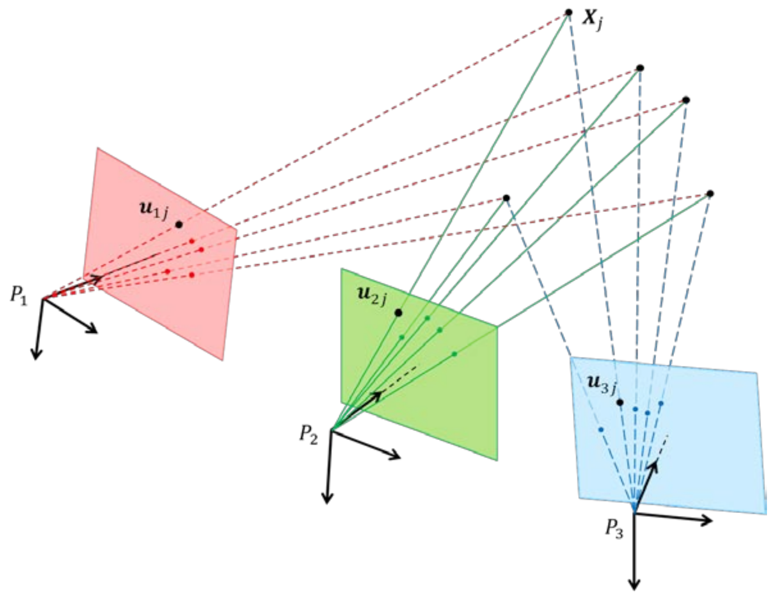
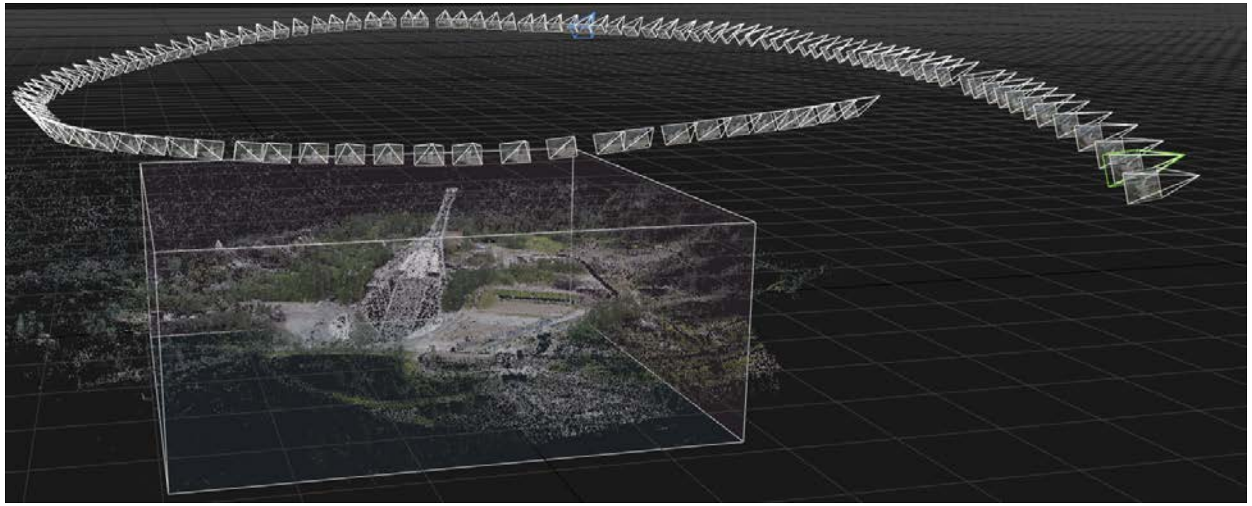
Lecture 8.1
Multiple-View Geometry

Thomas Opsahl

Weekly overview





- Multiple-view geometry
  - Correspondences
- Structure from Motion (SfM)
  - Sparse 3D reconstruction
- Multiple-view stereo
  - Dense 3D reconstruction

