Solutions to exercises from Lecture 4 Swarm robotics 1

TEK5010 Multiagent systems 2020

Question)

a) Explain USL

$$R(N) = C \frac{N}{1 + \alpha (N-1) + \beta N (N-1)}$$

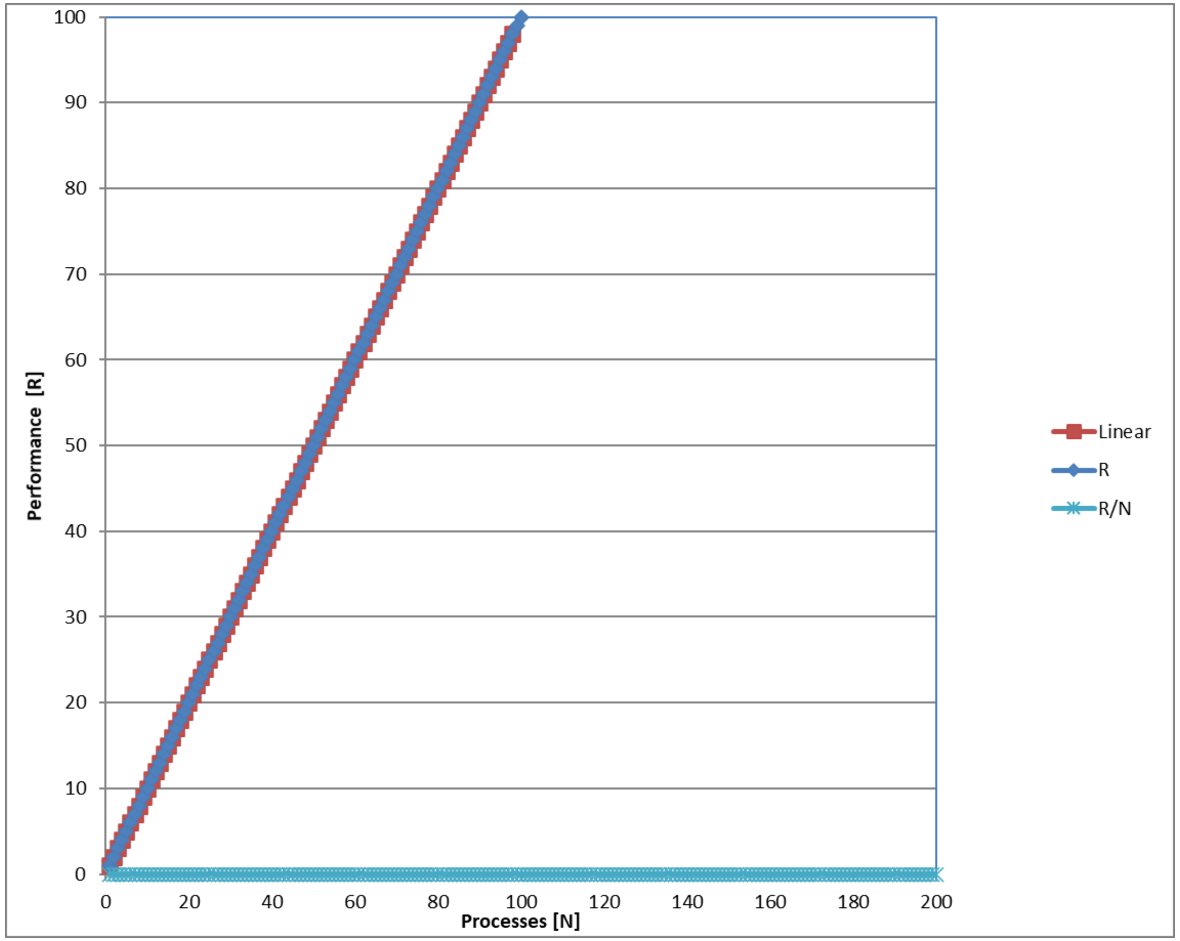
from parallel systems research

where R is the performance measure
C is a scalar
N is number of processes/agents/robots
α is contention (limited resources)
β is coherency (lack of communication)

