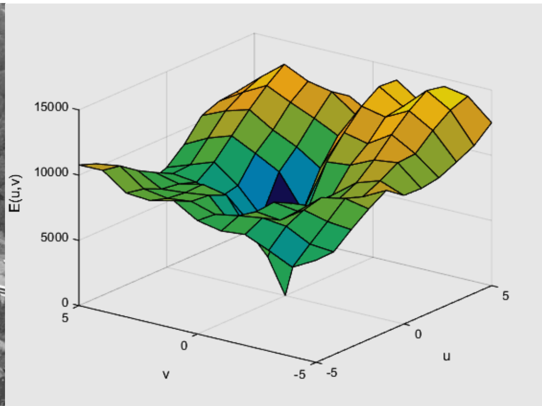
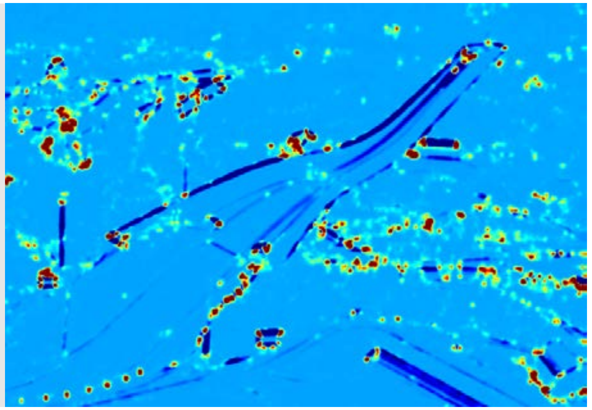




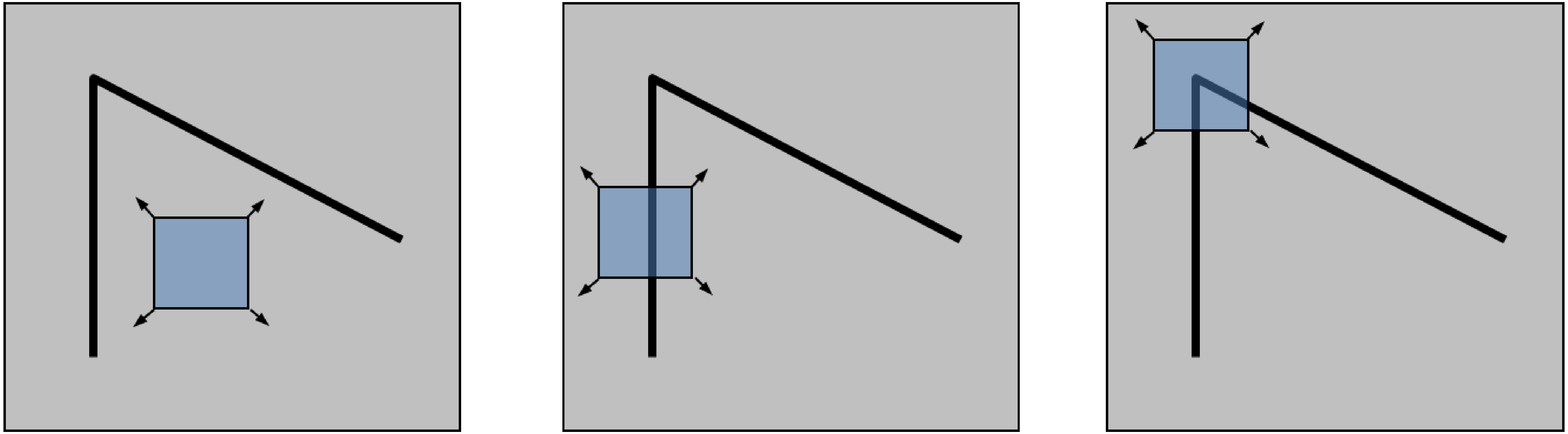
Lecture 3.2.1
Corner features

Trym Vegard Haavardsholm

Slides from Rick Szeliski, S. Seitz. Svetlana Lazebnik, Derek Hoiem, Grauman&Leibe, James Hayes and Noah Snavely

Local measure of feature distinctiveness

- Consider a small window of pixels around a feature
- How does the window change when you shift it?



"Flat" region:
No change in all directions

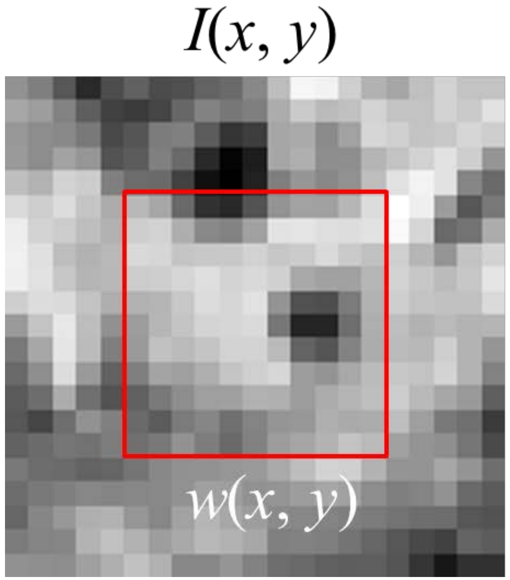
"Edge":
No change along edge

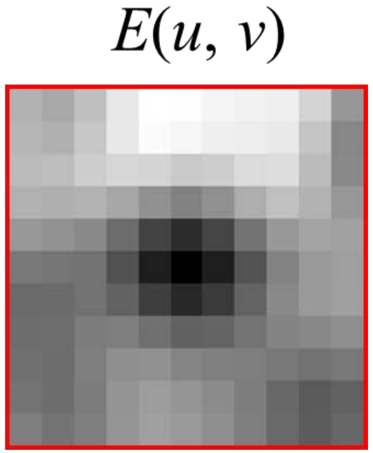
"Corner":
Change in all directions

Local measure of feature distinctiveness

- Change in appearance of window *w*(*x*,*y*) for the shift [*u*,*v*]:

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$



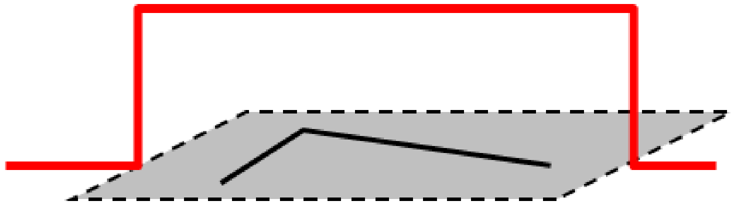


Local measure of feature distinctiveness

- Change in appearance of window *w*(*x*,*y*) for the shift [*u*,*v*]:

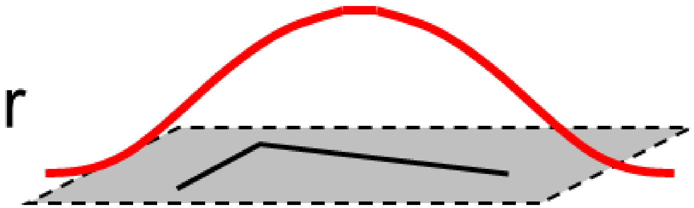
$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window function $w(x,y)$ =



