

# Solutions to exercises from Lecture 11 Auctions

TEK5010 Multiagent systems 2020

# Question 1

a) What is the valuation of all bundles?

	$A_1$	$A_2$	$A_3$
$v(\{a\})$	$\{a\} = 3$	0	0
$v(\{b\})$	0	0	$\{b\} = 3$
$v(\{c\})$	$\{c\} = 1$	$\{c\} = 5$	$\{c\} = 4$
$v(\{ab\})$	$\{ab\} = 5$	$\{ab\} = 6$	$\{ab\} = 11$
$v(\{ac\})$	$\{ac\} = 3$	$\{c\} = 5$	$\{c\} = 4$
$v(\{bc\})$	$\{c\} = 1$	$\{c\} = 5$	$\{bc\} = 15$
$v(\{abc\})$	$\{abc\} = 7$	$\{abc\} = 14$	$\{bc\} = 15$

b) Who is winner if max SW (80) is assumed?

	$A_{S_1}$	$A_{S_2}$	$A_{S_3}$	SW
$\{a\} \{b\} \{c\}$	$\{a\} = 3$	$\{b\} = 5$	$\{c\} = 3$	11
$\{ab\} \{c\}$	0	$\{c\} = 5$	$\{ab\} = 11$	16
$\{ac\} \{b\}$	0	$\{ac\} = 5$	$\{b\} = 3$	8
$\Rightarrow \{bc\} \{a\}$	$\{a\} = 3$	0	$\{bc\} = 15$	18
$\{abc\}$	0	0	$\{abc\} = 15$	15

$$v_{A_{S_1}}(\{a\}) = 3, \quad v_{A_{S_2}}(\{\emptyset\}) = 0, \quad v_{A_{S_3}}(\{bc\}) = 15$$

c) What is the price of VCG?

$\overline{A_{S_1}}$	$A_{S_1}$	$A_{S_2}$	$A_{S_3}$	SW
$\{a\} \{b\} \{c\}$	0	$\{c\} = 5$	$\{b\} = 3$	8
$\Rightarrow \{ab\} \{c\}$	0	$\{c\} = 5$	$\{ab\} = 11$	16
$\{ac\} \{b\}$	0	$\{ac\} = 5$	$\{b\} = 3$	8
$\{bc\} \{a\}$	0	0	$\{bc\} = 15$	15
$\{abc\}$	0	0	$\{abc\} = 15$	15

$$P_{A_{S_1}} = \Delta A_{S_2} + \Delta A_{S_3} = (5 - 0) + (11 - 15) = 5 - 4 = \underline{\underline{1}}$$

* $\overline{A_{S_2}}$	$A_{S_1}$	$A_{S_2}$	$A_{S_3}$	SW
{a} {b} {c}	{a} = 3	0	{c} = 4	7
{ab} {c}	{c} = 1	0	{ab} = 11	14
{ac} {b}	{ac} = 3	0	{b} = 3	6
$\Rightarrow$ {bc} {a}	{a} = 3	0	{bc} = 15	18
{abc}	0	0	{abc} = 15	15

$$P_{A_{S_2}} = \triangle A_{S_1} + \triangle P_{A_{S_3}} = (3-3) + (15-15) = \underline{\underline{0}}$$

Dummy player

$* \overline{A_{53}}$	$A_{51}$	$A_{52}$	$A_{53}$	SW
$\{a\} \{b\} \{c\}$	$\{a\} = 3$	$\{c\} = 5$	0	8
$\{ab\} \{c\}$	$\{ab\} = 5$	$\{c\} = 5$	0	10
$\{ac\} \{b\}$	0	$\{ac\} = 5$	0	5
$\{bc\} \{a\}$	$\{a\} = 3$	$\{bc\} = 5$	0	8
$\Rightarrow \{abc\}$	0	$\{abc\} = 14$	0	14

$$P_{A_{53}} = 0A_{51} + 0A_{52} = (0 - 3) + (14 - 0) = -3 + 14 = \underline{\underline{11}}$$