Solutions to exercises from Lecture 8 Non-cooperative game theory

TEK5010 Multiagent systems 2020

avestion

a) Identify all(pure mostegy) North, PodSo

North (0,0) PO (CD) ~ (DC) ~ (C,C) 80 (c/c)

* Nach, + Po, 1 So

Brmpn

H T

H I,-1 b 1,-1 b

T -1,1 b 1,-1 b

No Noon AU outcomes are Po AU outcomes are 80

* NOM, + PO, DSD

* Non, + PO, USO

Noth
$$(0,0) \vee (F,F)$$

Po $(0,0) \vee (F,F)$
So $(6,0) \vee (F,F)$

y What is Nam's Leoven?

Every game in aMich every player hes a finite set of persibilities hes a Nach equilibrium in mixed strategies.

Offen MSNE is hard to compute?

Mixed Brotegy Noch spirihlim: Find approach probabilities of playing the various ornategies. How the do you play a postionlar objector?

A mixed Arabegy over (M, D21..., SM) strategies is to find a probability distribution (P, P21..., Pm) of playing the strategis (so apporends become in different in their choice of strategy) () Find MSNE in the gomes p is probability of i planging D (1-p) is probability of i playing C 9 å probability of j panjing D (1-2) is probatility of j playing c >>> p+(1-p)=1 q+(1-g)=1 Mat is pada it $E_i(U_D) = E_i(U_C) \wedge E_i(U_D) = E_i(U_C)$

$$\Pi \qquad U_{S} = p2 + (1-p)U = U_{C}(1-q) = p \cdot 1 + (1-p)3$$

$$\Rightarrow 2p + 4 - 4p = p + 3 - 3p$$

$$2p - 4p - p + 3p = 3 - 4$$

$$0p = -1 \quad \emptyset$$

$$II E_{j}(U_{0}) = E_{j}(u_{c}) U_{c} = U_{(1-8)}$$

$$T = q^{2} + (1-q) \cdot 4 = q^{1} + (1-q)^{3}$$

$$2q + 4 - 4q = q + 3 - 3q$$

$$2q - 4q - 2 + 3q = 3 - 4$$

$$0q = -1$$

No MSNE, but pure shtepy esoit so Nan's theorem holds V

$$J U_{p} = 9.1 + (1-9)(-1) = U_{1-p} = 9(-1) + (1-9)1$$

$$9-1+9 = -9+1-9$$

$$9+9+9+9=1+1$$

$$49=2 \Rightarrow 9=\frac{1}{2}$$

II & (UH) = E; (UT)

II
$$P(-1)+(1-p)|_{2}p|_{+}(1-p)(-1)$$
 MSNE
 $-p+1-p=p-1+p$ $p=\frac{1}{2}, q=\frac{1}{2}$
 $-p-p-p=-1-1$
 $-4p=-2$ $p>\frac{1}{2}$

I
$$Ei(u_o) = Ei(u_F)$$

II $Ei(u_o) = Ei(u_F)$

I
$$Ei(u_o) = Ei(u_F)$$

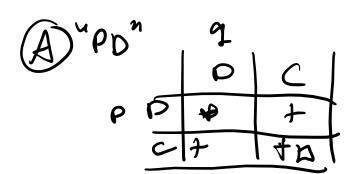
II $Ei(u_o) = Ei(u_F)$

II
$$2p + (i-p)0 = 0p + (1-p)3$$

 $2p = 3 - 3p$
 $5p = 3$
 $p = \frac{3}{5}$

MSNE

Summary of the 3 games



No MSNE?

- In PD it is not persolite to rench a takend out come that is also so

- Nather ferrem bolds he to gave NE

B	MP	9 H) T]
P	H	+ 13	40
•	T	+4	10

No pure NE?

MSNE p= 1/2/2

- Neel MSNE

to some MP

(C) 7º	Bsh	9		
		0	E	
P	0	*+0		
	F		*+0	

Mattiple pure NEP MSNE p= 3 A S= 3

- Need MSNE to chose between multiple pre NE?

* Pure Nah, & PO, DSO

$$I = E_i(u_c) = E_i(u_p)$$

$$I = E_i(u_c) = E_i(u_p)$$

$$\begin{aligned}
& = g(puop+(i-p)uco) + (i-g)(pup(+(i-p)ucc)) \\
& = g(puop+(i-p)uco) + (i-g)(pup(+(i-p)ucc)) \\
& = g(l+l-g)(l-g)(l-g)(l-g)(l-g) \\
& = l(g+(l-g)) \\
& = l(g+(l-g))
\end{aligned}$$