

## Lecture 6.2

# Stereo imaging

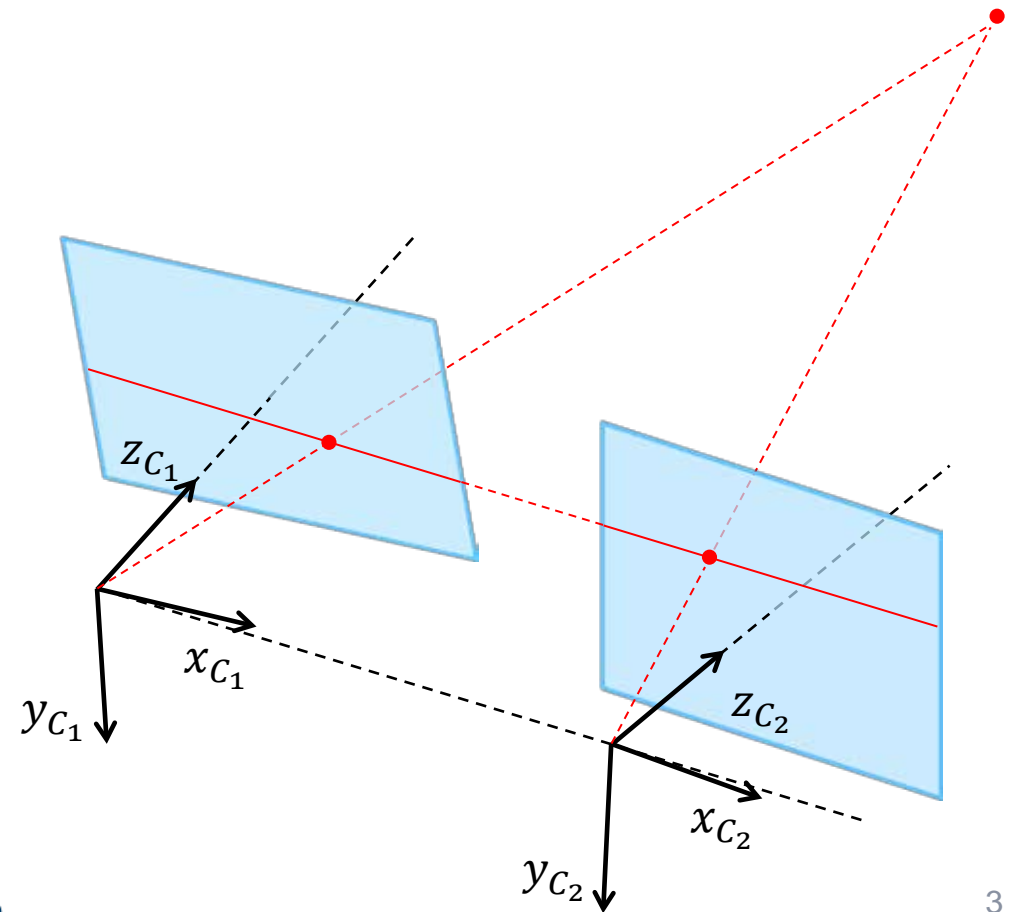
Trym Vegard Haavardsholm

# World geometry from correspondences

	Structure (scene geometry)	Motion (camera geometry)	Measurements
Pose estimation	Known	Estimate	3D to 2D correspondences
Triangulation, Stereo	Estimate	Known	2D to 2D correspondences
Reconstruction, Structure from Motion	Estimate	Estimate	2D to 2D correspondences

# Stereo vision

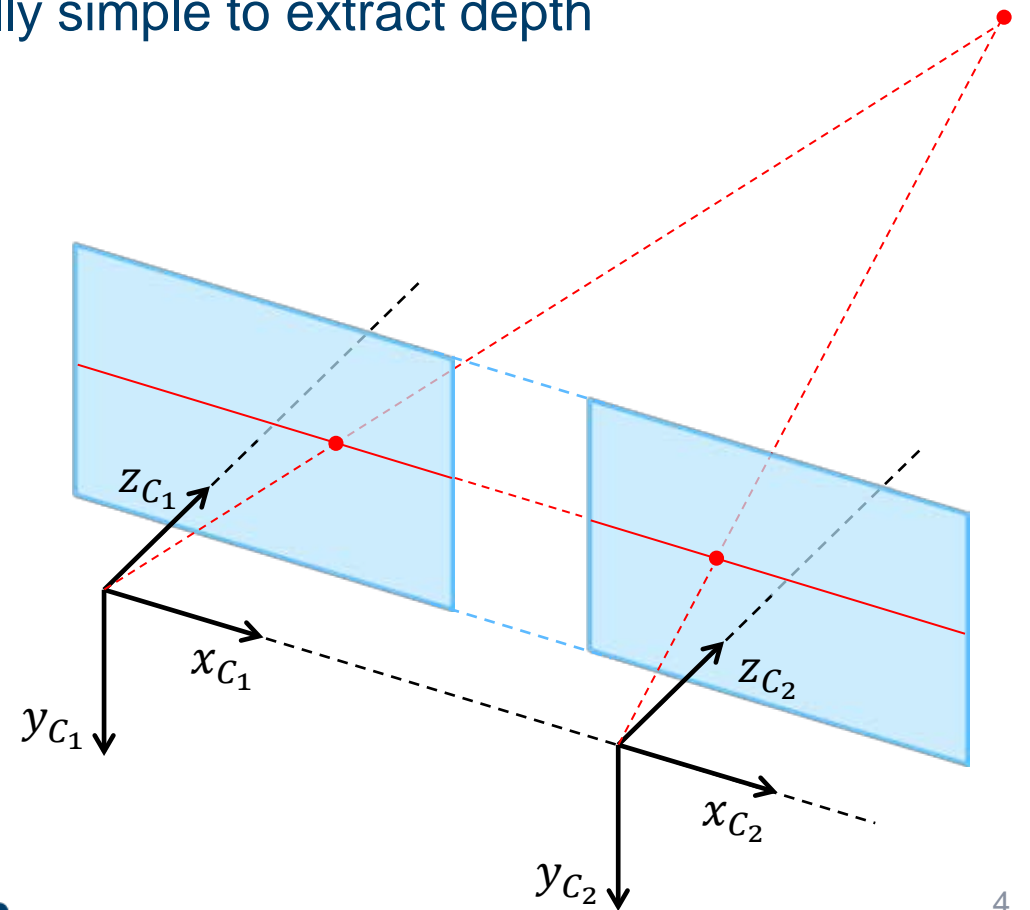
- Depth from two views with known viewpoints



