

Exercise 4

Question 1

Given a marginal contribution net

$$a \vee b \rightarrow 7$$

$$b \rightarrow 4$$

$$c \rightarrow 6$$

$$b \wedge c \rightarrow 3$$

a) (calculate the characteristic function

$$v(\emptyset) = 0 \quad \text{empty coalition}$$

$$v(\{a\}) = 0 \quad \text{No rule apply}$$

$$v(\{b\}) = 4$$

$$v(\{c\}) = 6$$

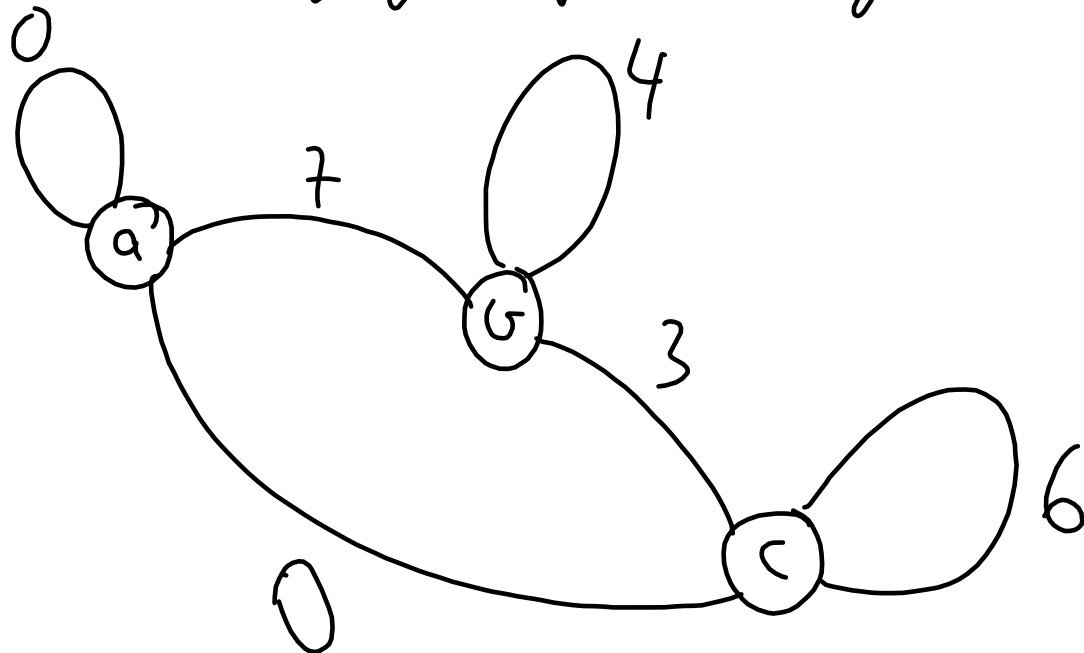
$$\begin{aligned} v(\{a, b, c\}) &= 0 + 4 + 6 + 7 + 3 \\ &= 20 \end{aligned}$$

$$v(\{a, b\}) = 0 + 4 + 7 = 11$$

$$v(\{a, c\}) = 0 + 6 = 6$$

$$v(\{b, c\}) = 4 + 6 + 3 = 13$$

6) Weighted graph of this game



c) Is this game stable? Do we have a non-empty core?

	a	b	c
level 1	0	4	6
	└──────────┘		
level 2	11	13	
	└── 76 ──┘		
	└── 6 ──┘		
level 3	└──────────┘		
	20		

a = 0

a	b	c
0	20	0
0	19	1
:	:	:
0	14	6
:	:	:
0	11	9
0	10	10
:	:	:
0	0	20

c objects

core is nonempty

at objects

	a	b	c	
a = 1	1	19	0	c objects
	:	:	:	
	1	13	6	} core
⇒ There exist	1	10	9	
some non-empty	1	9	10	at objects
core, is the	:	:	:	
game in state	:	:	1	
	1	0	19	

d) Shapley for the grand coalition

$$Sh_i = \frac{1}{|A_S|!} \sum_{O \in \Pi(A_S)} M_i(C_i(O))$$

Two ways of calculating Shapley

1) The naive way

2) The weighted graph

Naive way

Maximal contribution

Different coalition orderings

a b c

a c b

b a c

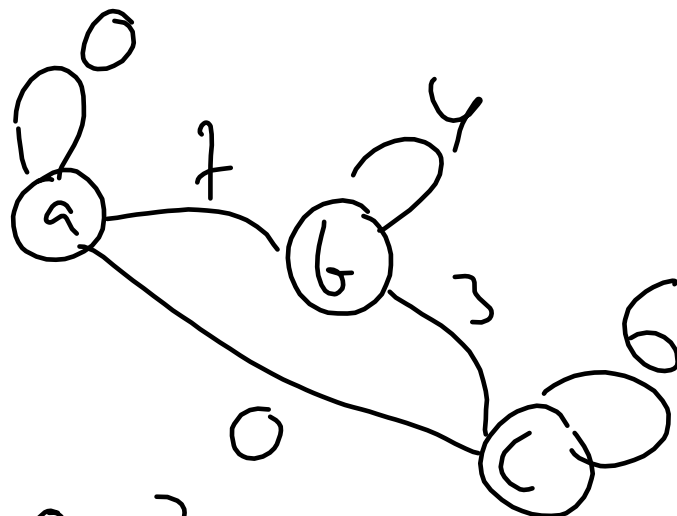
b c a

c a b

c b a

a	b	c
0	11	9
0	14	6
7	4	9
7	4	9
0	14	6
7	7	6
$21/6 = 3.5$	$54/6 = 9$	$45/6 = 7.5$

the weighted graph



$$Sh_a = 0 + \frac{7}{2} + 0 = 3,5$$

$$Sh_b = 4 + \frac{7}{2} + \frac{3}{2} = 9$$

$$Sh_c = 6 + \frac{3}{2} + 0 = 7,5$$

