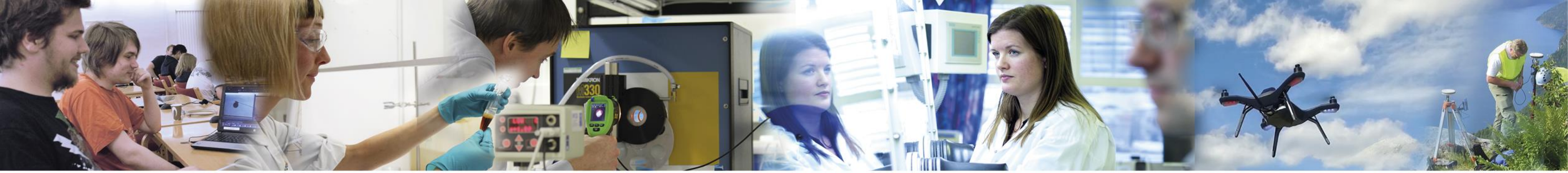


UiO : Department of Technology Systems
University of Oslo

TEK5010/9010 - Multiagent systems 2023
Lecture 6

Task allocation and self-assembly in swarms

Jonas Moen

Highlights lecture 6 -
Task allocation and self-assembly in swarms*

- Task allocation and division of labour
  - Models using response thresholds
- Nest building and self-assembly
  - Discrete stigmergy

*Bonabeau et *al*., 1999: chapter 3 and 6

Task allocation and division of labour

Introduction:

Many species of social insects have divison of labour, i.e. specialization of workers in order to perform coordinated tasks efficiently.

The behavioural repertoire of workers can be stretched back and forth in response to perturbations.

Task allocation and division of labour

Basic idea:

A model based on response thresholds of individual agents that connects individual-level placticity with colony-level resilience.

Task allocation and division of labour

Response thresholds:

Response thresholds refer to the likelihood of individuals reacting to task-associated stimuli. If stimuli is above a threshold the agent will most likely perform that task:

1. Low threshold individuals perform task at a lower level of stimuli than,
2. Higher threshold individuals.

Task allocation and division of labour

Extentions of model:

Extenstions of this threshold model using a simple form of learning. Within individual workers, performing a task induces a decrease in corresponding threshold, and not performing task induce an increase of the same threshold.

This double reinforcement leads to specialization of workers from a group of intially homogenous workers.

Task allocation and division of labour

Comparison of models:

The fixed response threshold model for task-allocation is similar to market-based models, i.e. auctions and bargaining.

Models with learning are more robust to perturbations compared to fixed threshold systems.

Division of labour in social insects

Definition [Oster & Wilson, 1978; Robinson; 1994]:

Different activities performed simultaneously by specialized individuals.

1. Believed to be more efficient than sequential tasks performed by unspecialized workers [Jeanne, 1986; Oster & Wilson, 1978].
2. Parallelism avoids task-switching.
3. Specialization is efficient due to individuals «know» the task at hand.

Division of labour in social insects

All social insects exhibit reproductive division of labour.

Other forms of division of labour may take 3, possibly coexisting, basic forms.

Division of labour in social insects

1. Temporal polyethism
   Age cast, individuals of same age do identical sets of tasks.
2. Worker polymorphism
   Workers belong to different morphological or physical castes that do different tasks.
3. Individual variability
   Behavioural cast (among age and morphological cast) describes groups of individuals that perform the same set of tasks within a given periode.

Division of labour in social insects

Division of labour is rarely rigid but rather characterized by its plasticity in relation to internal and external perturbations [Robinson, 1992].

Division of labour in social insects

Wilson [Wilson, 1984] experimented with *Pheidole* ants. When ratios of minor ants became small, major ants (e.g. soldiers) engaged in tasks usually performed by minors.

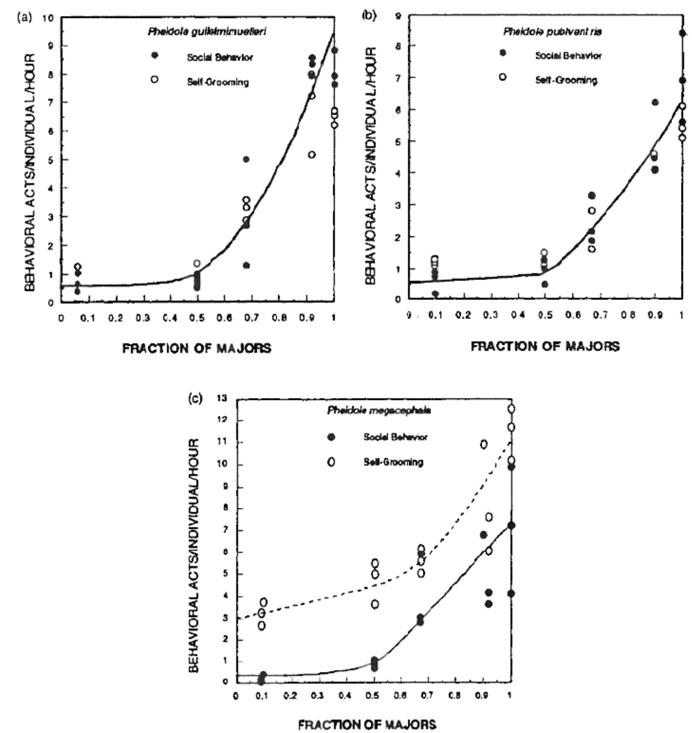


Image: Figure 3.1, Bonabeau et *al*., 1999

Division of labour in social insects

Response thresholds:

Example of larva feeding. Stimuli above threshold makes individuals engage in task. Removal of low threshold individuals highten the stimulus, e.g. increase in pheromones stimulating larva feeding, until it reaches the high threshold individuals. Feeding the larvae reduces the larval demand.

