TEK5010 Multiagent systems

Lecture 9: Voting

Exercise: Majority graph

Question 1

a) What does a majority graph describe?

A majority graph describes in a compact way all outcomes in paiwise elections between the possible candidates.

A majority graph is a directed graph constructed from voter preferences with nodes as outcomes and edges as majority outcomes in pairwise node elections.

b) Is there a clear winner here?

There is no clear winner here, no Condorcet winner, since all nodes have at least one edge pointing into node.

c) Are there any outcomes that can't be a winner?

Since all outcomes have at least one edge pointing from node, there is no outcome that is not a possible winner.

d) Could you write up all pairwise elections representing the given majority graph?
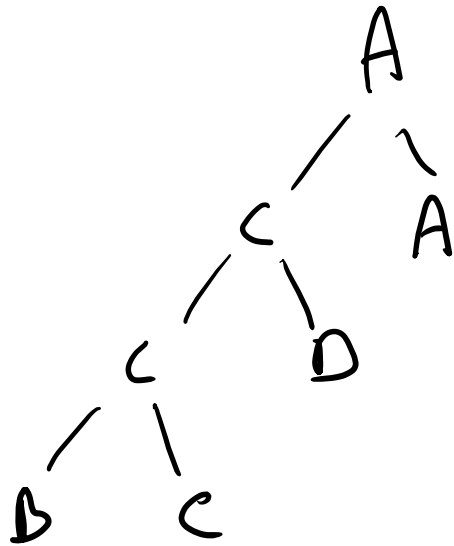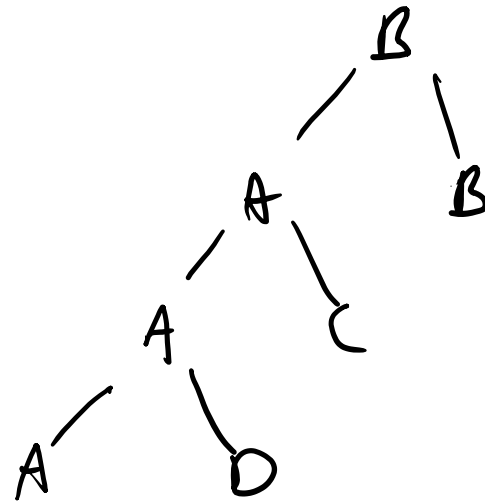
A < B   B < C   C > D

A > C   B < D

A > D

e) For each of the possible winner candidates give an example of a linear agenda that would lead to the respective candidate winning.
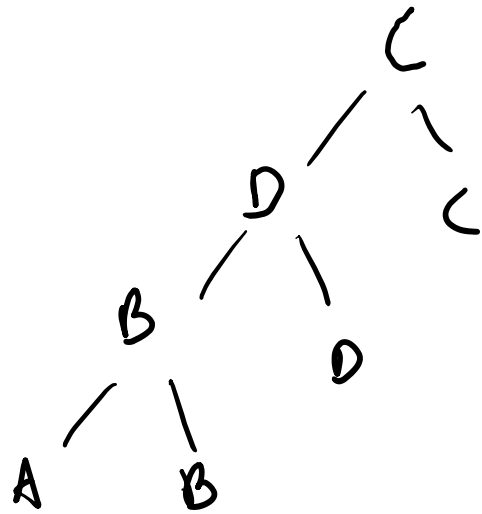
Ⓐ

```
        A
       / \
      C   A
     / \  
    C   D
   / \
  B   C
```
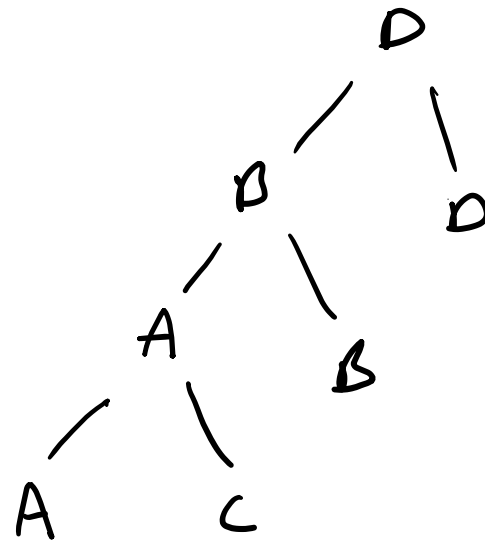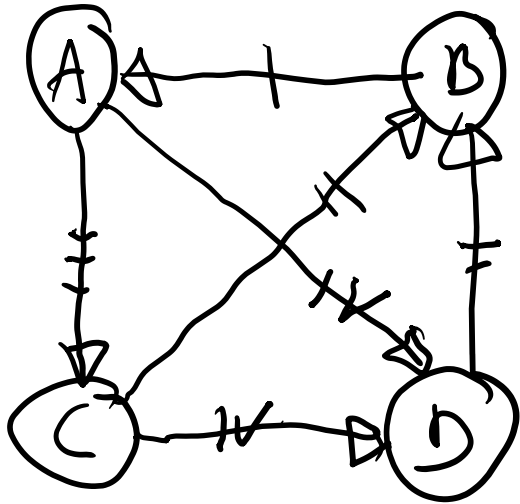
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```
        B
       / \
      A   B
     / \
    A   C
   / \
  A   D
```

C