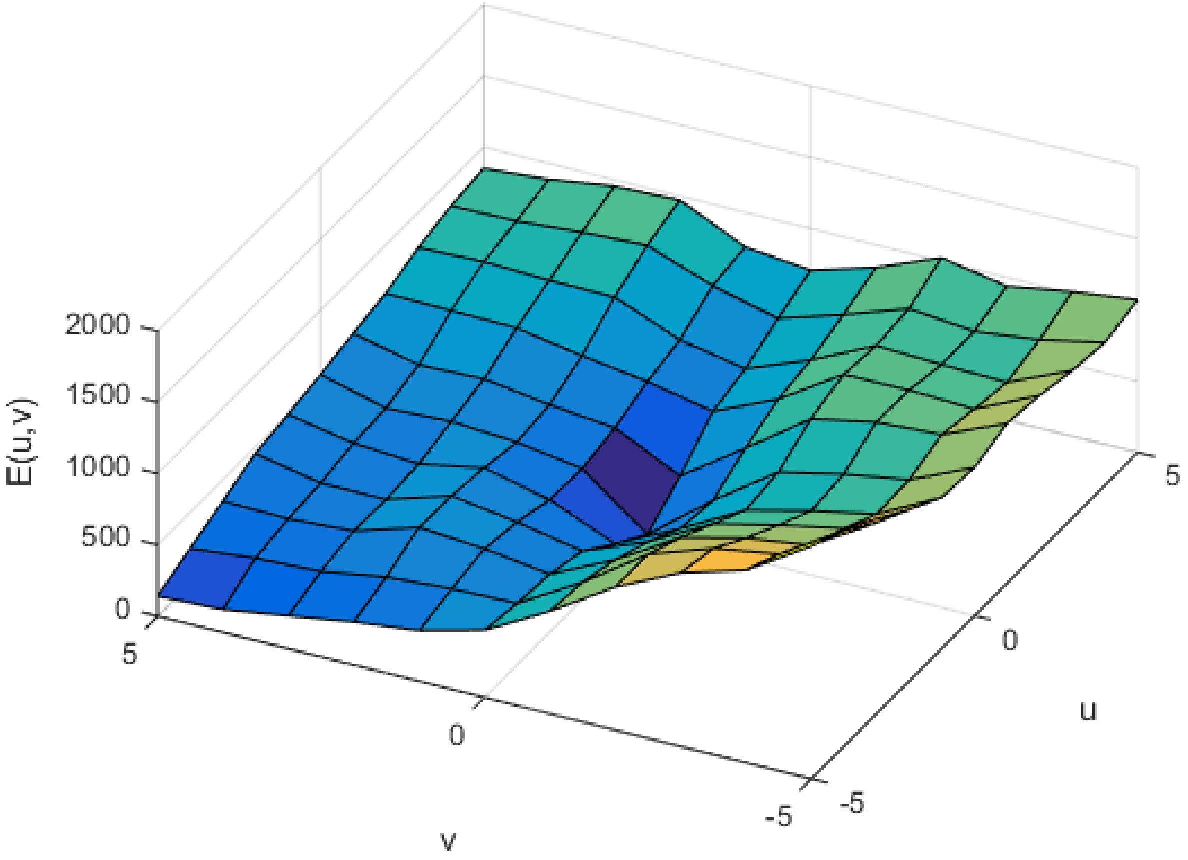
1 in window, 0 outside

or

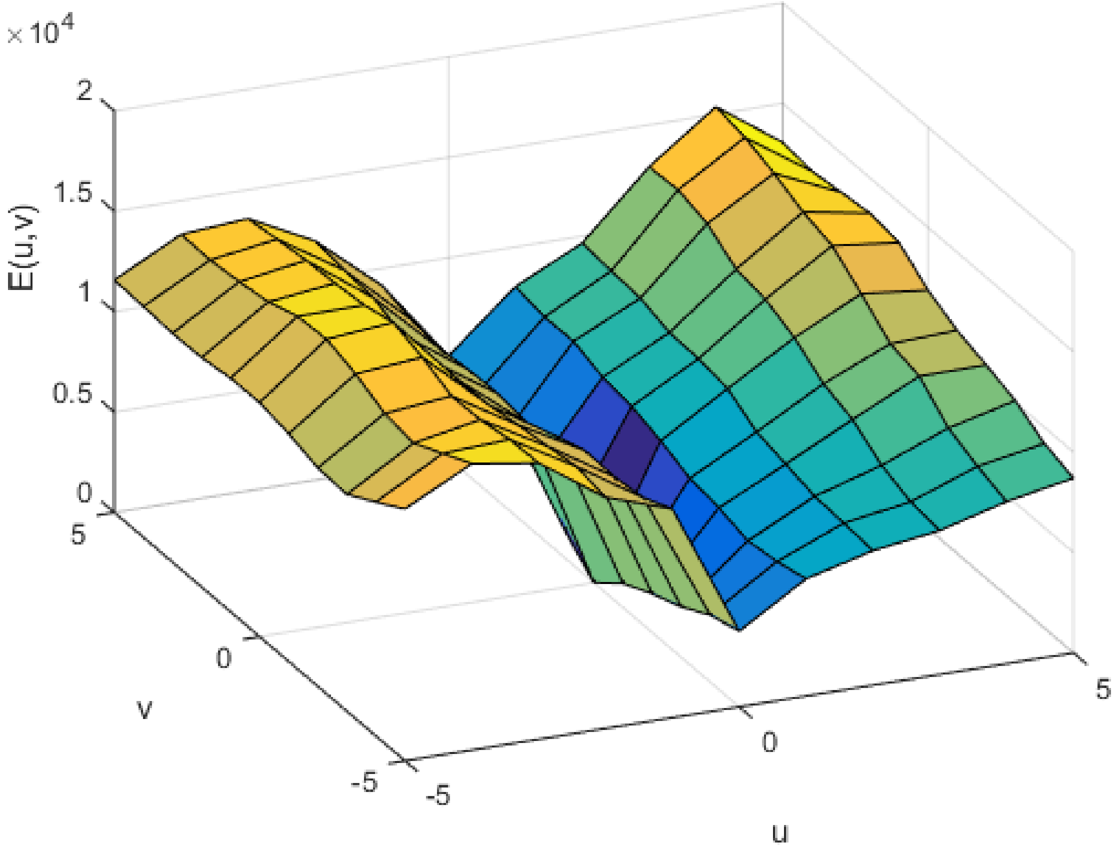


Gaussian

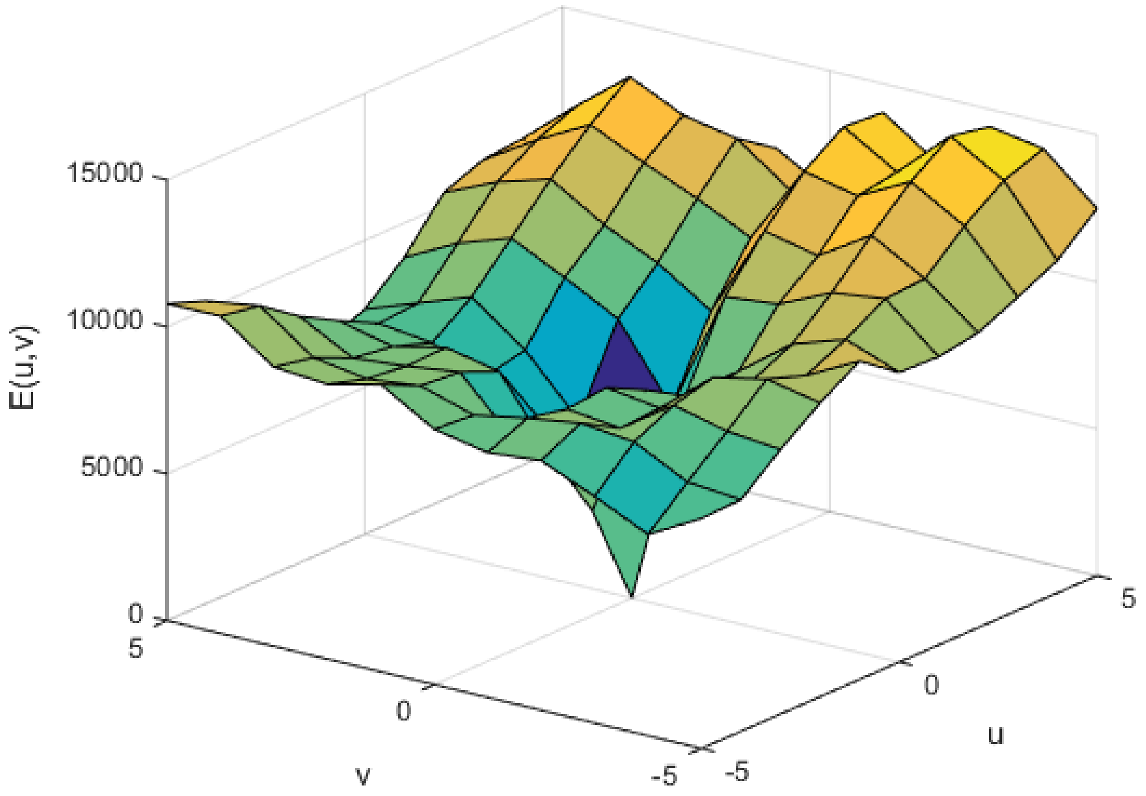
Examples Holmenkollen



Examples Holmenkollen



Examples Holmenkollen



Simplifying the measure

- Local first order Taylor Series expansion of *I*(*x*,*y*):

$$I(x+u, y+v) \approx I(x, y) + I_x u + I_y v$$

- Local quadratic approximation of *E*(*u*,*v*):

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

$$\approx \sum_{x,y} w(x,y)\left[I_x u + I_y v\right]^2$$

$$\approx Au^2 + 2Buv + Cv^2$$

Simplifying the measure

- Local quadratic approximation of the surface *E*(*u*,*v*):

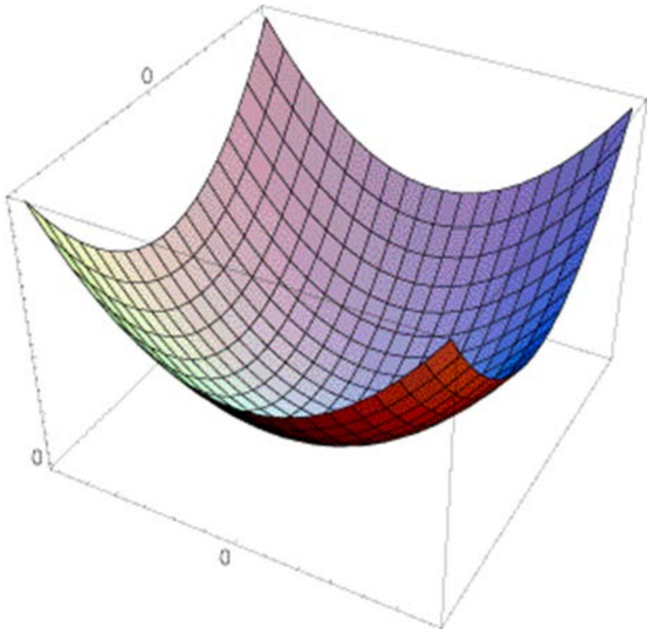
$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{x,y} w(x,y) I_x^2$$

$$B = \sum_{x,y} w(x,y) I_x I_y$$

$$C = \sum_{x,y} w(x,y) I_y^2$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \sum_{x,y} w(x,y) \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} I_x \\ I_y \end{bmatrix}$$



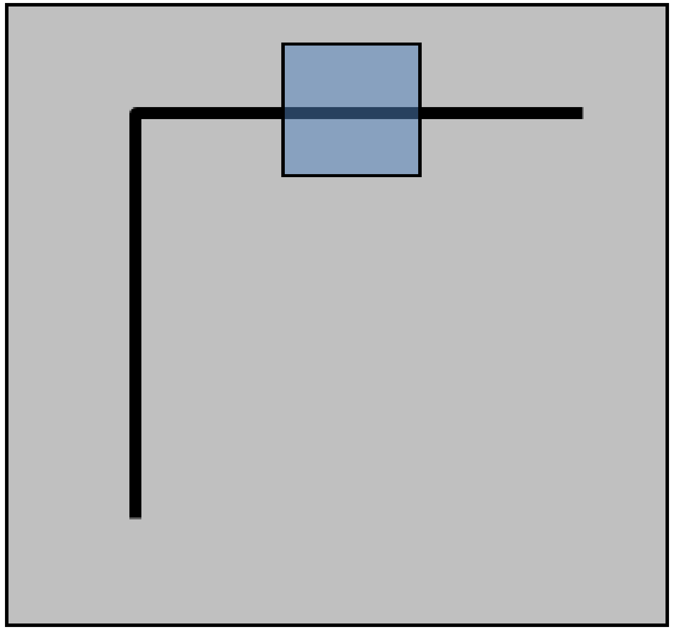
Interpreting the second moment matrix

$$E(u,v) \approx [u \quad v]\begin{bmatrix} A & B \\ B & C \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = [u \quad v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{x,y} w(x,y) I_x^2$$

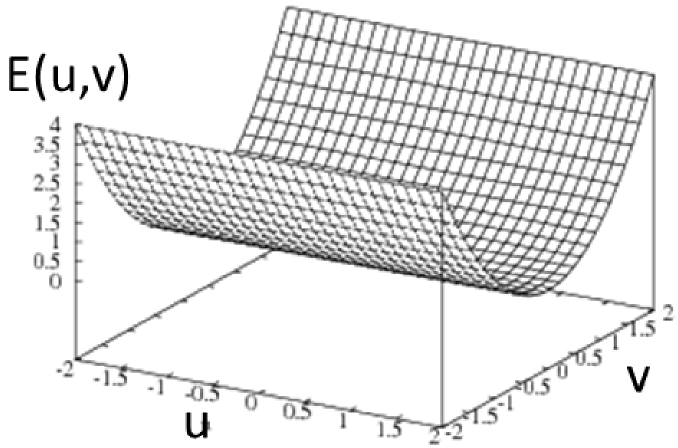
$$B = \sum_{x,y} w(x,y) I_x I_y$$

$$C = \sum_{x,y} w(x,y) I_y^2$$



Horizontal edge: $I_x = 0$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$



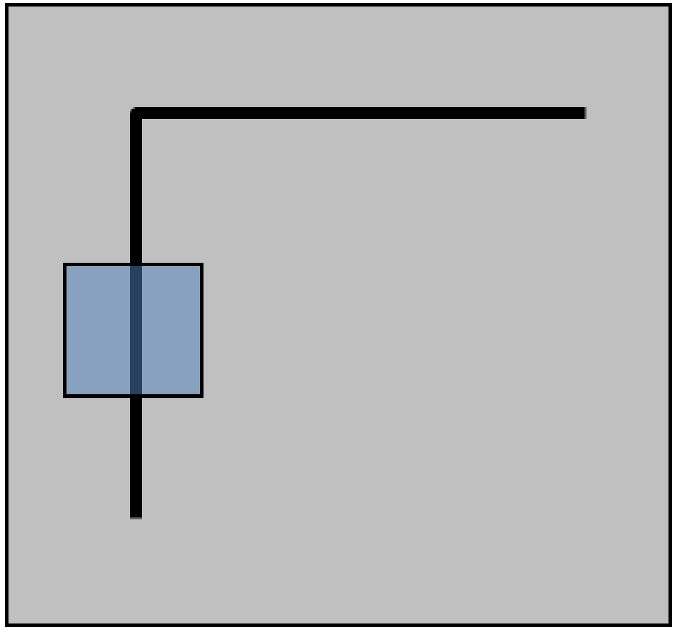
Interpreting the second moment matrix

$$E(u,v) \approx [u \quad v] \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = [u \quad v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{x,y} w(x,y) I_x^2$$

