

**Question 1**

The Universal Scalability Law (Gunther, 1993) is given by the following equation:

$$R(N) = C \frac{N}{1 + \alpha(N - 1) + \beta N(N - 1)}$$

a) Given the following measured data set from a swarm system:

$N$	0	15	30	60	100	125	150
$R_1(N)$	0	7	24	22	25	29	26

what numerical parameter values would be a reasonable assumption for “manually” fitting a USL curve to the data? You can assume that  $C=1$  but  $\alpha$  and  $\beta$  are unknown.

We also have data from a second swarm system:

$N$	0	15	30	60	100	125	150
$R_2(N)$	0	20	50	65	30	26	15

what would be the corresponding parameter values in this case?

- b) Why must we use non-linear regression methods for curve fitting the USL? And why does this complicate the analysis?
- c) Could you use the Python library SciPy to curve fit the USL to the given data?

Hint, you could base your answer on tweaking the examples given in

<https://hernandis.me/2020/04/05/three-examples-of-nonlinear-least-squares-fitting-in-python-with-sciipy.html>

- d) Optional: Could you explain the underlying mathematics for curve fitting the USL using non-linear least squares?

Hint, a good explanation on non-linear regression for curve fitting is given in the following link

<http://www.incertitudes.fr/book.pdf>