TEK 5010 MAS

Lecture: 3

Exercise: ACO

Question 1

a) What is the transition rule (the probability of going to city j) in AS? Explain the variables and the parameters

$$p_{ij}^{k} = \frac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\sum_{c_{il} \in N(s^p)} \tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}}$$ normalization

where k is ant

i is start city, j is next city

$c_{il} \in N(s^p)$ is a city not visited yet

α, β are non linear parameters

$\eta_{ij} = \frac{1}{d_{ij}}$ is inverse length between city i and city j.

τ_{ij} is pheromone concentration on edge between city i and j

b) What is the pheromone update rule in AS? Also explain the variables and parameters

$$\tau'_{ij} \leftarrow (1-\rho)\tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

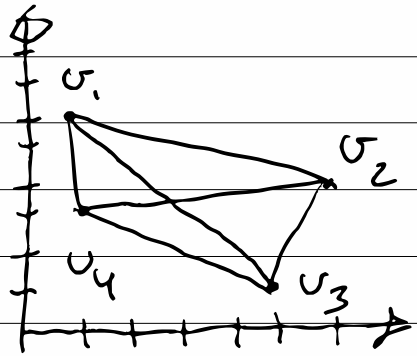
where τ_{ij} is pheromone concentration on edge between city i and j

ρ is evaporation rate

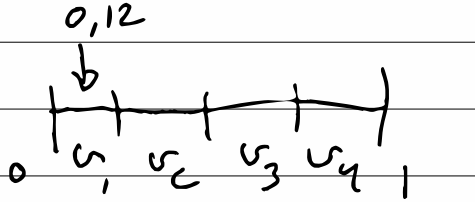
$\Delta\tau_{ij}$ is pheromones laid by ant k on edge ij if part of tour

$$= \begin{cases} Q/L_k & \text{if ant k used edge ij on tour of length } L_k \\ 0 & \text{if not part of tour} \end{cases}$$

c) Calculate a tour of one of the ants in TSP using ACO-AS.



$d_{12} = 5,099$ $\eta_{12} = 0,196$
$d_{23} = 3,16$ $\eta_{23} = 0,316$
$d_{34} = 4.47$ $\eta_{34} = 0,224$
$d_{14} = 2$ $\eta_{14} = 0.5$
$d_{13} = 5,652$ $\eta_{13} = 0,177$
$d_{24} = 5,099$ $\eta_{24} = 0,196$



$\Rightarrow k$ in v_1

* $v_i = v_1 \Rightarrow v_j \in \{v_2, v_3, v_4\}$

$$\sum \tau_{ij}^{\alpha} \eta_{ij}^{\beta} = \sum \tau_{1j}^{1} \eta_{1j}^{5} = \tau_0 \left(\eta_{12}^{5} + \eta_{13}^{5} + \eta_{14}^{5}\right)$$

$$= 10^{-6}\left(0,196^5 + 0,177^5 + 0.5^5\right) = 3,17 \cdot 10^{-8}$$

boring to v_2

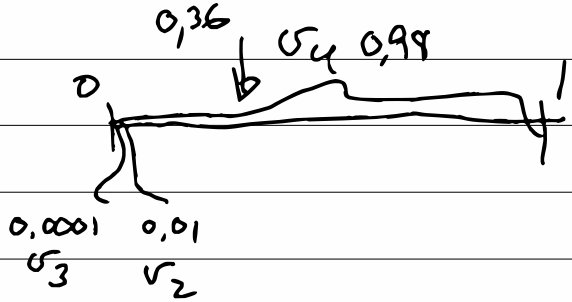
$$p_{12} = \frac{10^{-6} \cdot 0.196^5}{3.12 \cdot 10^{-8}} = 0.01$$

Going to v_3

$$p_{13} = \frac{10^{-6} \cdot 0,1725}{3,17 \cdot 10^{-8}} = 0,0001$$

Going to v_4

$$p_{14} = \frac{10^{-6} 0,5^5}{3,17 \cdot 10^{-8}} = 0,98$$



and k=1 chose city 4 (after city 1)

* $v_i = v_4 \quad v_j \in \{v_2, v_3\}$

$$\sum \pi_{4j} n_{4j}^5 = \pi_0 \left(n_{42}^5 + n_{43}^5\right)$$

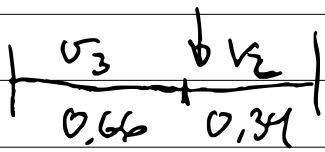
$$= 10^{-6}\left(0.196^5 + 0,224^5\right) = 8.53 \cdot 10^{-10}$$

Going to v_3

$$p_{43} = \frac{10^{-6} \cdot 0.224^5}{8.53 \cdot 10^{-10}} \approx 0.66$$

Going to v_2

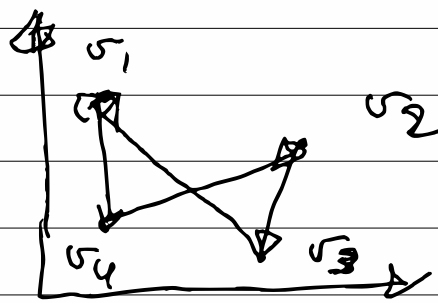
$$p_{42} = \frac{10^{-6} \cdot 0.172^5}{8.53 \cdot 10^{-10}} \approx 0.34$$



ant k=1 chose city 2 (after city 4)

* $v_i = v_2 \quad v_j \in \{v_3\}$

must go to v_3



Tour length

$$L = 2 + 5,099 + 3,161 + 5.66$$
$$= 15.92$$

d) Calculate the tours of the rest of the ants assuming $m = n$ where m is the number of ants and n is number of cities.

Simulate by computer

e) Apply the AS pheromone update rule to the system. What is best tour now?

$$\tau'_{ij} \leftarrow (1-\rho)\tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

$$\tau'_{14} \leftarrow \left(1-\frac{1}{2}\right)\cdot 10^{-6} + \frac{100}{15,96} + \ldots +$$
$$0,5\cdot 10^{-6} + 6.28 + \ldots +$$