Solutions to exercises from Lecture 10 Cooperative game theory

TEK5010 Multiagent systems 2020

Question 1

a) Calculate $v(C)$ for all coalitions

Possible coalitions:

$\{a\} = 4$

$\{b\} = 5$

$\{c\} = 0$

$\{ab\} = 4 + 5 + 10 = 19$

$\{ac\} = 4 + 0 = 4$

$\{bc\} = 5 + 0 + 2 = 7$

$\{abc\} = 4 + 5 + 0 + 10 + 2 = 21$

Rules:

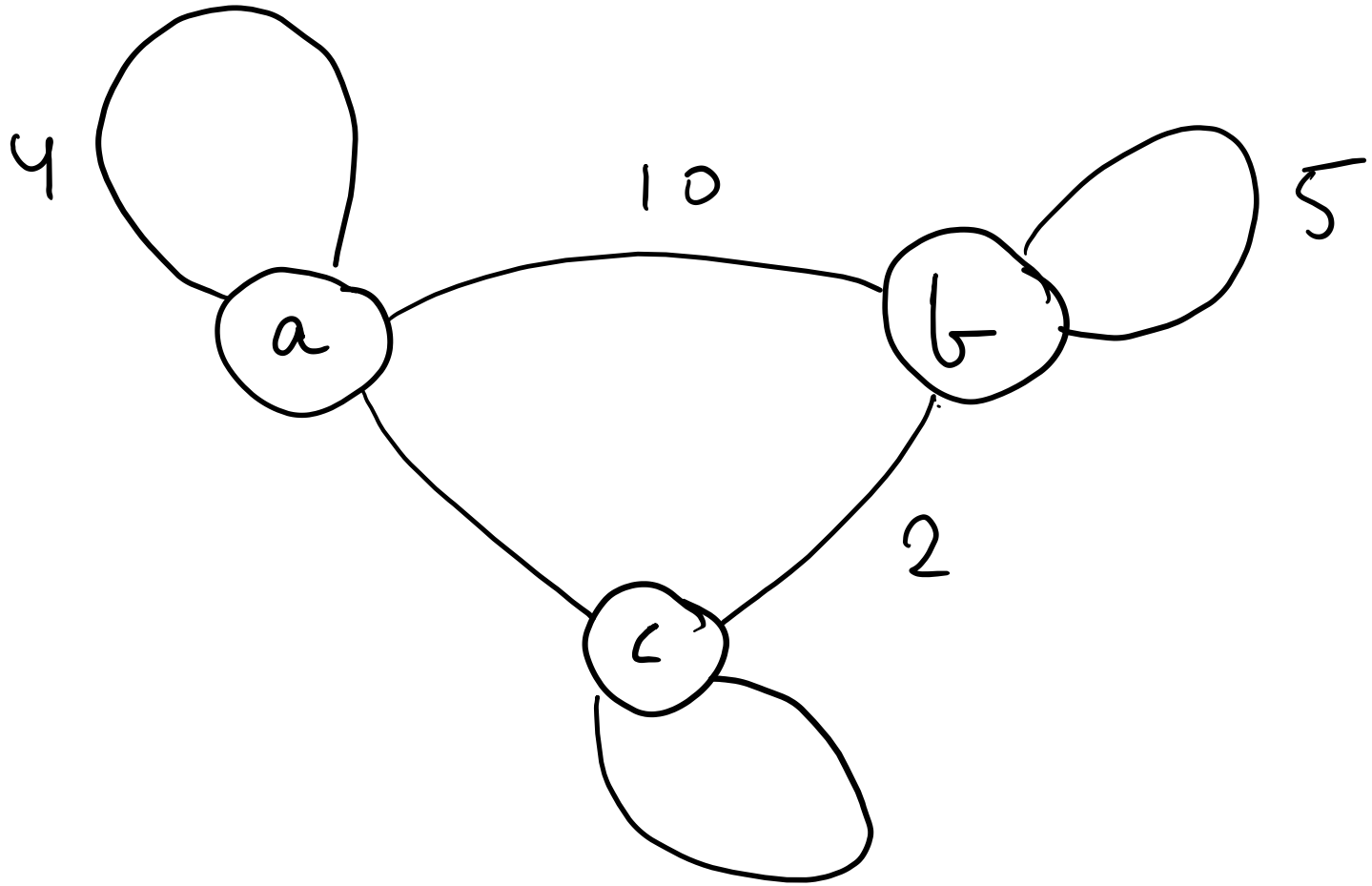
$a \rightarrow 4$

$b \rightarrow 5$

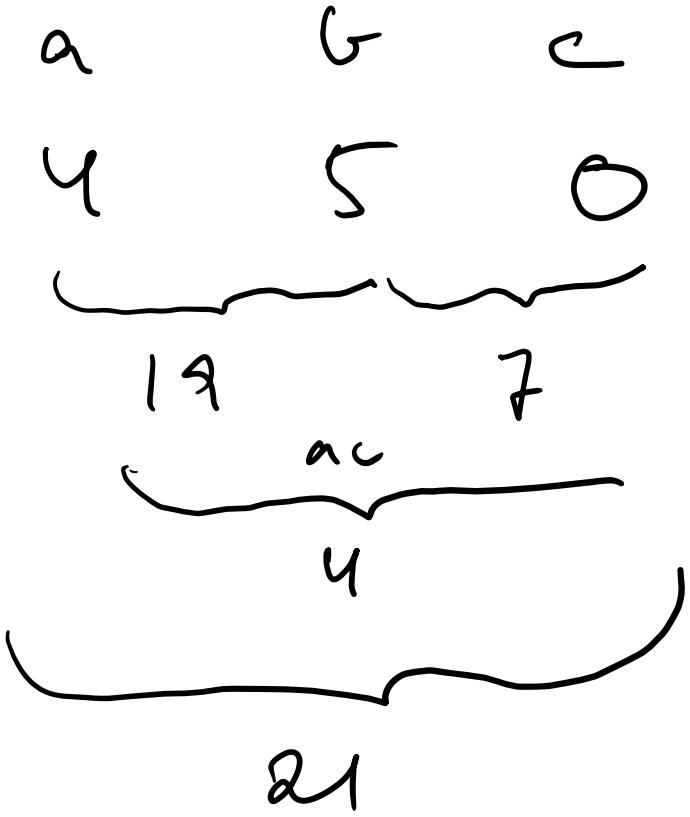
$a \wedge b \rightarrow 10$

$b \wedge c \rightarrow 2$

b) Draw the weighted graph.



() Is this game stable?



level 1 $a \geq 4, b \geq 5, c \geq 0$

level 2 $ab \geq 19, ac \geq 4, bc \geq 7$

level 3 $abc = 21$

$\text{test}\{abc\} = 21$