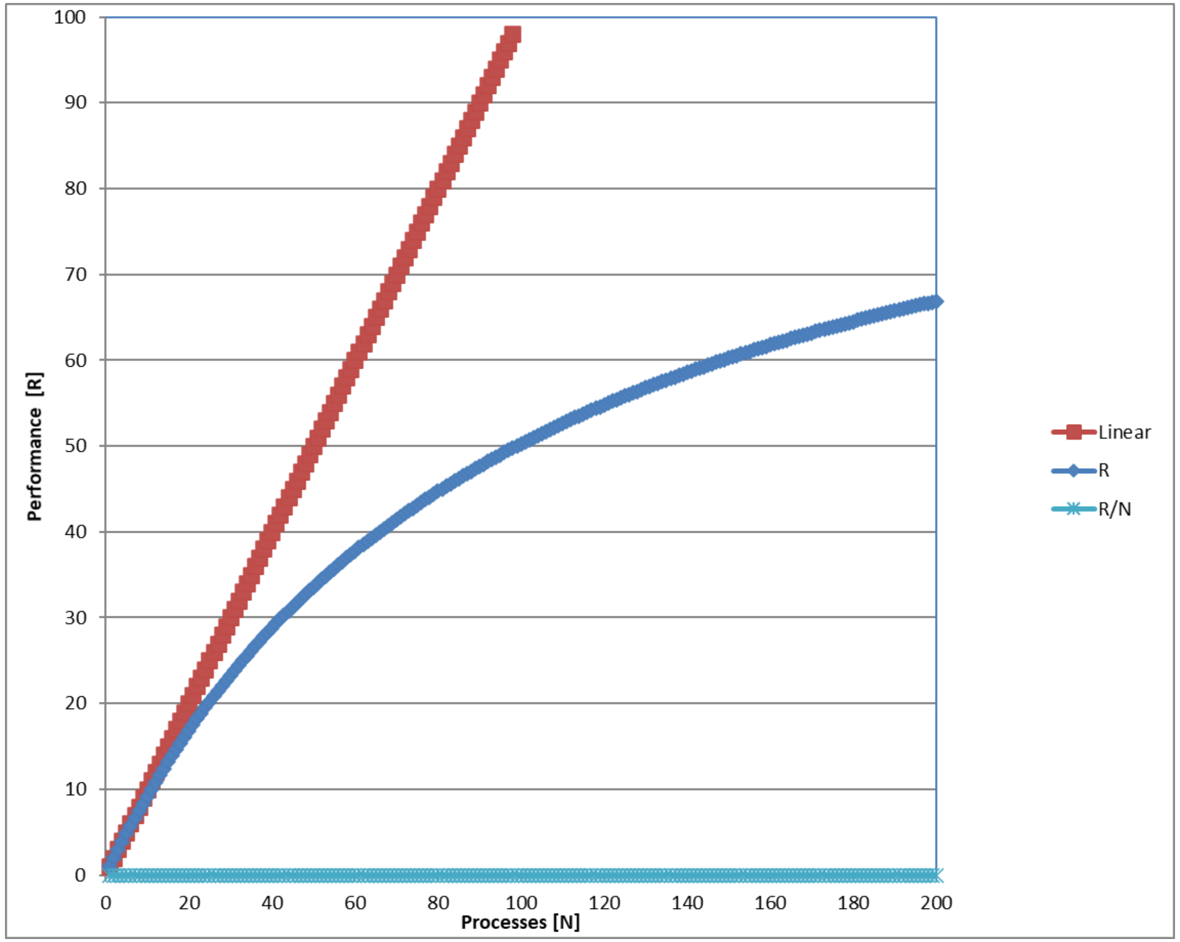
Typically fit equation to performance data

b) Describe different types of swarm performance:

