Solutions to exercises from Lecture 9 Voting

TEK5010 Multiagent systems 2020

Question 1

a) What outcome is winner in plurality?

No winner since ω_1, ω_2 and ω_3 all get one vote each.

b) Condorcet's paradox?

* If w_1 is winner: Ag_1 gets preferred outcome
  Ag_2 would prefer w_2 and w_3 over w_1
  Ag_3 would prefer w_3 and w_4 over w_1

$\Rightarrow w_3$ is preferred by 2/3 of voters compared to w_1

* If w_2 is winner: Ag_1 would prefer w_1 over w_2
  Ag_2 gets preferred outcome
  Ag_3 would prefer w_3, w_4 and w_1 over w_2

$\Rightarrow w_1$ is preferred by 2/3 of voters compared to w_2

* If w_3 is winner : Ag_1 would prefer w_1 and w_2 over w_3

Ag_2 would prefer w_2 over w_3

Ag_3 gets preferred outcome

$\Rightarrow$ w_2 is preferred by $2/3$ of voters compared to w_3

* If w_4 is winner : Ag_1 would prefer w_1, w_2, w_3 to w_4

Ag_2 would prefer w_2, w_3, w_1 to w_4

Ag_3 would prefer w_3 to w_4

$\Rightarrow$ w_3 is preferred by all voters compared to w_4

⇒ We have a condorcet's paradox!
No matter outcome we pick as winner, majority of voters would prefer another outcome.

c) Is this election possible to manipulate?

Tactical voting is when an agent i can unilaterally change the outcome of the social order in a favourable way by misrepresenting its preferences.

$$f(\bar{\omega}_1, \bar{\omega}_2, \ldots, \bar{\omega}_i', \ldots, \bar{\omega}_N) \succ_i f(\bar{\omega}_1, \bar{\omega}_2, \ldots, \bar{\omega}_i, \ldots, \bar{\omega}_N)$$

This election is manipulable.

If Ag_1 change preference list to

$\overline{w_1}'(w_2, w_1, w_3, w_4)$

outcome w_2 is elected in plurality, and w_2 is preferred by w_3 and w_4 for Ag_1.

* How can we safeguard against strategic manipulation?

All elections are manipulable according to the Gibbard - Satterthwaite theorem.

But, we can make elections hard to manipulate by increasing complexity of calculation.

e.g. second-order Copeland is NP-hard to manipulate.

d) Calculate Borda for all outcomes.

$$BC_{\omega_j} = \sum_{i=1}^{N} k - \text{rank}(\overline{\omega}_i(\omega_j))$$

where $k = |\Omega|$ number of outcomes

N is number of voters

$\overline{\omega}_i$ is preference list of voter i

	$\overline{w_1}$	$\overline{w_2}$	$\overline{w_3}$	Borda
w_1	3	1	1	5
w_2	2	3	0	5
w_3	1	2	3	6
w_4	0	0	2	2

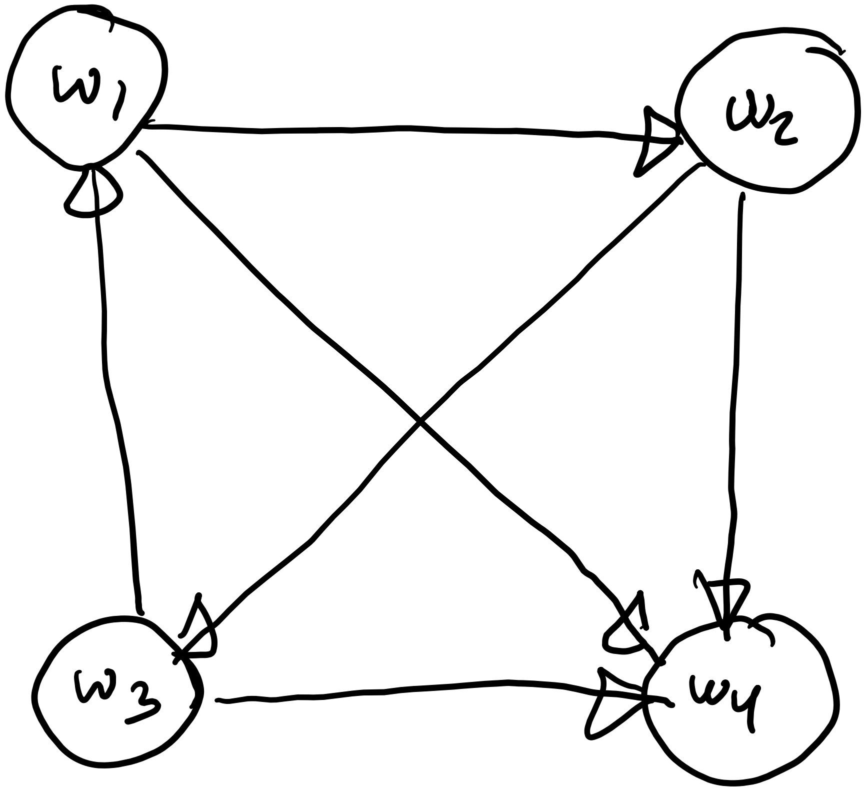
e) Write up all pairwise elections and draw the majority graph.

