TEK5010 MAS

Lecture 11 : Auction

Exercise: Auction 1

Question 1

a) What is the valuation function for bidders in terms of the following bundles of good $Z = \{a, b, c\}$ given the bids of $A_5 = \{1, 2, 3\}$

$$B_1 = (\{a\}, 3) \text{ XOR } (\{c\}, 1) \text{ XOR } (\{a, b\}, 5) \text{ XOR } (\{a, b, c\}, 7)$$

$$B_2 = (\{c\}, 5) \text{ XOR } (\{a, b\}, 6) \text{ XOR } (\{a, b, c\}, 14)$$

$$B_3 = (\{b\}, 3) \text{ XOR } (\{c\}, 4) \text{ XOR } (\{a, b\}, 11) \text{ XOR } (\{b, c\}, 15)$$

PS Only one bundle per agent

	1	2	3
$v(\{a\})$	3	0	0
$v(\{b\})$	0	0	3
$v(\{c\})$	1	5	4
$v(\{a,b\})$	5	6	11
$v(\{a,c\})$	3	5	4
$v(\{b,c\})$	1	5	15
$v(\{a,b,c\})$	7	14	15

4 Determine the winner in this auction assuming auctioneer is maximizing social welfare. (Hint, enumerate all possible bundles)

	1	2	3	T
$\{a\}\{b\}\{c\}$	$\{a\}=3$	$\{c\}=5$	$\{b\}=3$	11
$\{ab\}\{c\}$	0	$\{c\}=5$	$\{ab\}=11$	16
$\{ac\}\{b\}$	0	$\{ac\}=5$	$\{b\}=3$	8
$\{bc\}\{a\}$	$\{a\}=3$	0	$\{bc\}=15$	18
$\{abc\}$	0	0	$\{abc\}=15$	15

⇒ We would give $\{a\}$ to $A_{3_1}=3$, 0 to $A_{3_2}=0$ and $\{bc\}$ to $A_{3_3}=15$ if max SO

c) What is the price each agent must pay if we use the VCG mechanism instead?

$\overline{Ag_1}$	1	2	3	T
$\{a\}\{b\}\{c\}$	∞	$\{c\}=5$	$\{b\}=3$	8
$\{ab\}\{c\}$	∞	$\{c\}=5$	$\{ab\}=11$	16
$\{ac\}\{b\}$	∞	$\{ac\}=5$	$\{b\}=3$	8
$\{bc\}\{a\}$	∞	0	$\{bc\}=15$	15
$\{abc\}$	∞	0	$\{abc\}=15$	15

$$p_i = \sum_{j \in A_3/\{i\}} \left(Ag_{j,\overline{Ag_i}} - Ag_{j,Ag_i} \right)$$

$$p_1 = (5-0)+(11-15) = 5-4 = 1$$

A_{S_2}

	1	2	3	T
$\{a\}\{b\}\{c\}$	$\{a\}=3$	$\times$	$\{c\}=4$	7
$\{ab\}\{c\}$	$\{c\}=1$	$\times$	$\{ab\}=11$	12
$\{ac\}\{b\}$	$\{ac\}=3$	$\times$	$\{b\}=3$	6
$\{bc\}\{a\}$	$\{a\}=3$	$\times$	$\{bc\}=15$	18
$\{abc\}$	0	$\times$	$\{abc\}=15$	15

$$p_2 = (3-3) + (5-15) = 0$$

Dummy player

A_{S_3}	1	2	3	T
$\{a\}\{b\}\{c\}$	$\{a\}=3$	$\{c\}=5$	✗	8
$\{ab\}\{c\}$	$\{ab\}=5$	$\{c\}=5$	✗	10
$\{ac\}\{b\}$	0	$\{ac\}=5$	✗	5
$\{bc\}\{a\}$	$\{a\}=3$	$\{bc\}=5$	✗	8
$\{abc\}$	○	$\{abc\}=14$	✗	(14)

$$P_3 = (0-3)+(14-0) = -3+14 = 11$$