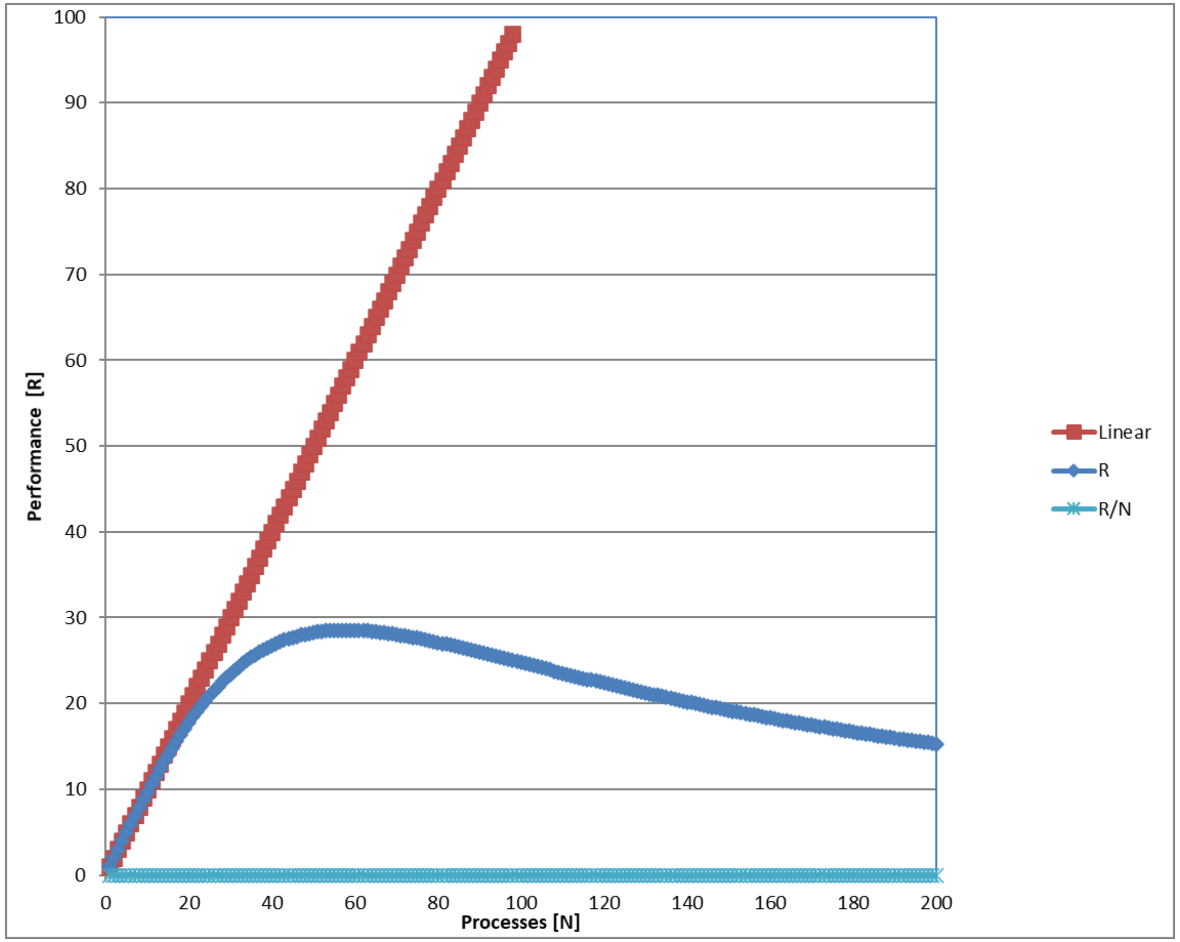
1) Linear speed up $\Rightarrow C = 1, \alpha = \beta = 0$