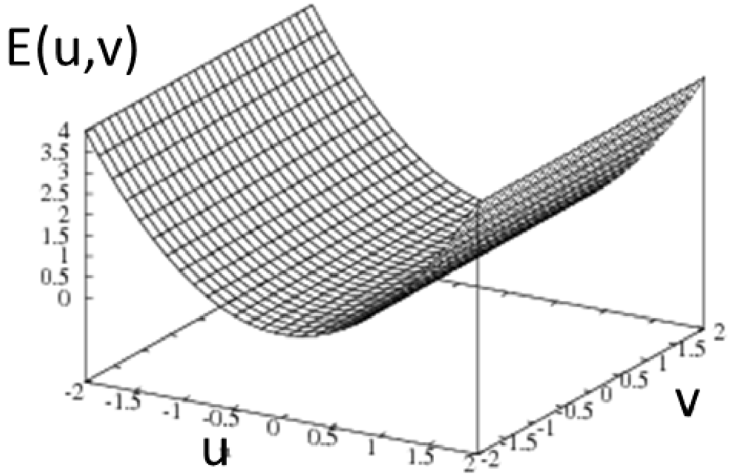
$$B = \sum_{x,y} w(x,y) I_x I_y$$

$$C = \sum_{x,y} w(x,y) I_y^2$$

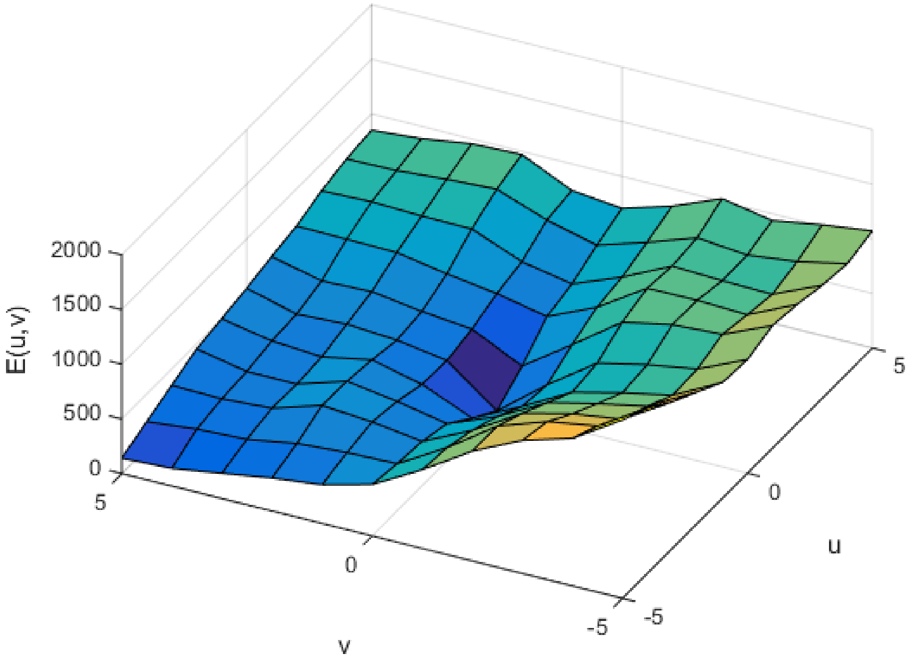


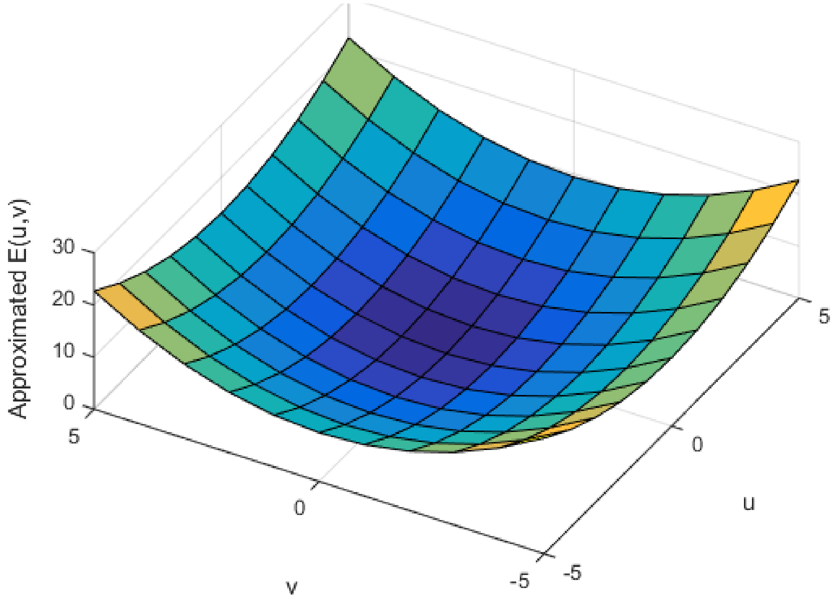
Vertical edge: $I_y = 0$

$$M = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$$

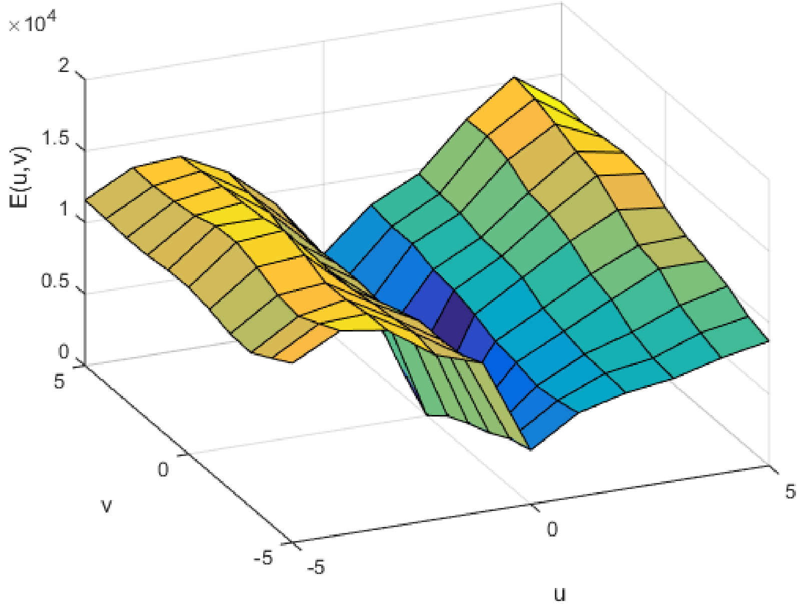


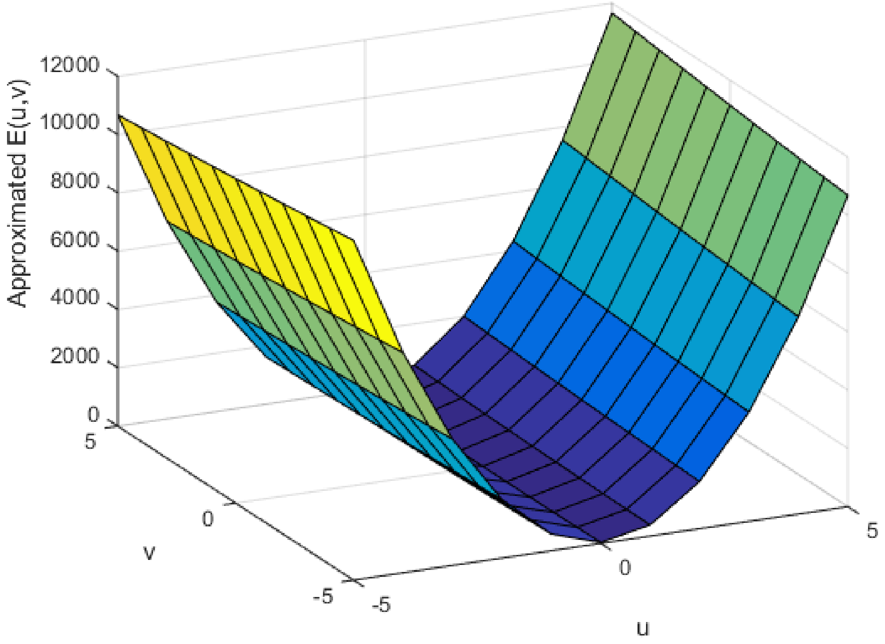
Examples Holmenkollen



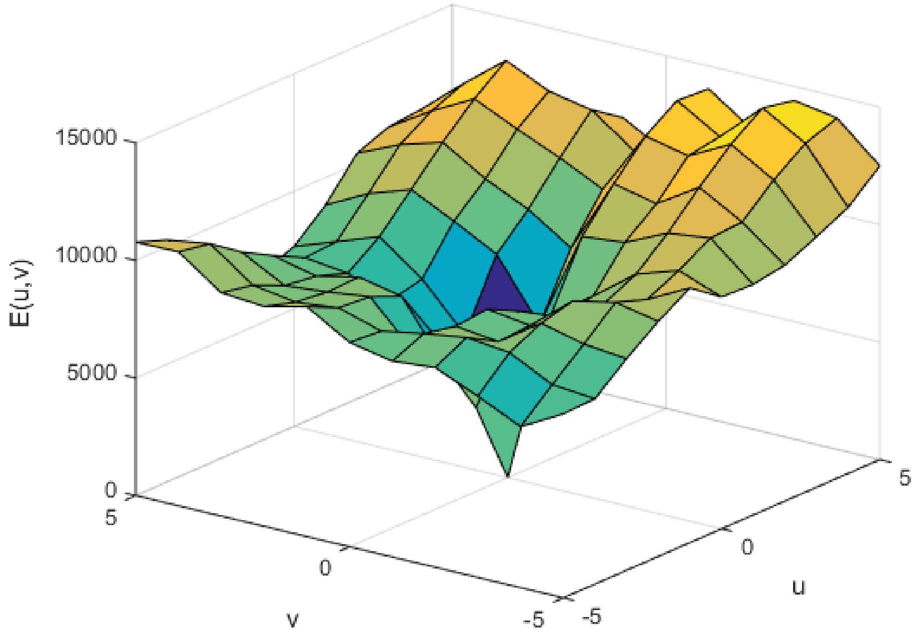


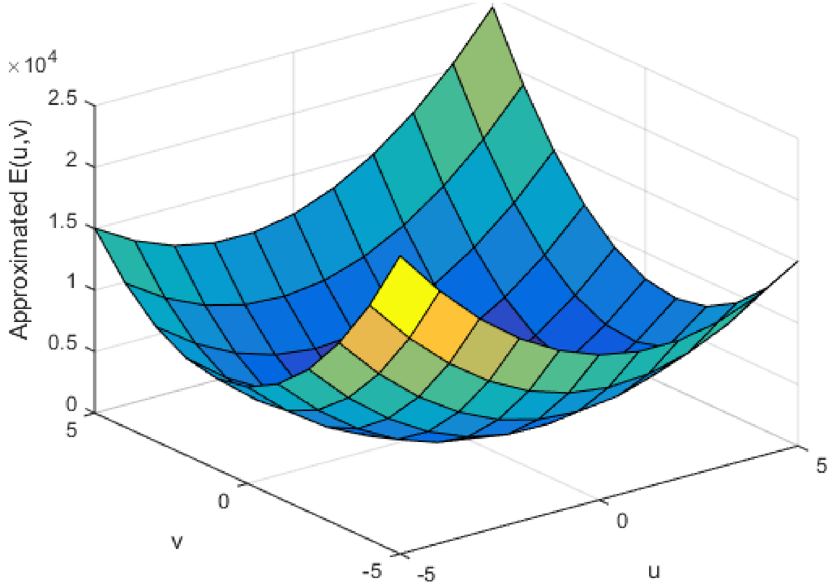
Examples Holmenkollen





Examples Holmenkollen



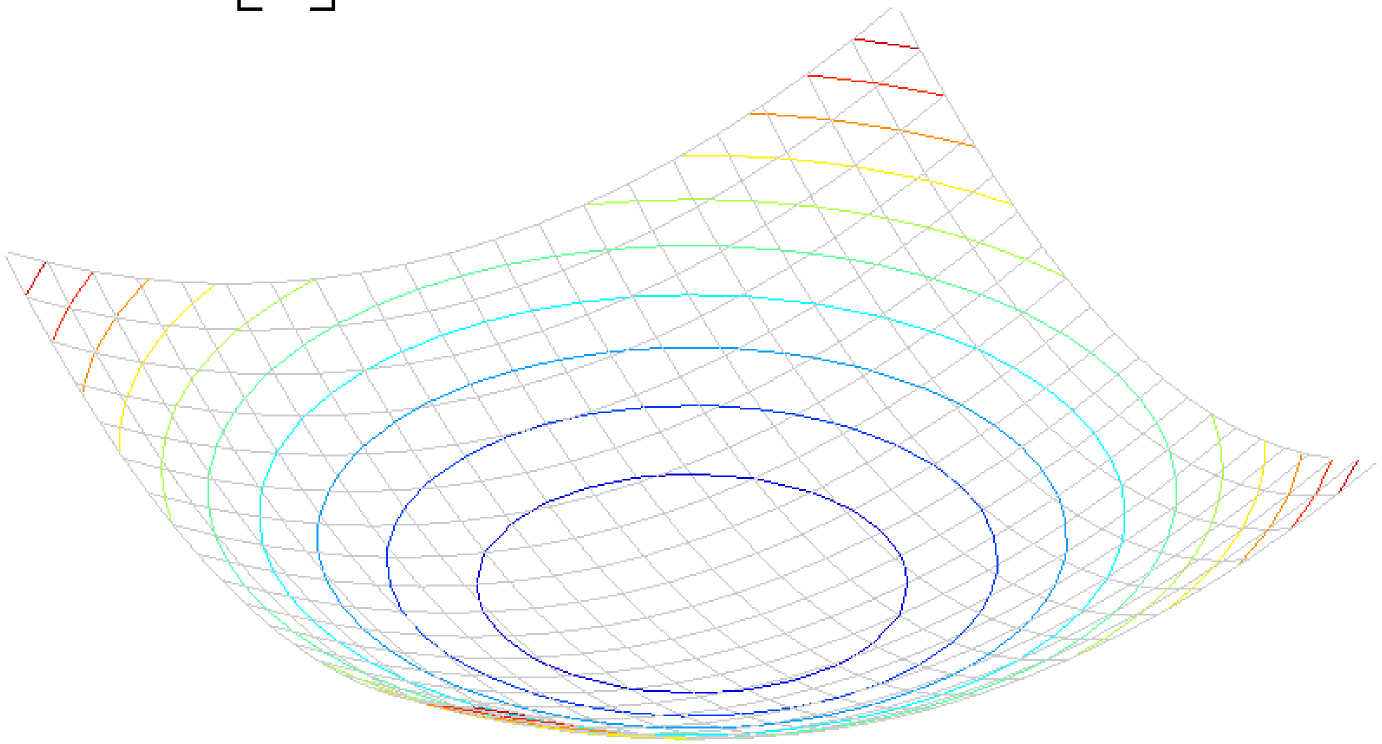


Simplifying the measure even further

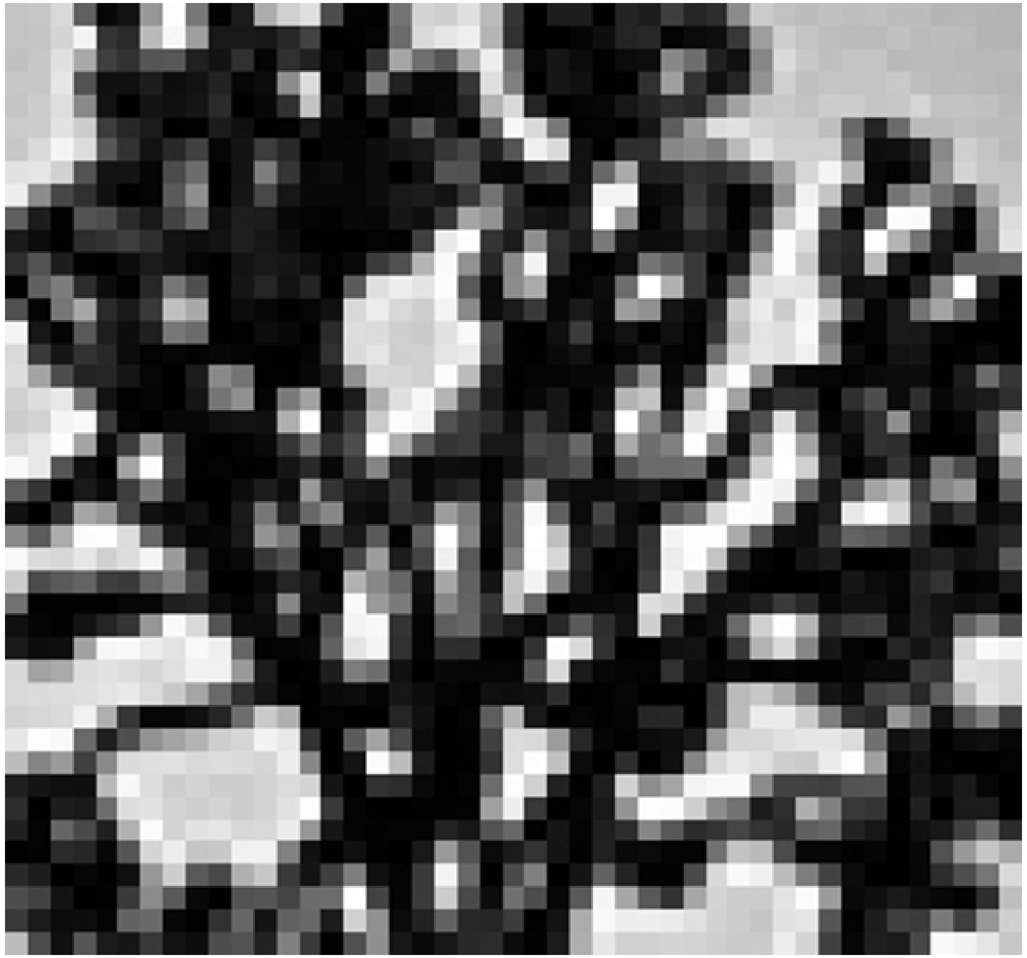
- Consider a horizontal "slice" of *E*(*u*,*v*):