This process can be modelled by a simple response threshold model [Bonabeau et *al.*, 1996].

Response thresholds

Response threshold I:

$$T_\theta(s) = \frac{s^n}{s^n + \theta^n}$$

where s is stimulus

T is probability of doing task T in response to s

θ is threshold

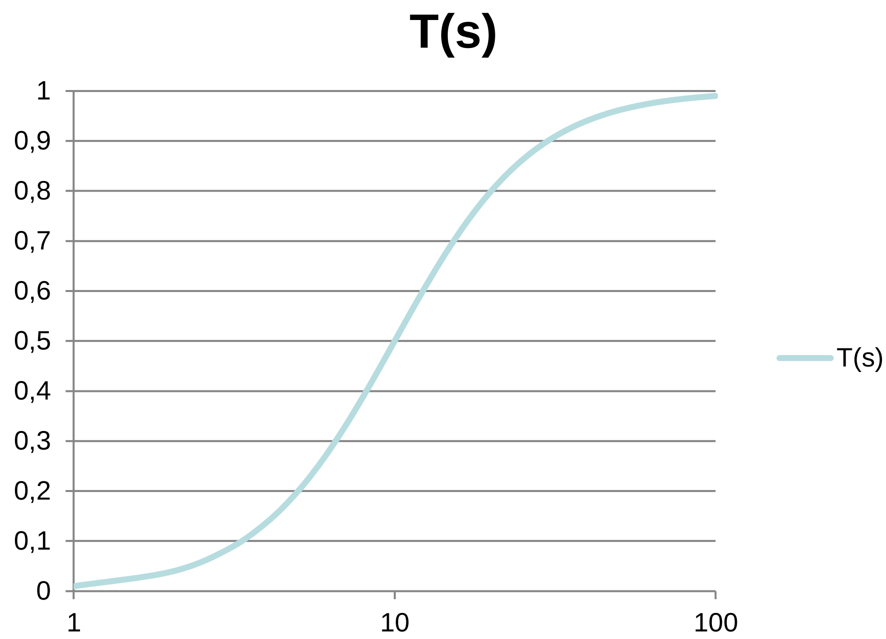
n is steepness of threshold ($n > 1$)

Response thresholds

Response threshold I:

If $s \ll \theta$ the probability of engaging in task T is close to zero.

If $s \gg \theta$ the probability of engaging in task T is close to 1.

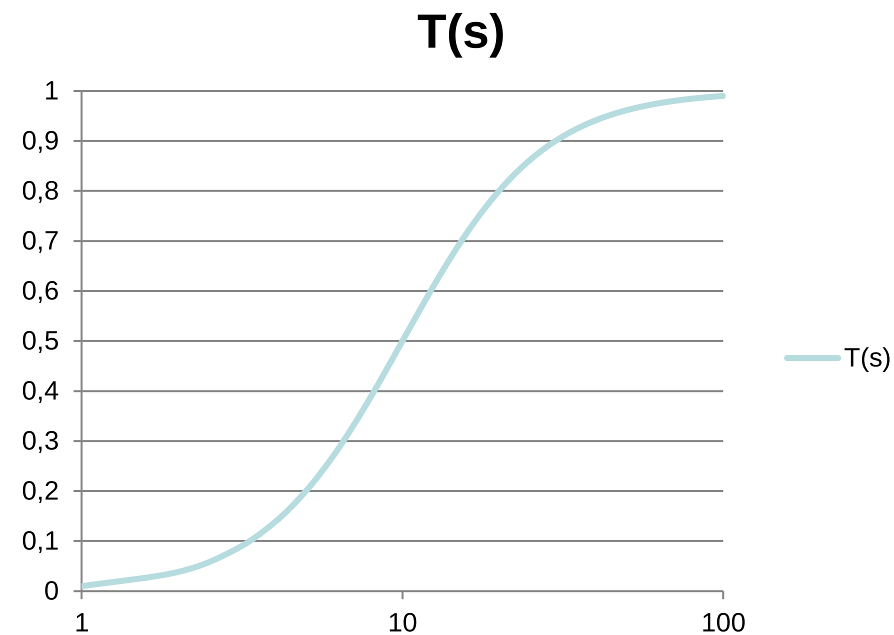


Response thresholds

Response threshold I:

If $s = \theta$ then $T = 0.5$.

Often $n = 2$ which in many cases give analytical solutions.