2) sub linear speed up → $C=1$, $\alpha=10^{-4}$, $\beta=0$

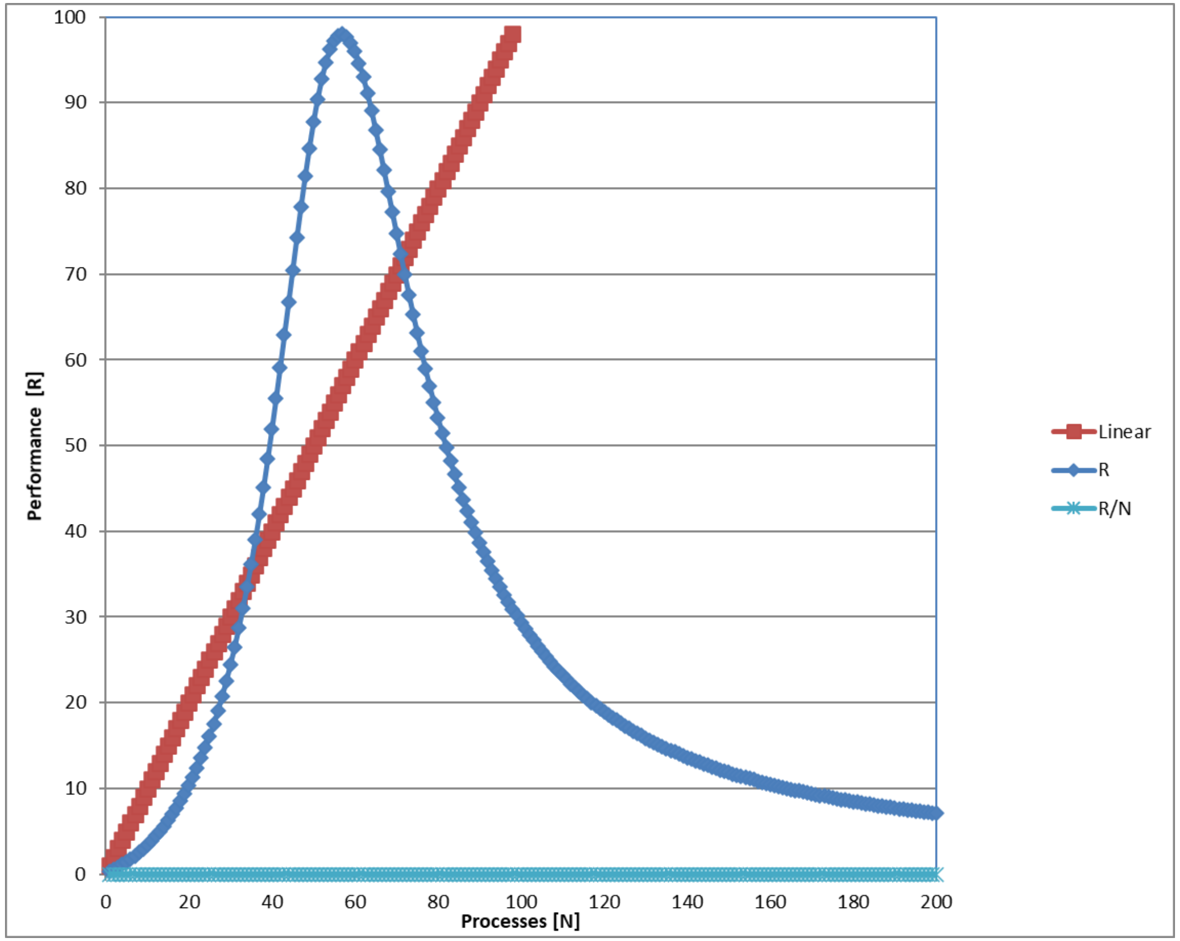


3) Decrease ⇒ $c=1$, $\alpha = 2\cdot 10^{-4}$ $\beta = 3\cdot 10^{-4}$

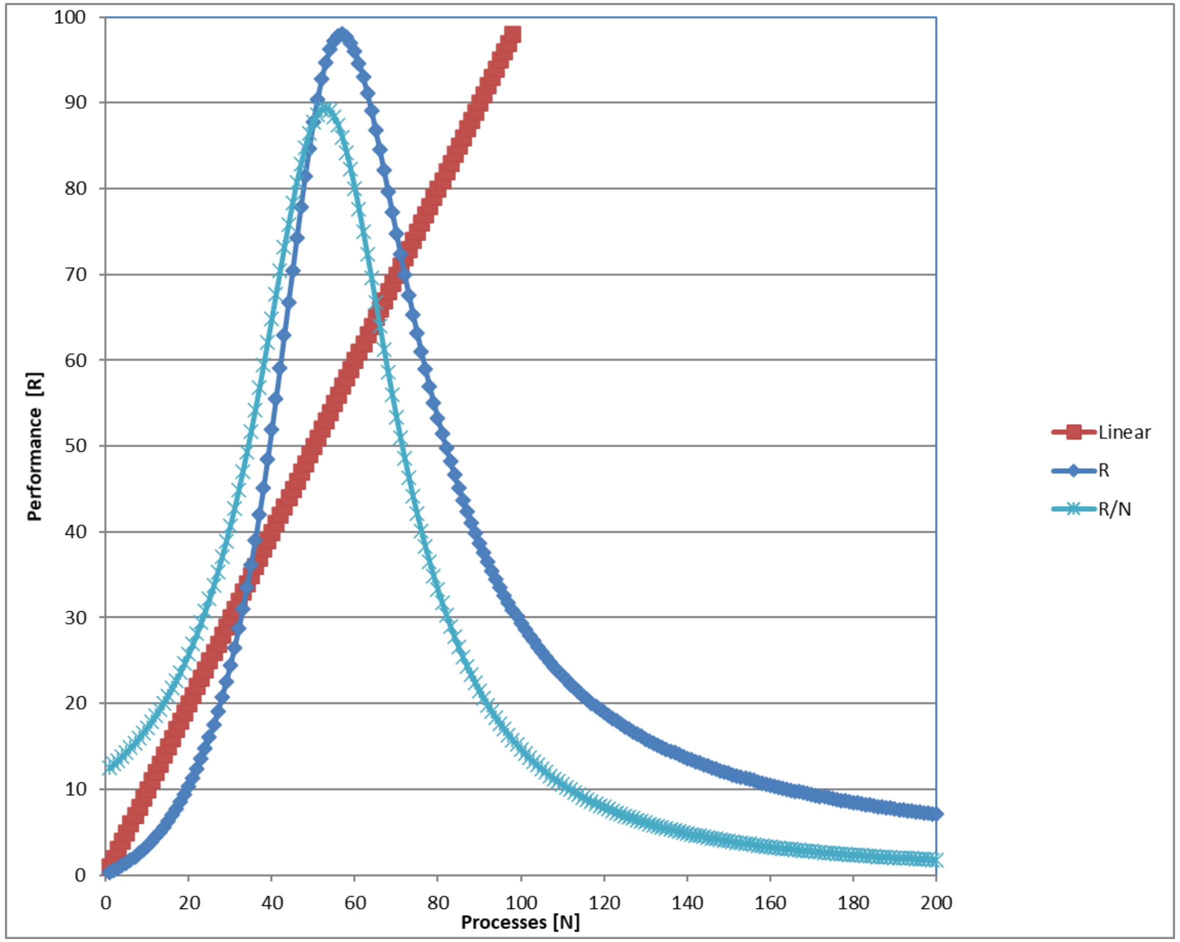


4) sub, super, optimal, inference regions

$\Rightarrow C \approx 0,25,\ \alpha = -0,0335,\ \beta = 0,00032$



c) Optimal performance derived from graph



* Optimal performance derived from graph

1) When N is unlimited

$$\frac{dR}{dN} = 0 \Rightarrow \text{max performance level at } N \approx 57$$

2) When N is limited and expect a constant probability of loss of agents

$$\frac{d\left(\frac{R}{N}\right)}{dN} = 0 \Rightarrow \text{optimal performance level at } N \approx 53$$

* Analytic method for calculating optimality using the chain rule:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

The case of unlimited supply of robots

$$\frac{dR}{dN} = 0 \qquad R = \frac{C \cdot N}{1 + \alpha(N-1) + \beta N(N-1)} = \frac{f}{g}$$

$$f = C \cdot N \qquad f' = C$$

$$g = 1 + \alpha(N-1) + \beta N(N-1)$$

$$= 1 + \alpha N - \alpha + \beta N^2 - \beta N \qquad g' = \alpha + 2\beta N - \beta$$

$$\frac{dR}{dN} = \frac{C \cdot (1 + \alpha N - \alpha + \beta N^2 - \beta N) - (\alpha + 2\beta N - \beta)\, C \cdot N}{\left[1 + \alpha(N-1) + \beta N(N-1)\right]^2} = 0$$