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = const$$

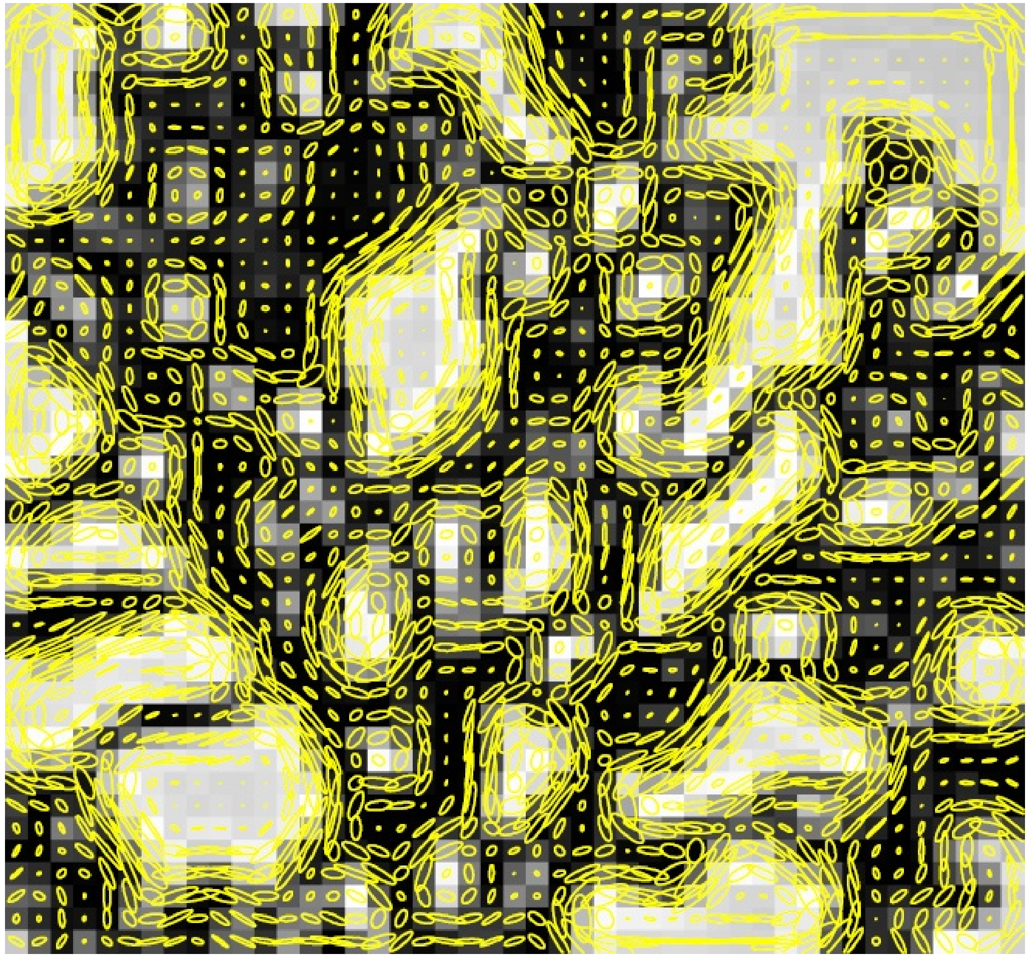
- This is the equation of an ellipse



Visualization of second moment matrices



Visualization of second moment matrices

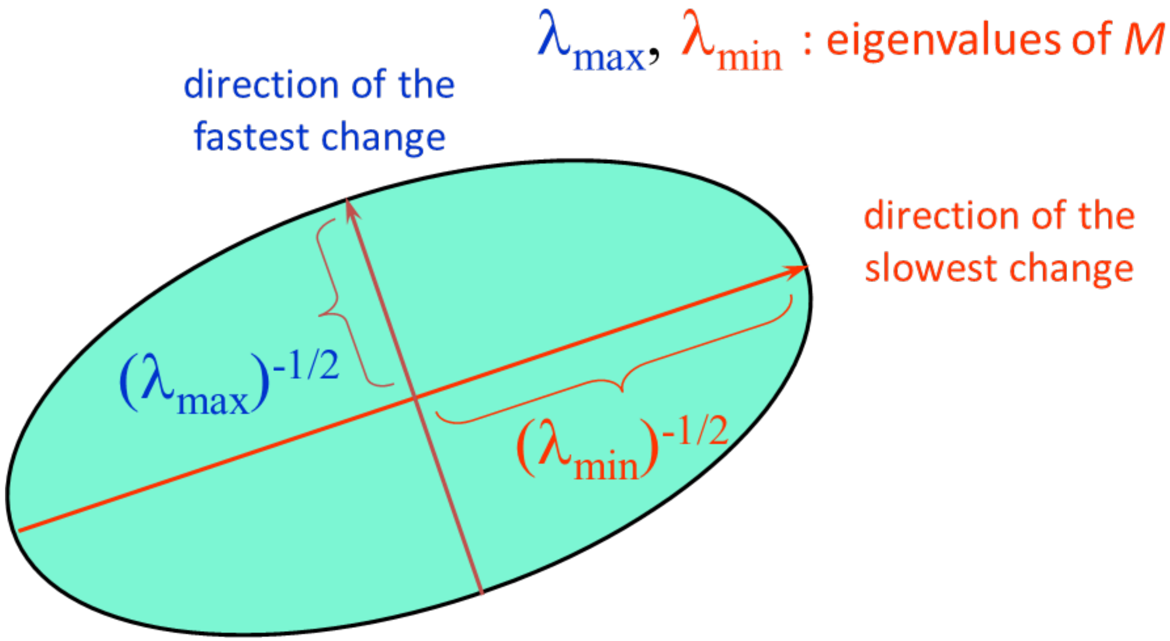


Simplifying the measure even further

- Consider a horizontal "slice" of *E*(*u*,*v*):

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = const$$

- This is the equation of an ellipse
  - Describe the surface using the eigenvalues of M



The eigenvalues and eigenvectors of M

- The eigenvalues:

$$\lambda = \frac{1}{2}\left[(A+C) \pm \sqrt{4B^2 + (A-C)^2} \right]$$

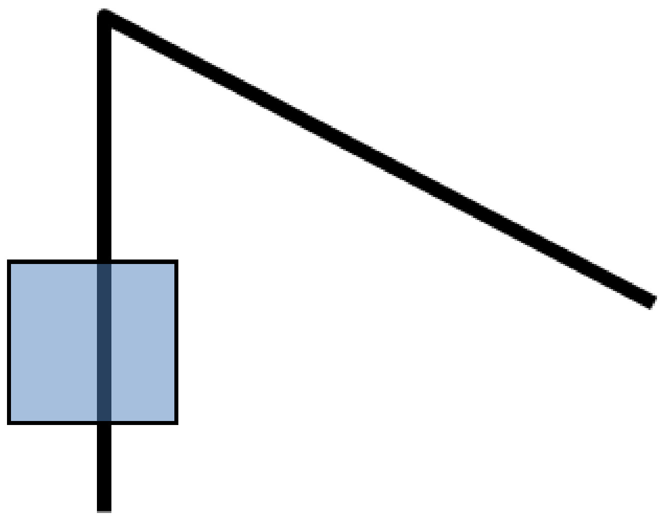
- Once you know λ, you find the eigenvectors **x** by solving

$$\begin{bmatrix} A-\lambda & B \\ B & C-\lambda \end{bmatrix} \mathbf{x} = 0$$

The eigenvalues and eigenvectors of M

- Define shift directions with the smallest and largest change in error

- $\mathbf{x}_{max}$ = direction of largest increase in E
- λ_{max} = amount of increase in direction $\mathbf{x}_{max}$
- $\mathbf{x}_{min}$ = direction of smallest increase in E
- λ_{min} = amount of increase in direction $\mathbf{x}_{min}$

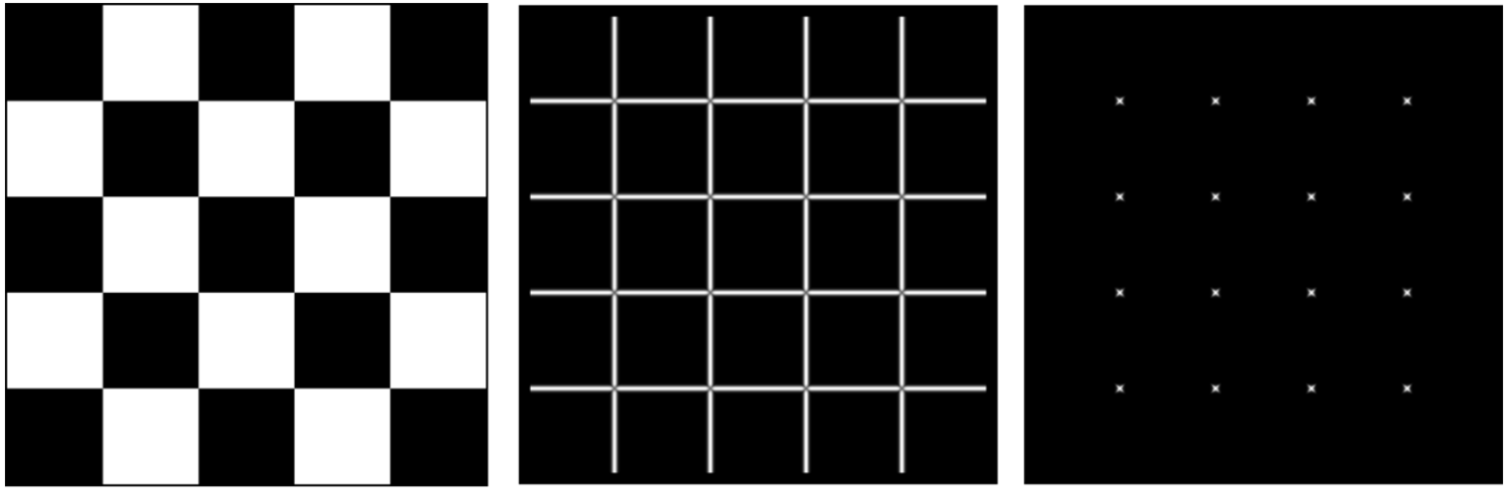


Local measure of feature distinctiveness

- How are λ_{max}, $\mathbf{x}_{max}$, λ_{min}, $\mathbf{x}_{min}$ relevant for feature detection?
  - What is our feature scoring function?

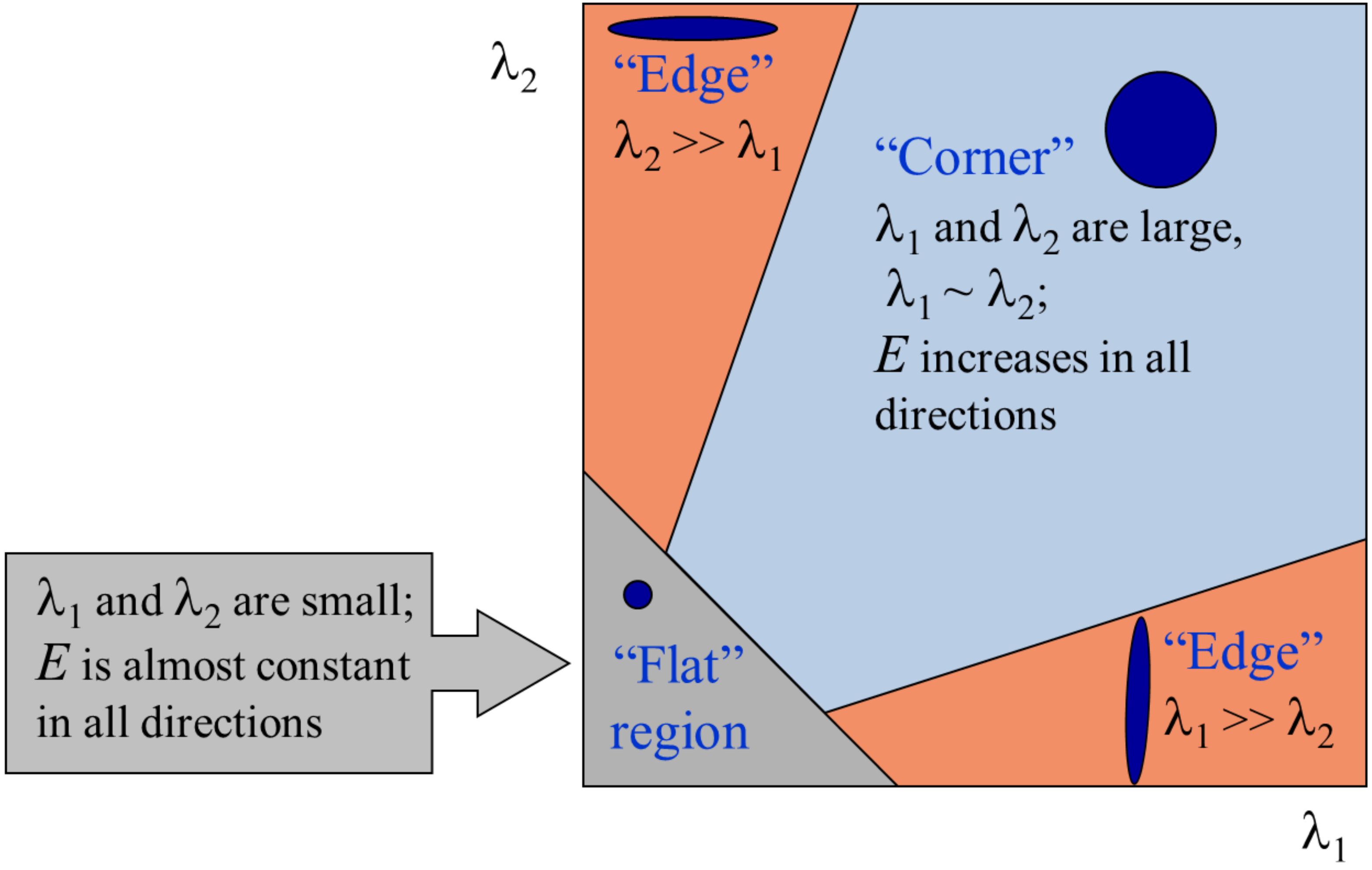
Local measure of feature distinctiveness

- How are λ_{max}, $\mathbf{x}_{max}$, λ_{min}, $\mathbf{x}_{min}$ relevant for feature detection?
  - What is our feature scoring function?
- Want $E(u,v)$ to be large for small shifts in all directions
  - the minimum of $E(u,v)$ should be large, over all unit vectors $[u\ v]$
  - this minimum is given by the smaller eigenvalue (λ_{min}) of M