	a	b	$c = 1$	
$a \geq 4$	4	16	1	No coalition objects
$b \geq 5$	5	15	1	
$c \geq 0$	6	14	1	
$ab \geq 19$	⋮			
$ac \geq 4$				
$bc \geq 7$	14	6	1	
	15	5	1	$\{bc\}$ objects
	16	4	1	$\{bc\} \cup \{b\}$ objects

$\Rightarrow$ This game is stable, non-empty core!

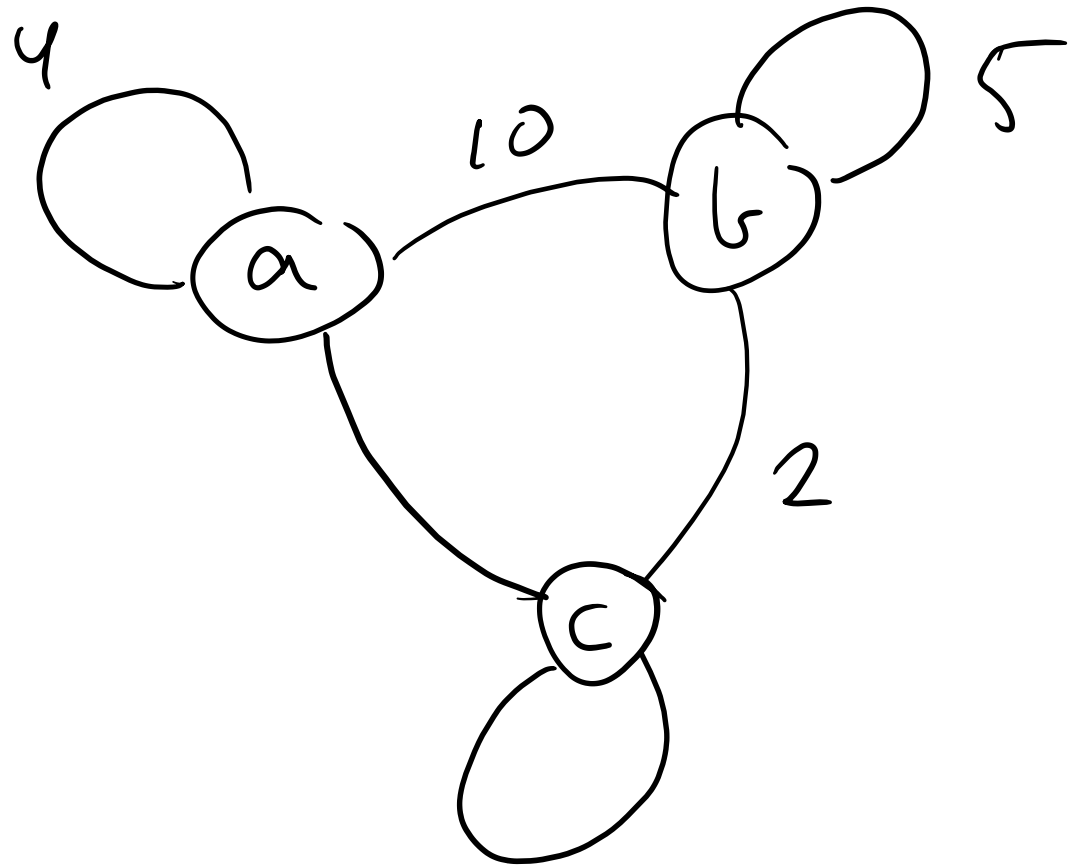
d) Calculate Shapley for $\{a, b, c\}$

* Direct calculation

Permutations	Marginal Contribution a	b	c
a b c	4	15	2
a c b	4	17	0
b a c	14	5	2
b c a	14	5	2
c a b	4	17	0
c b a	14	7	0

$Sh_a = 54/6 = 9$ $Sh_b = 66/6 = 11$ $Sh_c = 6/6 = 1$

* Shapley from graph



$Sh_a = 4 + \frac{10}{2} + 0 = 9$

$Sh_b = 5 + \frac{10}{2} + \frac{2}{2} = 11$

$Sh_c = 0 + \frac{2}{2} + 0 = 1$