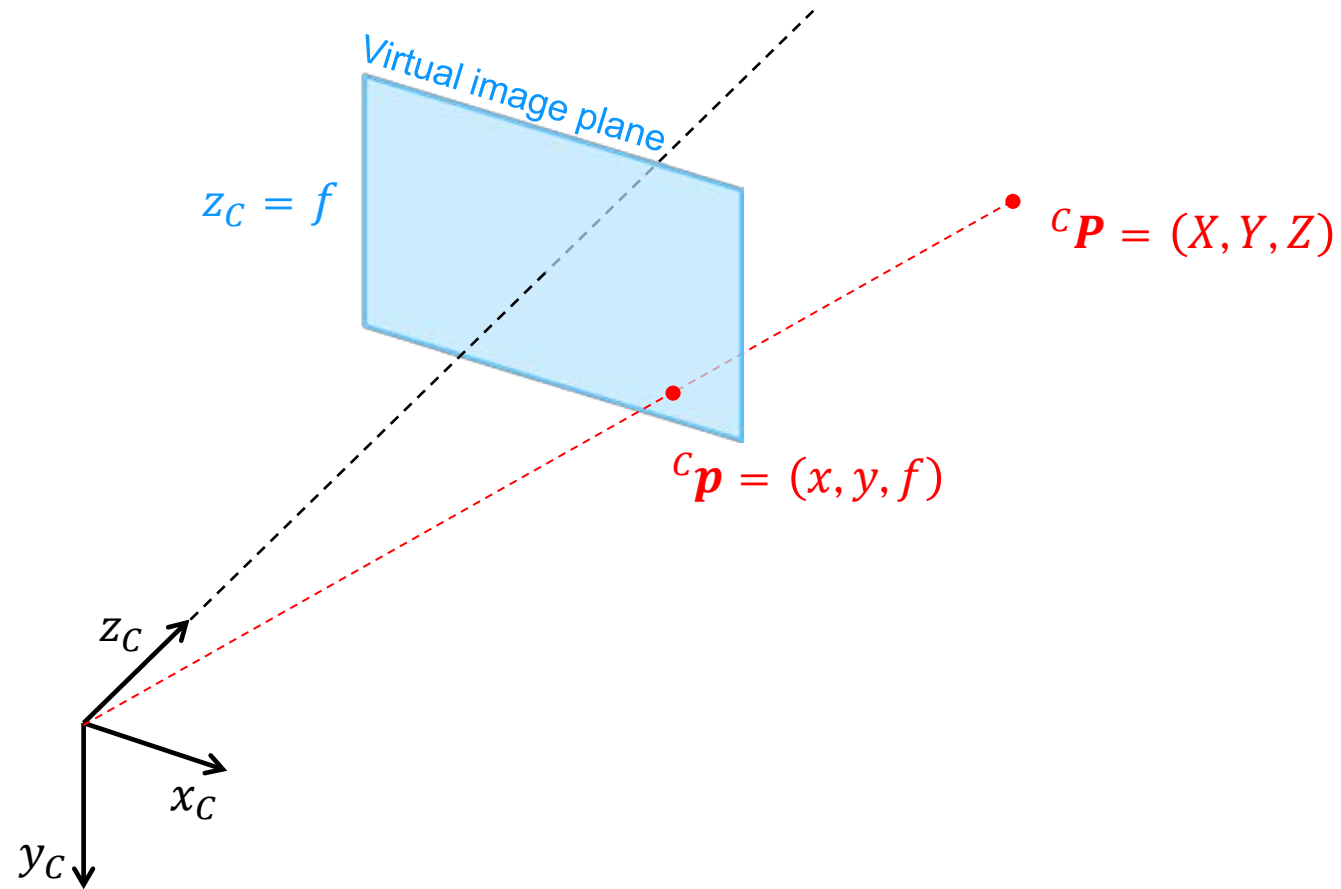
# Stereo vision

- Depth from two views with known viewpoints

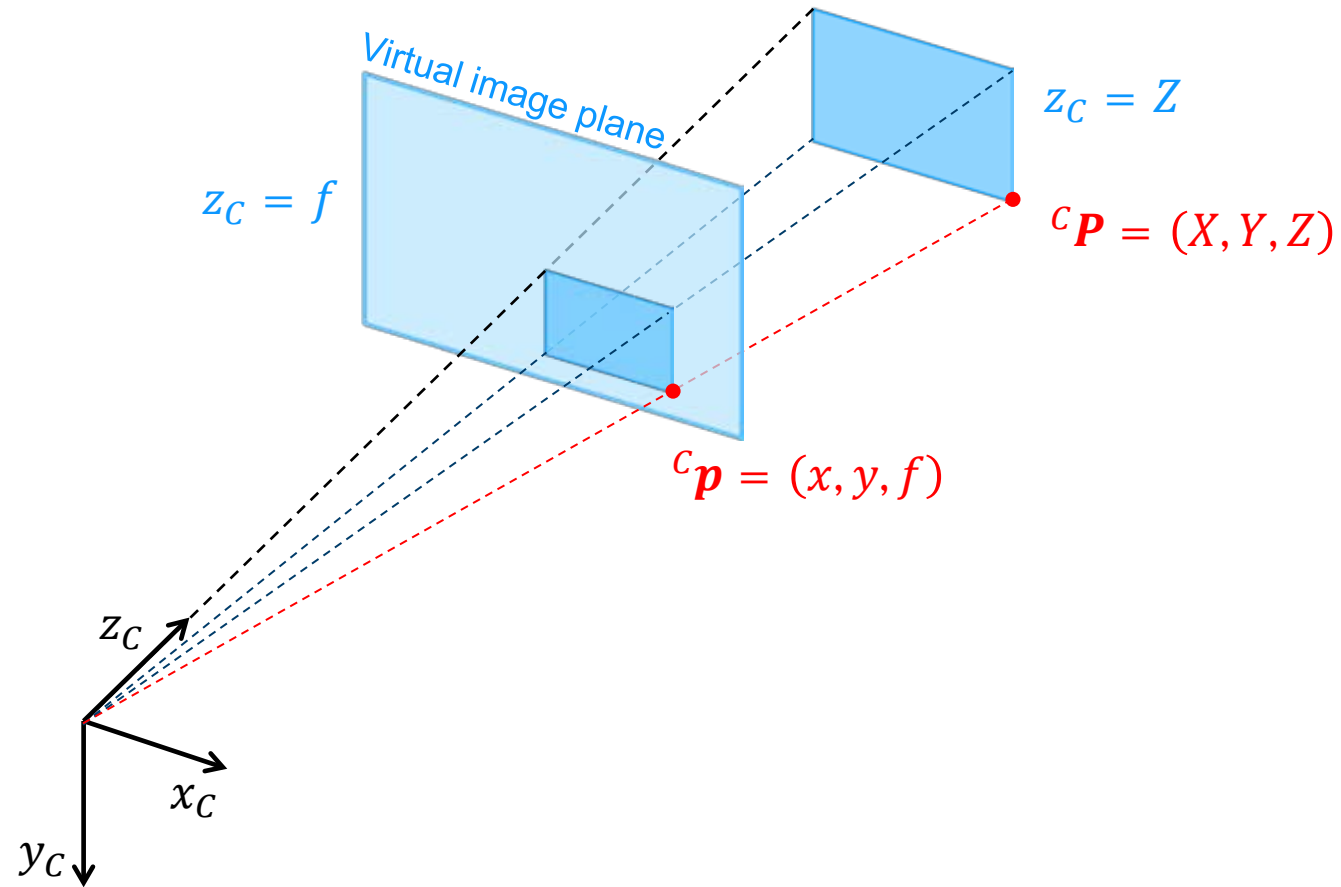
...with an imaging geometry that makes it especially simple to extract depth