Recap on two-view geometry

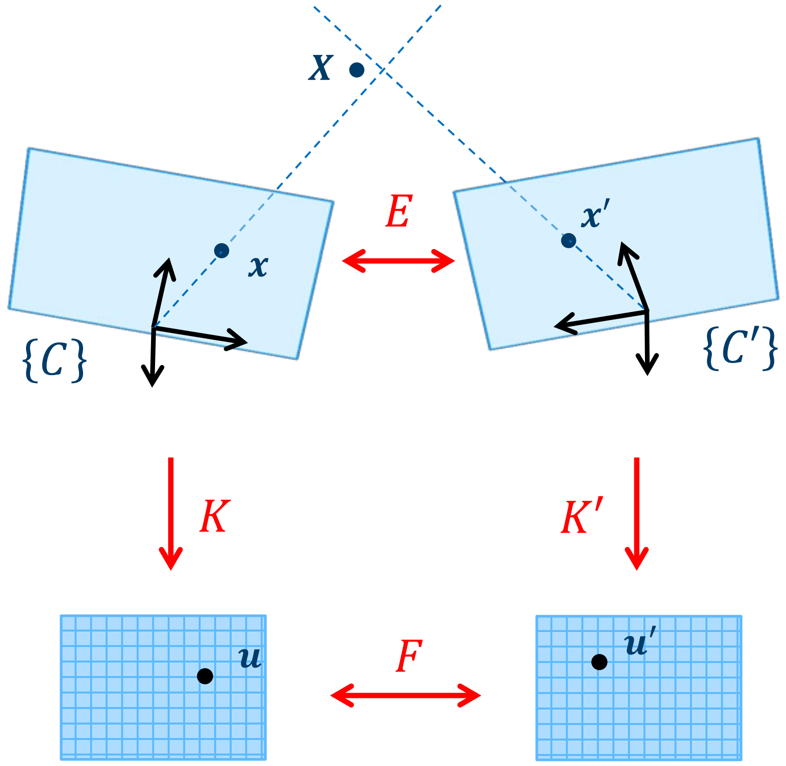
- Epipolar geometry
  - The essential matrix $E = [\boldsymbol{t}]_\times R$

    $$\tilde{\boldsymbol{x}}'^T E \tilde{\boldsymbol{x}} = 0$$

  - The fundamental matrix $F = K'^{-T} E K^{-1}$

    $$\tilde{\boldsymbol{u}}'^T F \tilde{\boldsymbol{u}} = 0$$

  - Estimating F from 7 or 8 correspondences $\boldsymbol{u}_i \leftrightarrow \boldsymbol{u}_i'$
  - Estimating E from 5 correspondences $\boldsymbol{x}_i \leftrightarrow \boldsymbol{x}_i'$



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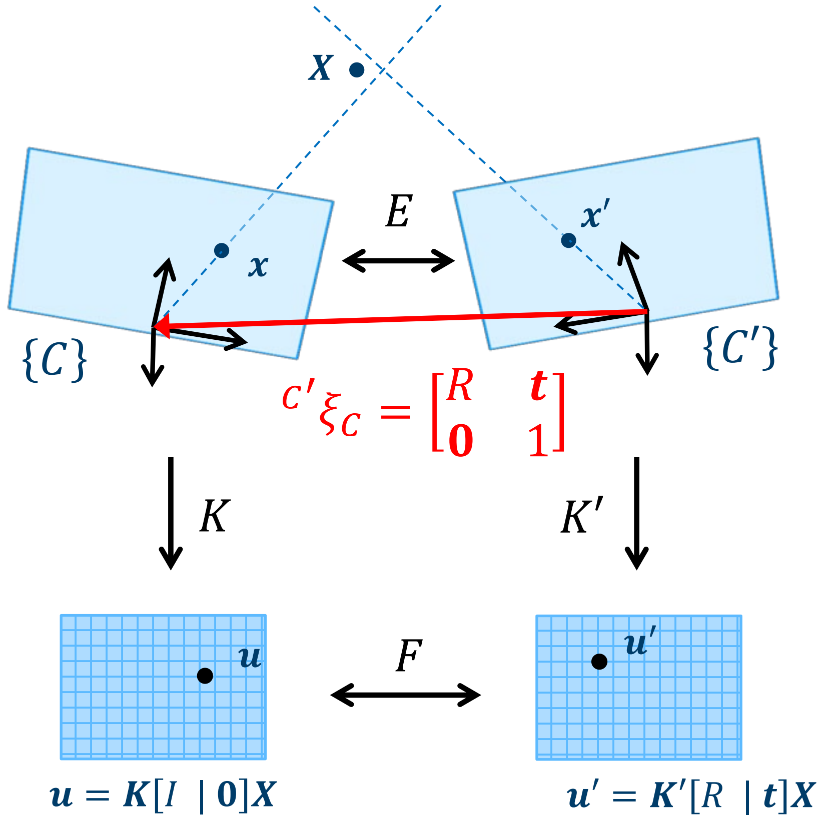
- Pose from epipolar geometry
  - Decomposing E into R and $\boldsymbol{t}$ (up to scale)