```
            C
           /|
          / |
         D  C
        /|
       / |
      B  D
     /|
    / |
   A  B
```

D

```
        D
       /|
      / |
     B  D
    /|
   / |
  A  B
  |
  C
```

f) What edges in the majority graph must be flipped in order to have a Condorcet winner?

(I) A winner one flip

(II) B winner two flips

(III) C winner one flip

(IV) D winner two flips

g) What is the best outcome in terms of the slater rule? Why is the slater ranking problematic to use in the general case? In terms of the analysis above, in which sequence should the agents select the 4 outcomes?

Slater rule: given a particular social order, how many edges must be flipped in the majority graph in order to produce that social ordering?

We start analysis with A and C since B and D are at least 2+ flips.

A > B > C > D   3 flips
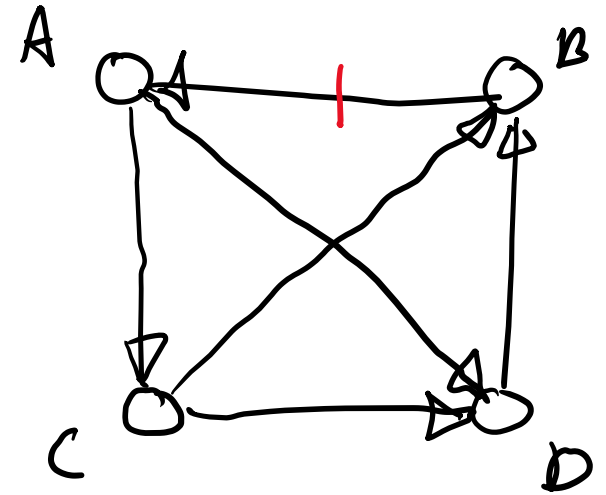
A > B > D > C   4 flips

A > C > B > D   2 flips

A > C > D > B   1 flip

A > D > B > C   3 flips

A > D > C > B   2 flips
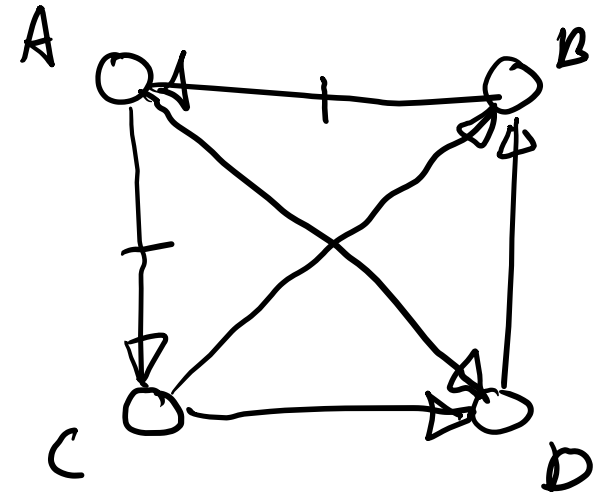
C > A > B > D   3 flips

C > A > D > B   2 flips

C > B > A > D   2 flips

C > B > D > A   3 flips

C > D > A > B   3 flips

C > D > B > A   2 flips

A   B

C   D

$\Rightarrow$ Slater rule/ranking gives the sequence of

A > C > D > B

as the social ranking of the possible outcomes.

The slater rule is NP-hard!