft
Intended status: Informational
Expires: April 27, 2010

J. Solinas
National Security Agency
P. Hoffman
VPN Consortium
October 24, 2009

Additional PRF Inputs for TLS
draft-solinas-tls-additional-prf-input-01

Status of this Memo

[Search] [txt|pdf|bibtex] [Tracker] [WG] [Email] [Diff1] [Diff2] [Nits]

Versions: [00](#) [01](#)
Network Working Group
Internet-Draft
Intended status: Standards Track
Expires: August 15, 2010

Standards Track
P. Hoffman
VPN Consortium
February 11, 2010

Additional Random Extension to TLS
draft-hoffman-tls-additional-random-ext-01

Abstract

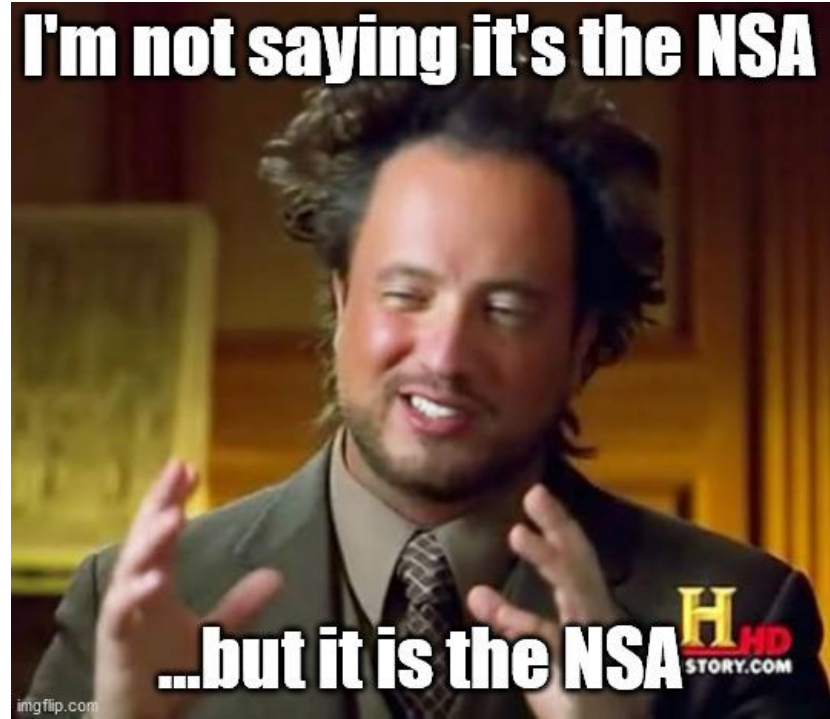
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RSA Security – BSAFE

- Turns out Juniper weren't the only one using Dual EC
- RSA Security
 - Big computer and network security company
 - Creator of BSAFE cryptographic library



- **2004** – accepted \$10 million from the NSA in order to make Dual EC the *default* in BSAFE
- **2014** – adapted the TLS Extended Random extension



Next week

- More Diffie-Hellman
- Implementation issues