Response thresholds

Response threshold II:

$$T_\theta(s) = 1 - e^{-s/\theta}$$

where s is stimulus

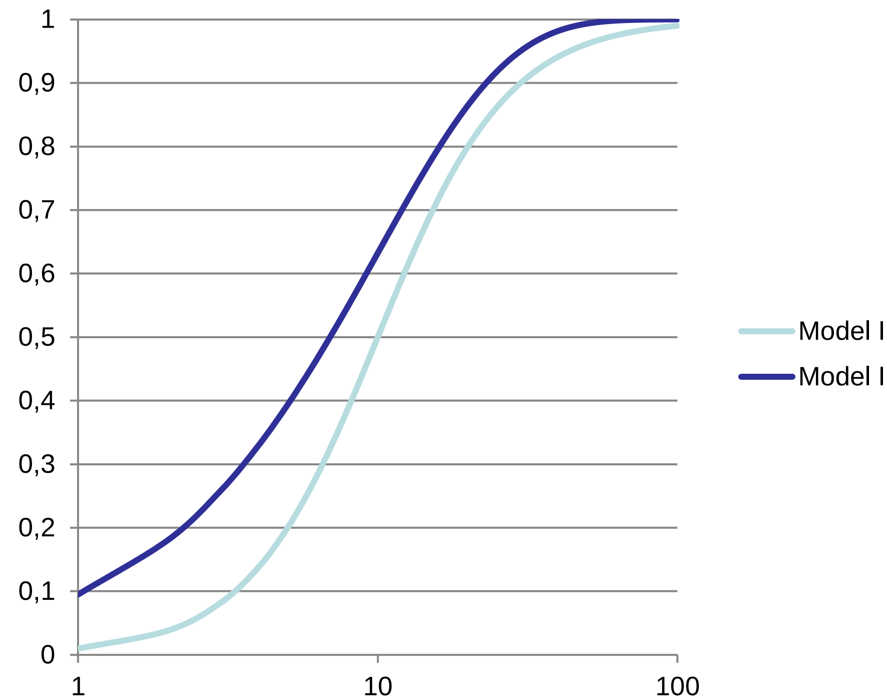
T is probability of doing task T in response to s

θ is threshold

Response thresholds

Response threshold II:

This model II encompass a exponential response function, which is essential in modelling real biological systems.



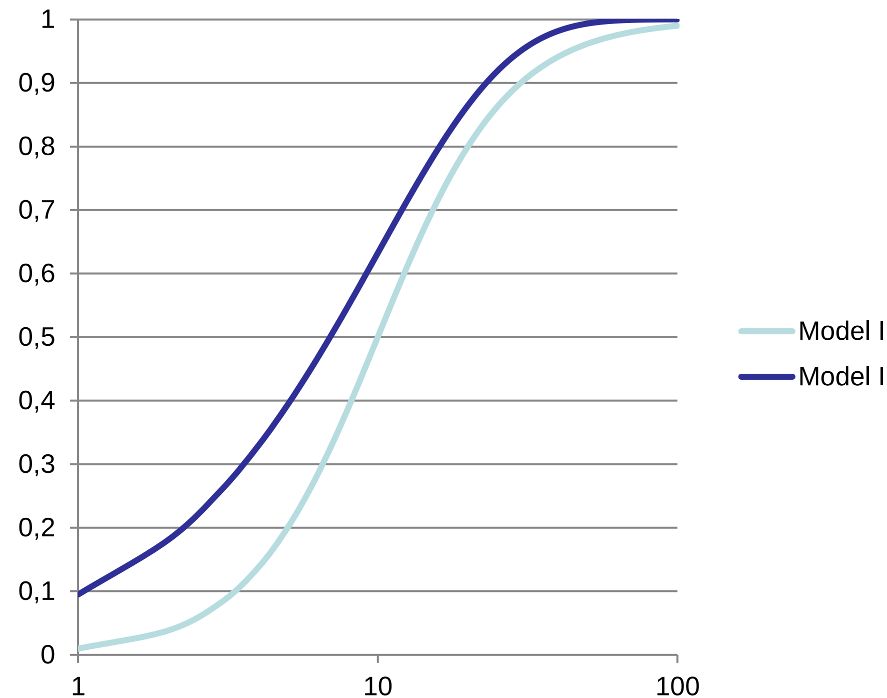
Response thresholds

Response threshold II:

The probability of assigning a task T after N encounters:

$$P(N) = 1 - (1 - \varrho)^N$$

where ϱ is probability of an individual doing task T at each encounter.

