

Exercises from L5 (loop games)

Question 1

q) The characteristic function is hard to represent, two possible representations

- 1, Marginal contribution nets,  
Complete and succinct
- 2, Induced subgraphs are succinct  
but not complete

- The marginal contribution net represent the characteristic function as a set of rules

$$V_n(x) = \sum_{\varphi \rightarrow x \in RS} x \quad \text{where } \varphi \rightarrow x \text{ is a rule in the rule set } RS$$

$$U(\{a\}) = 3$$

$$a \rightarrow 3$$

$$U(\{b\}) = 0$$

$$b \rightarrow 0$$

$$U(\{c\}) = 7$$

$$c \rightarrow 7$$

$$U(\{ab\}) = a \rightarrow 3 + b \rightarrow 0 + a \wedge b \rightarrow x = 13$$
$$a \wedge b \rightarrow 10$$

$$U(\{a, c\}) = a \rightarrow 3 + c \rightarrow 7 + a \wedge c \rightarrow x = 16$$

$$a \wedge c \rightarrow 6$$

$$U(\{b, c\}) = b \rightarrow 0 + c \rightarrow 7 + b \wedge c \rightarrow x = 7$$

$$b \wedge c \rightarrow 0$$

$$U(\{a, b, c\}) = a \rightarrow 3 + b \rightarrow 0 + c \rightarrow 7 + a \vee b \rightarrow 10 + a \wedge c \rightarrow 6 + b \wedge c \rightarrow 0 + a \wedge b \wedge c \rightarrow y = 26$$

