Exercise 3

Question 1

Given $A_s = \{1, 2, 3\}$

$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$

$$\overline{\omega}_1 = (\omega_1, \omega_2, \omega_3, \omega_4) \Leftrightarrow \omega_1 \succ \omega_2 \succ \omega_3 \succ \omega_4$$

$$\overline{\omega}_2 = (\omega_2, \omega_3, \omega_4, \omega_1)$$

$$\overline{\omega}_3 = (\omega_3, \omega_4, \omega_1, \omega_2)$$

a) No winner in plurality since different first place for all winners.

b) Condorcet's Paradox?

if w_1 as winner: $\overline{W_2}$ prefer w_2, w_3, w_4 (to w_1)

$2/3$ prefer w_3 to w_1 $\overline{W_3}$ prefer w_3, w_4

if w_2 as winner: $\overline{W_1}$ prefer w_1 (to w_2)

$2/3$ prefer w_1 to w_2 $\overline{W_3}$ prefer w_3, w_4, w_1

if w_3 as winner: $\overline{w_1}$ prefer w_1, w_2 (to w_3)
$2/3$ prefer w_2 to w_3 $\overline{w_2}$ prefer w_2

if w_4 as winner: $\overline{w_1}$ prefer w_1, w_2, w_3 (to w_4)
$3/3$ prefer w_3 to w_4 $\overline{w_2}$ prefer w_2, w_3
$\overline{w_3}$ prefer w_3

$\Rightarrow$ Yes, we do have a Paradox

c) Tactical voting is when

$$f(\bar{w}_1, \ldots, \bar{w}_i', \ldots, \bar{w}_N) >_i f(\bar{w}_1, \ldots, \bar{w}_i, \ldots, \bar{w}_N)$$

* $\bar{w}_1$ would rather have w_2 than w_3 or w_4
  $\bar{w}_2$ would rather have w_3 than w_1 or w_4
  $\Rightarrow$ Both could change 1 and 2 place in the ordering to secure w_2 or w_3 being chosen.
* $\bar{w}_3$ would rather have w_4 than w_1 or w_2
  $\Rightarrow$ Not so easy for $\bar{w}_3$ to manipulate

By making it NP-hard to manipulate elections we could possibly safeguard against tactical voting, e.g. Second-order Copeland.

d) Calculate Borda count BC

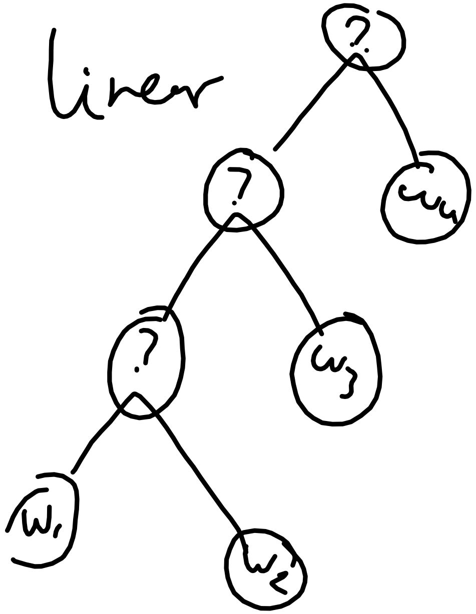
	$\overline{w_1}$	$\overline{w_2}$	$\overline{w_3}$		BC
w_1	3	0	1	=	4
w_2	2	3	0	=	5
w_3	1	2	3	=	6
w_4	0	1	2	=	3

$\Rightarrow$ w_3 has the highest BC, it is not possible to change outcome without making voter 3 be off

