

# TEK 5010 MAS

## Lecture 9: Voting

### Exercise: Group decisions

#### Question 1

a) What outcome is winner if voting is plurality?

$$\bar{w}_1 = \{w_1, w_2, w_3, w_4\}$$

$$\bar{w}_2 = \{w_2, w_3, w_4, w_1\}$$

$$\bar{w}_3 = \{w_3, w_4, w_1, w_2\}$$

No winner in plurality since different first place for all votes

b) Do we have a Condorcet's paradox here?

If  $w_1$  is winner  $\bar{w}_2$  prefer  $w_2, w_3, w_4$   
Yes  $\bar{w}_3$  prefer  $w_3, w_4$

if  $w_2$  is winner  
Yes  $\bar{w}_1$  prefer  $w_1$ ,  
 $\bar{w}_3$  prefer  $w_3, w_4, w_1$

if  $w_3$  is winner  
Yes  $\bar{w}_1$  prefer  $w_1, w_2$   
 $\bar{w}_2$  prefer  $w_2$

⇒ We do have a Condorcet's paradox? No matter what candidate we pick the majority would prefer another candidate.

c) What is tactical voting? Is it possible for voters to manipulate this election? How can we safeguard against strategic manipulation?

Tactical voting is when an agent  $i$  unilaterally can influence the ordering of outcomes in the social preference list

by misrepresenting their individual preference order

$$f(\bar{w}_1, \dots, \bar{w}'_1, \dots, \bar{w}_N) \succ_i f(\bar{w}_1, \dots, \bar{w}_1, \dots, \bar{w}_N)$$

\*  $\bar{w}_1$  would rather have  $w_2$  than  $w_3$  or  $w_4$

$\bar{w}_2$  would rather have  $w_3$  than  $w_1$  or  $w_4$

$\Rightarrow$  Both  $\bar{w}_1$  and  $\bar{w}_2$  could change 1 and 2 place to secure  $w_2$  or  $w_3$  being chosen.

\*  $\bar{w}_3$  would rather have  $w_4$  than  $w_1$  or  $w_2$  (or  $w_1$  over  $w_2$ )

$\Rightarrow$  Not possible for  $\bar{w}_3$  to manipulate election on its own

(unless 3 rank is considered)

\* By making it NP-hard to manipulate elections,  
e.g. Second-order Copeland.

d) What is the Borda count for each outcome? Apply the Borda rule. Is this outcome Pareto efficient (as in Arrow's theorem)? Explain.

Calculate Borda count

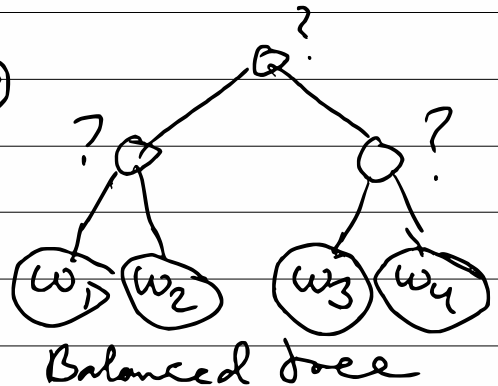
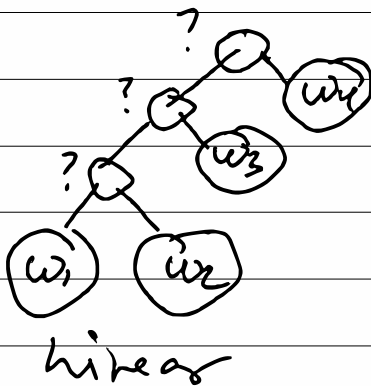
$$BC(w_j) = \sum_{i=1}^N k - \text{rank}(\bar{w}_i(w_j))$$

	$\bar{w}_1$	$\bar{w}_2$	$\bar{w}_3$	BC
$w_1$	3	0	1	4
$w_2$	2	3	0	5
$w_3$	1	2	3	6
$w_4$	0	1	2	3

⇒  $w_3$  is optimal outcome corresponding with the Borda rule.