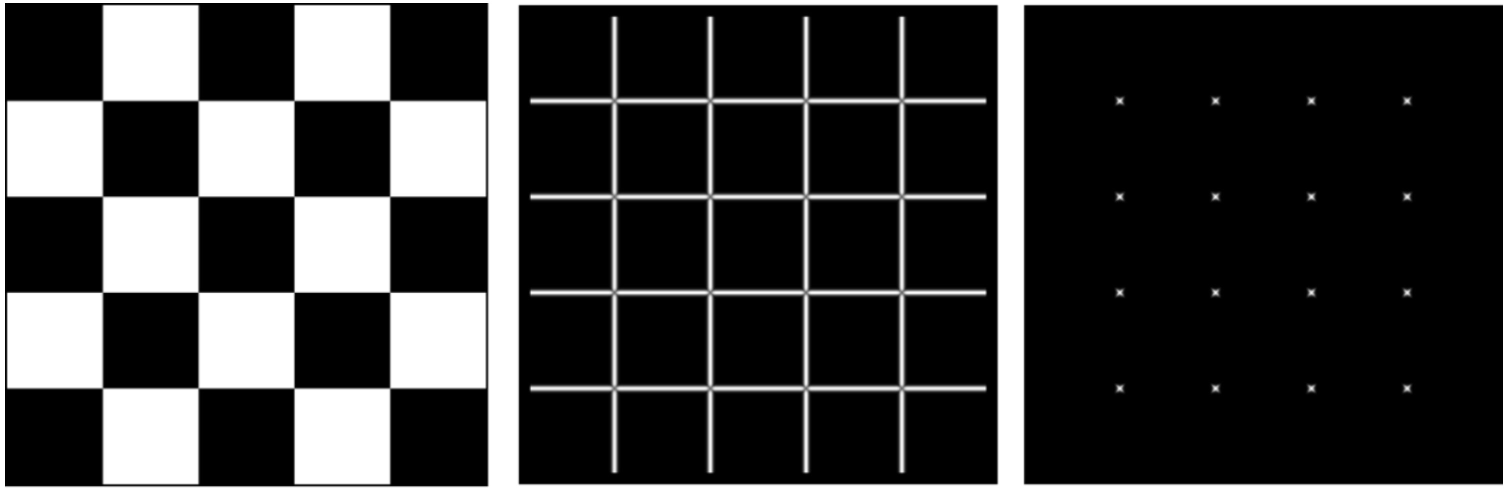
I λ_{max} λ_{min}

Interpreting the eigenvalues



Corner detection summary

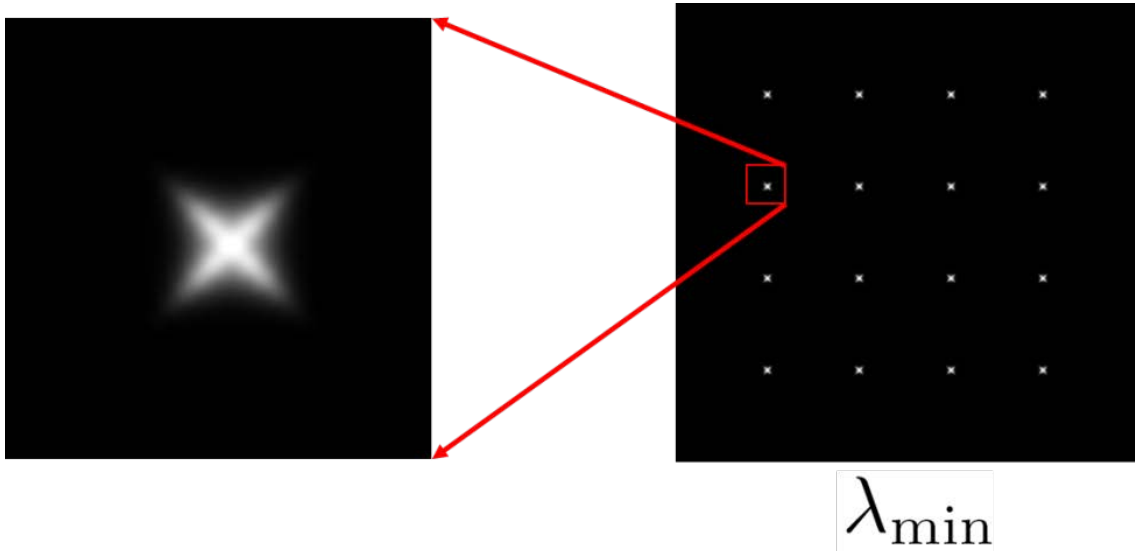
- Compute the gradient at each point in the image using derivatives of Gaussians
- Create the second moment matrix M from the entries in the gradient
- Compute the eigenvalues



I $\lambda_{\max}$ $\lambda_{\min}$

Corner detection summary

- Compute the gradient at each point in the image using derivatives of Gaussians
- Create the second moment matrix M from the entries in the gradient
- Compute the eigenvalues
- Find points with large response (λ_{min} > threshold)
- Choose those points where λ_{min} is a local maximum as features



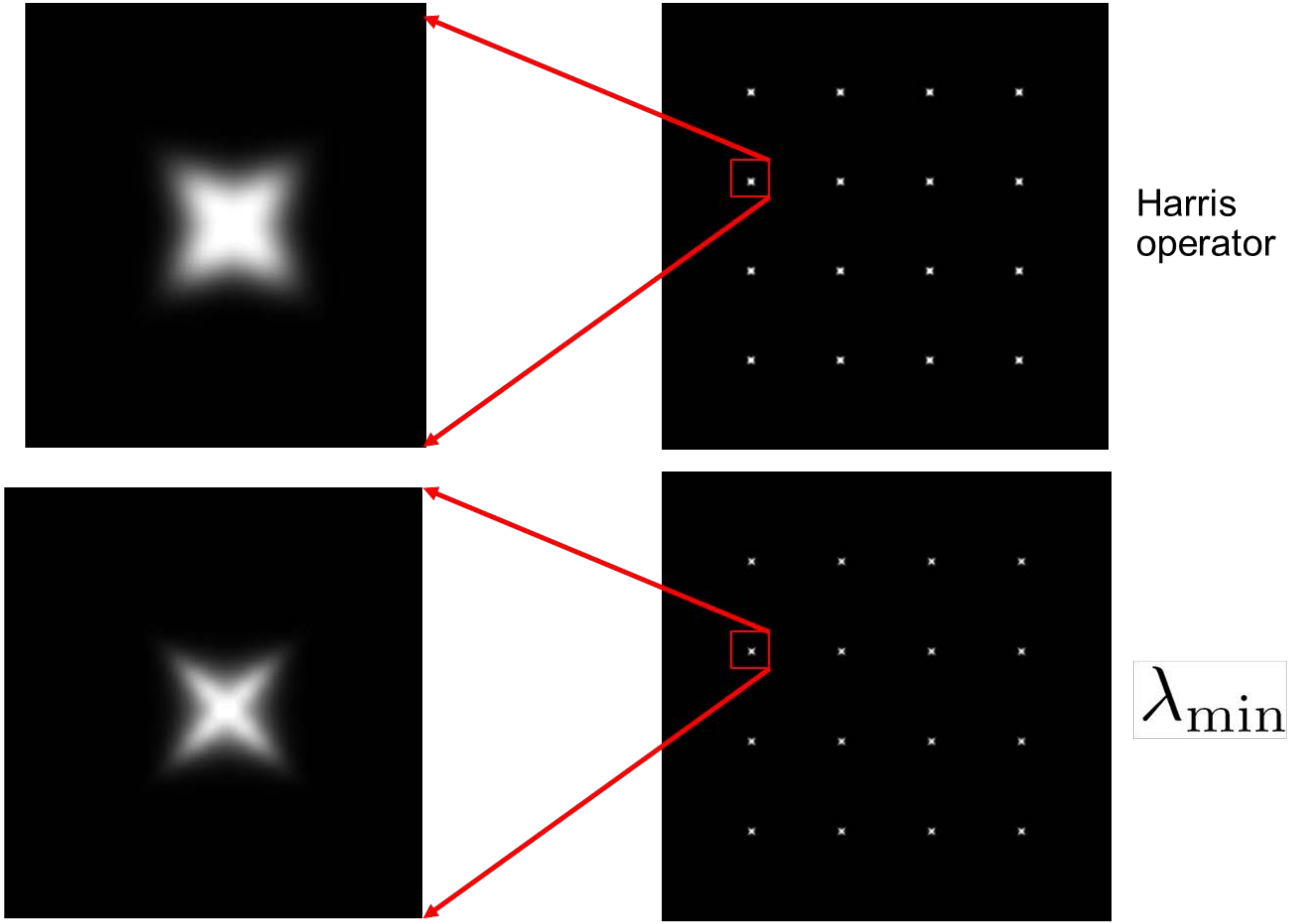
The Harris operator

- An alternative to λ_{min}:

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{\det(M)}{\text{trace}(M)}$$

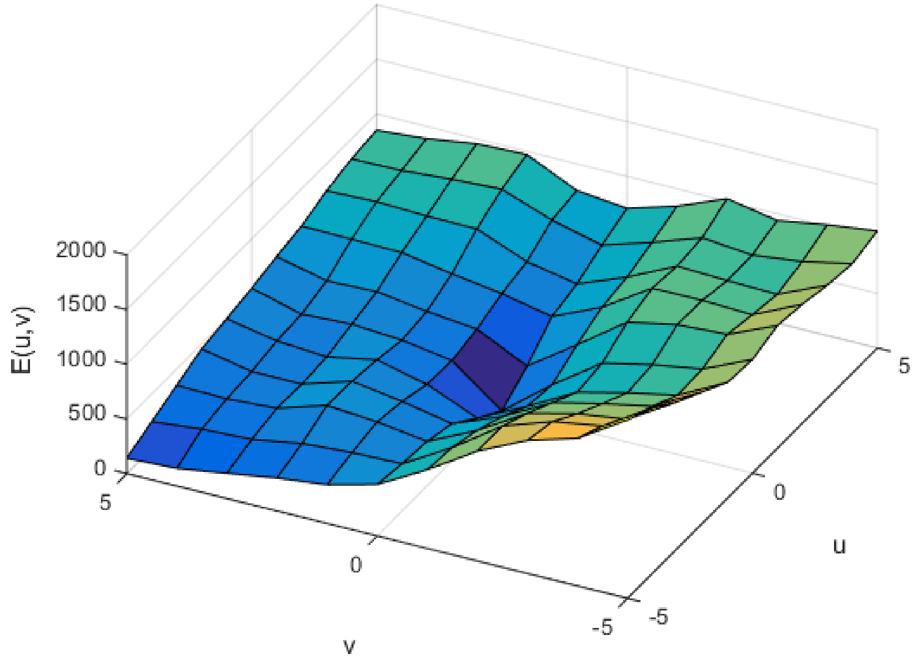
- Very similar to λ_{min} but less expensive (no square root)
- Called the “Harris Corner Detector” or “Harris Operator”
- Lots of other detectors, this is one of the most popular

