Introduction to Cryptography

TEK 4500 (Fall 2023)
Problem Set 4

Problem 1.

Read Chapter 7 in [BR] (Section 7.8 can be skipped, as can the proof of Theorem 7.5).

Problem 2.

The hex-string

b393d9ab1410d9c73660a1b18b18e56e0b60d664cb547f31

represents a ciphertext of the ASCII-encoded message $M =$ "`Transfer: $000010 to Bob`", encrypted using the OTP. Modify the ciphertext so that it decrypts to $123456.

Note: For reference, the above ciphertext was created using the following Python 3 code.

```
import secrets

m = bytes("Transfer: $000010 to Bob", 'ascii')
k = secrets.token_bytes(len(m))
c = bytearray([a ^ b for (a,b) in zip(m,k)])
print(c.hex())
```

Problem 3. [Problem 10.1 in [Ros]]

Consider the following MAC scheme, where $F\colon \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$ is a secure PRF.

Σ.KeyGen:

1: $K \xleftarrow{\$} \{0,1\}^k$
2: **return** K

Σ.Tag$(K, M_1 \| \cdots \| M_\ell)$: // each M_i is n bits

1: $M \leftarrow 0^n$
2: **for** $i = 1, \ldots, \ell$ **do**
3: $\quad M \leftarrow M \oplus M_i$
4: **return** $F(K, M)$

Show that the scheme is *not* a secure MAC. Describe an adversary and compute its UF-CMA advantage (see Fig. 1 for the formal definition).

$\underline{\mathbf{Exp}^{\text{uf-cma}}_{\Sigma}(\mathcal{A})}$

$K \xleftarrow{\$} \Sigma.\mathsf{KeyGen}$
$\mathsf{won} \leftarrow 0$
$S \leftarrow [\,]$
$\mathcal{A}^{\textsc{Tag}(\cdot),\textsc{Vf}(\cdot)}$
return won

$\underline{\textsc{Tag}(M)}$:

$S.\mathsf{add}(M)$
$T \leftarrow \Sigma.\mathsf{Tag}(K, M)$
return T

$\underline{\textsc{Vf}(M, T)}$:

$d \leftarrow \Sigma.\mathsf{Vrfy}(K, M, T)$
if $d = 1$ **and** $M \notin S$ **then**
    won $\leftarrow 1$
return d

$\mathbf{Adv}^{\text{uf-cma}}_{\Sigma}(\mathcal{A}) = \Pr[\mathbf{Exp}^{\text{uf-cma}}_{\Sigma}(\mathcal{A}) \Rightarrow 1]$

Figure 1: UF-CMA security experiment and UF-CMA-advantage definition.

Problem 4. [Problem 10.3 in [Ros]]

Suppose $\mathsf{MAC} = (\mathsf{Tag}, \mathsf{Vrfy})$ is a secure MAC algorithm. Create a new MAC algorithm $\mathsf{MAC}' = (\mathsf{Tag}', \mathsf{Vrfy}')$, where $\mathsf{Tag}'(K, M) = \mathsf{Tag}(K, M) \| \mathsf{Tag}(K, M)$. Define the Vrfy algorithm for MAC' and explain why MAC' is also a secure MAC algorithm.

Note: Tag' is not a secure PRF (why?). This illustrates that MAC security is different from PRF security.

Problem 5. [Problem 7.1 and 7.2 in [BR]]

Consider the following variants of CBC-MAC, intended to allow one to MAC messages of arbitrary length.[1] The domain for all MACs is $\{0, 1\}^{n \cdot \ell}$ for $\ell = 0, 1, \ldots$ where n is the block length of the underlying blockcipher used by CBC-MAC (thus, the MACs takes as input arbitrary multiples of the block length n). Show that all these variants are completely insecure according to the UF-CMA definition (Fig. 1): break them with a constant number of queries.

a) $\mathsf{CBCv1}(K, M) = \mathsf{CBC}(K, M \| |M|)$, where $|M|$ is the length of M, written in n bits.

Hint: You can do this with 3 Tag queries: two queries having 1 (message) block, and one query having 3 (message) blocks. The Vf query should have 3 (message) blocks and use elements from all 3 Tag queries.

b) $\mathsf{CBCv2}((K, K'), M) = \mathsf{CBC}(K, M) \oplus K'$, where K' has n bits.

[1] We're slightly abusing notation and use CBC, CBCv1, ..., to denote both the MAC algorithm and the Tag function.

c) $\mathsf{CBCv3}(K, M) = (IV, \mathsf{CBC}(K, IV \| M))$, where IV is drawn at random from $\{0,1\}^n$. That is, this an IV-based version of CBC-MAC.

Problem 6. [Problem 6.1 in [BS]]

Consider the following MAC (a variant of this was used for WiFi encryption in 802.11b WEP), where $F\colon \{0,1\}^{128} \times \{0,1\}^{128} \to \{0,1\}^{32}$ is a PRF. Let CRC32 be a simple and popular error-detecting code meant to detect random errors; CRC32 is a function that takes as input $M \in \{0,1\}^*$ and outputs a 32-bit string. Define the following scheme Σ:

Σ.KeyGen:	Σ.Tag(K, M):	Σ.Vrfy$(K, M, (R, T))$:
1: $K \xleftarrow{\$} \{0,1\}^{128}$	1: $R \xleftarrow{\$} \{0,1\}^{128}$	1: $T' \leftarrow F(K, R) \oplus \mathsf{CRC32}(M)$
2: **return** K	2: $T \leftarrow F(K, R) \oplus \mathsf{CRC32}(M)$	2: **if** $T' = T$ **then**
	3: **return** (R, T)	3: **return** 1
		4: **else**
		5: **return** 0

Show that this MAC system is insecure

Hint 1: One possible adversary creates a forgery by making one $\textsc{Tag}(\cdot)$ query and running for about 2^{32} time.

Hint 2: Another possible adversary makes one $\textsc{Tag}(\cdot)$ query, but runs in virtually no time using the following property of CRC32: $\mathsf{CRC32}(M \oplus M') = \mathsf{CRC32}(M) \oplus \mathsf{CRC32}(M')$.

Problem 7.

Let $F\colon \mathcal{K} \times \{0,1\}^{2n} \to \{0,1\}^n$ be an UF-CMA secure MAC.[2]

a) Define another MAC $F'\colon \mathcal{K} \times \{0,1\}^{2n} \to \{0,1\}^{2n}$ as follows:

$$F'_K(M \| N) \stackrel{\text{def}}{=} F_K(M \| N) \| M,$$

where $M, N \in \{0,1\}^n$. That is, F' first applies F to the entire message, then appends the *first half* of the input (M) to the *end* of the output. Show that F' is UF-CMA secure.

b) Suppose instead of a secure PRF, CBC-MAC was instantiated with a fixed-length UF-CMA secure MAC as its internal building block. Show that this variant of CBC-MAC is not necessarily a secure MAC (even for fixed-length messages).

Hint: Use a).

[2] We're slightly abusing notation and use F to denote both the MAC algorithm and the Tag function.

References

[BR] Mihir Bellare and Phillip Rogaway. *Introduction to Modern Cryptography*. https://web.cs.ucdavis.edu/~rogaway/classes/227/spring05/book/main.pdf.

[BS] Dan Boneh and Victor Shoup. *A Graduate Course in Applied Cryptography*, (version 0.5, Jan. 2020). https://toc.cryptobook.us/.

[Ros] Mike Rosulek. *The Joy of Cryptography*, (draft Feb 6, 2020). https://web.engr.oregonstate.edu/~rosulekm/crypto/crypto.pdf.