TER5010 MAS

Lecture 4: SR1

Exercise: Analysis of USL performance

Question 1

a) In terms of using USL for modelling the performance of swarm systems, could you explain the variables in the model?

USL is from parallel processing systems

$$R(N) = C \frac{N}{1+\alpha(N-1)+\beta N(N-1)}$$

where R is the performance measure
N is the number of processes/agents
C is a scalar
α is contention (limited resources)
β is coherency (lack of communication)

b) Could you describe the performance of the swarm system when best fit of model is given by the parameters $C=1$ and $\alpha=\beta=0$? Make a plot of the US2 spanning at least $N=200$ processes.

$C=1, \alpha=\beta=0 \Rightarrow$ linear speedup

c) How would that change if best fit is given by $C=1, \alpha=0{,}0001$ and $\beta=0$?

$C=1, \alpha=0.0001, \beta=0 \Rightarrow$ Sublinear speedup

d) Or $C=1, \alpha=0{,}0007, \beta=0{,}0003$?

$C=1, \alpha=0.0007, \beta=0.0003 \Rightarrow$ Decrease

e) Now, given the parameters $C=0.25$, $\alpha=-0.0335$, $\beta=0.00032$, how would you characterize the performance of this swarm system?
What is the optimal performance?

$C=0.25$, $\alpha=-0.0335$, $\beta=0.00032$
$\Rightarrow$ sub, super, optimal, inference regions
$\Rightarrow$ Optimal performance at $N \approx 57$

f) What would constitute an optimal operation level for this swarm system considering that a loss of processes would have to be replaced with a limited number of robots available?

* optimal performance derived from graph

1) When N is unlimited
$\frac{dR}{dN} = 0 \Rightarrow$ optimal at $N \approx 57$

2) When N is limited and expect a constant probability of loss of agents
$\frac{d(R/N)}{dN} = 0 \Rightarrow$ optimal at $N \approx 53$

* Analytic method for calculating optimality using the chain rule:

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

1, The case of unlimited supply of robots

$$\frac{dR}{dN} = 0 \quad R = \frac{C \cdot N}{1+\alpha(N-1)+\beta N(N-1)} = \frac{f}{g}$$

$$f = C \cdot N$$

$$g = 1+\alpha(N-1)+\beta N(N-1)$$

$$= 1+\alpha N-\alpha+\beta N^2-\beta N$$

$$f' = C$$

$$g' = \alpha+2\beta N-\beta$$

$$\frac{dR}{dN} = \frac{C\cdot(1+\alpha N-\alpha+\beta N^2-\beta N)-(\alpha+2\beta N-\beta)CN}{[1+\alpha(N-1)+\beta N(N-1)]^2} = 0$$

$\Rightarrow C + C\alpha N - C\alpha + C\beta N^2 - C\beta N - C\alpha N - 2C\beta N^2$
$+ C\beta N = 0$

$C - C\alpha - C\beta N^2 = 0$
$C(1 - \alpha - \beta N^2) = 0$

$C = 0 \quad \vee \quad 1 - \alpha - \beta N^2 = 0$

$$N^2 = \frac{1-\alpha}{\beta}$$

$$N = \pm\sqrt{\frac{1-\alpha}{\beta}}$$

$$= \pm\sqrt{\frac{1-(-0{,}0335)}{0{,}00032}} = \pm 56{,}83$$

$$\underline{\underline{N \approx 57}}$$

2) The case of limited supply of robots

$$\frac{d(R/N)}{dN} = 0 \qquad \frac{R}{N} = \frac{C \cdot N}{1+\alpha(N-1)+\beta N(N-1)} \cdot \frac{1}{N}$$

$$= \frac{C}{1+\alpha(N-1)+\beta N(N-1)} = \frac{f}{g}$$

$f = C$

$g = 1+\alpha(N-1)+\beta N(N-1)$

$f' = 0$

$g' = \alpha + 2\beta N - \beta$ (same as unlimited)

$$\frac{d(R/N)}{dN} = \frac{0 \cdot (1+\alpha(N-1)+\beta N(N-1)) - (\alpha+2\beta N-\beta)C}{[1+\alpha(N-1)+\beta N(N-1)]^2} = 0$$

$$\frac{d(R/N)}{dN} = \frac{-(\alpha+2\beta N-\beta)C}{[1+\alpha(N-1)+\beta N(N-1)]^2} = 0$$

$\Rightarrow \; -(\alpha+2\beta N-\beta)C = 0$

$$L(\alpha + 2\beta N - \beta) = 0$$

$$L = 0 \quad \text{r} \quad \alpha + 2\beta N - \beta = 0$$

$$2\beta N = \beta - \alpha$$

$$N = \frac{\beta - \alpha}{2\beta}$$

$$= \frac{0{,}00032 - (-0{,}0335)}{0{,}00032}$$

$$= 52{,}84$$

$$\underline{\underline{N \approx 53}}$$