It is not possible to change outcome that makes someone better off without making someone else less off, this is in accordance with Arrow's theorem, BC is PO.

e) We are now going to look at this voting in terms of sequential majority voting. What types of sequential majority voting is there? Sketch the different types.



⇒ Sequence matters!

§ Write up all winners in pairwise elections and draw a majority graph representing the results. What is the preferred social ordering based on this graph?

Pairwise elections

$w_1 \succ w_2$

$w_2 \succ w_3$

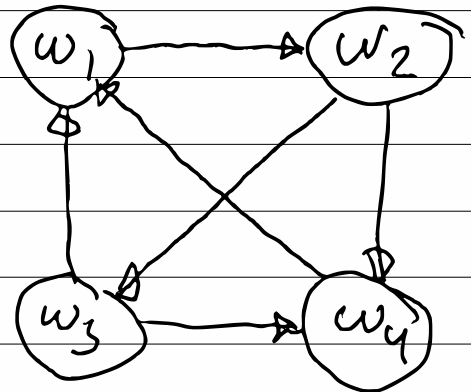
$w_3 \succ w_4$

$w_1 \succ w_3$

$w_2 \succ w_4$

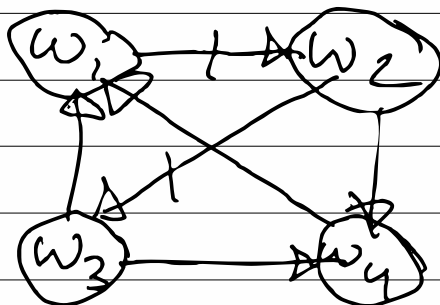
$w_1 \succ w_4$

All outcomes are possible winners!



g) What edges in the majority graph must be flipped in order to have a social ordering with a Condorcet's winner (assuming minimal number of edges to flip)?  
 What is the best outcome in terms of the Slater rule?  
 Why is the Slater rule problematic in the general case?

We could change  $w_3 \succ w_2$  to  $w_3 \succ w_1$   
 $w_2 \succ w_1$  to  $w_2 \succ w_3$ ,  
 to have  $w_2$  or  $w_3$  as Condorcet's winner with only one edge flip



Is this the least SC?  
 (All other Condorcet's winners  
 must have a SC >)

\* Lets look at  $w_2$

$w_2 \succ w_1 \succ w_3 \succ w_4$	3
$w_2 \succ w_1 \succ w_4 \succ w_3$	4
$w_2 \succ w_3 \succ w_1 \succ w_4$	2
$w_2 \succ w_3 \succ w_4 \succ w_1$	1
$w_2 \succ w_4 \succ w_1 \succ w_3$	3
$w_2 \succ w_4 \succ w_3 \succ w_1$	2

\* Lets look at  $w_3$

$w_3 \succ w_1 \succ w_4 \succ w_2$	3
$w_3 \succ w_1 \succ w_2 \succ w_4$	2
$w_3 \succ w_2 \succ w_1 \succ w_4$	3
$w_3 \succ w_2 \succ w_4 \succ w_1$	2
$w_3 \succ w_4 \succ w_1 \succ w_2$	2
$w_3 \succ w_4 \succ w_2 \succ w_1$	2



⇒ According to the Slater rule  
the social outcome

$$\omega_2 \succ \omega_3 \succ \omega_4 \succ \omega_1$$

is the most socially acceptable

⇒ SE/SC is generally NP-hard!

h, In terms of the analysis  
above, which task should the  
robots do first?

Summary of analysis:

Plurality	No ranking possible
Borda	$\omega_3 \succ \omega_2 \succ \omega_1 \succ \omega_4$
Slater	$\omega_2 \succ \omega_3 \succ \omega_4 \succ \omega_1$

For low  $|\Omega|$  Slater

For high  $|\Omega|$  Borda