$w_1 \succ w_2$ $\quad$ $w_2 \succ w_3$ $\quad$ $w_3 \succ w_4$

$w_1 \prec w_3$ $\quad$ $w_2 \succ w_4$

$w_1 \succ w_4$

$$\left(\begin{array}{l} \bar{w}_1 = (w_1, w_2, w_3, w_4) \\ \bar{w}_2 = (w_2, w_3, w_1, w_4) \\ \bar{w}_3 = (w_3, w_4, w_1, w_2) \end{array} \right)$$



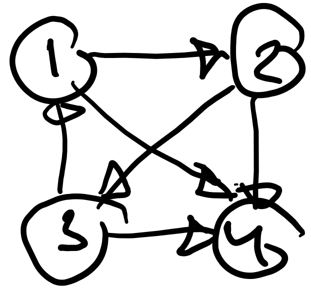
* No condorcet's winner

* w_1, w_2 and w_3 are all possible winners

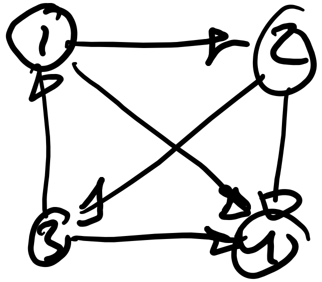
* w_4 always loose

f) What is best out come if Slater rule is used?

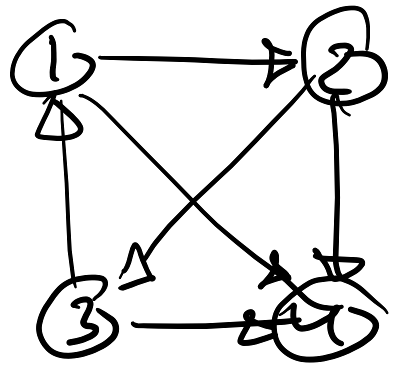
order	flips	
$w_1 \succ w_2 \succ w_3 \succ w_4$	1	$= 1$
$w_1 \succ w_2 \succ w_4 \succ w_3$	$1+1$	$= 2$
$w_1 \succ w_3 \succ w_2 \succ w_4$	$1+1$	$= 2$
$w_1 \succ w_3 \succ w_4 \succ w_2$	$1+1+1$	$= 3$
$w_1 \succ w_4 \succ w_2 \succ w_3$	$1+2$	$= 3$
$w_1 \succ w_4 \succ w_3 \succ w_2$	$1+2+1$	$= 4$



	order	flips	
	$w_2 \succ w_1 \succ w_3 \succ w_4$	$1+1$	$= 2$
	$w_2 \succ w_1 \succ w_4 \succ w_3$	$1+1+1$	$= 3$
*	$w_2 \succ w_3 \succ w_1 \succ w_4$	1	$= 1$
	$w_2 \succ w_3 \succ w_4 \succ w_1$	$1+1$	$= 2$
	$w_2 \succ w_4 \succ w_1 \succ w_3$	$1+2+1$	$= 4$
	$w_2 \succ w_4 \succ w_3 \succ w_1$	$1+2$	$= 1$



order	flips	
$w_3 > w_1 > w_2 > w_4$	1	$= 1$
$w_3 > w_1 > w_4 > w_2$	$1+1$	$= 2$
$w_3 > w_2 > w_1 > w_4$	$1+1$	$= 2$
$w_3 > w_2 > w_4 > w_1$	$1+1+1$	$= 3$
$w_3 > w_4 > w_1 > w_2$	$1+2$	$= 3$
$w_3 > w_4 > w_2 > w_1$	$1+2+1$	$= 4$



$w_4 > \ldots$ requires at least 3 flips

So, according to the Slater ranking

$w_1 \succ w_2 \succ w_3 \succ w_4$

$w_2 \succ w_3 \succ w_1 \succ w_4$

$w_3 \succ w_1 \succ w_2 \succ w_4$

are all social orderings that require one edge flip in the majority graph, and we are undicided in terms of the Slater rule.

NP-hard to calculate Slater!

g) Summary

Plurality : No clear winner

Borda : $w_3 > w_2 = w_1 > w_4$

Slater : 3 social orders are equivalent

$(w_1 > w_2 > w_3 > w_4) \vee (w_2 > w_3 > w_1 > w_4) \vee$
$(w_3 > w_1 > w_2 > w_4)$

$\Rightarrow$ $\underline{\underline{w_3 > w_1 > w_2 > w_4}}$ is probably best overall !