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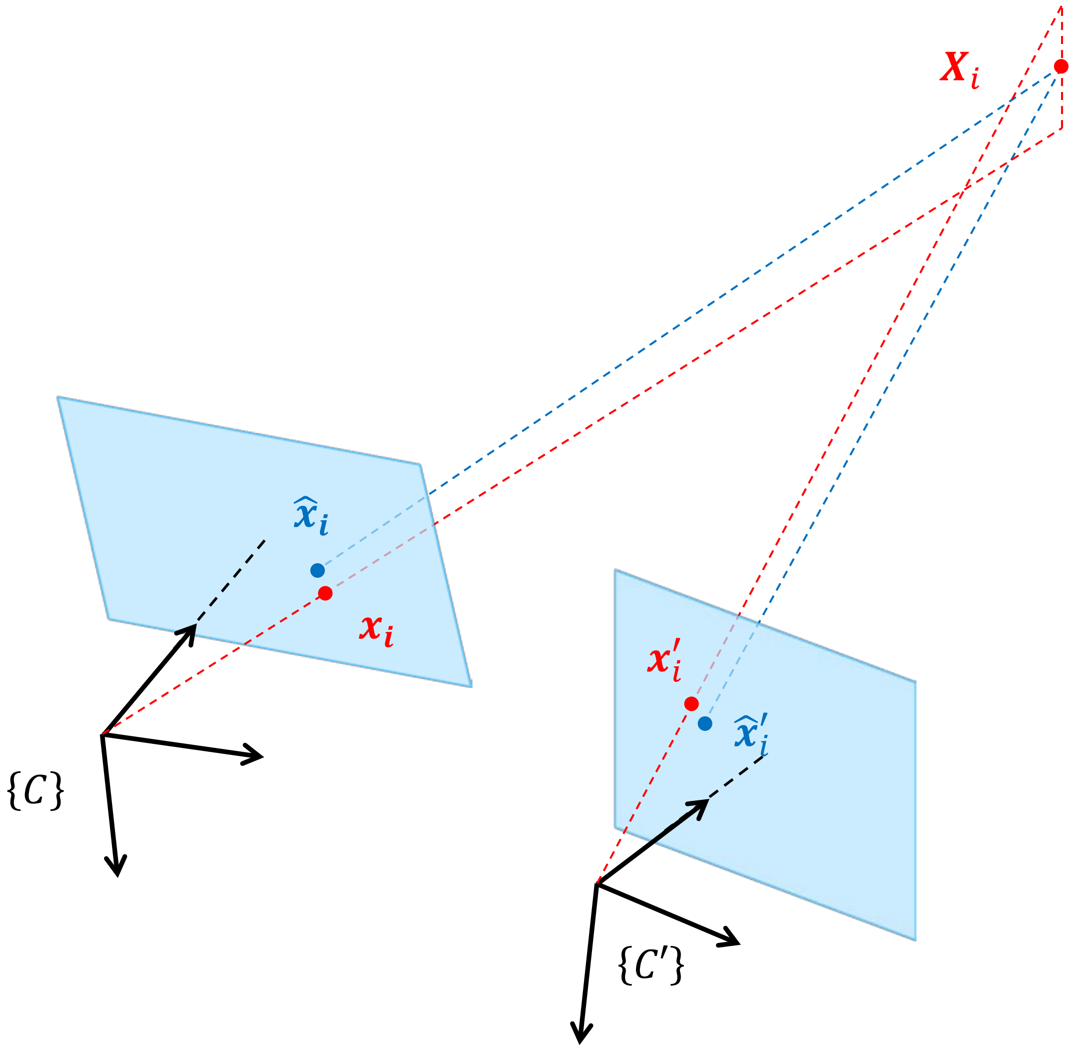
- 3D structure from epipolar geometry
  - Triangulation based on known camera matrices