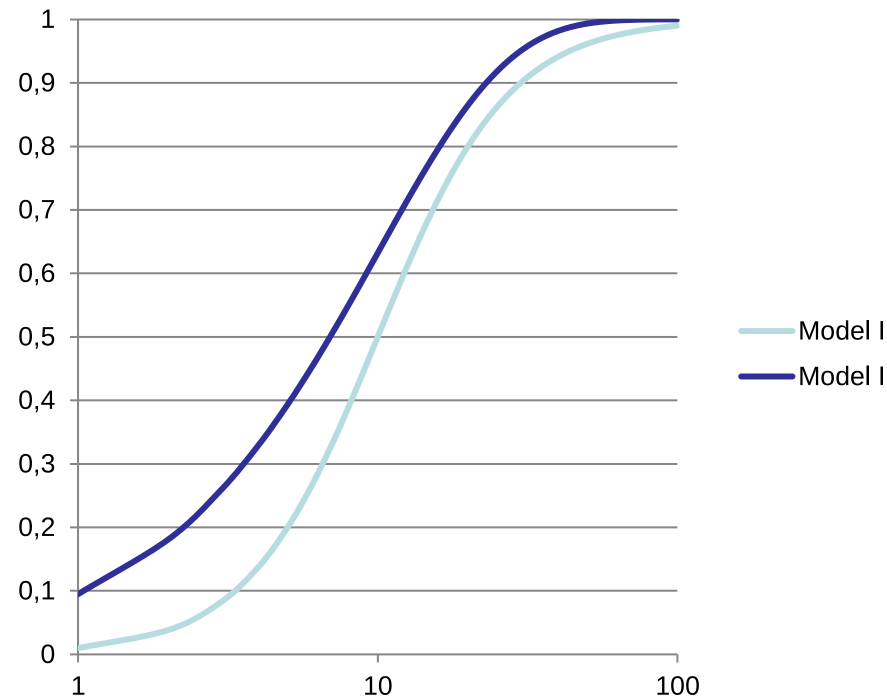


Response thresholds

Response threshold II:

$$P(N) = 1 - (1 - \varrho)^N$$
$$= 1 - e^{N ln(1-\varrho)}$$
$$= 1 - e^{-s/\theta}$$

If $s = N$ and $\theta = -{}^{1}/_{ln(1-\varrho)}$

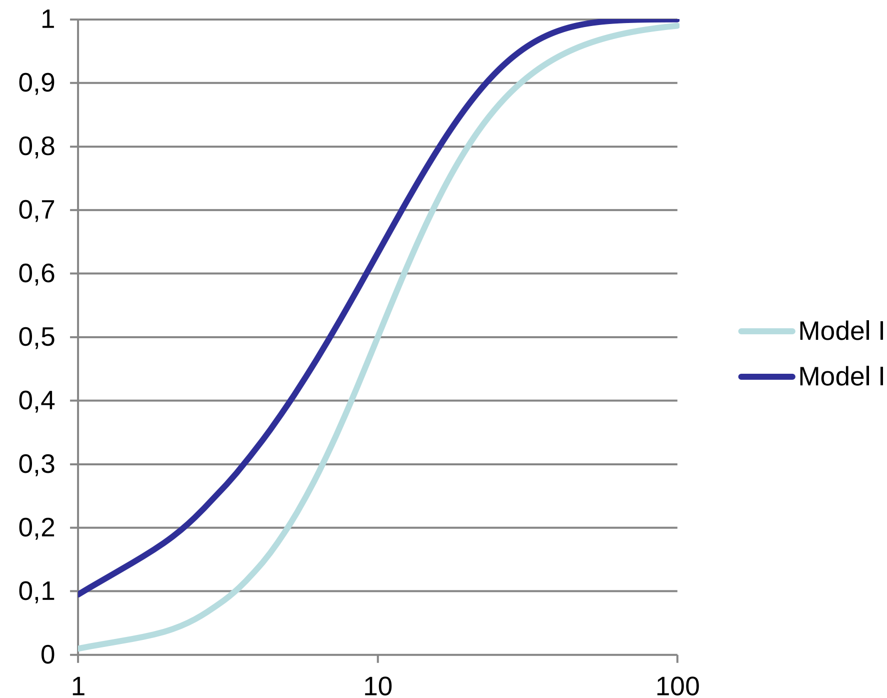


Response thresholds

Response threshold II:

Experiments verify the exponential threshold function in honey bees, ant cemeteries, etc.

[Chretien, 1996; Page & Robinson, 1991; Robinson & Page, 1988; Seeley, 1992]



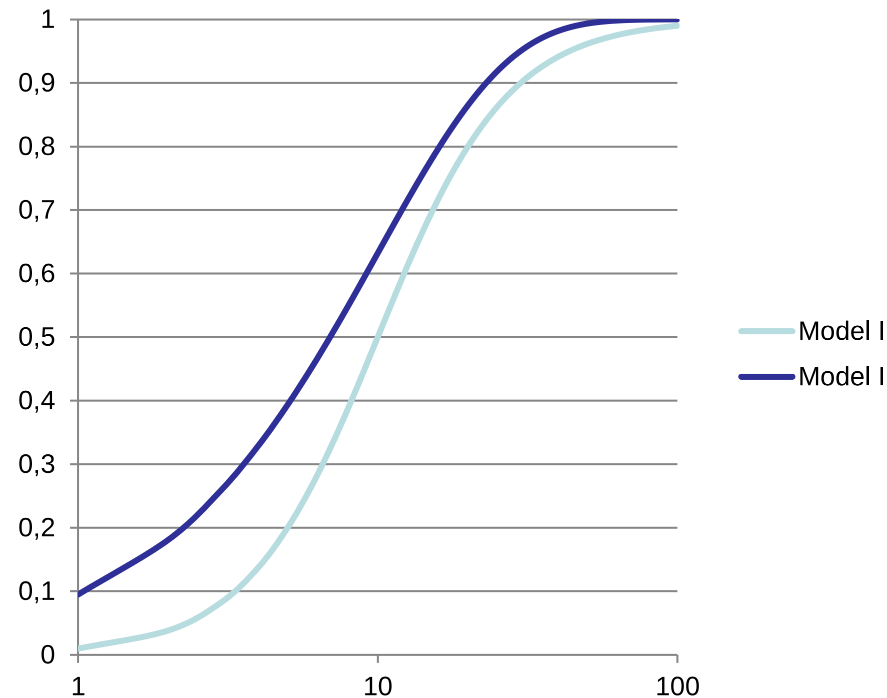
Response thresholds

Comparison:

The two functions are quite similar.

