Exercise 2

Question 1

a) This is a strategic game since optimal strategy of each player is mutually dependent on other players action.

This game can be written in
Strategic or normal form since
player 2 is unknown to player 1
first action (dashed line)

		Player 2 "Odd"	
		H	T
"Even" Player 1	H	$(1,-1)$	$(-1,1)$
	T	$(-1,1)$	$(1,-1)$

"Matching pennies" p. 231 in W

b) This is a zero-sum game since

$$U_{P_1} + U_{P_2} = 0 \quad \text{for } \Omega$$

c) i) Nash equilibrium is when two strategies s_1 and s_2

1, given that player i plays s_1, agent j can do no better than play s_2, and

2, given that player j plays s_2, agent i can do no better than play s_1

$\Rightarrow$ s_1 and s_2 are best response to each other, players do not regret choice of strategy.

(ii) A solution is Pareto efficient/optimal if no improvement is possible without making someone else worse off.

iii) Maximizing social welfare means to choose the strategy/outcome that gives the highest aggregated utility among agents

$$sw(\omega_i) = \sum_{j \in Ag} u_j(\omega_i)$$

Where sw is social welfare of outcome ω_i

$u_j(\omega_i)$ is utility of agent j of outcome ω_i

player 2

player 1

	H	T
H	(1, -1)	(-1, 1)
T	(-1, 1)	(1, -1)

* Nash No pures Nash

△ Pareto All are PO

□ Social Welfare $u_{pl1} + u_{pl2} = 0$

d) Nash's theorem

Every game in which every player has a finite set of possible strategies has a Nash equilibrium in mixed strategies.

A mixed strategy over set $\{s_1, s_2, \ldots, s_m\}$ is to give a probability distribution $\{p_1, p_2, \ldots, p_m\}$ of playing the different strategies.

$\Rightarrow$ Randomize your strategies!

MS Nash eq. is Nash for randomized strategies.

e) Find the MS Nash eq.

Player 2 "Odd"

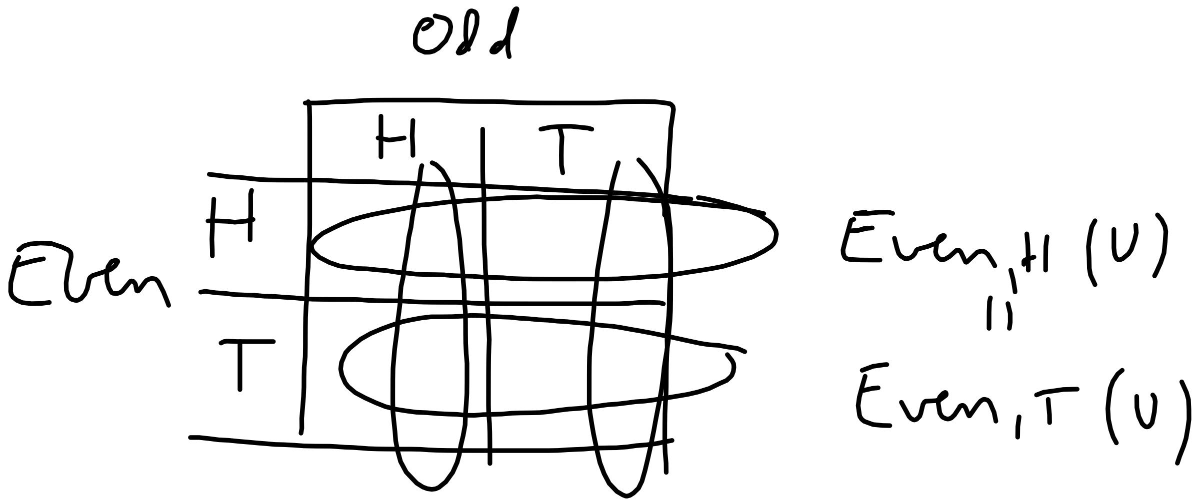
			H	T
			p_O	$(1-p_O)$
Player 1 "Even"	H	p_E	(1,-1)	(-1,1)
	T	$(1-p_E)$	(-1,1)	(1,-1)

So we want to calculate:

I $E_{odd,H}(U) = E_{odd,T}(U)$

II $E_{even,H}(U) = E_{even,T}(U)$

which can be drawn



$$E_{odd,H}(U) = E_{odd,T}(U)$$

which gives

$$\mathrm{I} \quad p_E(-1) + (1-p_E)1 = p_E \cdot 1 + (1-p_E)(-1)$$

$$\mathrm{II} \quad p_O \cdot 1 + (1-p_O)(-1) = p_O(-1) + (1-p_O)\cdot 1$$

$$\mathrm{I} \quad -p_E + 1 - p_E = p_E - 1 + p_E$$

$$-2p_E + 1 = 2p_E - 1$$

$$\Rightarrow 4p_E = 2 \qquad \Rightarrow p_E = \frac{2}{4} = \underline{\frac{1}{2}}$$

II $p_0 - 1 + p_0 = -p_0 + 1 - p_0$

$2p_0 - 1 = -2p_0 + 1$

$\Rightarrow 4p_0 = 2 \Rightarrow p_0 = \frac{2}{4} = \underline{\underline{\frac{1}{2}}}$

Question 2

a) Why is PD "proof" of cooperation is impossible?

i \ j	D	C
D	$\underline{2},\underline{2}$ *	$\underline{5},0$
C	$0,\underline{5}$	$3,3$ □

* Nash (2,2) (D,D) SO = 4

□ SO (3,3) (C,C) SO = 6

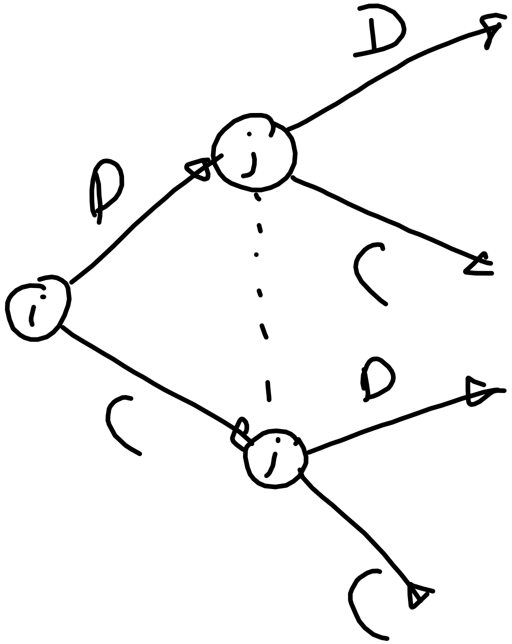
⇒ Social welfare seems wasted in PD but not eg (C,C)

In an evolutionary perspective
the D strategy has a higher fitness/payoff
producing more offspring, out competing
C strategy agents in the long run making
C strategy impossible to survive (and
be created/started again.

Program Equilibria might solve this by a moderator comparing strategies or programs.

if $p_1 == p_2$ do C else do D

Write PD in extensive form



D,D ~ CC (3,3)
or ~ DD (?,?)

DC ~ DD (?,?)

CD ~ DD (?,?)

CC ~ CC $\boxed{(3,3)}$

$\Rightarrow$ Now C becomes rational to play for self-interested players.

(But we have altered PD and Introduced a moderator!)