TEK5010 Multiagent systems

Lecture 10: Cooperative game theory

Exercise: Cooperative games 2

Question 1

a) 3 agents Ag={a,b,c} are evaluating their expected utility in terms of collaborating in different groups. However, coalitional games present several problems from a computational perspective. Explain what you understand these problems to be by giving two examples of representation of the characteristic function of the 3 agents:

$v(\emptyset)=0$
$v(\{a\})=3$, $v(\{b\})=0$, $v(\{c\})=7$
$v(\{a,b\})=13$, $v(\{a,c\})=16$, $v(\{b,c\})=7$
$v(\{a,b,c\})=26$

The characteristic function is hard to represent, two possible representations:

1) Marginal contribution nets
   Complete and succinct

2) Induced subgraphs
   Succinct but not complete

The marginal contribution net represent the characteristic function as a set of rules:

$$v_{rs}(C) = \sum_{\varphi \rightarrow x \in rs_C} x$$

Where $\varphi \longrightarrow x$ is a rule in the rule set rs_C

$\nu(\emptyset)=0$ no rule

$\nu(\{a\})=3$ gives rule $a \longrightarrow 3$

$\nu(\{b\})=0$ gives rule $b \longrightarrow 0$

$\nu(\{c\})=7$ gives rule $c \longrightarrow 7$

$\nu(\{a,b\})=13$ using rule $a \longrightarrow 3 + b \longrightarrow 0 + a \wedge b \longrightarrow x = 13$
gives rule $a \wedge b \longrightarrow 10$

$\nu(\{a,c\})=16$ using rule $a \longrightarrow 3 + c \longrightarrow 7 + a \wedge c \longrightarrow x = 16$
gives rule $a \wedge c \longrightarrow 6$

$\nu(\{b,c\})=7$ using rule $b \longrightarrow 0 + c \longrightarrow 7 + b \wedge c \longrightarrow x = 7$
gives rule $b \wedge c \longrightarrow 0$

$$v(\{a,b,c\})=26 \text{ using rule } a \rightarrow 3 + b \rightarrow 0 + c \rightarrow 7$$
$$+ a \wedge b \rightarrow 10 + a \wedge c \rightarrow 6 + b \wedge c \rightarrow 0$$
$$+ a \wedge b \wedge c \rightarrow x = 26$$

giving rule $a \wedge b \wedge c \rightarrow 0$

Which gives the following rule set:

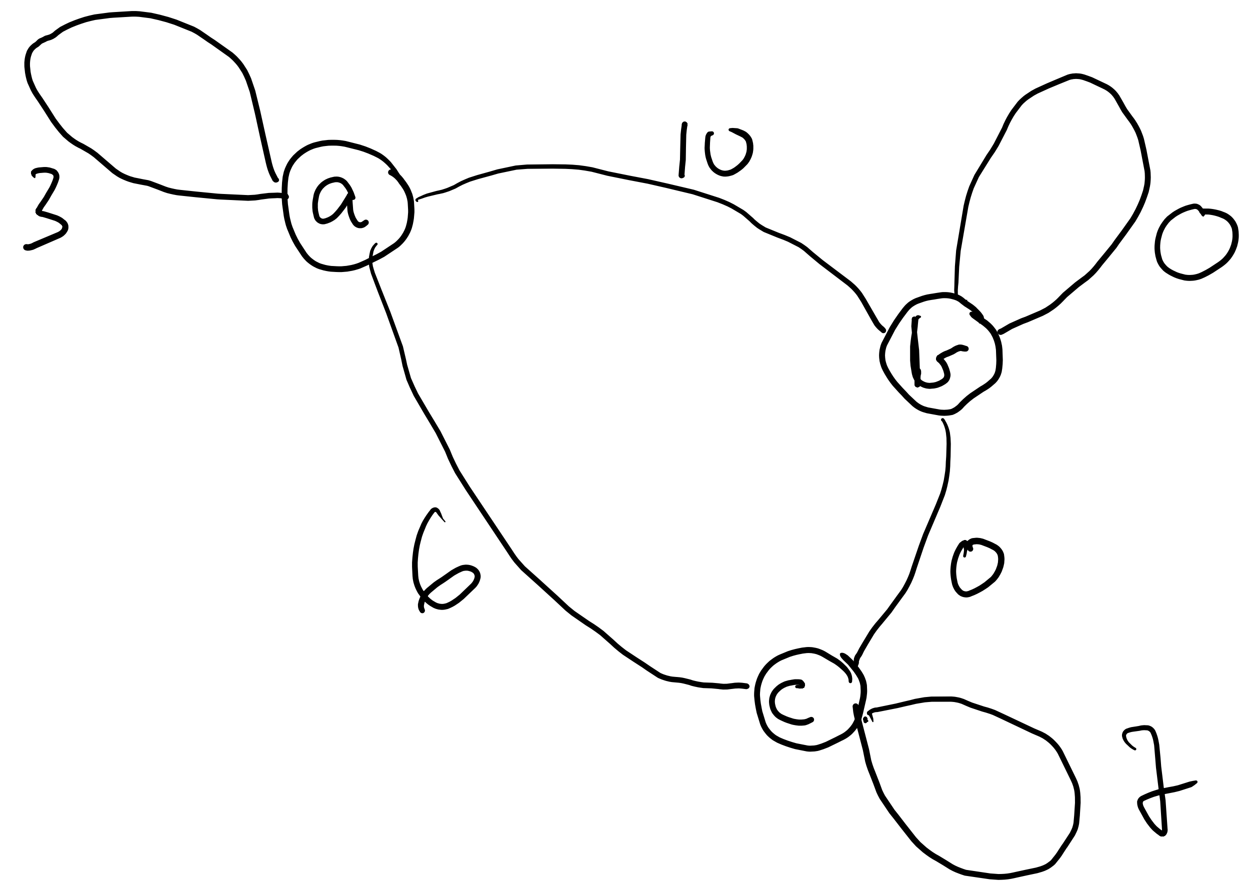
$a \rightarrow 3$

$c \rightarrow 7$

$a \wedge b \rightarrow 10$

$a \wedge c \rightarrow 6$

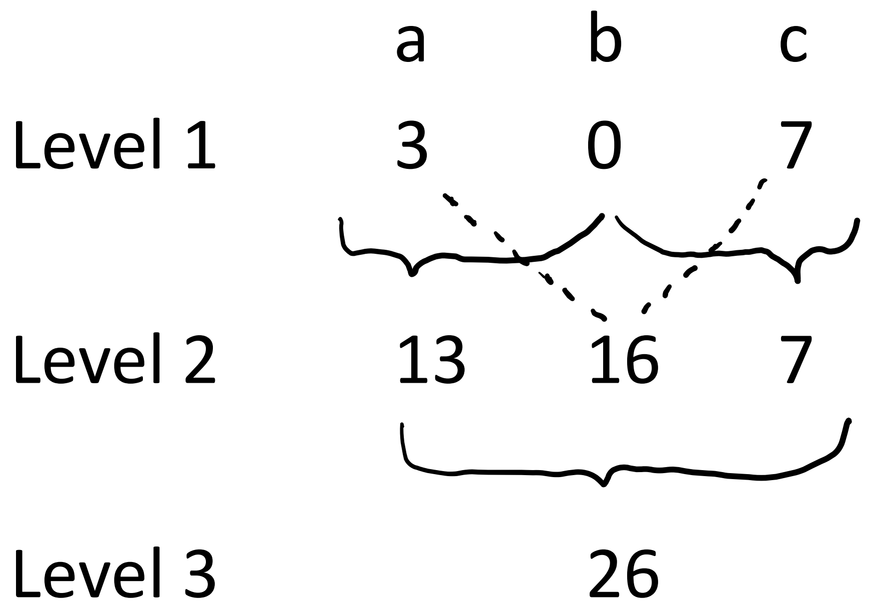
The induced subgraph is an undirected weighted graph, nodes are given by the agents Ag = {a, b, …., N} and edges are weights corresponding to the rules in the marginal contribution net.



b) Solve the game.

The game is solved by checking if the core is non-empty.

The core is the set of coalitions that no other coalition objects to.


a b c
Level 1 3 0 7
Level 2 13 16 7
Level 3 26

b = 0

a	b	c	
26	0	0	{c} objects
25	0	1	{c} objects
⋮	⋮	⋮	
13	0	7	Core is non-empty
⋮	⋮	⋮	
13	0	13	
12	0	14	{ab} objects
⋮	⋮	⋮	
0	0	26	{a} {ab} objects

b = 1

a	b	c	
25	1	0	{c} objects
24	1	1	{c} objects
⋮	⋮	⋮	
18	1	7	Core is non-empty
⋮	⋮	⋮	
12	1	13	
11	1	14	{ab} objects
⋮	⋮	⋮	
0	1	25	{a} {ab} objects

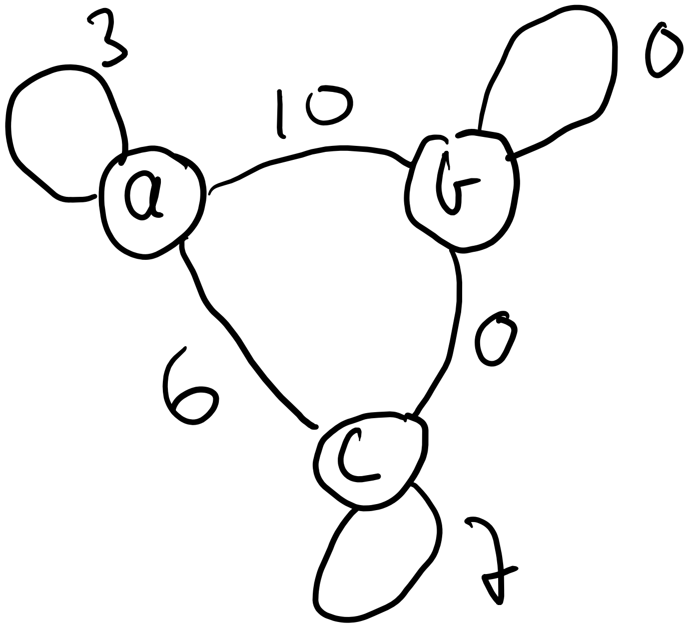
$\Rightarrow$ There exist some non-empty core,
i.e. the game is solvable or stable.

c) Calculate the Shapley value for each player in this game.

Direct calculation of Shapley:

	a	b	c
{a b c}	3	10	13
{a c b}	3	10	13
{b a c}	13	0	13
{b c a}	19	0	7
{c a b}	9	10	7
{c b a}	19	0	7
	a=66/6	b=30/6	c=60/6
	=11	=5	=10

Shapley from induced subgraph:



$\text{Shapley}_a = 3 + 10/2 + 6/2 = 11$

$\text{Shapley}_b = 0 + 10/2 + 0/2 = 5$

$\text{Shapley}_c = 7 + 0/2 + 6/2 = 10$