Model II is hard to manipulate analytically.

Thus, model I is used in the text with $n = 2$.



Response thresholds with one task

The transition probabilities (discrete-time dynamics):

$$P(X_i = 0 \rightarrow X_i = 1) = T_{\theta_i}(s)$$

$$P(X_i = 1 \rightarrow X_i = 0) = p$$

where i is worker type

$X_i = 0$ means that agent of type i is inactive in task T

$X_i = 1$ means that agent of type i is active in task T

p is probability of an agent of type i gives up task T

$1/p$ is average time spent on task T, independent of s

Response thresholds with one task

Stimulus of task (discrete-time dynamics)

$$s(t+1) = s(t) + \delta - \alpha \frac{N_{act}}{N}$$

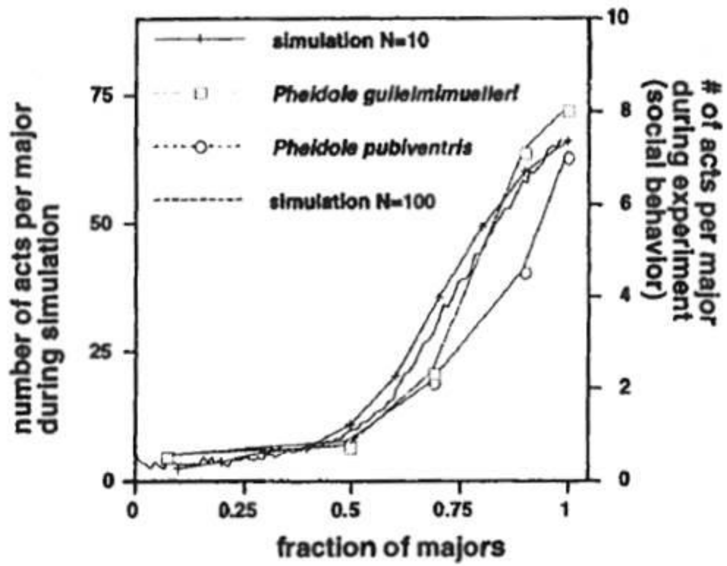
where s is stimulus of task T at time t

δ is the increase in stimulus intensity per unit time

α is a scale factor measuring the efficiency of task performance

N_{act} is number of active individuals

Response thresholds with one task



Good fit for Wilson when $\theta_1 = 8$, $\theta_2 = 1$, $\alpha = 3$, $\delta = 1$ and $p = 0.2$

Image: Figure 3.9, Bonabeau et *al*., 1999

Response thresholds with one task

Transition dynamics (continuous-time):

$$\underbrace{\delta_t x_i}_{\text{Change in active workers}} = \underbrace{T_{\theta_i}(s)(1 - x_i)}_{\text{Inactive workers recruited}} - \underbrace{p x_i}_{\text{Retired workers}}$$

where n_i is number of workers of type i, i.e. $N = \sum n_i$

N_i is number of workes of type i engaged in task T

$x_i = \frac{N_i}{n_i}$ is fraction of workers type i doing task T

$f = \frac{n_i}{N}$ is fraction of workers type i in colony

Response thresholds with one task

Stimulus dynamics (continuous-time):

$$\delta_t s = \delta - \alpha \frac{N_1 + N_2}{N} = \delta - \alpha f x_1 - \alpha(1 - f) x_2$$

since $(N_1 + N_2)/N = f x_1 + (1 - f) x_2$.

Using $z = \theta_1^2/\theta_2^2$ it can be shown analytically that the solution to this set of differential equations is given by:

Response thresholds with one task

Continuous-time model of one task allocation:

$$x_1^s = \frac{\chi + \left(\chi^2 + 4f(p+1)(z-1)(\delta/\alpha)\right)^{1/2}}{4f(p+1)(z-1)}$$

where $\chi = (z-1)\left(f + (p+1)(\delta/\alpha)\right) - z$

x_1^s is the fraction of majors ($\theta_1 > \theta_2$) involved in task *T* per time unit a function of f with δ/α, p and z as parameters.

Response thresholds for several tasks

Transition dynamics of several tasks (continuous-time):

$$\delta_t x_{ij} = \frac{s_j^2}{s_j^2+\theta_{ij}^2}\left(1 - \sum_{k=1}^{m} x_{ik}\right) - p x_{ij}$$