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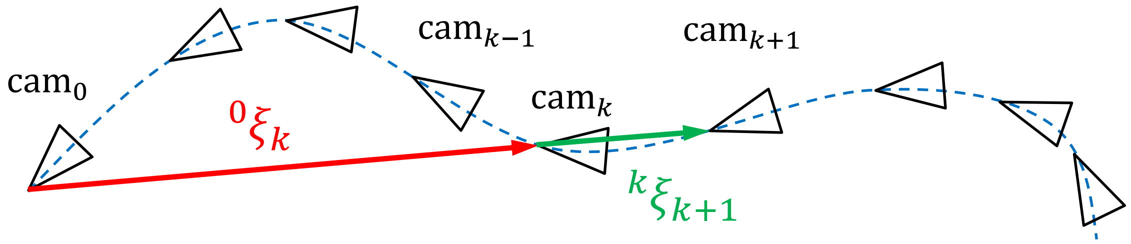

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- Sequential visual odometry
  - $\boldsymbol{x}_i{}^{(k)} \leftrightarrow \boldsymbol{x}_i{}^{(k+1)} \longrightarrow E_{k,k+1} \longrightarrow {}^k\xi_{k+1}$
  - ${}^0\xi_{k+1} = {}^0\xi_k \; {}^k\xi_{k+1}$

Recap on two-view geometry

Correspondences (matching)

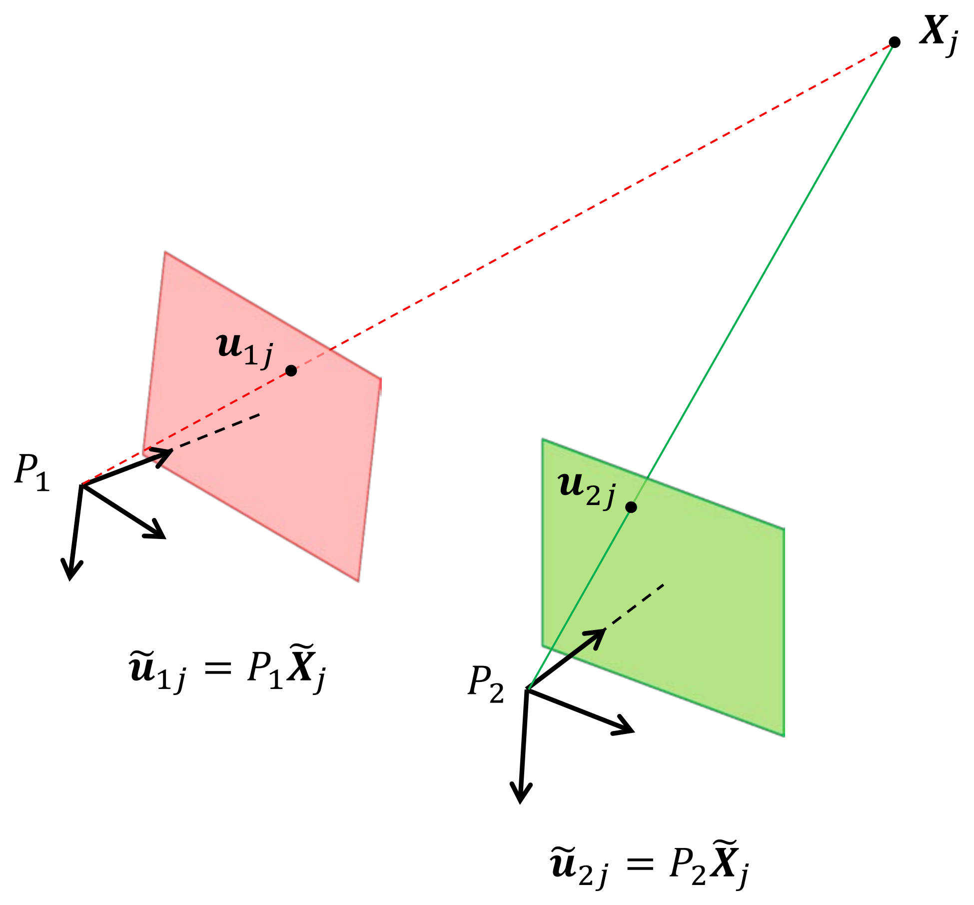
- Correspondences must satisfy the epipolar constraint represented by the fundamental matrix
$$\tilde{\boldsymbol{u}}'^T F \tilde{\boldsymbol{u}} = 0$$
- Useful for reducing the number of mismatches