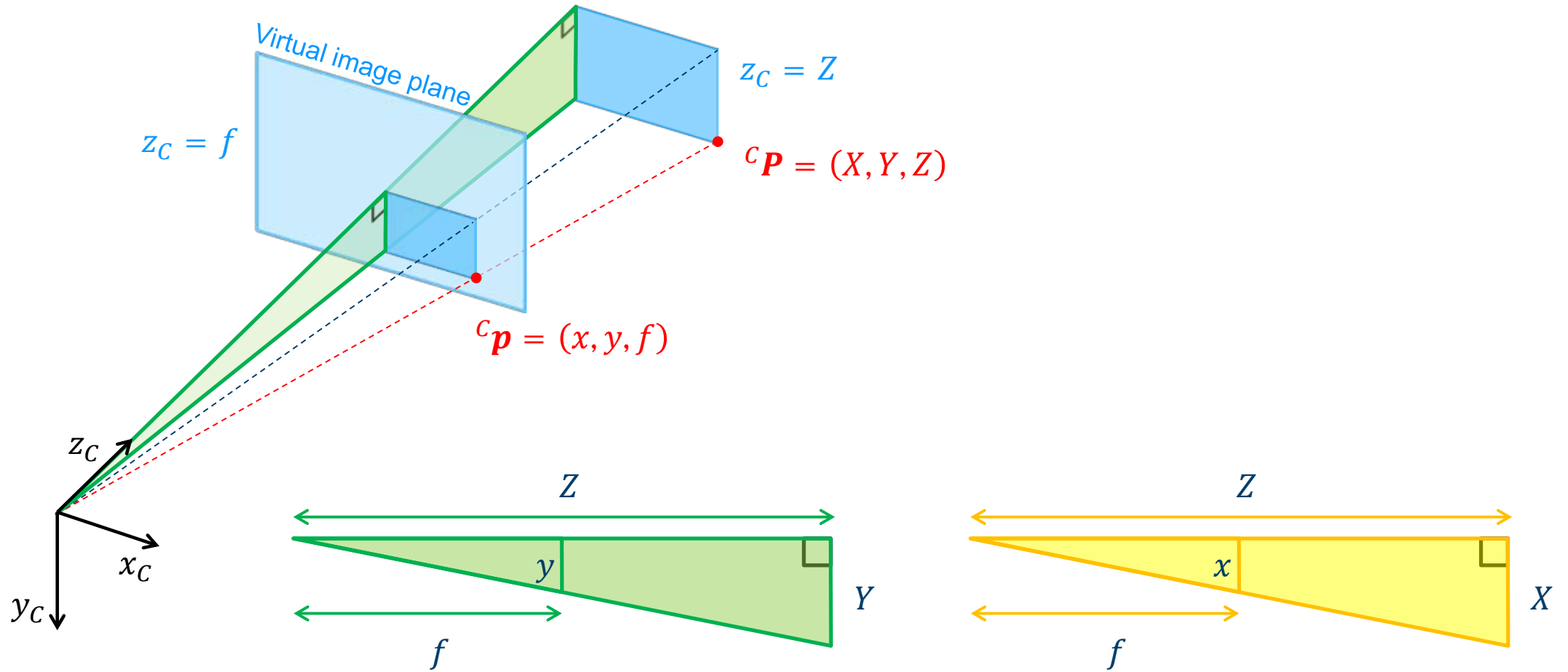
# Depth from images



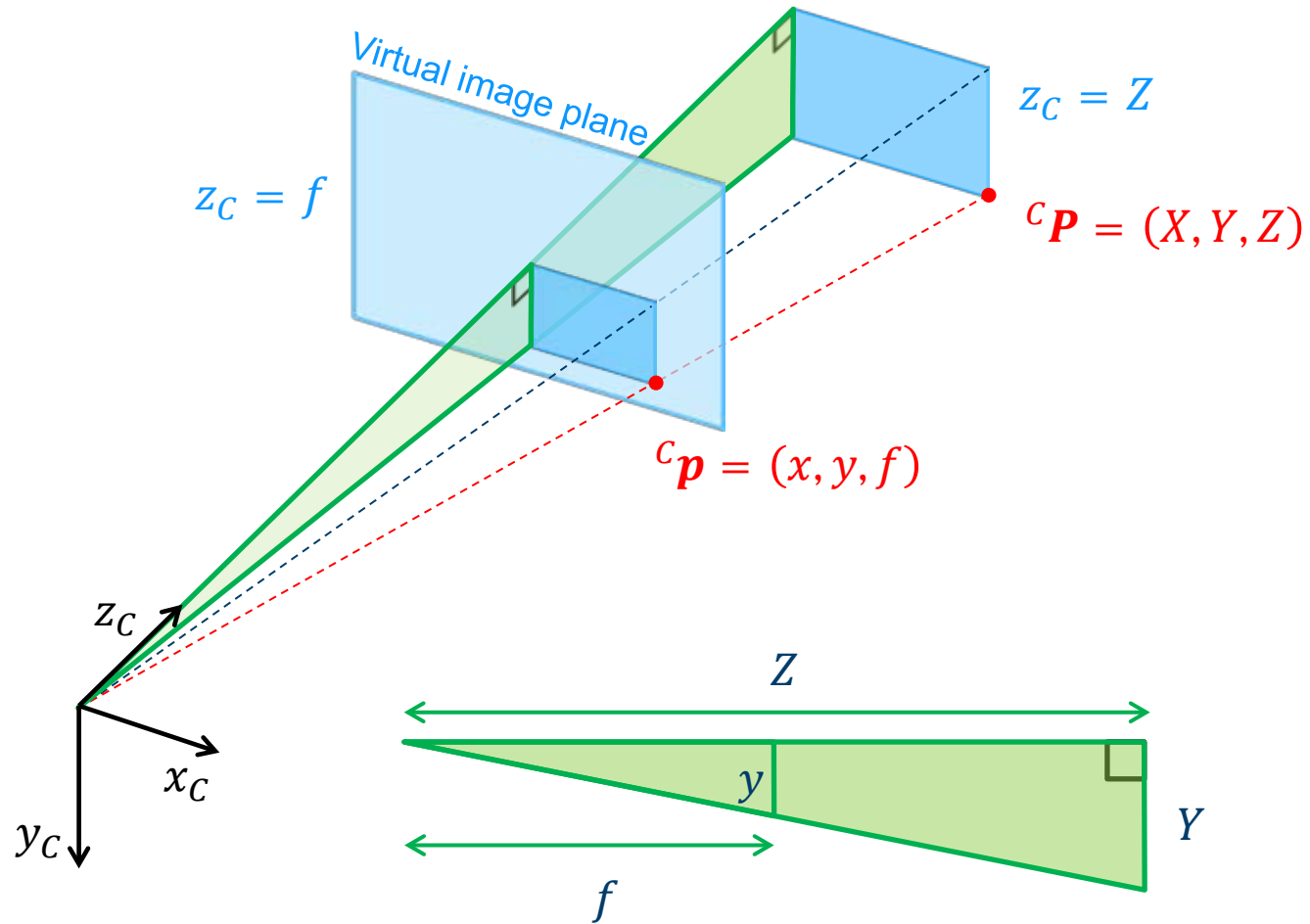
# Depth from images



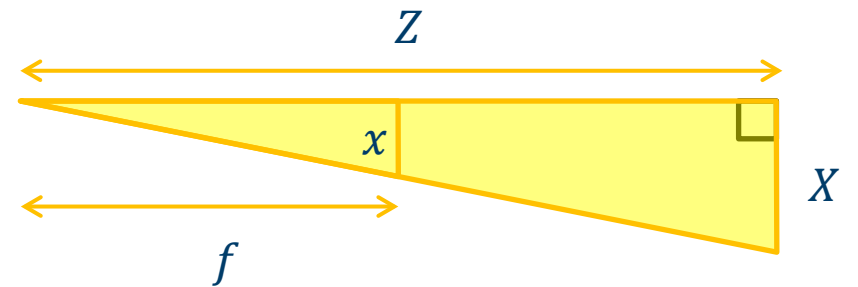
# Depth from images



# Depth from images

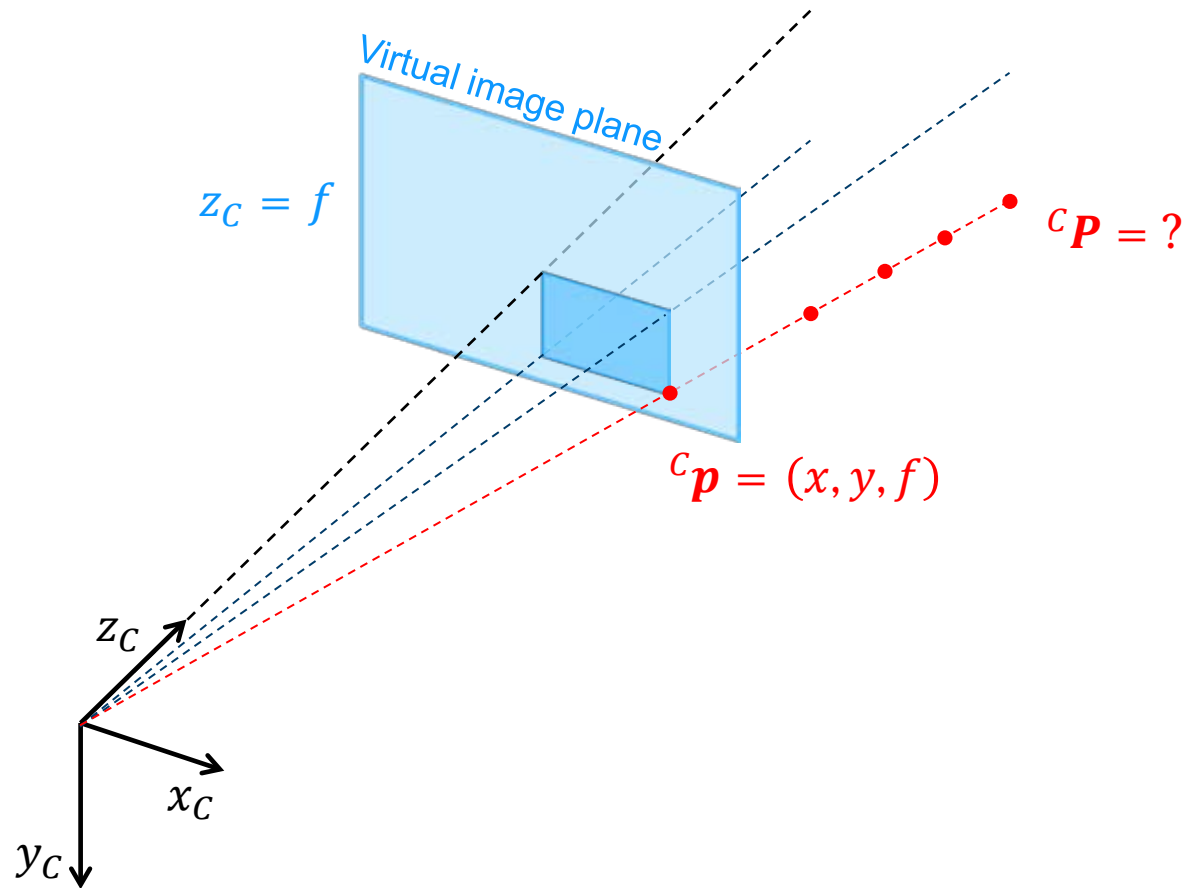


$$x = f \frac{X}{Z}$$
$$y = f \frac{Y}{Z}$$





# Depth from images

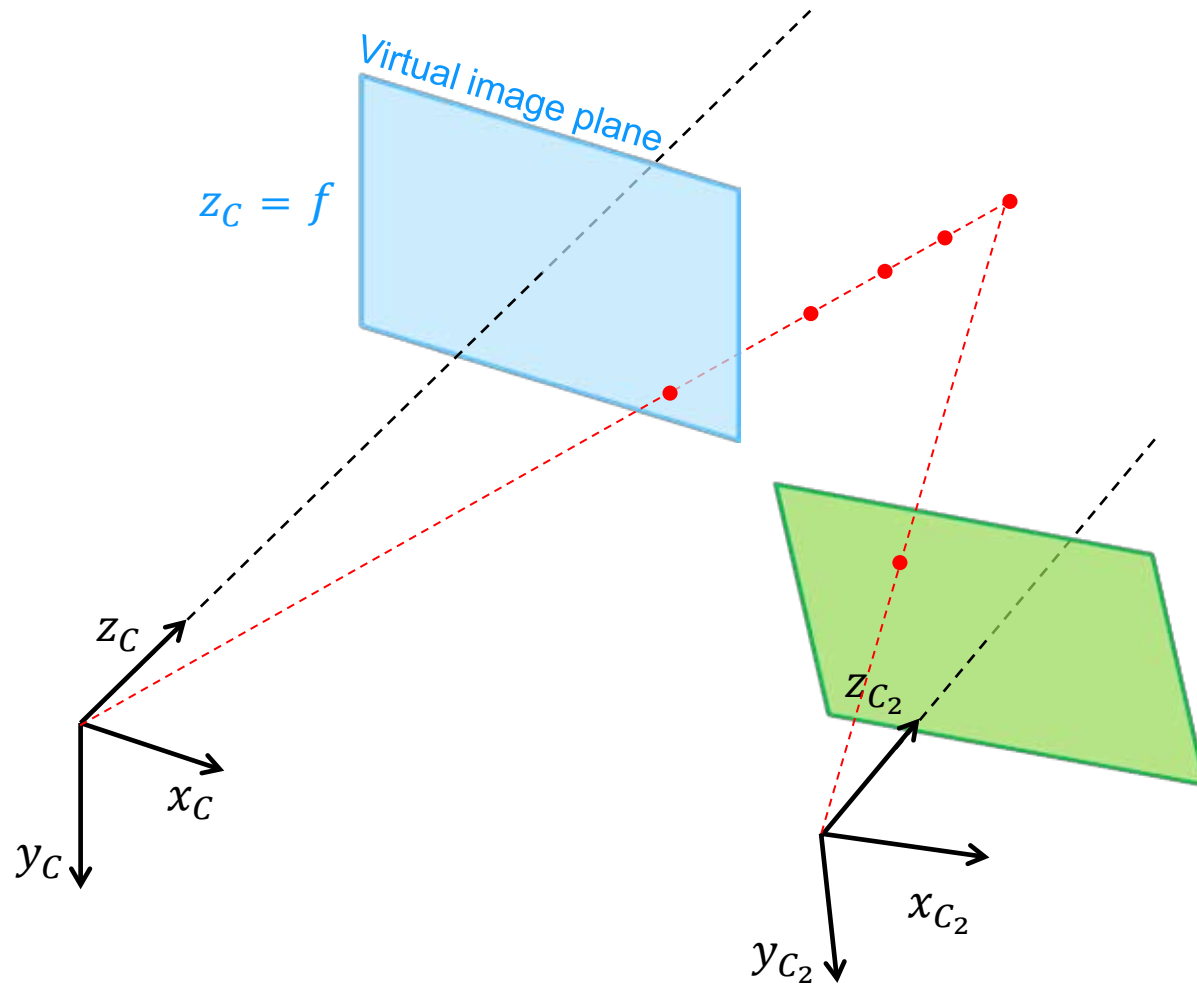


$$x = f \frac{X}{Z} = f \frac{kX}{kZ}$$

$$y = f \frac{Y}{Z} = f \frac{kY}{kZ}$$

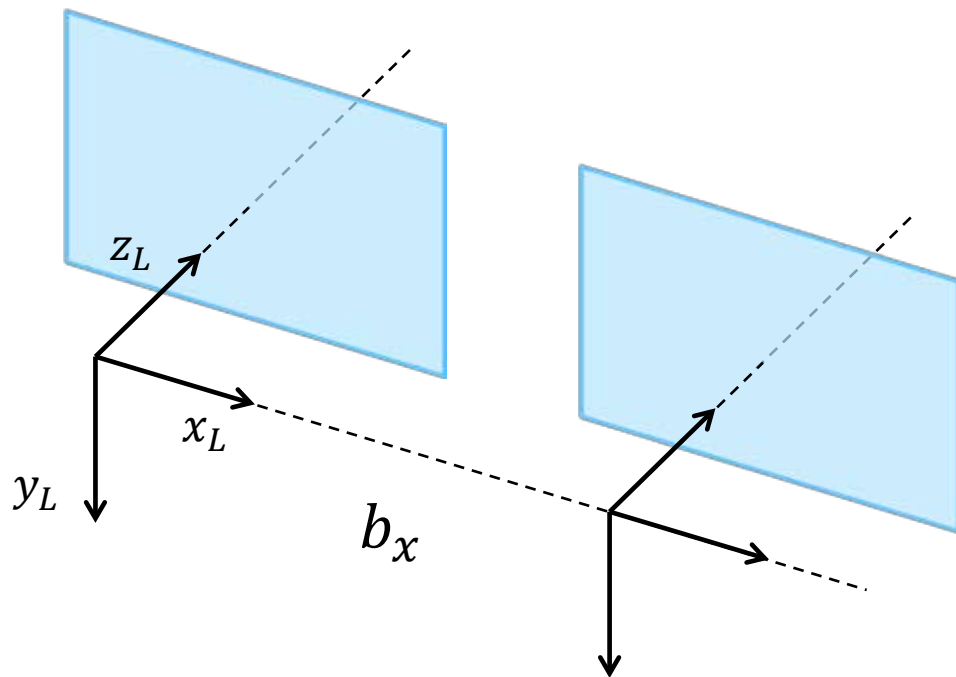
Any point on the ray has image  $c_p$