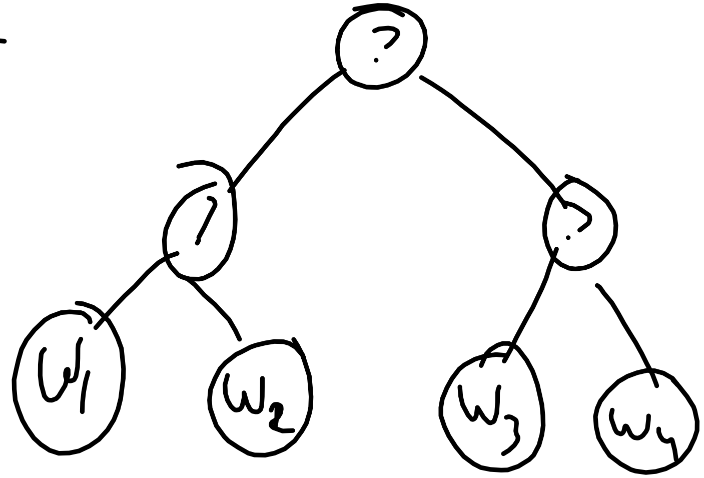
Borda is PO

e) Sequential majority voting

linear



balanced tree



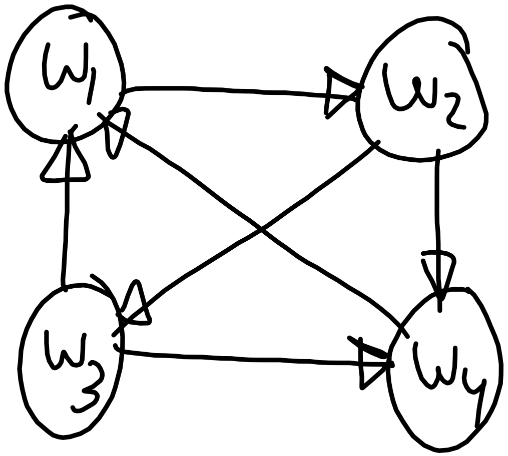
Sequence matters!

f) Pairwise election

$w_1 > w_2$ $\quad$ $w_2 > w_3$ $\quad$ $w_3 > w_4$

$w_1 < w_3$ $\quad$ $w_2 > w_4$

$w_1 < w_4$



$\Rightarrow$ All outcomes are possible winners

9) we could

$w_2 > w_3$ to $w_2 < w_3$

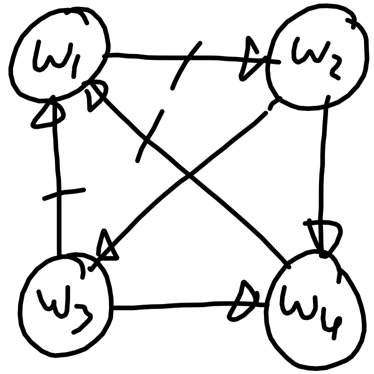
or

$w_1 > w_2$ to $w_1 < w_2$

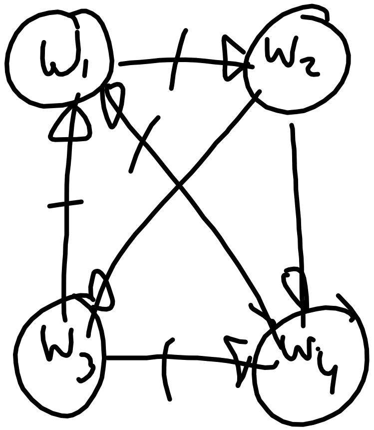
lets us look at $w_2^* \succ$

	SC
$w_2 \succ^* w_1 \succ^* w_3 \succ^* w_4$	3
$w_2 \succ^* w_1 \succ^* w_4 \succ^* w_3$	4
$w_2 \succ^* w_3 \succ^* w_1 \succ^* w_4$	2
$w_2 \succ^* w_3 \succ^* w_4 \succ^* w_1$	1
$w_2 \succ^* w_4 \succ^* w_1 \succ^* w_3$	3
$w_2 \succ^* w_4 \succ^* w_3 \succ^* w_1$	2

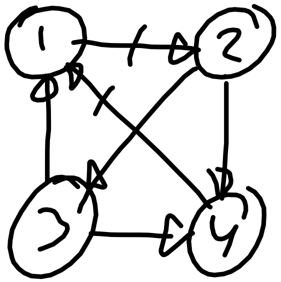
1 flip



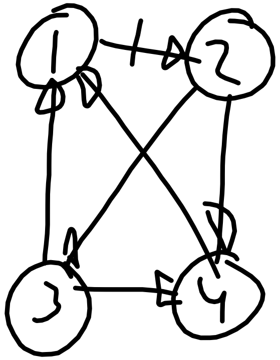
3 $\quad \omega_2 \succ^* \omega_1 \succ^* \omega_3 \succ^* \omega_4$



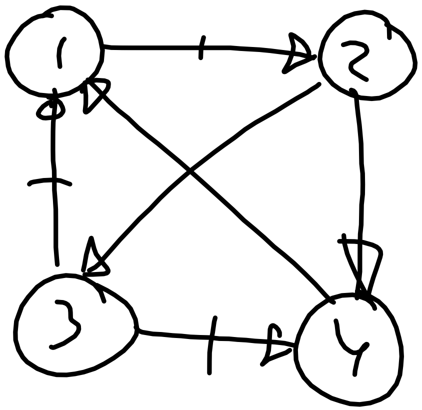
4 $\quad \omega_2 \succ^* \omega_1 \succ^* \omega_4 \succ^* \omega_3$