where j is one out of m possible tasks

n_i is number of workers of type i, i.e. $N = \sum n_i$

N_{ij} is number of workes of type i engaged in task T_j

$x_{ij} = \frac{N_{ij}}{n_i}$ is fraction of workers type i doing task T_j

Response thresholds for several tasks

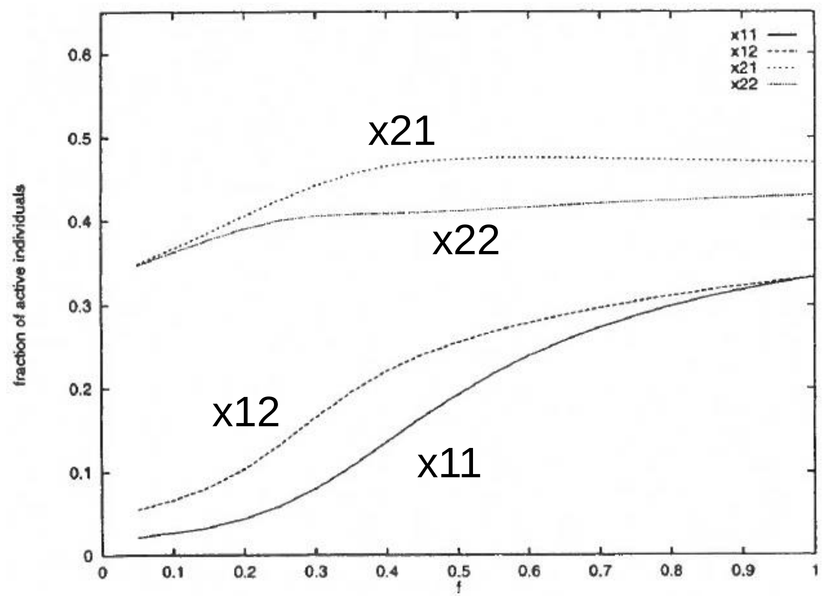
Stimulus dynamics of two tasks (continuous-time):

$$\delta_t s_j = \delta - \alpha f x_{1j} - \alpha(1-f)x_{2j}$$

Numerical integration is necessary to find stationary values of x_{ij} as a function of the fraction f of type-1 workers, e.g. figure 3.15 and 3.16 in [Bonabeu et al., 1999].

Response thresholds for several tasks

Only workers of type 2

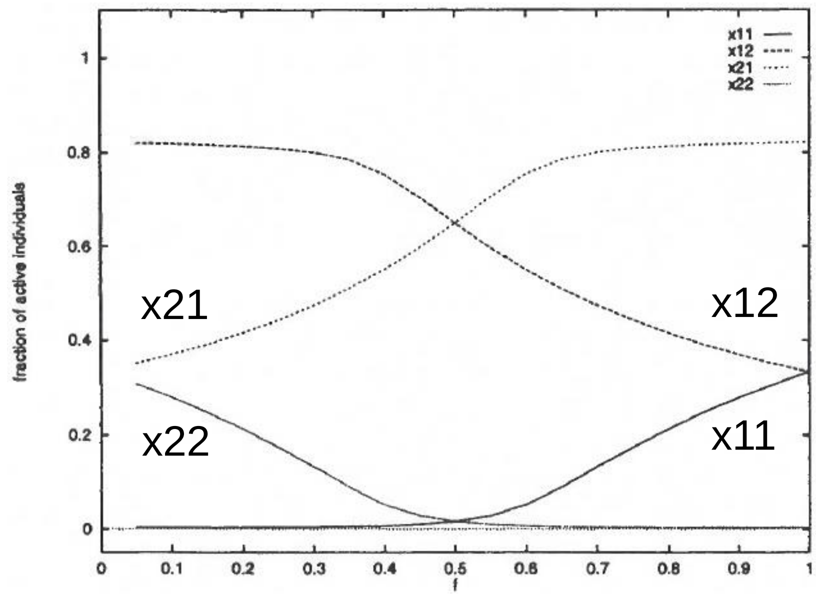


Only workers of type 1

Good fit for Wilson when $\theta_{11} = 8$, $\theta_{12} = 5$, $\theta_{21} = 1$, $\theta_{22} = 1$, $\alpha = 3$, $\delta = 1$ and $p = 0.2$

Image: Figure 3.15, Bonabeau et *al*., 1999

Response thresholds for several tasks



Only workers of type 2

Only workers of type 1

Good fit for Wilson when $\theta_{11} = 8$, $\theta_{12} = 1$, $\theta_{21} = 1$, $\theta_{22} = 8$, $\alpha = 3$, $\delta = 1$ and $p = 0.2$

Image: Figure 3.16, Bonabeau et *al*., 1999

Response thresholds for specialization

The fixed response threshold model cannot:

1. Account for genesis of task allocation, must assume preassigned castes.
2. Account for strong specialization within physical or temporal castes.
3. Valid only for as long as threshold are constants.
4. Not consistent with experiments on honey bees [Calderon & Page, 1996; Robinson et *al*., 1994] which indicate that learning is important in task allocation.

Response thresholds for specialization

Reinforcement of response threshold:

$$\theta_{ij} \leftarrow \theta_{ij} - \underbrace{x_{ij}\xi\Delta t}_{\text{Performing task } j} + \underbrace{(1 - x_{ij})\varphi\Delta t}_{\text{Not performing task } j}$$

where Δt is time interval of evaluation

ξ (ksi) is constant decreasing θ_{ij} if task j is performed

φ is constant increasing θ_{ij} if task j is not performed

x_{ij} is fraction of time spent on performing task j

Response thresholds for specialization

Reinforcement of response threshold:

$$\delta_t \theta_{ij} = \left[\left(1 - x_{ij} \right) \varphi - x_{ij} \xi \right] \Theta\left(\theta_{ij} - \theta_{min} \right) \Theta\left(\theta_{max} - \theta_{ij} \right)$$

where Θ (theta) is a step function for maintaining θ within bounds

Response thresholds for specialization

Transition dynamics of specialization (continuous-time):

$$\delta_t x_{ij} = \frac{s_j^2}{s_j^2+\theta_{ij}^2}\left(1-\sum_{k=1}^{m} x_{ik}\right) - p x_{ij} + \psi(i,j,t)$$

where $\psi(i,j,t)$ is a $N(0,\sigma)$ stochastic process simulating that individuals are in different environments.