# Depth from images



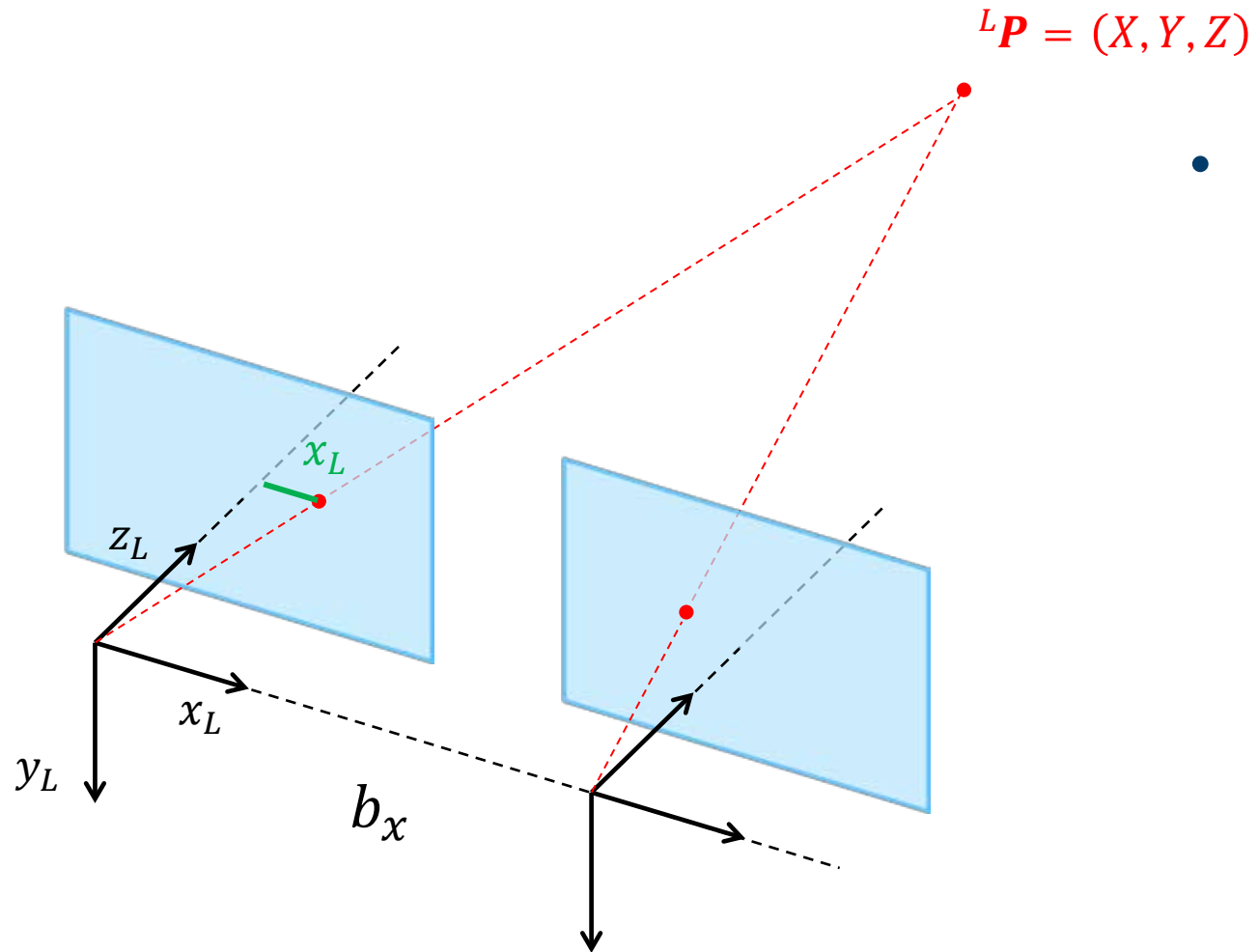
Add a second view!

# An ideal stereo system



- Left camera defines common coordinate system
- Right camera shifted  $b_x$  units along the  $x$ -axis
  - Baseline
- Otherwise identical
  - Orientation
  - Focal lengths

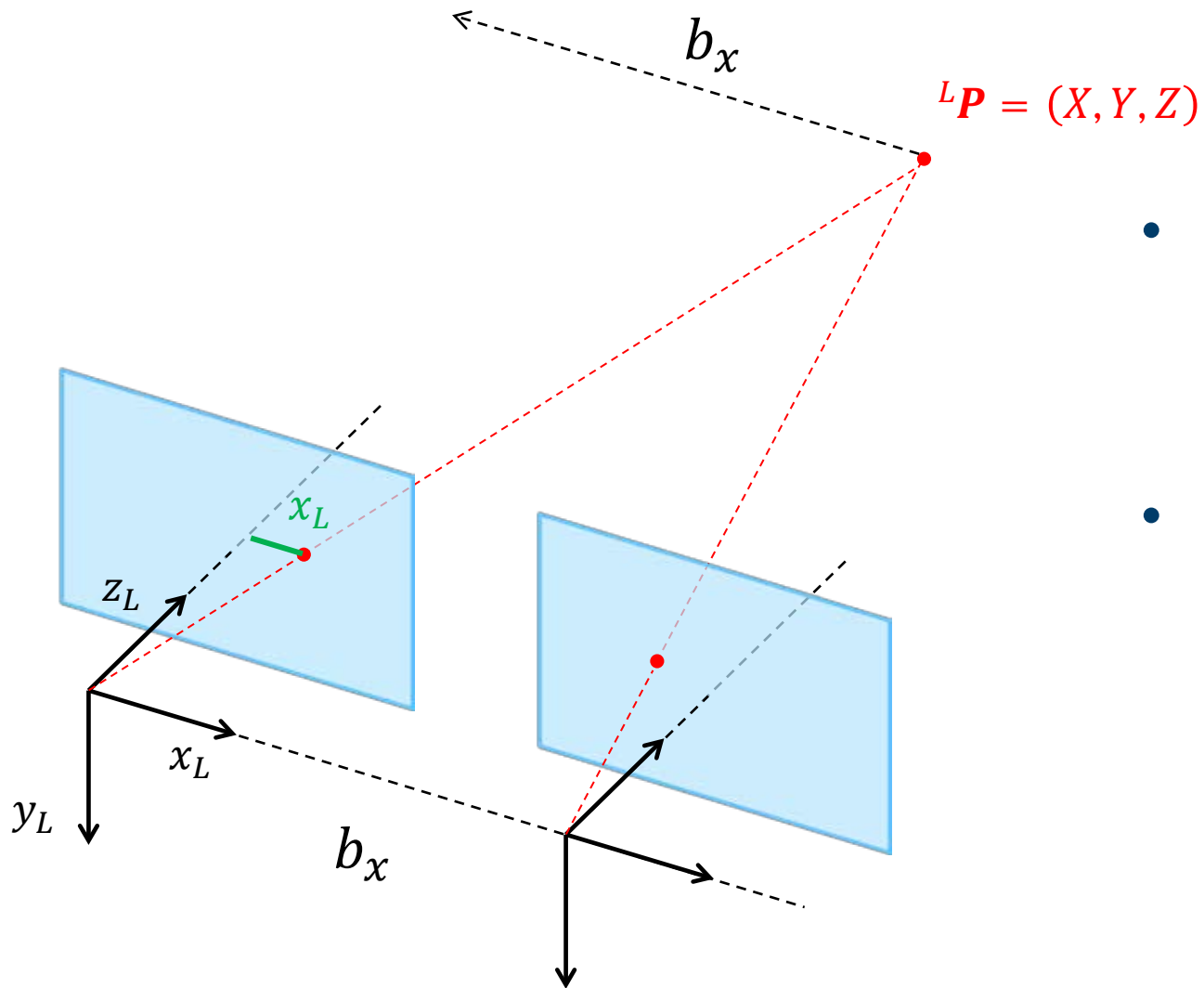
# An ideal stereo system



- Image of  $P$  in left camera:

$$x_L = f \frac{X}{Z}, \quad y_L = f \frac{Y}{Z}$$

# An ideal stereo system



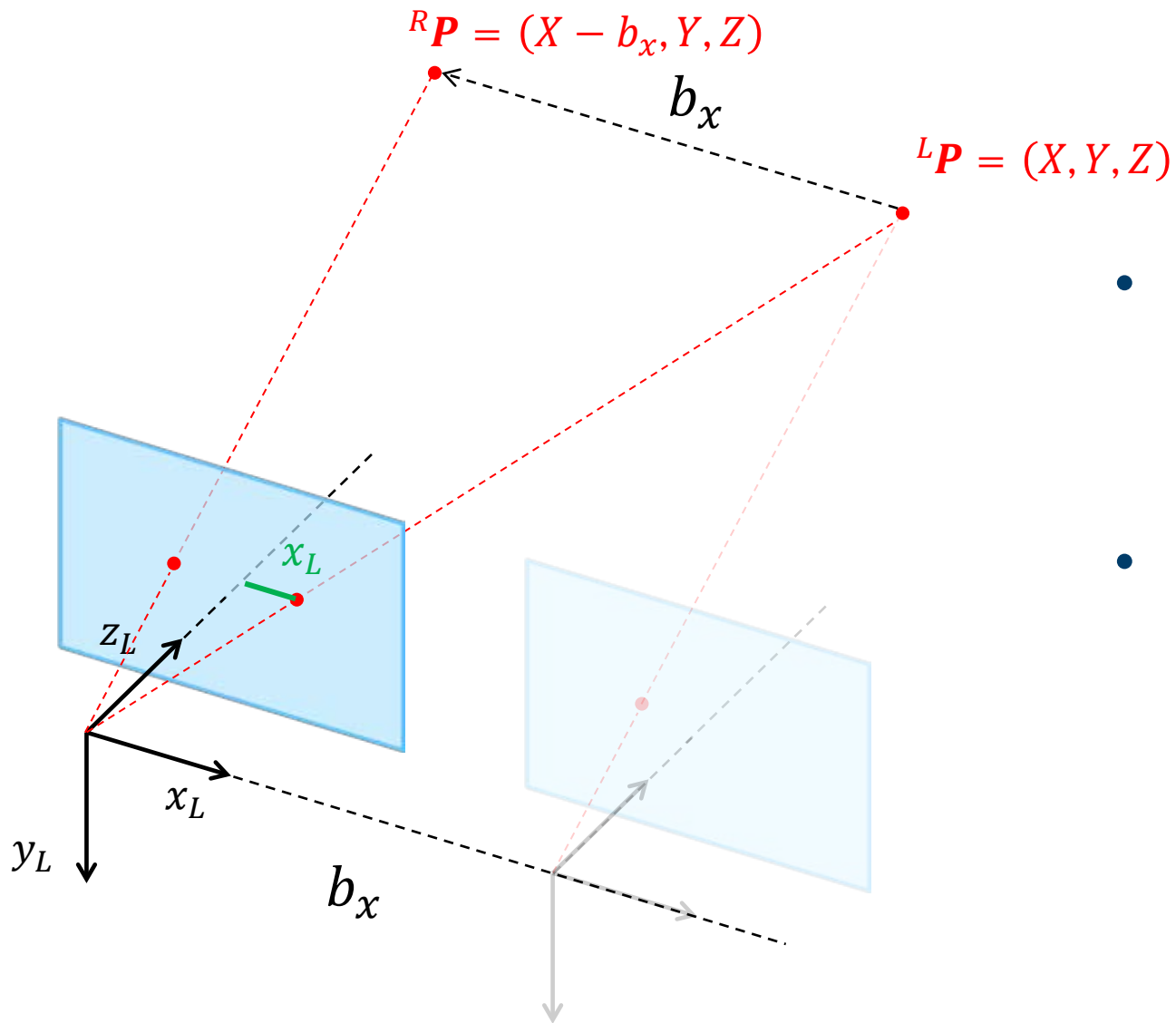
- Image of  $P$  in left camera:

$$x_L = f \frac{X}{Z}, \quad y_L = f \frac{Y}{Z}$$

- Image of  $P$  in right camera:

(translating the camera to the right is equivalent to translating the world to the left)

# An ideal stereo system



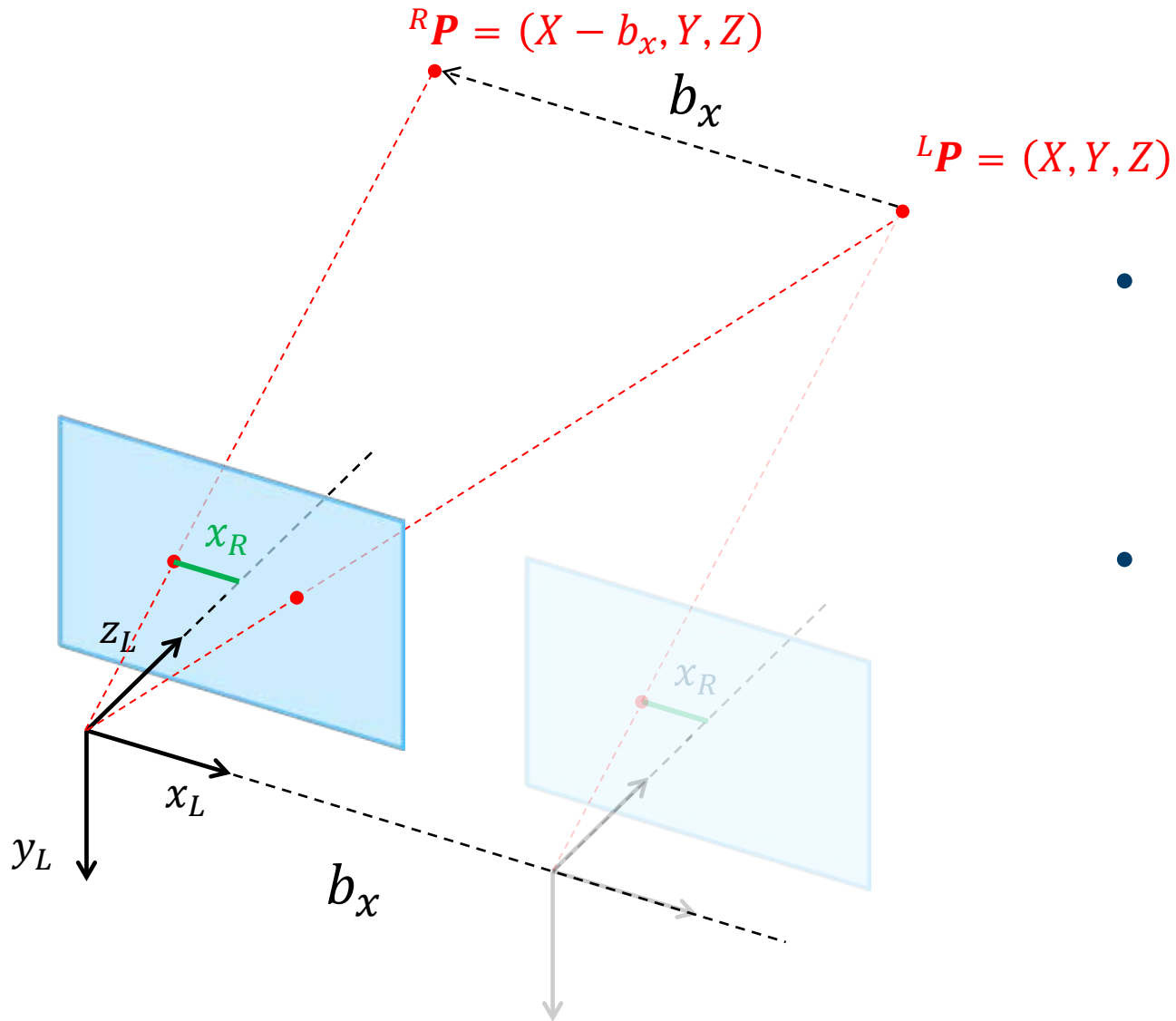
- Image of  $P$  in left camera:

$$x_L = f \frac{X}{Z}, \quad y_L = f \frac{Y}{Z}$$

- Image of  $P$  in right camera:

(translating the camera to the right is equivalent to translating the world to the left)

# An ideal stereo system



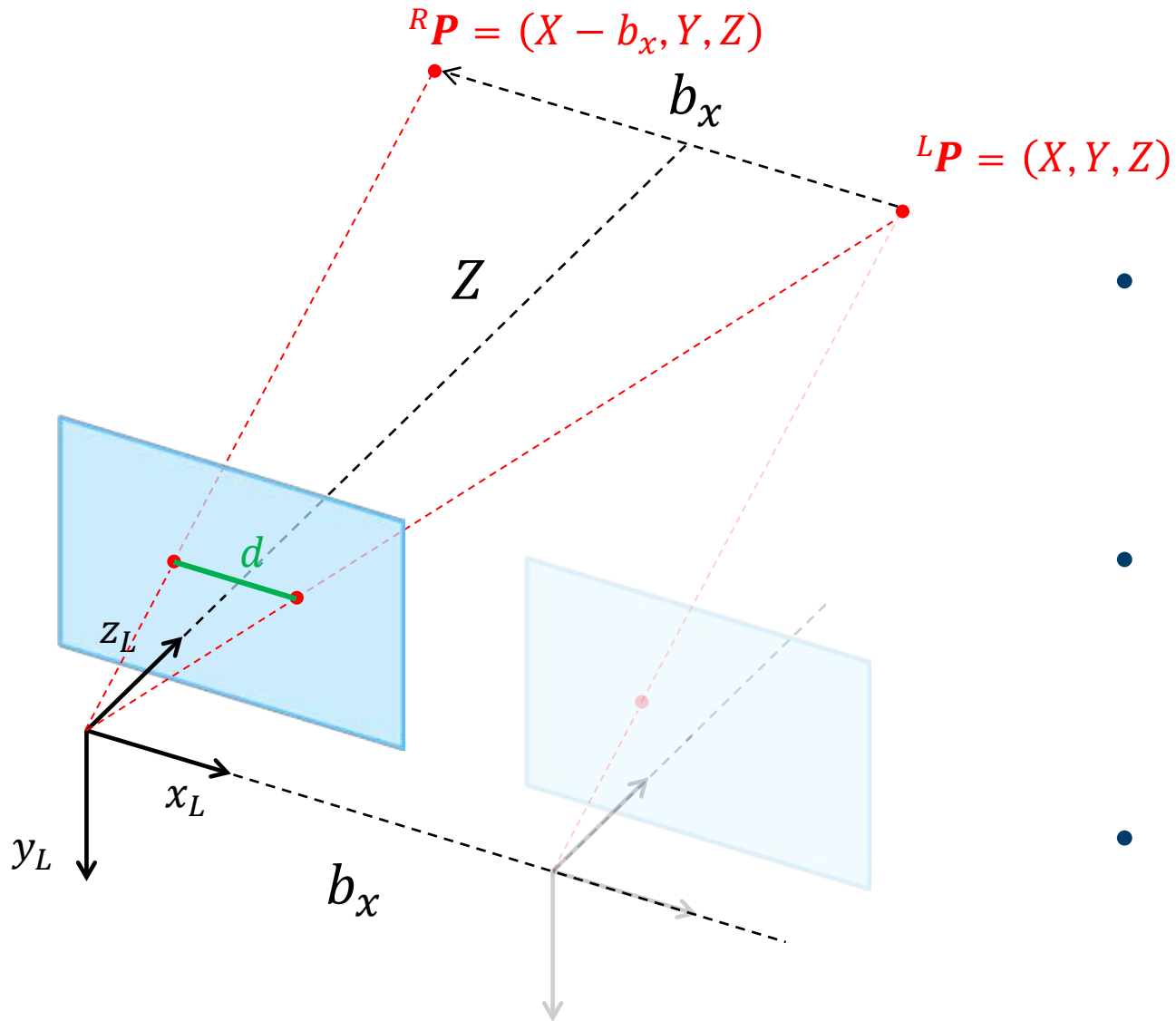
- Image of  $P$  in left camera:

$$x_L = f \frac{X}{Z}, \quad y_L = f \frac{Y}{Z}$$

- Image of  $P$  in right camera:

$$x_R = f \frac{X - b_x}{Z}, \quad y_R = f \frac{Y}{Z}$$

# An ideal stereo system



- Image of  $\mathbf{P}$  in left camera:

$$x_L = f \frac{X}{Z}, \quad y_L = f \frac{Y}{Z}$$

- Image of  $\mathbf{P}$  in right camera:

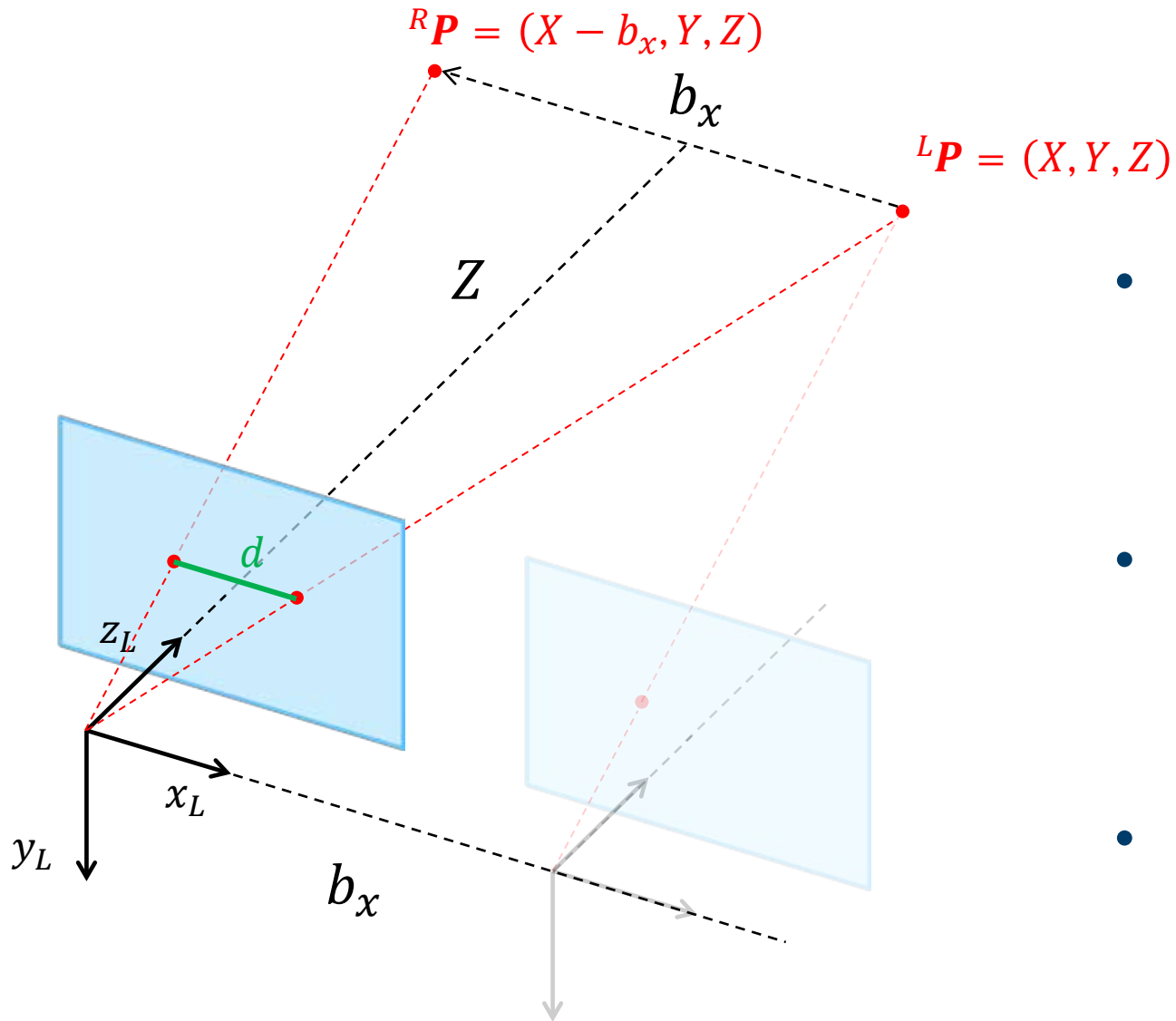
$$x_R = f \frac{X - b_x}{Z}, \quad y_R = f \frac{Y}{Z}$$

- Stereo disparity

$$d = x_L - x_R$$



# An ideal stereo system



- Image of  $P$  in left camera:

$$x_L = f \frac{X}{Z}, \quad y_L = f \frac{Y}{Z}$$

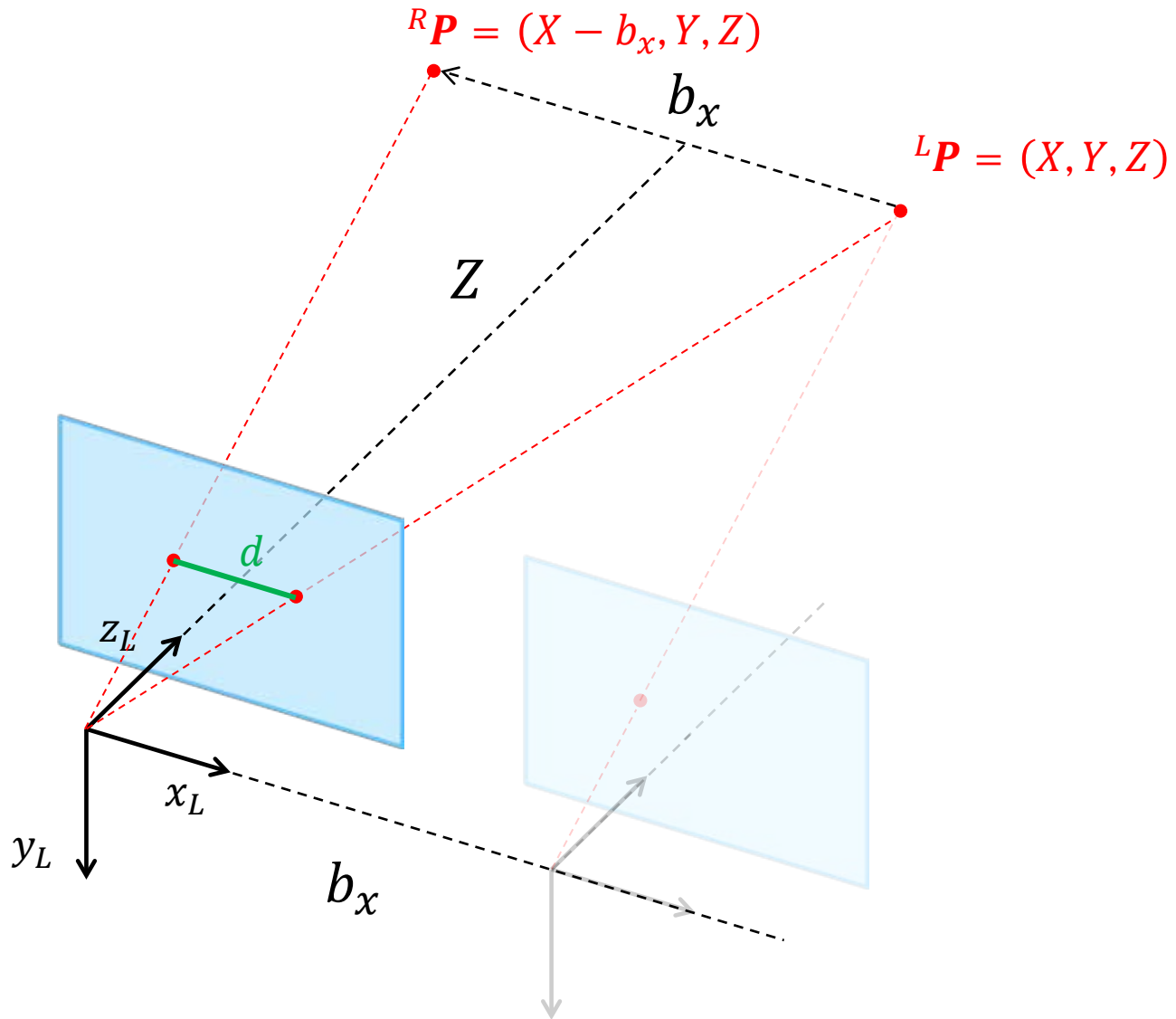
- Image of  $P$  in right camera:

$$x_R = f \frac{X - b_x}{Z}, \quad y_R = f \frac{Y}{Z}$$

- Stereo disparity

$$d = x_L - x_R = f \frac{X}{Z} - \left( f \frac{X}{Z} - f \frac{b_x}{Z} \right)$$