2 $w_2 \succ^* w_3 \succ^* w_1 \succ^* w_4$

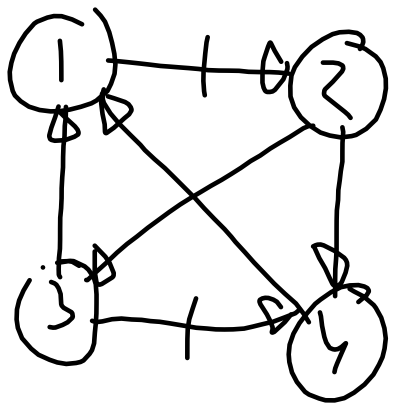


1 $w_2 \succ^* w_3 \succ^* w_4 \succ^* w_1$



3

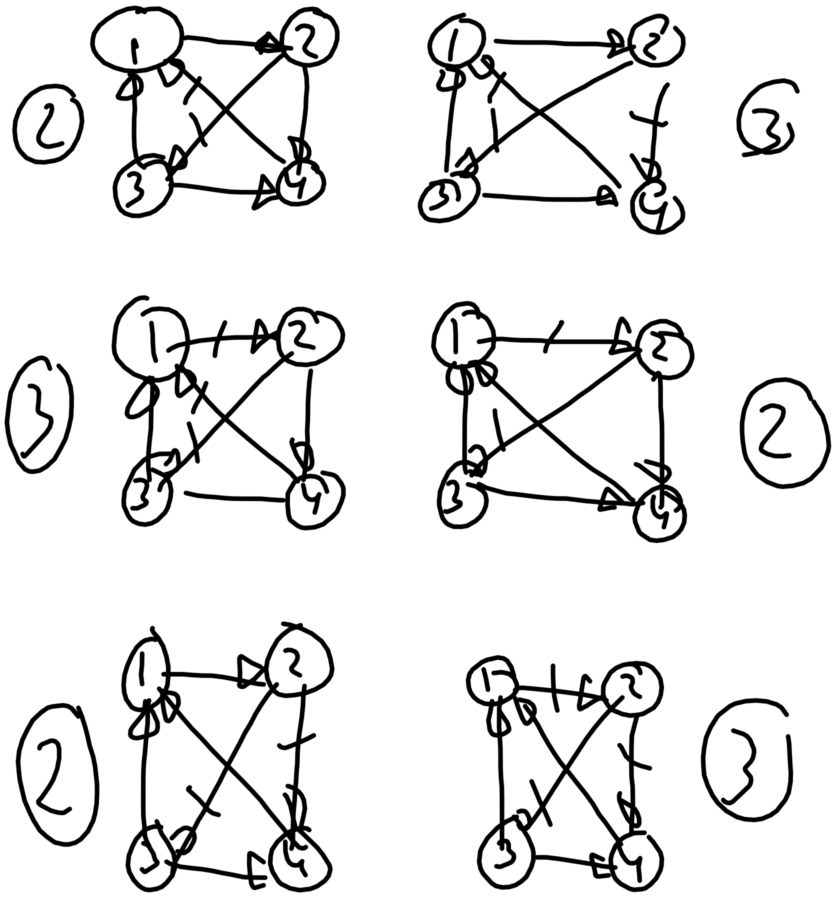
$\omega_2 \succ^{\phi} \omega_4 \succ^{\phi} \omega_1 \succ^{\phi} \omega_3$



2

$\omega_2 \succ^{\phi} \omega_4 \succ^{\phi} \omega_3 \succ^{\phi} \omega_1$

$w_3^* \succ$	SC
$w_3 \succ^* w_1 \succ^* w_2 \succ^* w_4$	2
$w_3 \succ w_1 \succ w_4 \succ w_2$	3
$w_3 \succ w_2 \succ w_1 \succ w_4$	3
$w_3 \succ w_2 \succ w_4 \succ w_1$	2
$w_3 \succ w_4 \succ w_1 \succ w_2$	2
$w_3 \succ w_4 \succ w_2 \succ w_1$	3



⇒ According to Slater rule the sequence

$w_2 \succ w_3 \succ w_4 \succ w_1$

is the most socially acceptable

⇒ In the general case the Slater count is NP-hard to calculate

h) Plurality gives no ranking

Borda $w_3 \succ w_2 \succ w_1 \succ w_4$ best for high N

Slater $w_2 \succ w_3 \succ w_4 \succ w_1$ best for low N