Response thresholds for specialization

Stimulus dynamics for specialization (continuous-time):

$$\delta_t s_j = \delta - \frac{\alpha_j}{N}\left(\sum_{i=1}^{N} x_{ij}\right)$$

The dynamics described here can lead to specialization out of a homogenous population.

Connection with «bidding» algorithms

«A high bid is similar to a low threshold.»

[Morley, 1996; Morley & Ekberg, 1998] describe examples exposing this connection.

Nest building and self-assembly

Hive of paper wasp*

Termite hive**

Image: *en.wikipedia.org and **inhabitat.com

Nest building and self-assembly

Social insect's nest architecture can be complex and intricate structures. Stigmergy is an important mechanism in nest construction in social insects. Stigmergy is the coordination of activities through the environment.

Nest building and self-assembly

Stigmergy

1. Qualitative or continuous
   The different stimuli that trigger behaviours are quantitatively different, e.g. emergence of pilars in termite hives are regulated by pheromone concentration.
2. Quantitative or discrete
   Stimuli can be classified into different classes that differ quantitatively, e.g. building behaviours of paper wasps depend on elementary building blocks and their configuration.

Nest building in social insects

Nest building demonstrate the greatest difference between individual and collective levels.

How can insects in a colony coordinate their behaviours in order to build these highly complex architectures?

Nest building in social insects

There is no evidence that the behaviour of an individual in a social species is more sophisticated than that of an individual of a solitary species.

The anthropomorphic model assumes that individual insects possesses a representation of the global structure to be produced and make decisions on the basis of that representation. Nest complexity would then result from the complexity of the insects individual behaviour.

Nest building in social insects

Swarm model assumes that social insect colonies are decentralized systems composed of cooperative, autonomous units that are distributed in the environment, that exhibits simple probabilistic stimulus-response behaviour and have access to local information only [Deneubourg and Goss, 1989; Bonabeau et *al.*, 1997].

Sensory system that singnals attractive or repulsive behaviours that varies in intensity and according to environmental context.

Nest building in social insects

Basic model:

1. The stimuli that initially trigger building behaviour may be quite simple and limited in number.
2. But as construction proceeds, these stimuli become more complex and numerous, inducing new types of behaviours.
3. A morphogenetic process follows where previous construction sets the stage for new building actions.
4. The larger the nest, the greater the variety of signals and cues it can encompass.

Nest building in social insects

Discrete stigmergy:

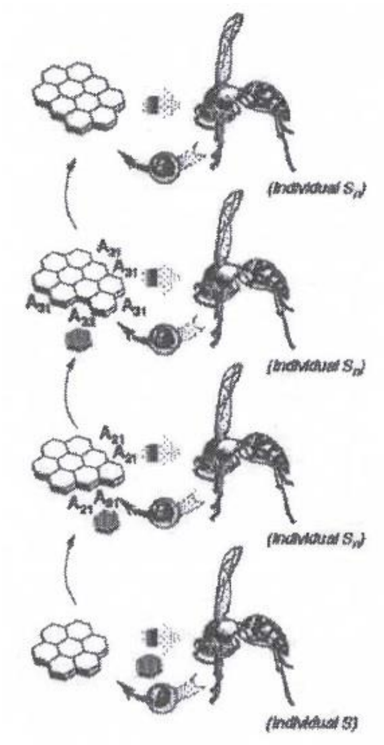


Image: Figure 6.1, Bonabeau et *al*., 1999

Nest building in social insects

Discrete stigmergy:

Stimuli are qualitatively different; stimulus of type-1 triggers an action A, which again transforms the type-1 stimulus into a type-2 stimulus that triggers an action B.

1. No positive feedback effect can amplify a stimulus to transform into a more intense version of the same stimulus
2. No such thing, in principle, as the intesity of a stimulus.
3. Continuous and discrete stigmergy are likely to coexist.
4. Parallelism must not destroy coordination.

Self-assembly

Discrete stigmergy model:

Agents move in a 3D grid, drop elementary building blocks depending on the configuration of blocks in their neighbourhood.

The fitness of the construction is then reviewed and used for exploring the space of possible architectures.