# Stereo disparity



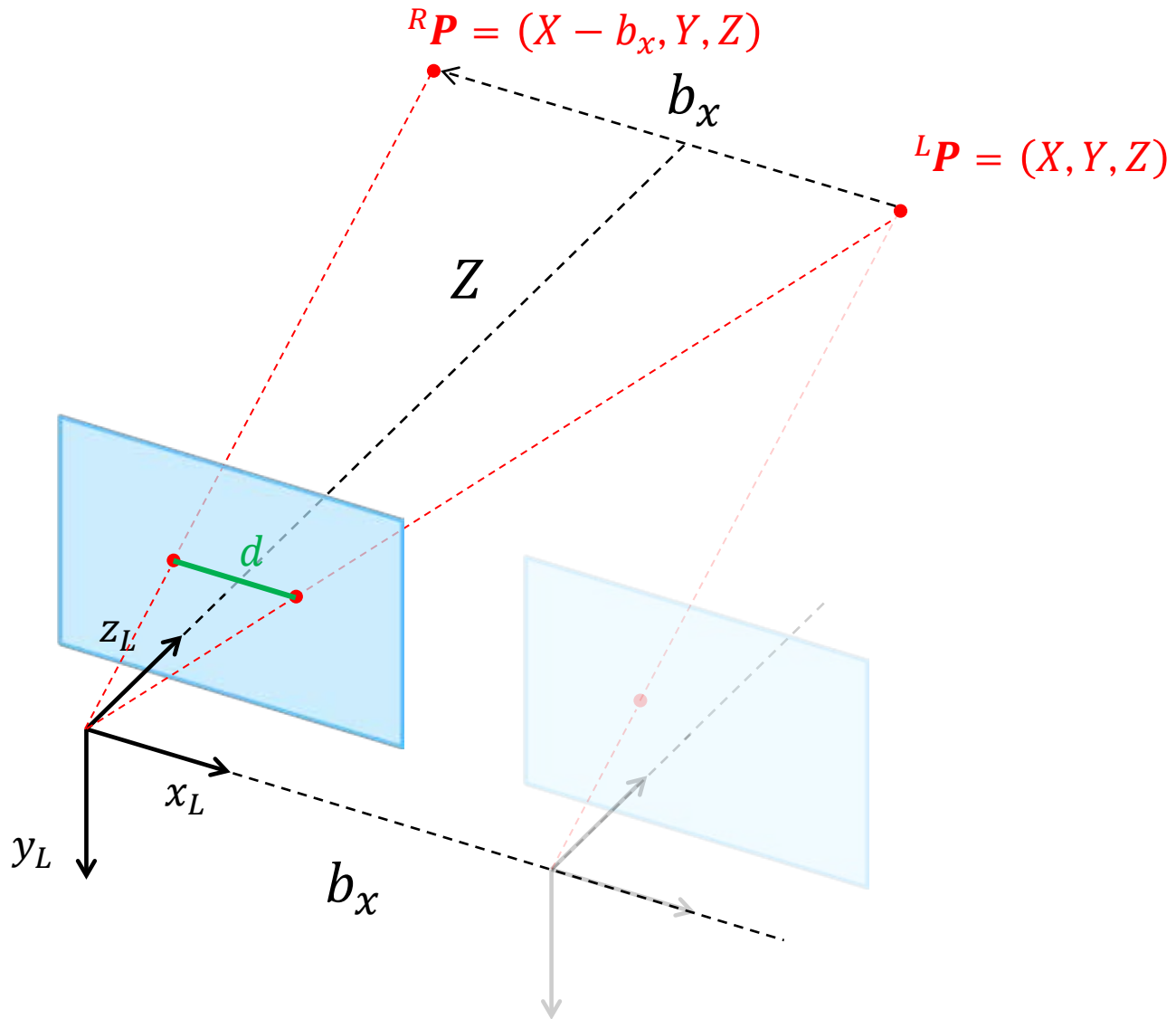
$$d = f \frac{b_x}{Z}$$

Baseline

Disparity

Depth

# Stereo disparity



$$\text{Depth } Z = f \frac{\text{Baseline } b_x}{\text{Disparity } d}$$

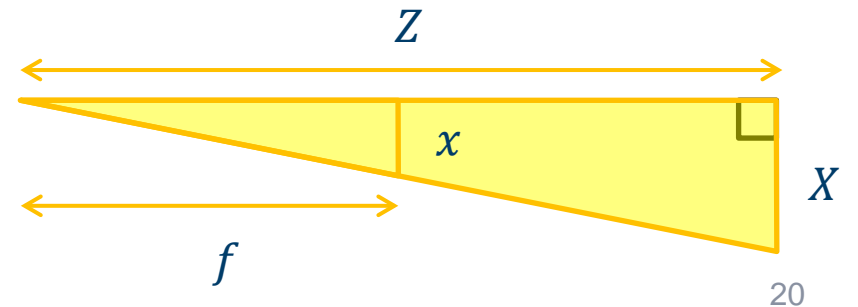
# 3D from stereo images

- From normalized coordinates

$$Z = f \frac{b_x}{d}$$

$$X = x_L \frac{Z}{f}$$

$$Y = y_L \frac{Z}{f}$$



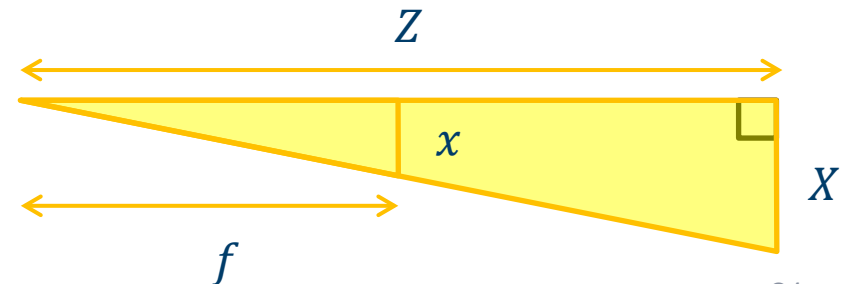
# 3D from stereo images

- From normalized coordinates

$$Z = f \frac{b_x}{d}$$

$$X = x_L \frac{b_x}{d}$$

$$Y = y_L \frac{b_x}{d}$$



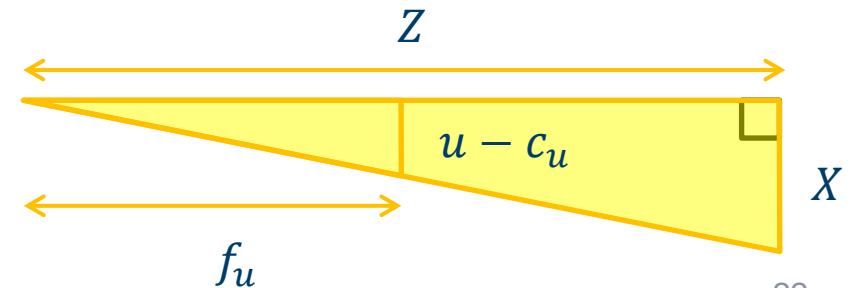
# 3D from stereo images

- From pixel coordinates

$$Z = f_u \frac{b_x}{d_u}$$

$$X = (u_L - {}^L c_u) \frac{b_x}{d_u}$$

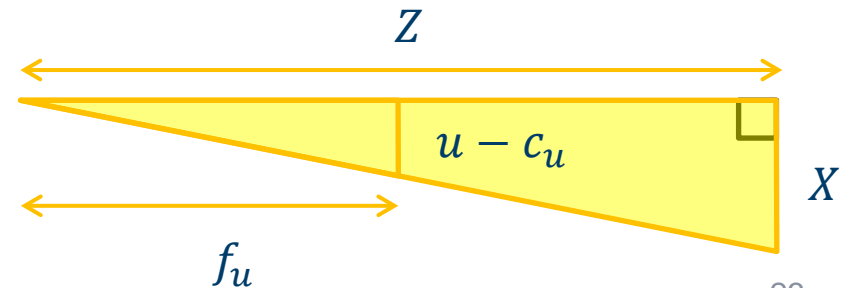
$$Y = (v_L - {}^L c_v) \frac{b_x}{d_u}$$



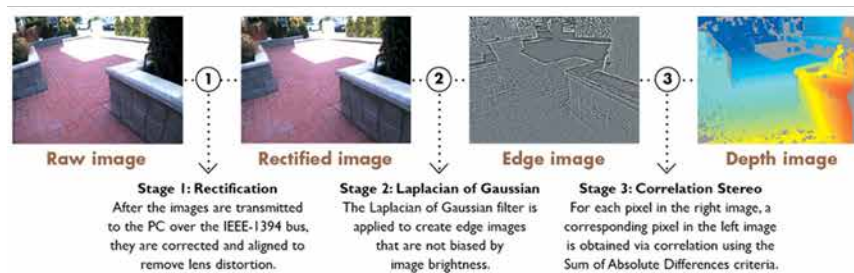
# 3D from stereo images

- From pixel coordinates

$$\begin{matrix}
 \hat{x} \\
 \hat{y} \\
 \hat{z} \\
 \hat{w}
 \end{matrix}
 Q_{L} =
 \begin{pmatrix}
 1 & 0 & 0 & -L c_u \\
 0 & 1 & 0 & -L c_v \\
 0 & 0 & 0 & f_u \\
 0 & 0 & \frac{1}{b_x} & \frac{R c_u - L c_u}{b_x}
 \end{pmatrix}
 \begin{pmatrix}
 u_L \\
 v_L \\
 d \\
 1
 \end{pmatrix}
 =
 \begin{pmatrix}
 X \\
 Y \\
 Z \\
 W
 \end{pmatrix}
 =
 \begin{pmatrix}
 X/W \\
 Y/W \\
 Z/W \\
 1
 \end{pmatrix}
 X$$



# Stereo cameras

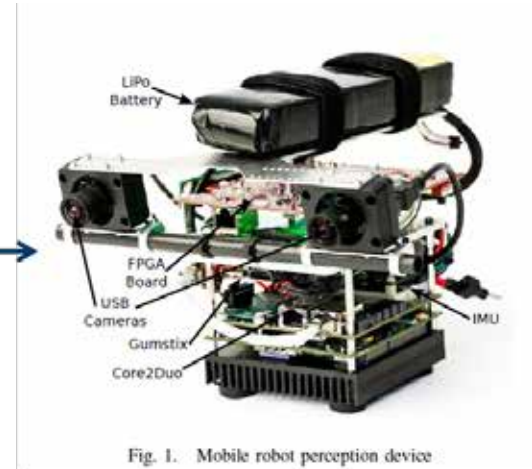


<http://www.ptgrey.com>

"Kinect2-ir-image" by User:Kolossos  
<http://commons.wikimedia.org/wiki/File:Kinect2-ir-image.png>



# Stereo cameras



**Towards Autonomous MAV Exploration in Cluttered Indoor and Outdoor Environments**

Korbinian Schmid

DLR, Robotics and Mechatronics Center (RMC)  
Perception and Cognition  
Mobile Robots (XRotor Group)

DLR Deutsches Zentrum für Luft- und Raumfahrt e.V.

The cover of a presentation titled "Towards Autonomous MAV Exploration in Cluttered Indoor and Outdoor Environments" by Korbinian Schmid. It features a background image of a MAV flying over a landscape. The text includes the author's name, affiliation with DLR, Robotics and Mechatronics Center (RMC), Perception and Cognition, and Mobile Robots (XRotor Group). The DLR logo is at the bottom left.

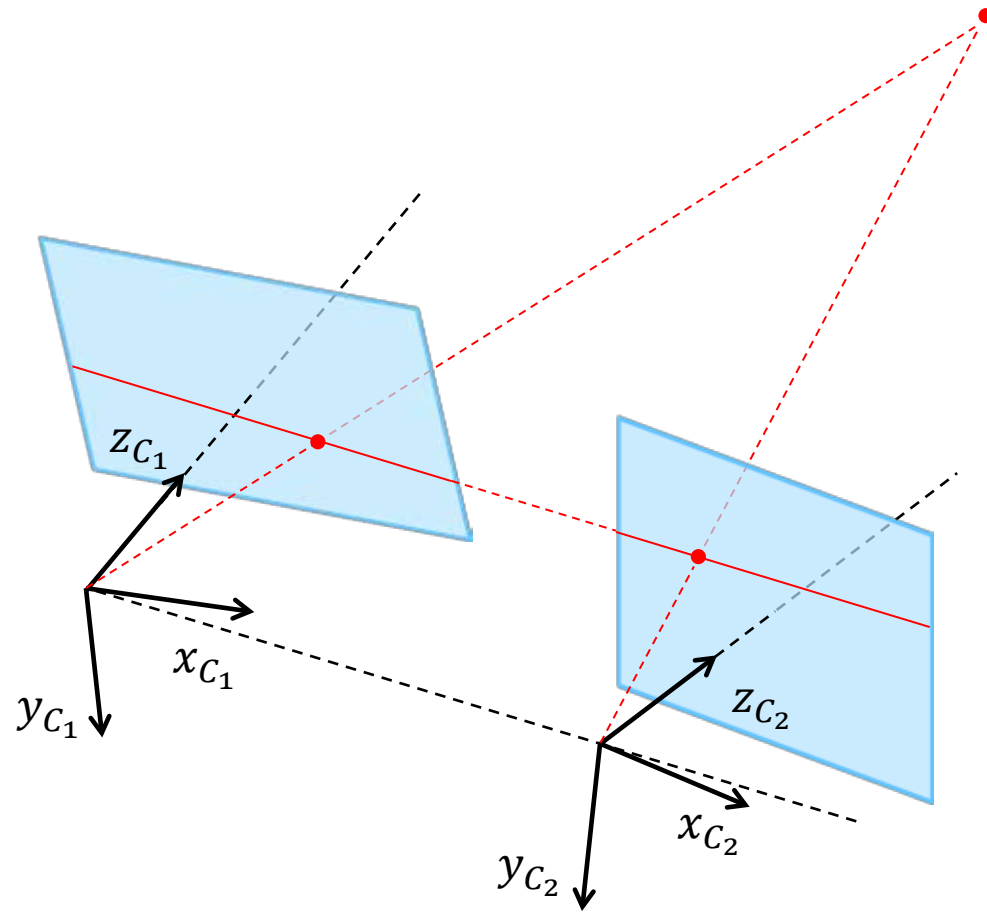
**Mixed indoor/outdoor exploration**

- Autonomous indoor/outdoor flight of 60m
- Mapping resolution: 0.1m
- Leaving through a window
- Returning through door

DLR Deutsches Zentrum für Luft- und Raumfahrt e.V.

