

## Exercises from lecture 9 (SI)

Question 1

$$a) \quad p_{ij}^k = \frac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\sum_{c_{il} \in N(s^p)} \tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}}$$

$\forall c_{il} \in N(s^p)$   
 i.e. a city not visited  
 Normalization

where  $\alpha, \beta$  are non-linear control parameters  
 Memory weight  
 distance weight  
 $\eta_{ij} = \frac{1}{d_{ij}}$  inverse distance between  $i$  and  $j$

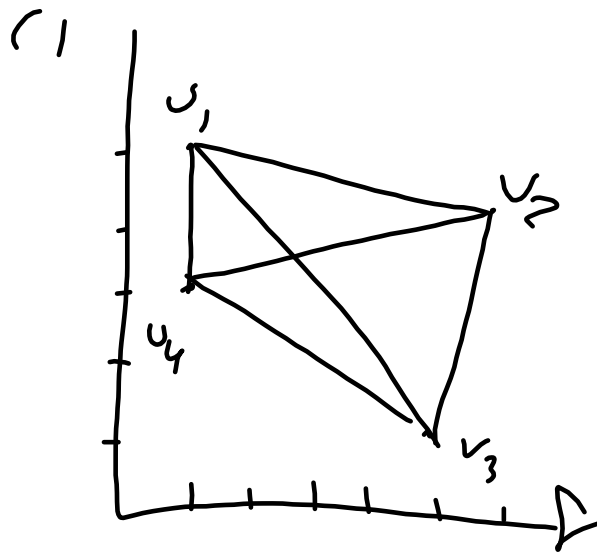
↳

$$\tau_{ij} \leftarrow (1 - \rho) \tau_{ij} + \sum_{k=1}^m \Delta \tau_{ij}^k$$

where  $\tau_{ij}$  is pheromone concentration

$\rho$  is evaporation rate

$\Delta \tau_{ij}$  is pheromone laid by ant  $k$   
 on edge  $(i, j)$  if part of tour  
 $= \begin{cases} 1/L_k & \text{if ant } k \text{ used edge } (i, j) \text{ on tour} \\ 0 & L_k \text{ is length of tour} \end{cases}$



$$d_{12} = 5,099$$

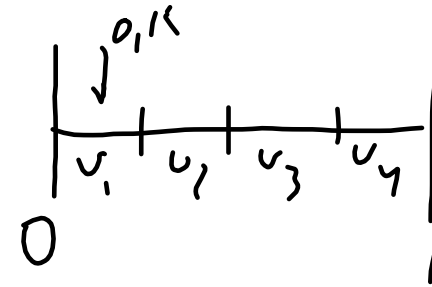
$$d_{23} = 3,16$$

$$d_{34} = 4,47$$

$$d_{14} = 2 = \sqrt{(1-1)^2 + (5-3)^2} = 2 \text{ in } v_1$$

$$d_{13} = 5,657$$

$$d_{24} = 5,099 = \sqrt{(6-1)^2 + (4-3)^2}$$



\*  $U_i = U_1$ ,  $U_j$  could be  $U_2, U_3$  or  $U_4$

$$\sum R_{ij} \eta_{ij}^{\beta} = 10^{-6} (0,176^5 + 0,177^5 + 0,17^5) = 3,17 \cdot 10^{-8}$$

going to  $U_2$

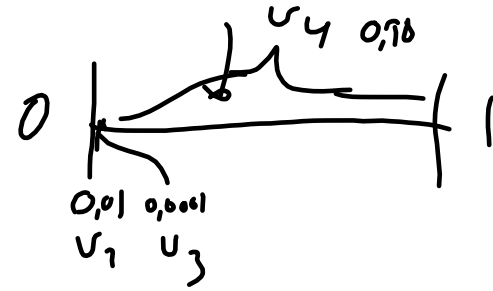
$$P_{12} = \frac{10^{-6} \cdot 0,176^5}{3,17 \cdot 10^{-8}} = 0,01$$

going to  $U_3$

$$P_{13} = \frac{10^{-6} \cdot 0,177^5}{3,17 \cdot 10^{-8}} = 0,0001$$

going to  $u_4$

$$P_{14} = \frac{10^{-6} \cdot 0,5^5}{3,17 \cdot 10^{-8}} = 0,98$$



and  $k=1$  chose city 4 (after city 1)

\*  $u_i = u_4$  could visit city  $u_2$  or  $u_3$

$$\sum P_{ij} \tau_{ij}^5 = 10^{-6} (2,196 \bar{5} + 9,274 \bar{5}) = 8,47 \cdot 10^{-10}$$

going to  $v_3$

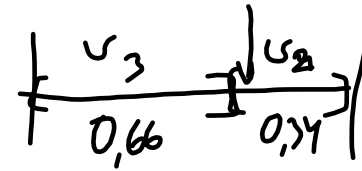
$$P_{43} = \frac{10^{-6} \cdot 0,774^5}{8,49 \cdot 10^{-10}} = 0,66$$

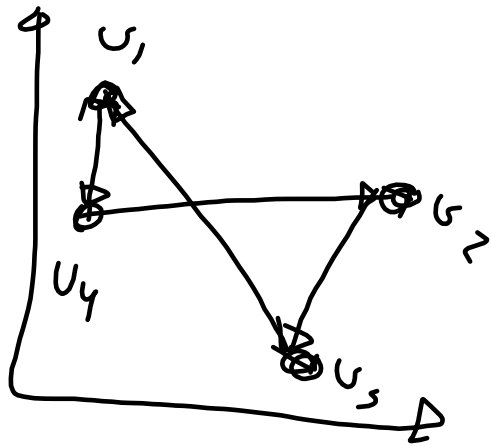
going to  $v_2$

$$P_{42} = \frac{10^{-6} \cdot 0,177^5}{8,47 \cdot 10^{-10}} = 0,34$$

and  $k=1$  chose city 2 (after 1)

\*  $v_i = v_2$  must go to  $v_3$





$$\begin{aligned} \text{Tour length } L &= 2 + 5,097 + 3,16 \\ &\quad + 5,66 \\ &= 15,92 \end{aligned}$$

d, Simulate by computer

e)

$$\hat{T}_{ij} = (1-g) \hat{T}_{ij} + \sum_{k=1}^n \Delta \hat{T}_{ij}^k$$

$$\hat{T}_{14} = \left(1 - \frac{1}{2}\right) 10^{-6} + \frac{100}{15,96} + \dots +$$

$$= 0,5 \cdot 10^{-6} + 6,28 + \dots$$