Self-assembly

Discrete stigmergy pseudocode [Bonabeau et *al.*, 2006]

Construct lookup table of rules and initialize all agents
for t=1 to t_{max} **do** //loop over all iterations
  for k=1 to m **do** //loop over all agents
    Sense local configuration
    if (local configuration is in lookup table)
      Deposit brick specified by lookup table
    Move randomly to unselected neighbour site
  end
end

Self-assembly

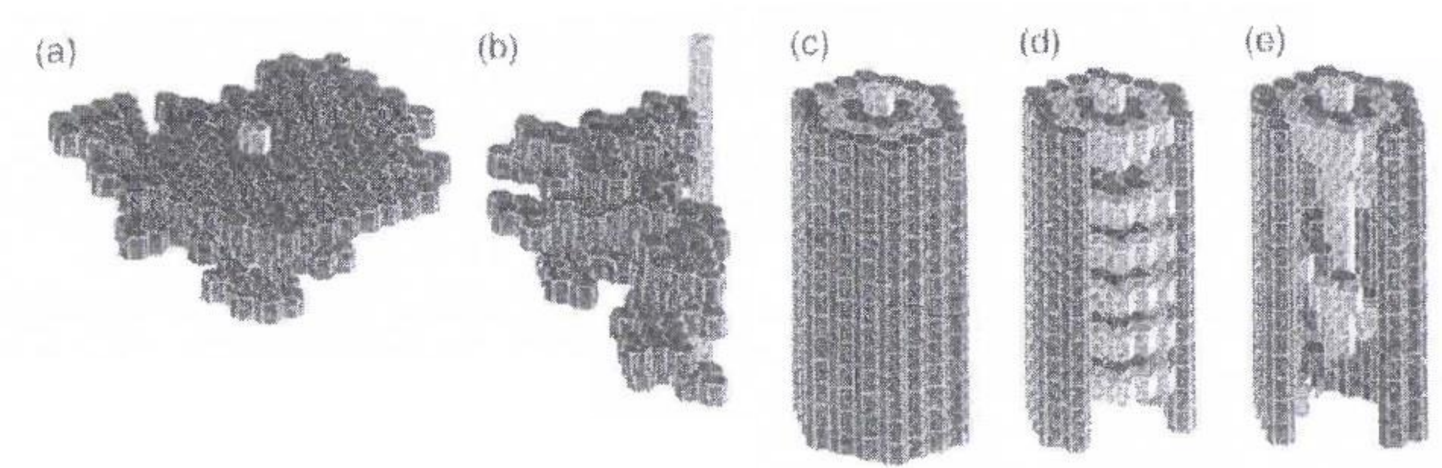


Image: Figure 6.7, Bonabeau et *al*., 1999

Self-assembly

Exploring the space of architecture:

The goal is to evolve microrules that produce interesting structures. A Genetic Algorithm (GA) [Forrest, 1993; Goldberg, 1989] is applied where microrules are genes and fitness is based on the following observations [Bonabeau et *al*., 1998]:

Self-assembly

Exploring the space of architecture:

1. Coherent architectures are the result of many microrules.
2. Building compactly requires collections of complementary correlated microrules.
3. Complex architectures are characterized by large patterns that repeat themselves.

Self-assembly

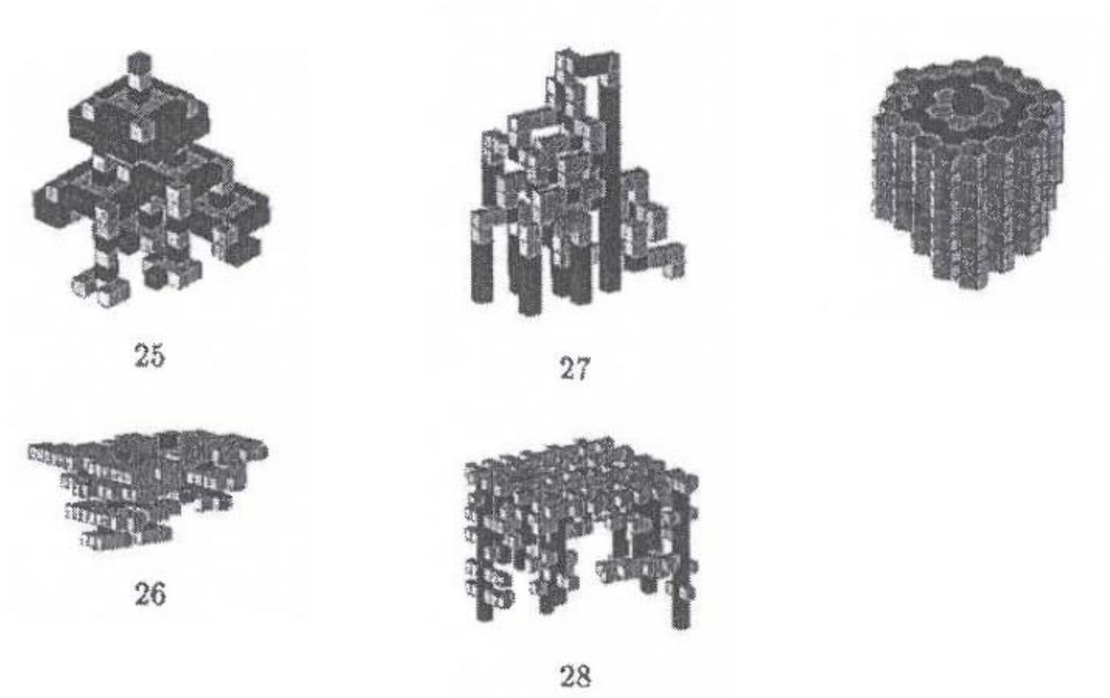


Image: Figure 6.13, Bonabeau et *al*., 1999

Highlights lecture 6 - Task allocation and self-assembly in swarms*

- Task allocation and division of labour
  - Models using response thresholds
- Nest building and self-assembly
  - Discrete stigmergy

*Bonabeau et *al*., 1999: chapter 3 and 6