$$Q \wedge \neg C \rightarrow \emptyset$$

which gives the following rule set

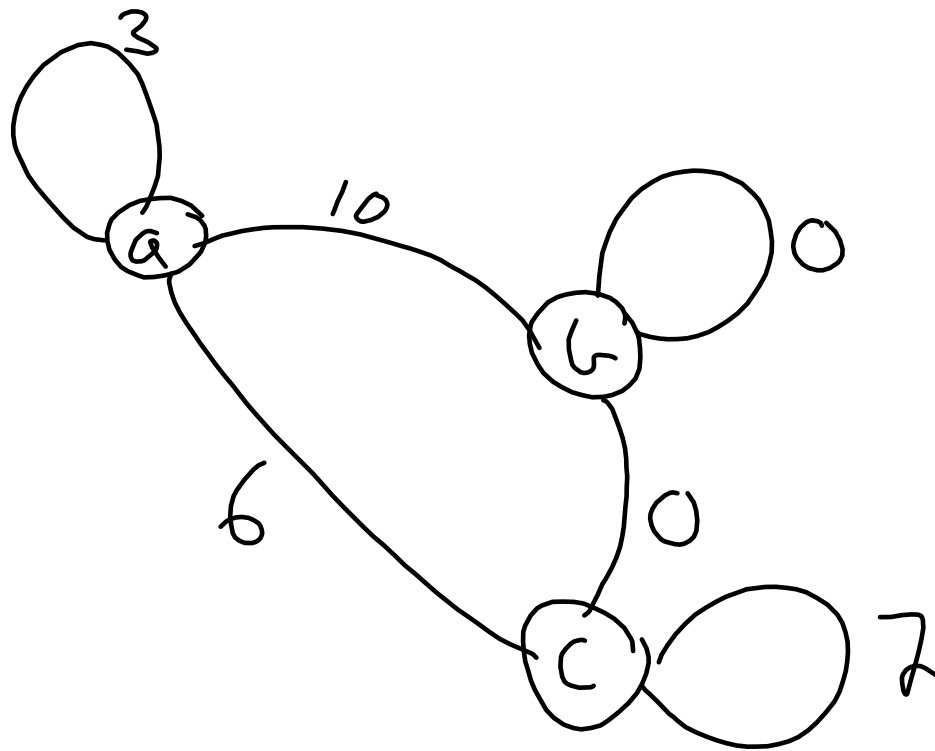
$$Q \rightarrow 3$$

$$C \rightarrow 7$$

$$Q \wedge C \rightarrow 10$$

$$Q \wedge \neg C \rightarrow 6$$

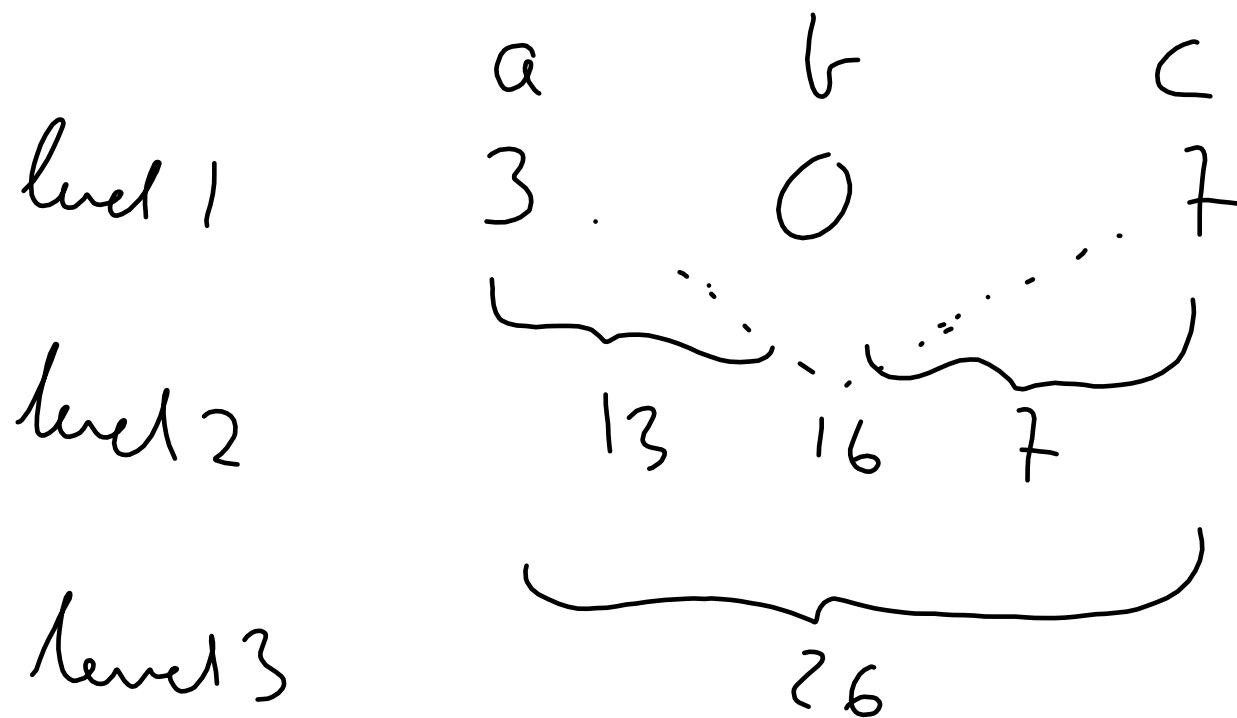
- Induced subgraph is an undirected weighted graph, nodes are  $A_g = \{a, b, \dots, N\}$  agents and edges are weights corresponding to the rules in marginal contribution nets.





b) Solve the game by checking if Core is non-empty.

The core is a coalition that no other coalition objects to.



b = 0

a	b	c	
26	0	0	{c} object
25	0	1	{c} object
⋮	⋮	⋮	
19	0	7	} Core is non-empty
13	0	13	
12	0	14	{a,b} object
⋮	⋮	⋮	
0	0	76	

$b = 1$

a  
25  
⋮  
18  
  
12  
11  
⋮  
0

b  
1  
⋮  
1  
  
1  
1  
  
1

c  
0  
⋮  
7  
  
13  
14  
  
25

{c} objects

} core is non-empty

{a,b} objects

$\Rightarrow$  There exist some non-empty  
core, i.e. the game is solvable or  
stable

Direct calculation of Shapley

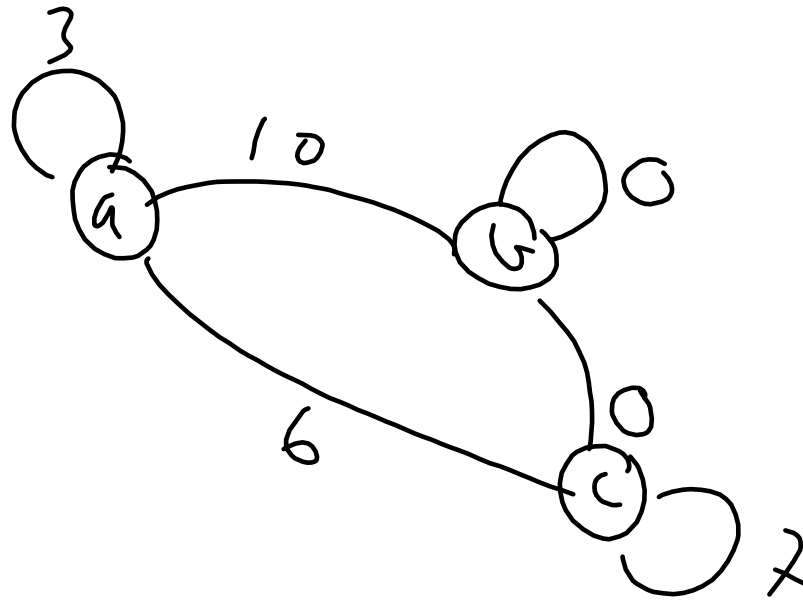
	a	b	c	a	b	c
a b c	3	10	13			
a c b	3	10	13			
b a c	13	0	13			
b c a	19	0	7			
c a b	9	10	7			
c b a	19	0	7			

$$a = \frac{66}{6}$$
$$= \underline{\underline{11}}$$

$$b = \frac{30}{6}$$
$$= \underline{\underline{5}}$$

$$c = \frac{60}{6}$$
$$= \underline{\underline{10}}$$

Shapley from induced subgraphs



$$Sh_1 = 3 + \frac{10}{2} + \frac{6}{2} = 11$$

$$Sh_2 = 0 + \frac{10}{2} + \frac{0}{2} = 5$$

$$Sh_3 = 7 + 0 + \frac{6}{2} = 10$$