Scene geometry (structure)

- Sparse 3D from triangulating correspondences
- Dense 3D from stereo processing

Camera geometry (motion)

- In the uncalibrated case, the camera matrices P and P' can be estimated from the fundamental matrix F up to a projective ambiguity
- In the calibrated case, the relative pose between cameras can be estimated up to scale by decomposing the essential matrix
$$(\boldsymbol{x}_j \leftrightarrow \boldsymbol{x}'_j) \xrightarrow{E=[\boldsymbol{t}]_\times R} (R, \lambda \boldsymbol{t})$$



More-than-two-view geometry

Correspondences (matching)

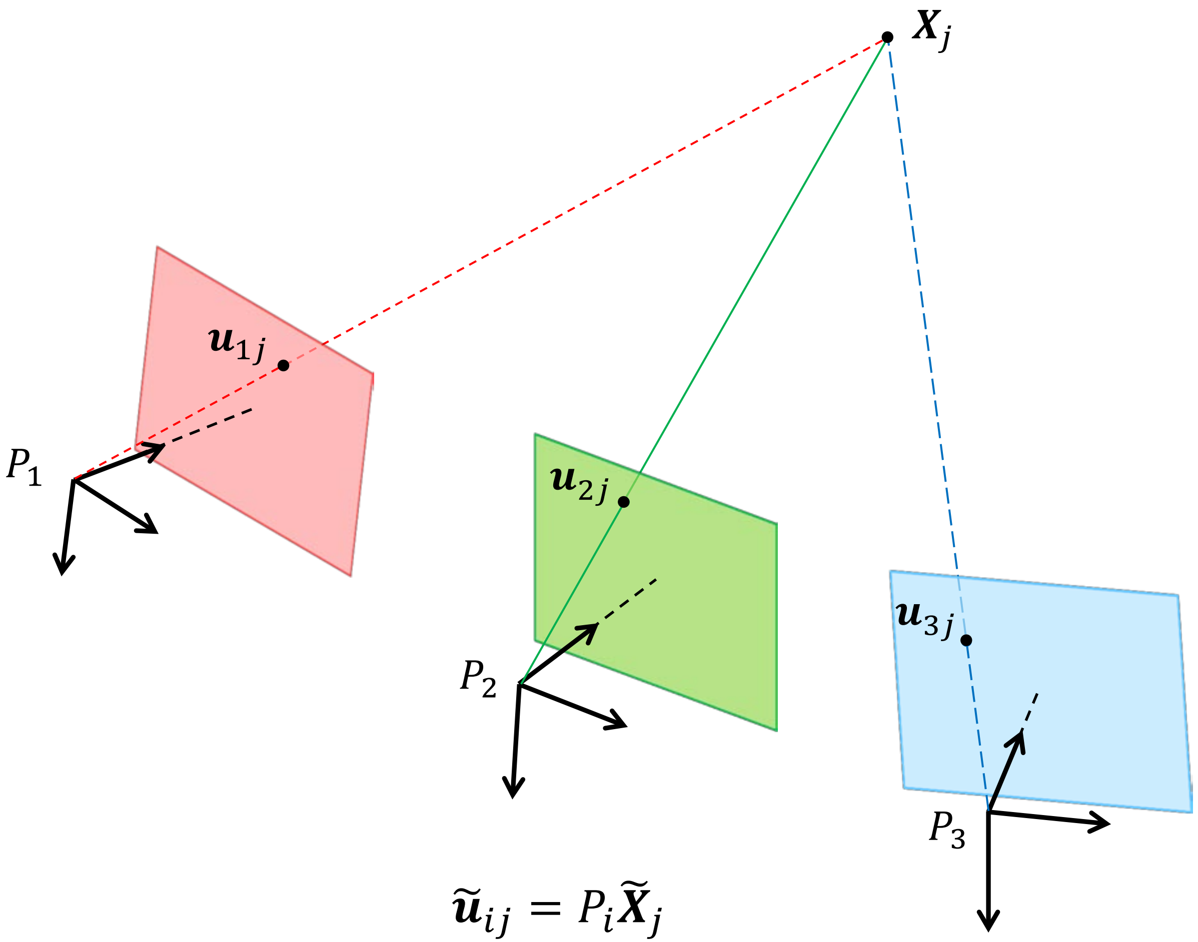
- How does “more-than-two-view geometry” constrain our 2D matches?
- Algebraic description?