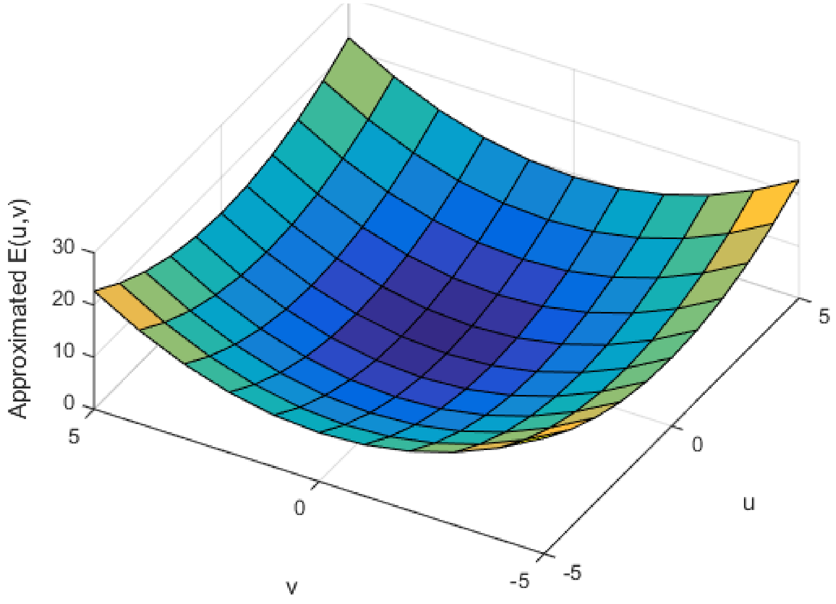
The Harris operator



Examples Holmenkollen

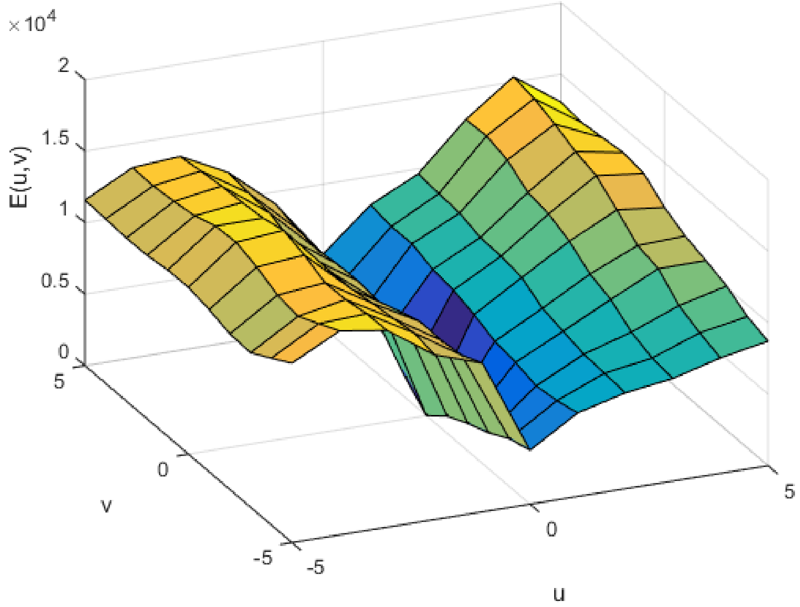
$\lambda_{min} = 0.4$

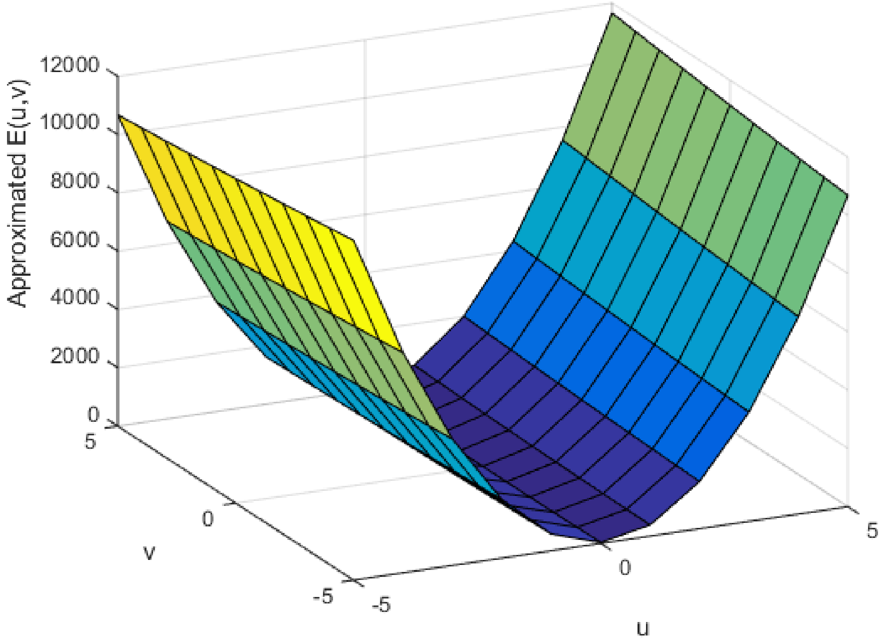




Examples Holmenkollen

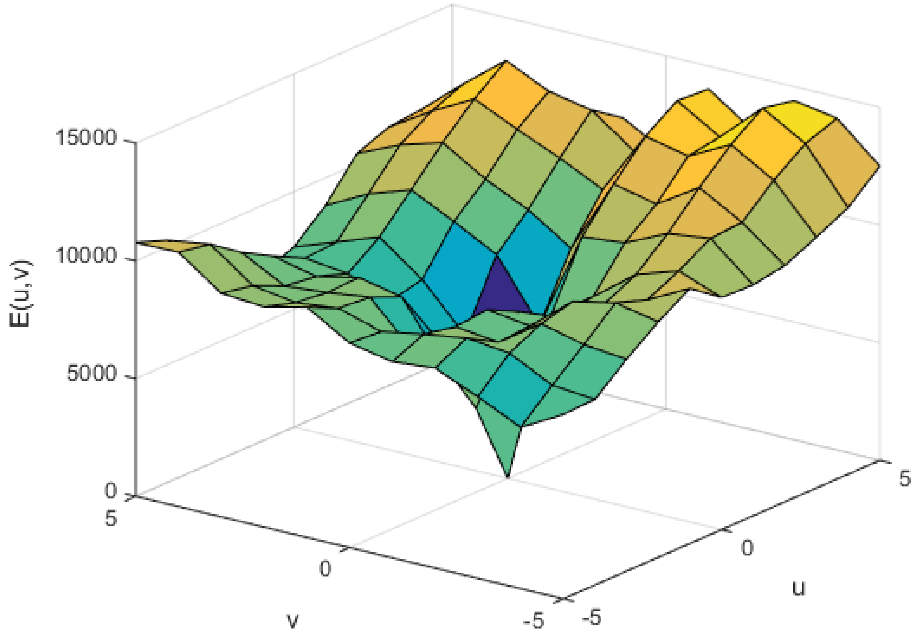
$\lambda_{min} = 1.2$

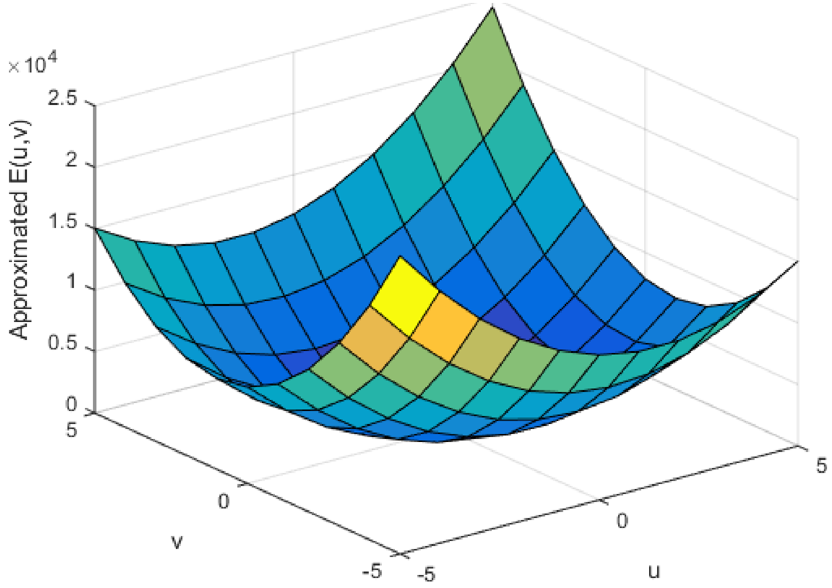




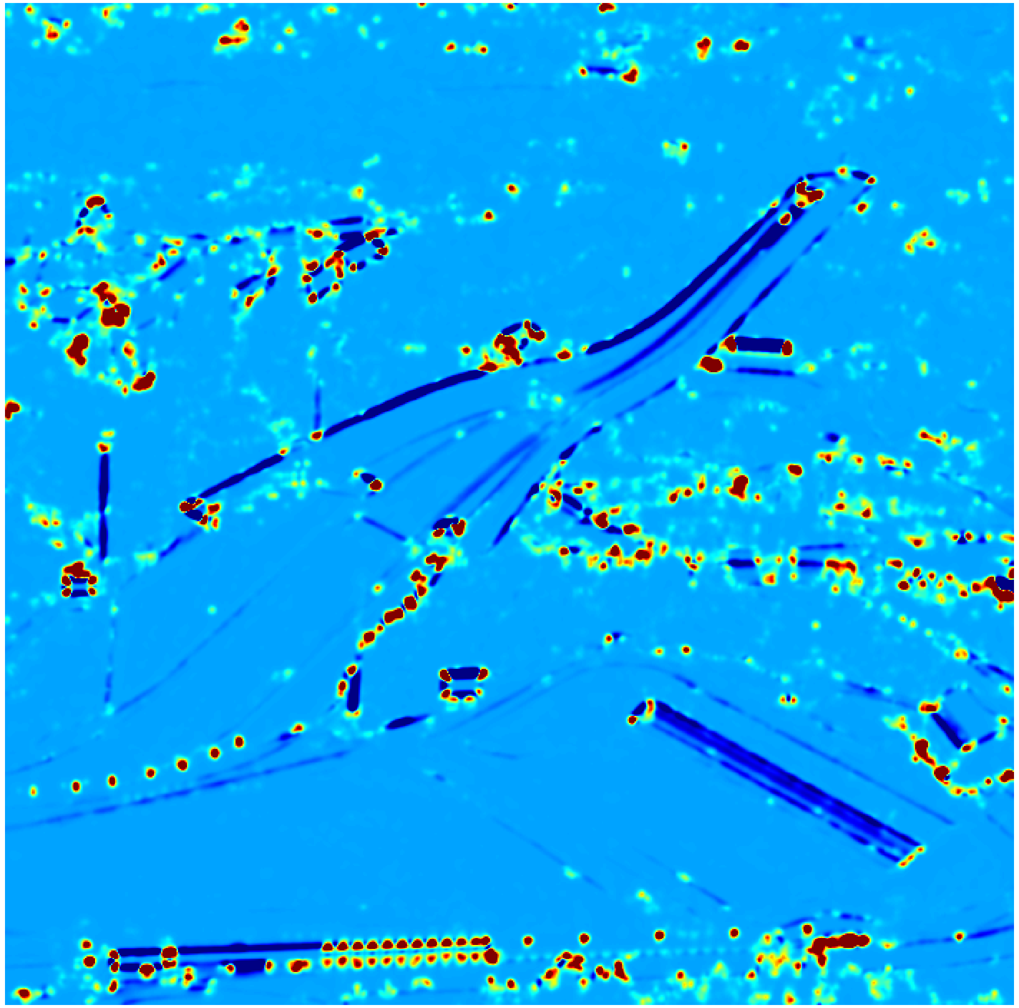
Examples Holmenkollen

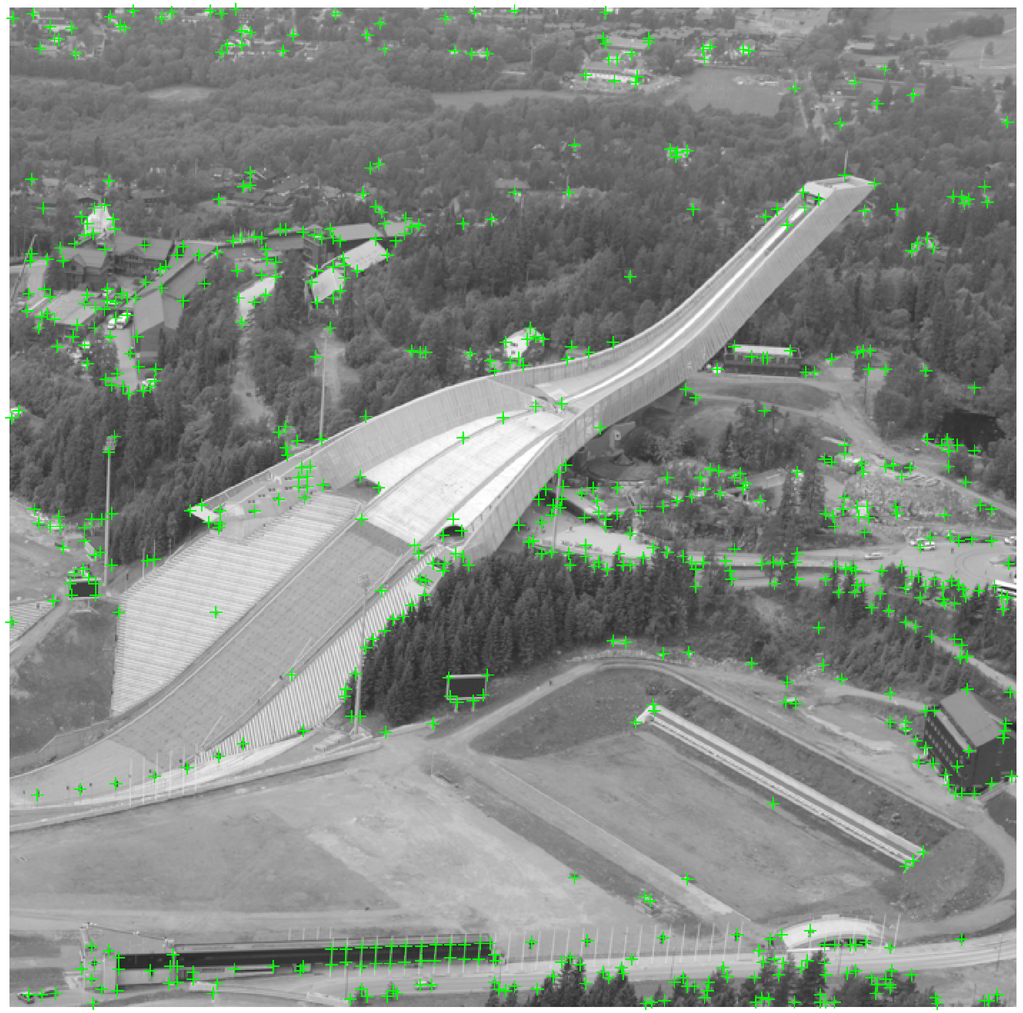
$\lambda_{min} = 272$





Holmenkollen example



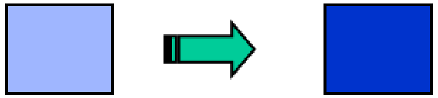


Invariance and covariance

- We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations
  - **Invariance**: image is transformed and corner locations do not change
  - **Covariance**: if we have two transformed versions of the same image, features should be detected in corresponding locations

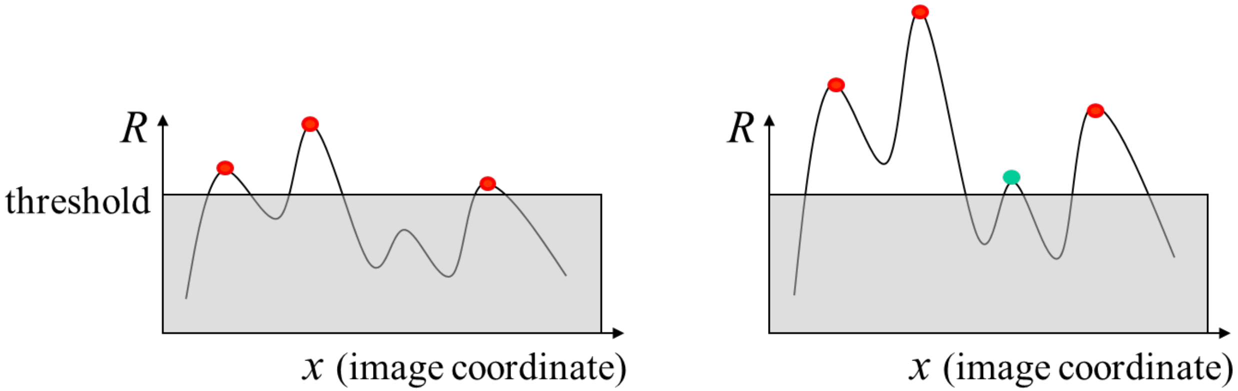
Harris detector properties

- **Affine intensity change**



$$I \rightarrow a\,I + b$$

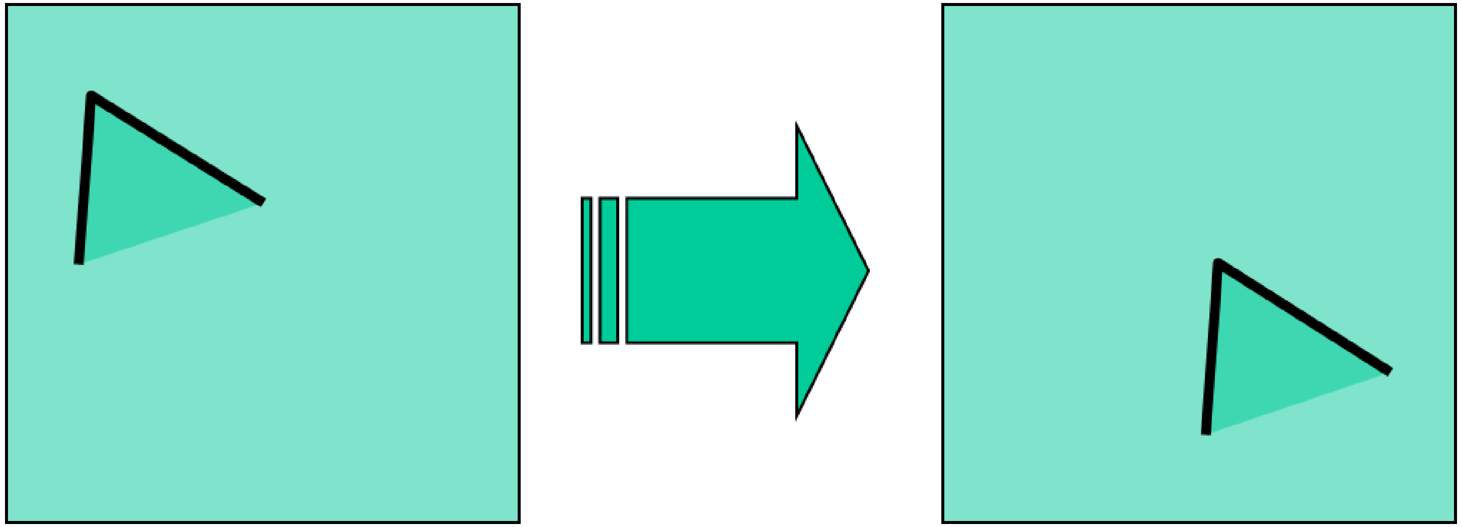
- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow a\,I$