$$\Rightarrow C + C\alpha N - C\alpha + C\beta N^2 - C\beta N - C\alpha N - 2C\beta N^2 + C\beta N = 0$$

$$C - C\alpha - C\beta N^2 = 0$$

$$C(1 - \alpha - \beta N^2) = 0$$

$$C(1-\alpha-\beta N^2)=0$$

$$C=0 \quad \vee \quad 1-\alpha-\beta N^2=0$$

$$N^2=\frac{1-\alpha}{\beta}$$

$$N=\pm\sqrt{\frac{1-\alpha}{\beta}}$$

$$=\pm\sqrt{\frac{1-(-0{,}0335)}{0{,}00032}} \quad =\pm 56{,}83$$

$$\underline{\underline{N\approx 57}}$$

The case of limited supply of robots

$$\frac{d\left(\frac{R}{N}\right)}{dN} = 0 \qquad \frac{R}{N} = \frac{c}{1+\alpha(N-1)+\beta N(N-1)} = \frac{f}{g}$$

$$f = c \qquad f' = 0$$

$$g = 1+\alpha N-\alpha+\beta N^2-\beta N \qquad g' = \alpha+2\beta N-\beta$$

$$\left(\frac{R}{N}\right)' = \frac{0\cdot(1+\alpha N-\alpha+\beta N^2-\beta N)-(\alpha+2\beta N-\beta)c}{\left[1+\alpha N-\alpha+\beta N^2-\beta N\right]^2} = 0$$

$$\left(\frac{R}{N}\right)' = \frac{\cancel{0 \cdot (1+\alpha N - \alpha + \beta N^2 - \beta N)} - (\alpha + 2\beta N - \beta)C}{[1+\alpha N - \alpha + \beta N^2 - \beta N]^2} = 0$$

$$\Rightarrow \quad -(\alpha + 2\beta N - \beta) \cdot C = 0$$

$$C(\alpha + 2\beta N - \beta) = 0$$

$$C = 0 \quad \vee \quad \alpha + 2\beta N - \beta = 0$$

$$2\beta N = \beta - \alpha$$

$$N = \frac{\beta - \alpha}{2\beta} = \frac{0{,}00032 - (-0{,}0335)}{2 \cdot 0{,}00032} \approx 52{,}84$$

$$\underline{\underline{N \approx 53}}$$