Scene geometry (structure)

- Effect of more views on determining the 3D structure of the scene?
- Next lecture

Camera geometry (motion)

- Effect of more views on determining camera poses?
- Next lecture



$$\tilde{\boldsymbol{u}}_{ij} = P_i \tilde{\boldsymbol{X}}_j$$

Correspondences

Two views

- Points $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ must satisfy the epipolar constraint

- The fundamental matrix F represents this constraint

$$\tilde{\boldsymbol{u}}_2{}^T F \tilde{\boldsymbol{u}}_1 = 0$$

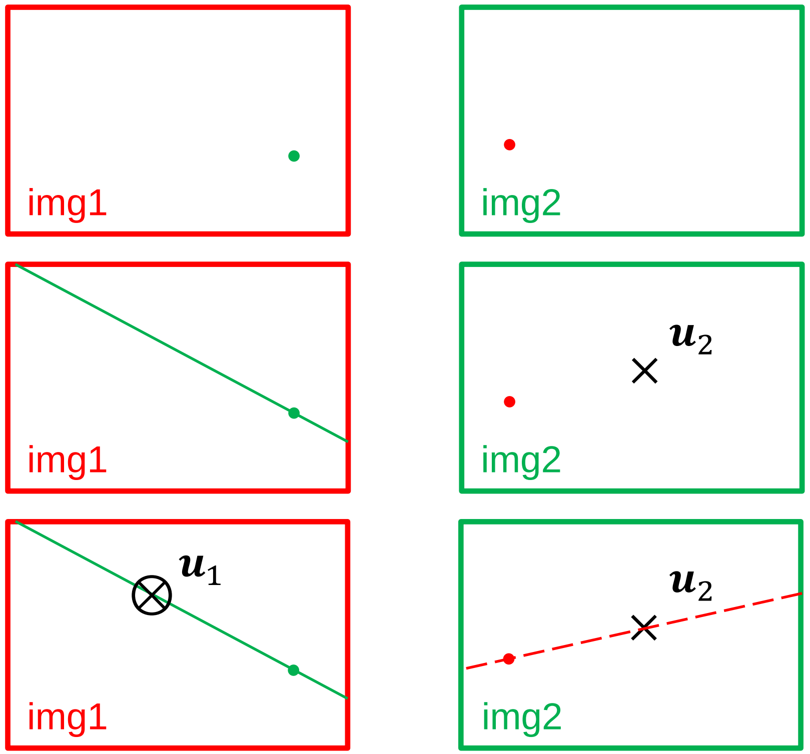
- F also describes the correspondence between points and epipolar lines

$$\tilde{\boldsymbol{l}}_2 = F \tilde{\boldsymbol{u}}_1$$
$$\tilde{\boldsymbol{l}}_1 = F^T \tilde{\boldsymbol{u}}_2$$