Partially invariant to affine intensity change

Harris detector properties

- **Image translation**

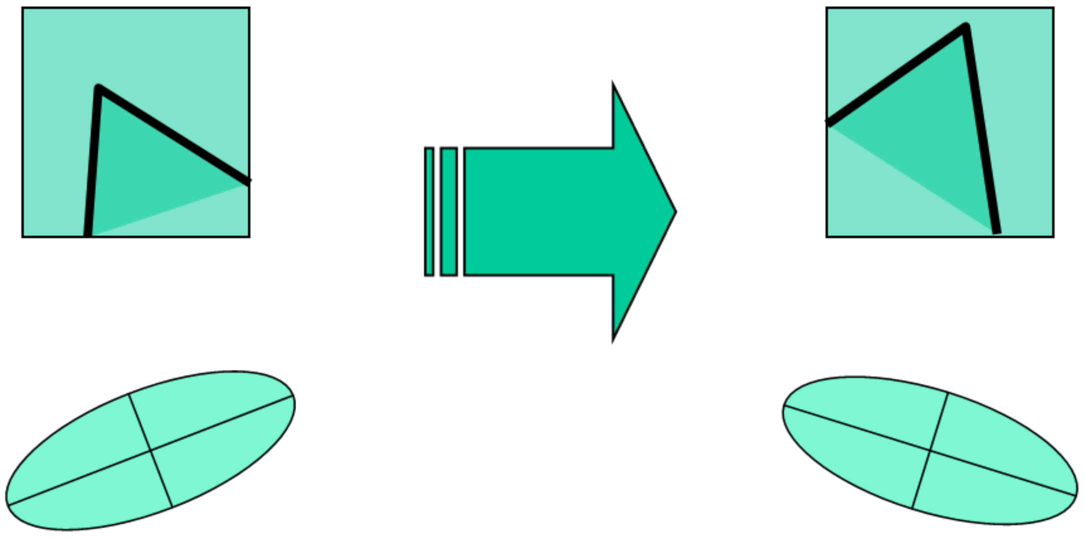


- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

Harris detector properties

- **Image rotation**

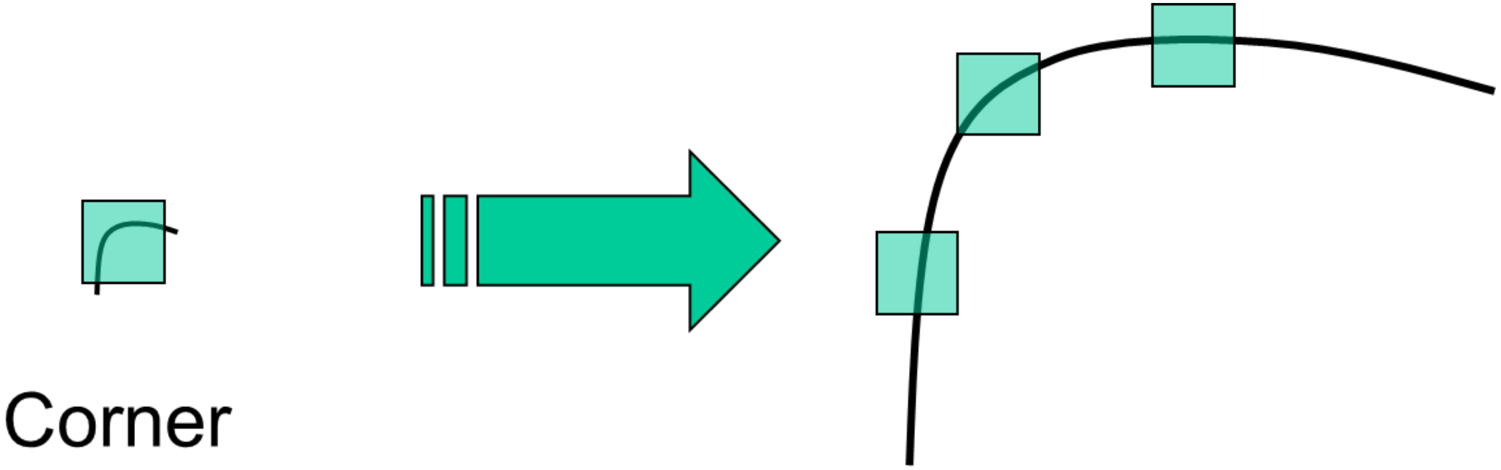


- Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Harris detector properties

- **Scaling**

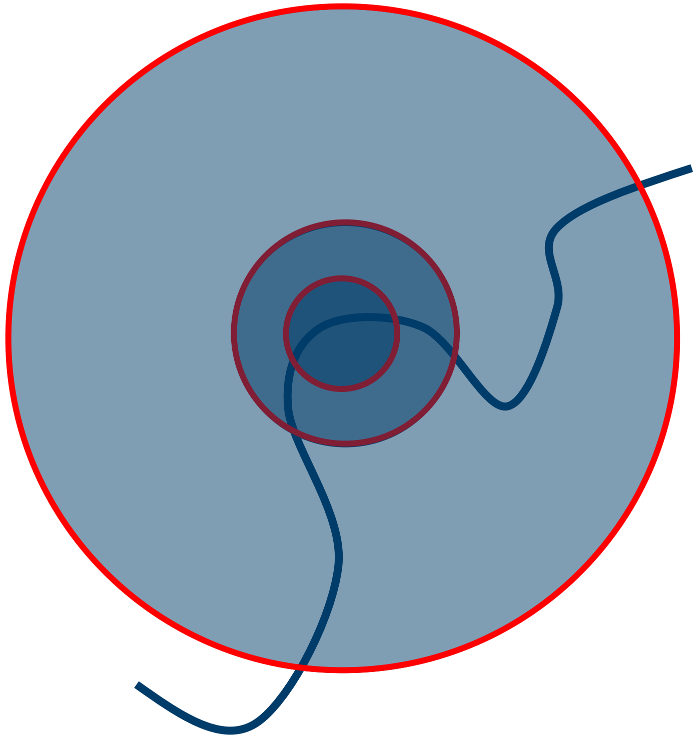


All points will be classified as edges

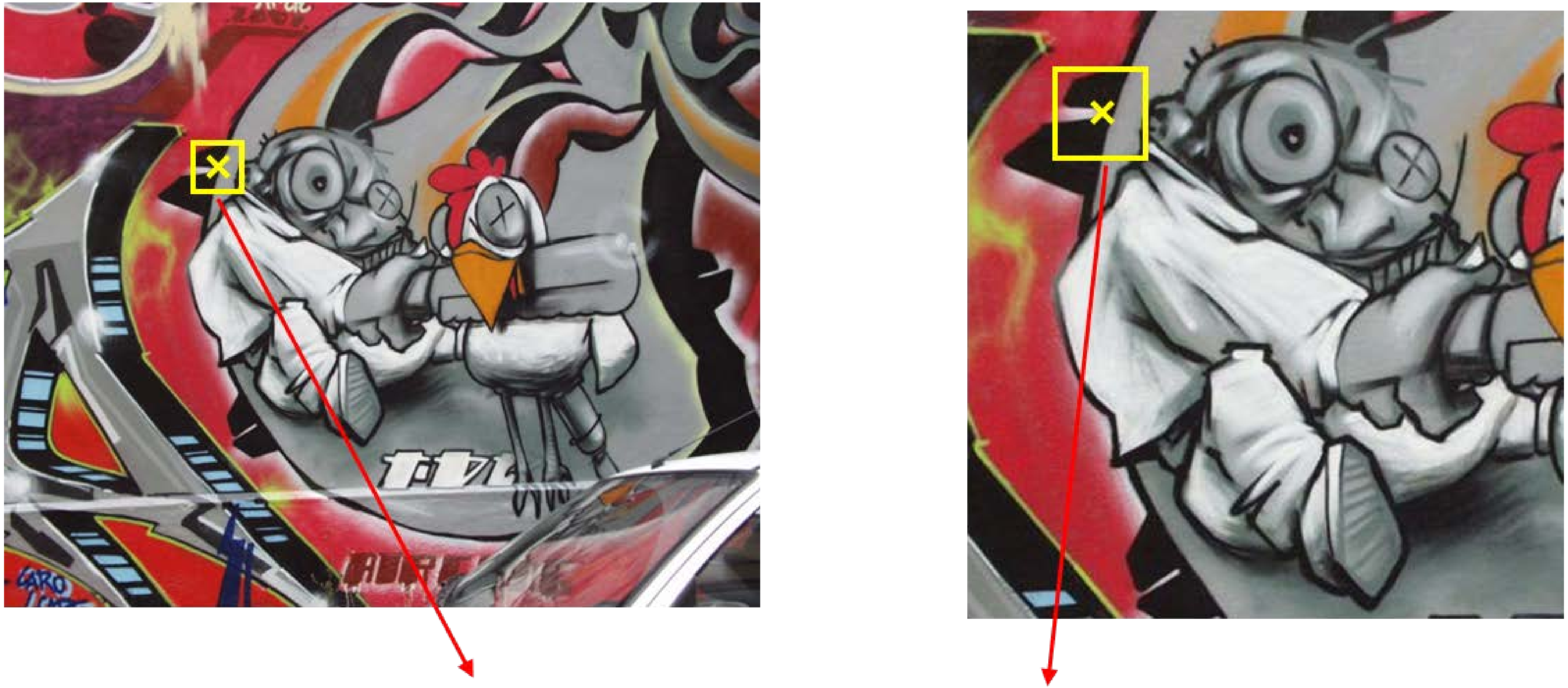
Corner location is not covariant to scaling!

Scale robust corner detection

- Find scale that gives local maximum of score *f*
  - In both position and scale

