Exercises from lecture 6 (Task allocation and self-assembly) TEK5010 Multiagent systems 2020

Question 1

Given 2 types of workers in a swarm with different response thresholds θ_1 and θ_2 reacting to a stimulus s. This can be modelled by the coupled differential equations:

$$\frac{\partial x_1}{\partial t} = T_{\theta_1}(s)(1 - x_1) - px_1$$

$$\frac{\partial x_2}{\partial t} = T_{\theta_2}(s)(1 - x_2) - px_2$$

$$\frac{\partial s}{\partial t} = \delta - \frac{\alpha}{N}(N_1 + N_2)$$

Where N_i is the number of workers of type i actively engaging in task related to stimuli and x_i is the fraction of worker of type i actively engaging in task associated with stimuli. N is the total number of workers of all types. T_{θ_i} is the stimuli response function of type i workers, α is a scaler related to the efficiency of doing task associated with stimuli and δ is a scales related to the increase in stimuli of not doing task associated with stimuli and p is the probability of a worker of any type stop doing task associated with stimuli (provided they already do the task).

a) Model and explain the stimuli response function T_{θ_i}. Are there any parameters that need to be fixed in order to solve the above differential equations analytically?

b) An analytic solution in terms of the probability of finding an active worker of type 1 of the above differential equations is given by:

$$x_1 = \frac{\chi + \left(\chi^2 + 4f(p+1)(z-1)\left(\frac{\delta}{\alpha}\right)\right)^{1/2}}{2f(p+1)(z-1)}$$

where $\chi = (z-1)\left(f + (p+1)\left(\frac{\delta}{\alpha}\right)\right) - z$ is a shift variable, $z = \theta_1^2/\theta_2^2$ and $f = n_1/N$ is the fraction of type 1 worker in the population.

Model the average fraction of active workers x_1 as a function of the fraction f of worker of type 1 in the population using parameters $\theta_1 = 8$, $\theta_2 = 1$, $p = 0.2$, $\delta = 1$ and $\alpha = 3$.

c) What happens if $\alpha \approx \delta$? And what happens when $\alpha \gg \delta$? Explain.