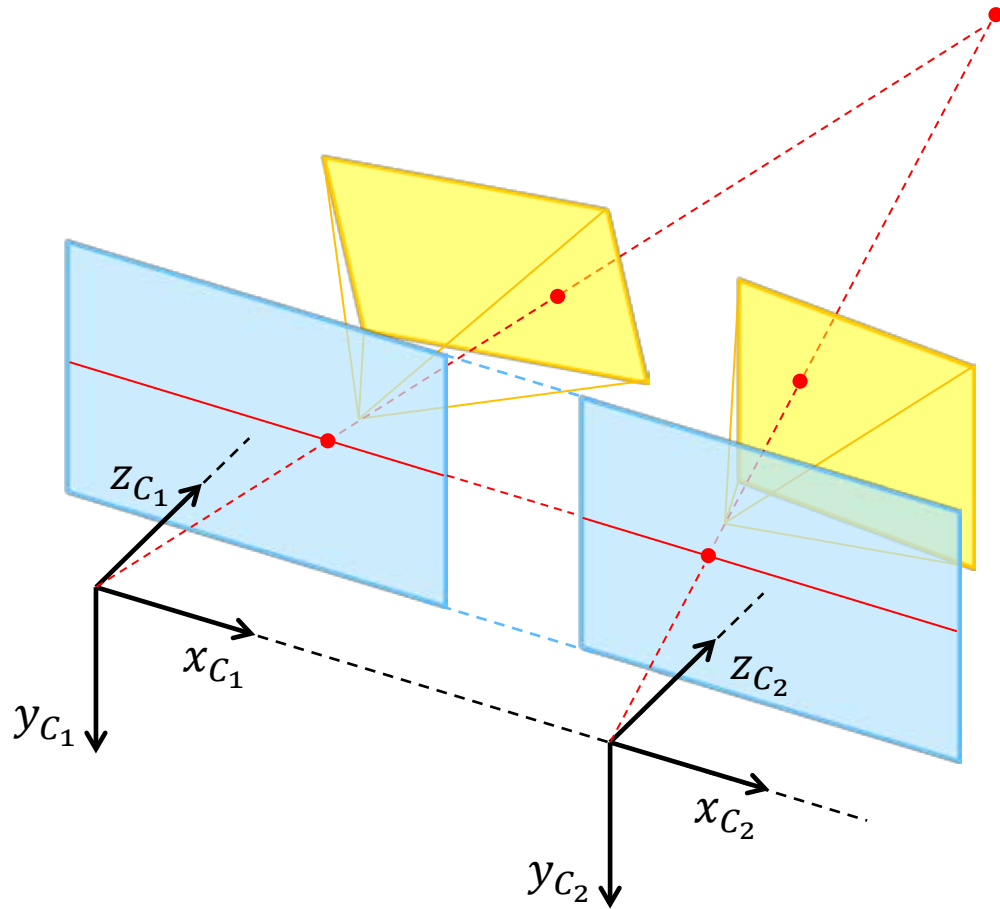
A slide titled "Mixed indoor/outdoor exploration" showing MAV flight paths and 3D maps. The slide lists four key achievements: autonomous indoor/outdoor flight of 60m, mapping resolution of 0.1m, leaving through a window, and returning through a door. It includes images of the MAV and 3D maps of the environment. The DLR logo is at the bottom left.

# What if we do not have parallel cameras?



- Epipolar lines no longer horizontal
- What does disparity mean now?
- How to reproject to 3D points?

# Stereo rectification



- Reproject image planes onto a common plane parallel to the line between the camera centers
- The epipolar lines are horizontal after this transformation
- Two homographies
- C. Loop and Z. Zhang. Computing Rectifying Homographies for Stereo Vision. IEEE Conf. Computer Vision and Pattern Recognition, 1999.

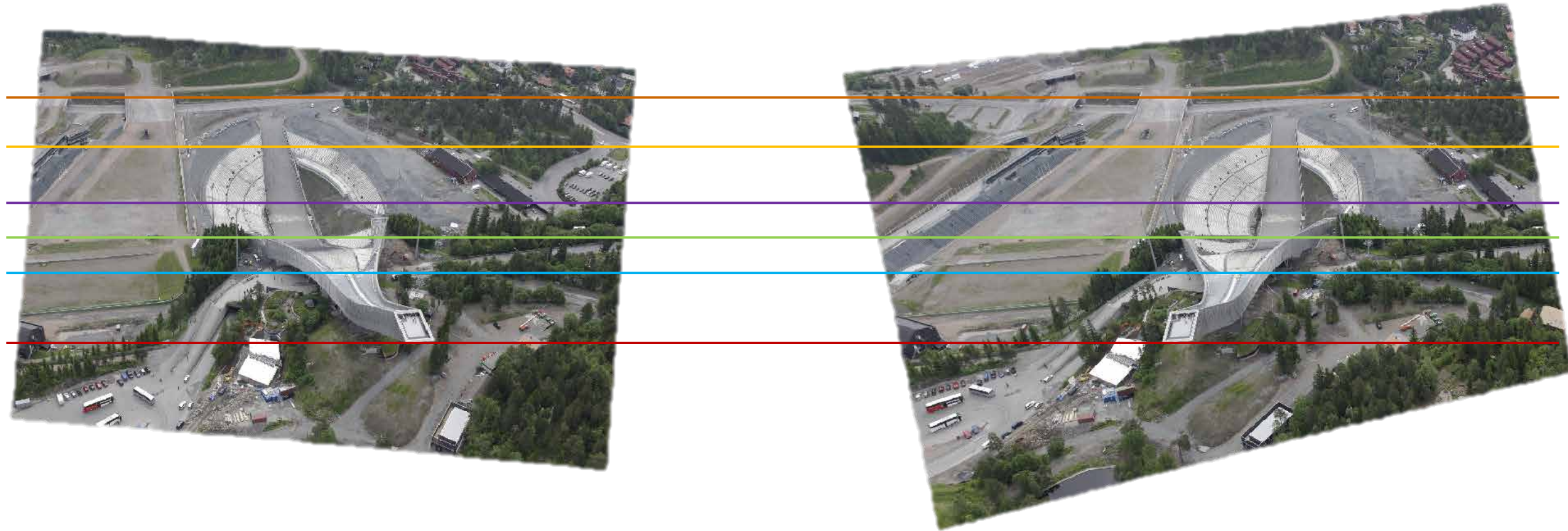
# Stereo rectification example



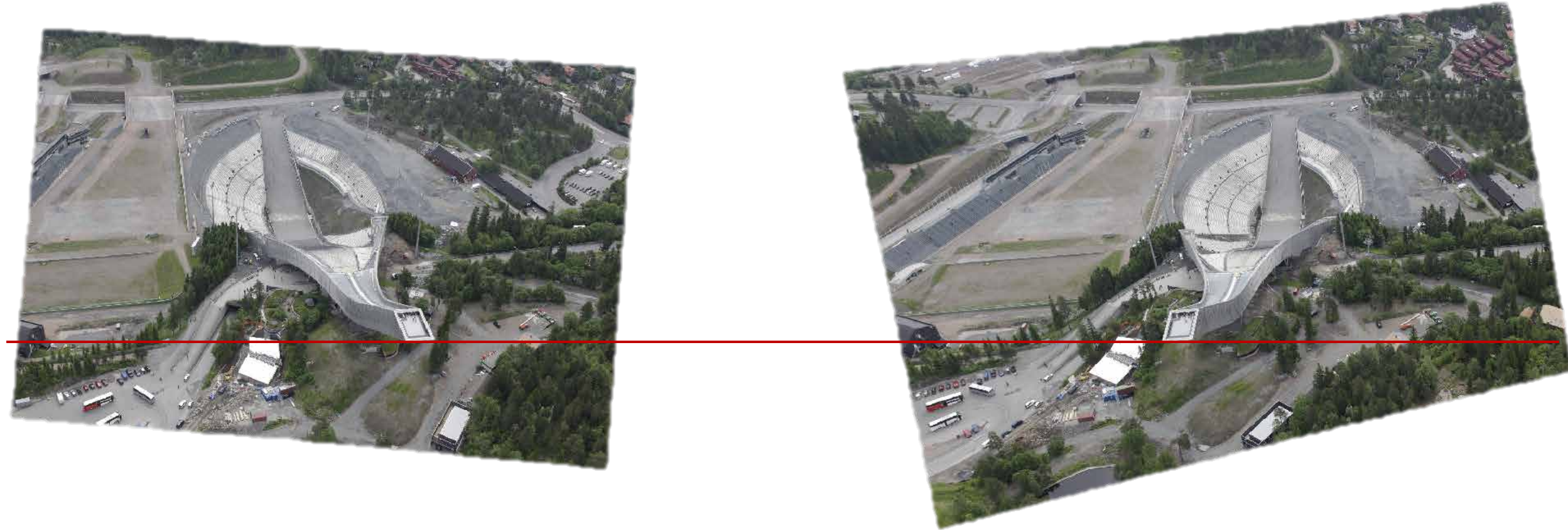
# Stereo rectification example



# Stereo rectification example



# Stereo rectification example



# Summary

- Stereo imaging
  - Horizontal epipolar lines
  - Disparity
  - 3D from disparity
  - Stereo rectification
- Next: Stereo processing