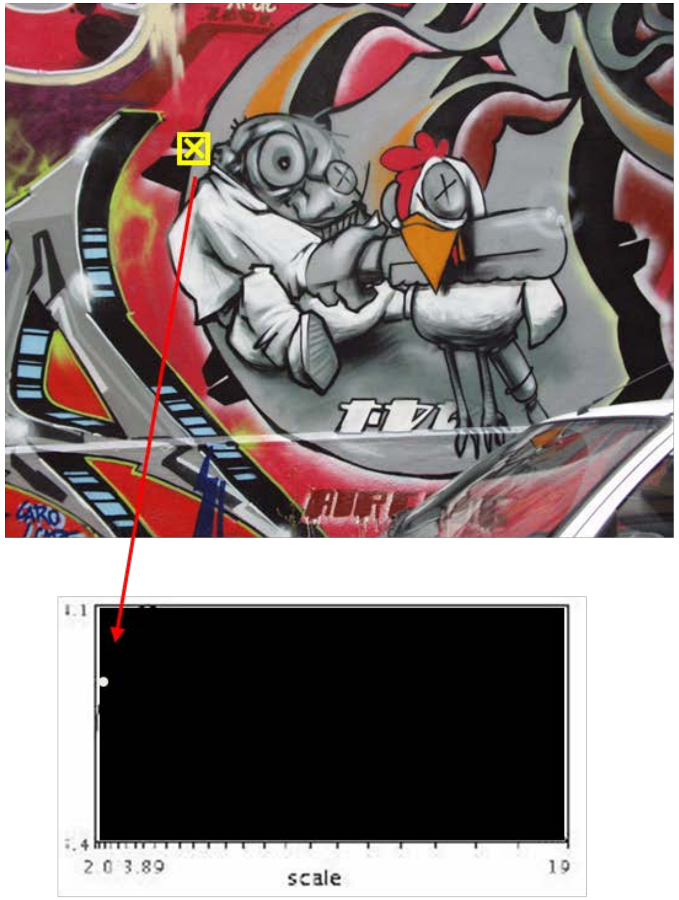


Automatic scale selection



$$f(I_{i_1 \dots i_m}(x, \sigma)) = f(I_{i_1 \dots i_m}(x', \sigma'))$$

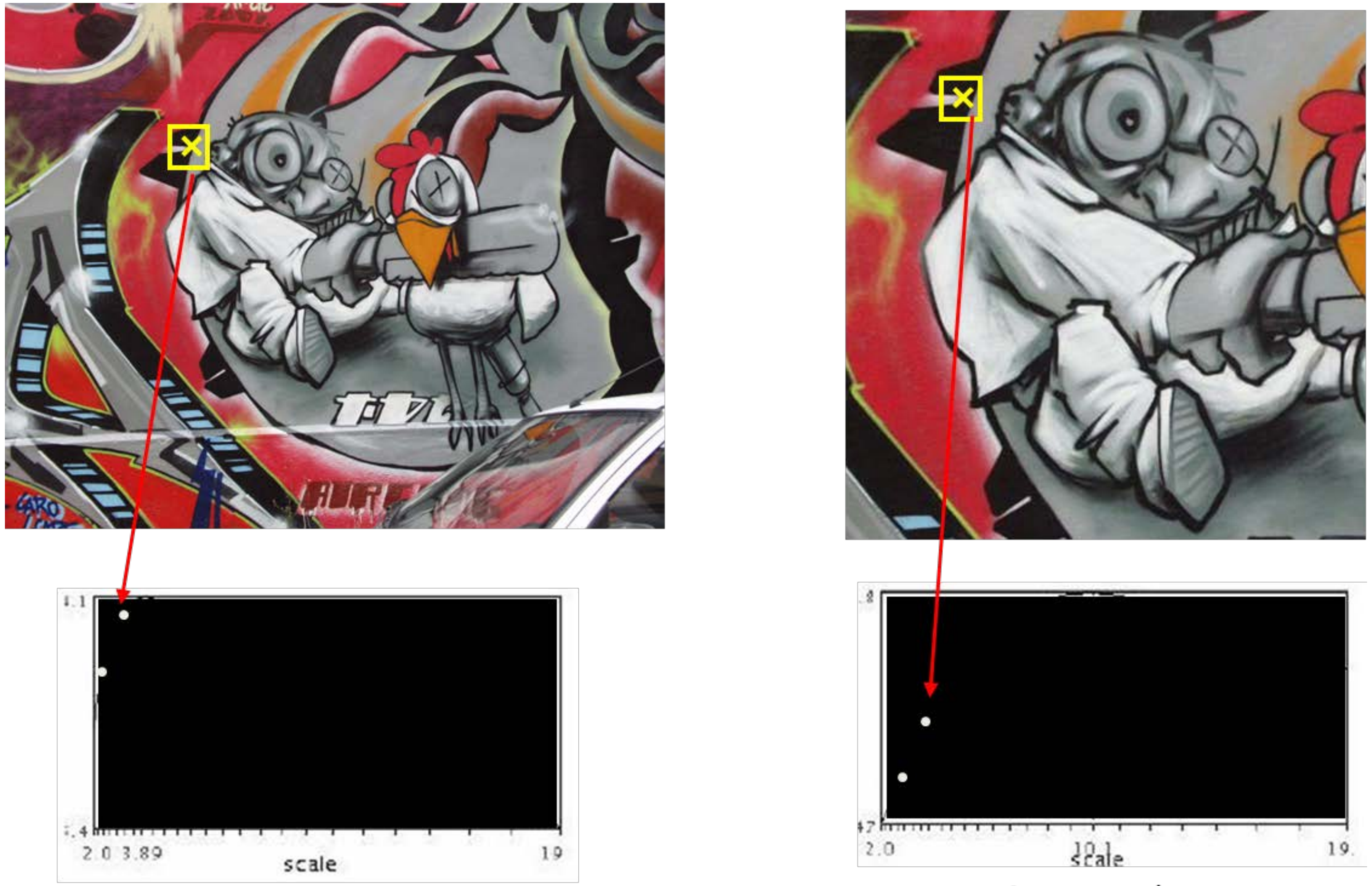
Automatic scale selection



$f(I_{i_1 \ldots i_m}(x, \sigma))$

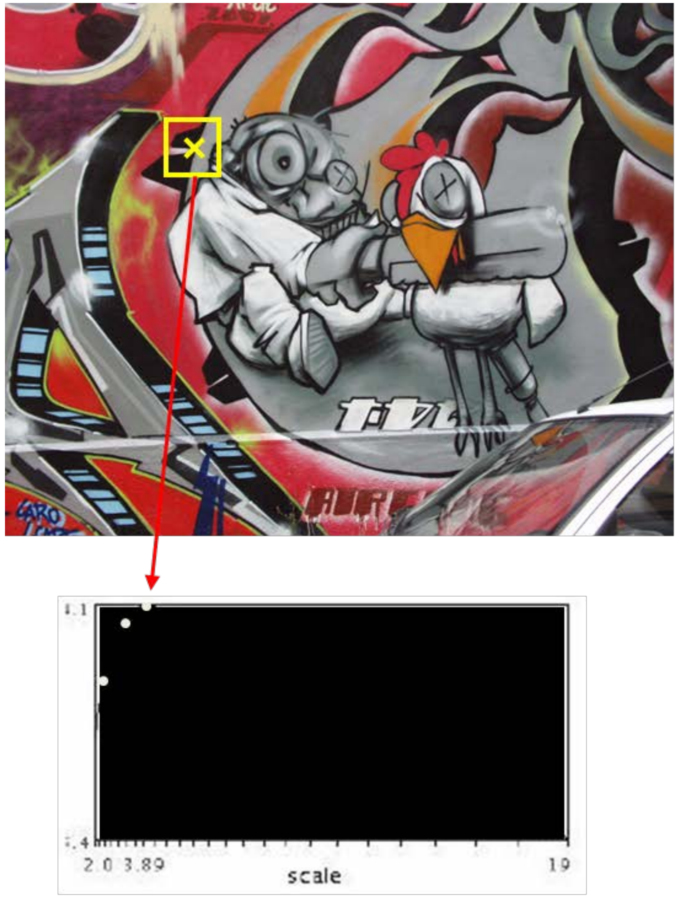
$f(I_{i_1 \ldots i_m}(x', \sigma))$

Automatic scale selection



$f(I_{i_1 \ldots i_m}(x, \sigma))$ $\quad$ $f(I_{i_1 \ldots i_m}(x', \sigma))$

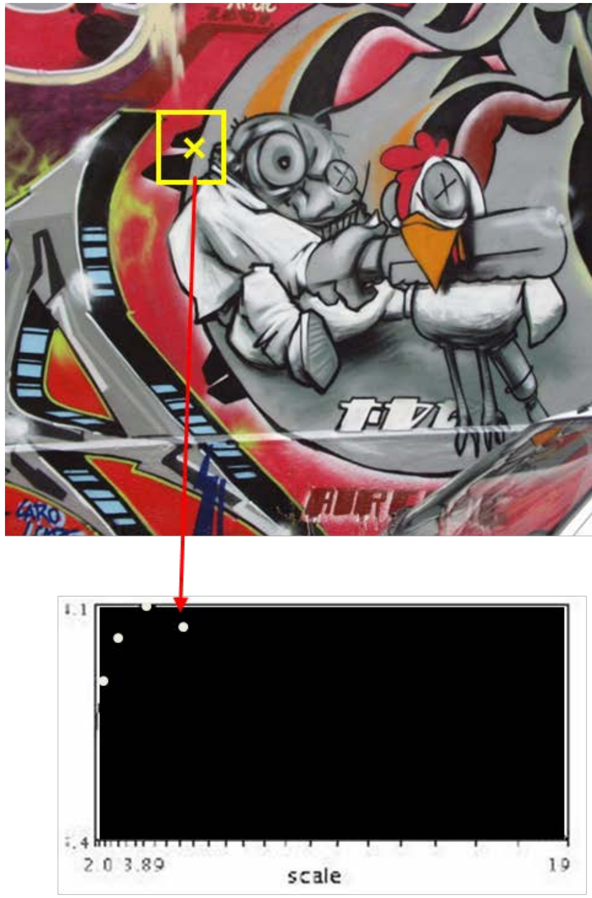
Automatic scale selection



$f(I_{i_1 \ldots i_m}(x,\sigma))$

$f(I_{i_1 \ldots i_m}(x',\sigma))$

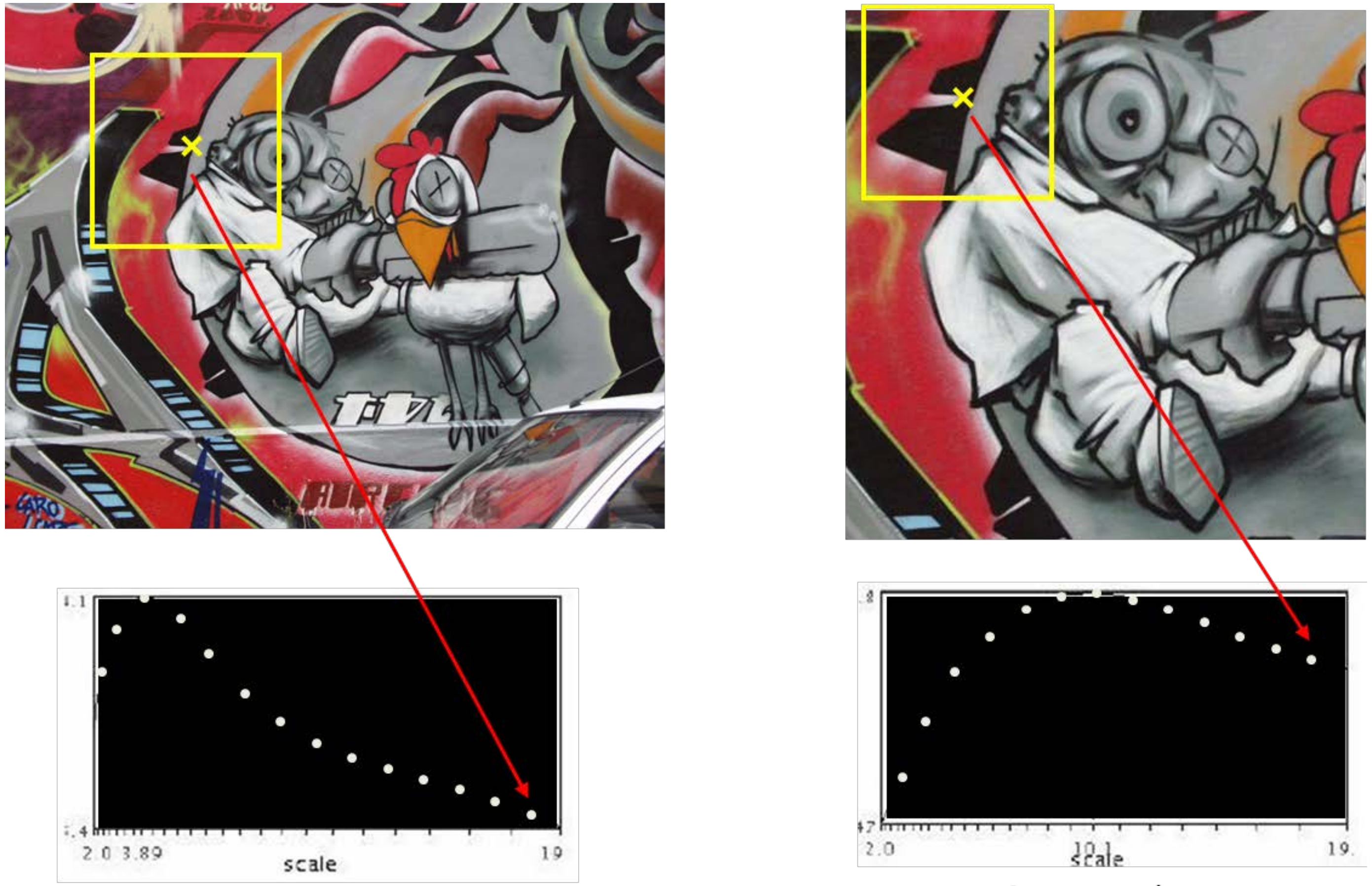
Automatic scale selection



$f(I_{i_1 \dots i_m}(x, \sigma))$

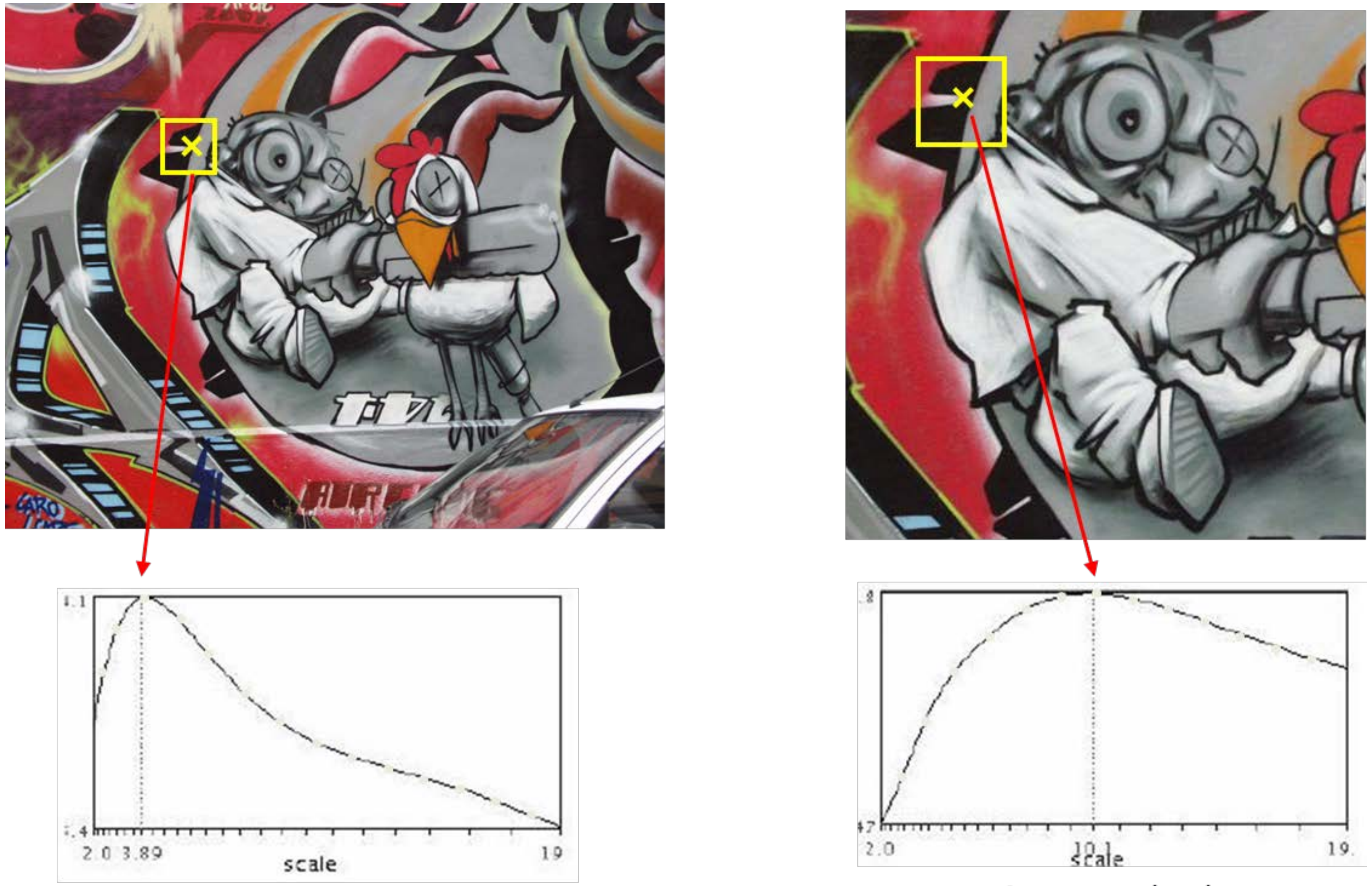
$f(I_{i_1 \dots i_m}(x', \sigma))$

Automatic scale selection



$f(I_{i_1 \ldots i_m}(x, \sigma))$ $\quad$ $f(I_{i_1 \ldots i_m}(x', \sigma))$

Automatic scale selection



$f(I_{i_1 \dots i_m}(x, \sigma))$ $f(I_{i_1 \dots i_m}(x', \sigma'))$

Next

- Blob detector: stable in space and scale