Epipoles

A point $\boldsymbol{u}_2$ in img2 correspond to a line in img1

A point $\boldsymbol{u}_1$ in img1 correspond to a line in img2



Correspondences

Epipoles

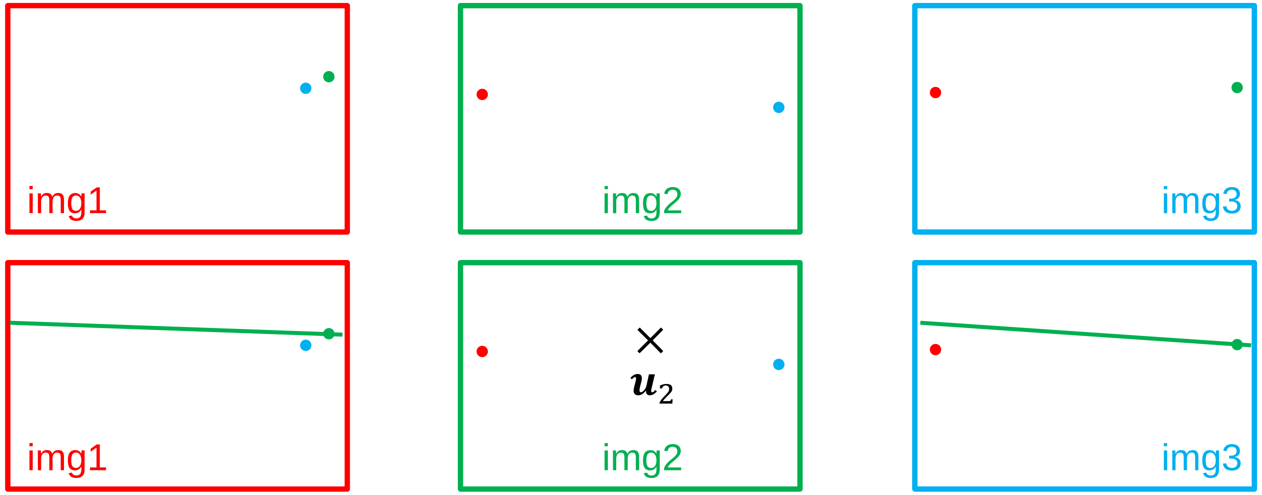
Three views

Correspondences

Three views

Epipoles

A point $\boldsymbol{u}_2$ in img2 correspond to lines in img1 and img3



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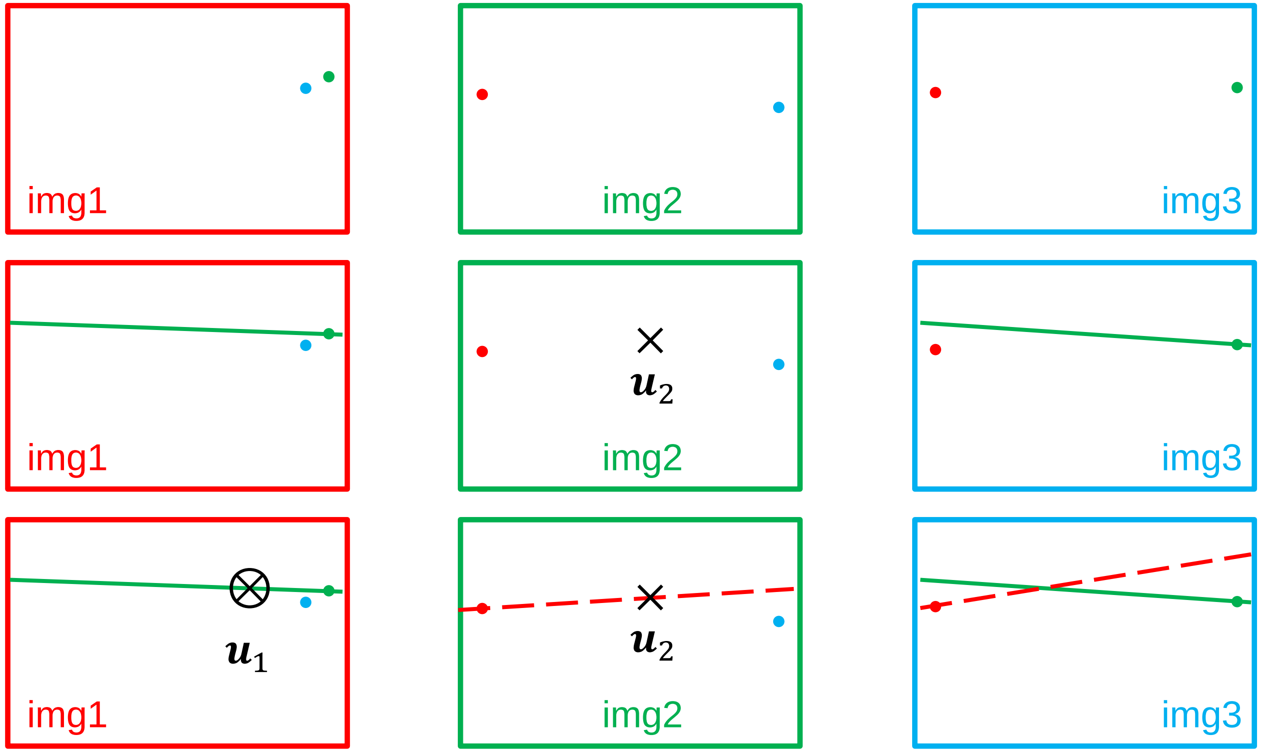


Correspondences

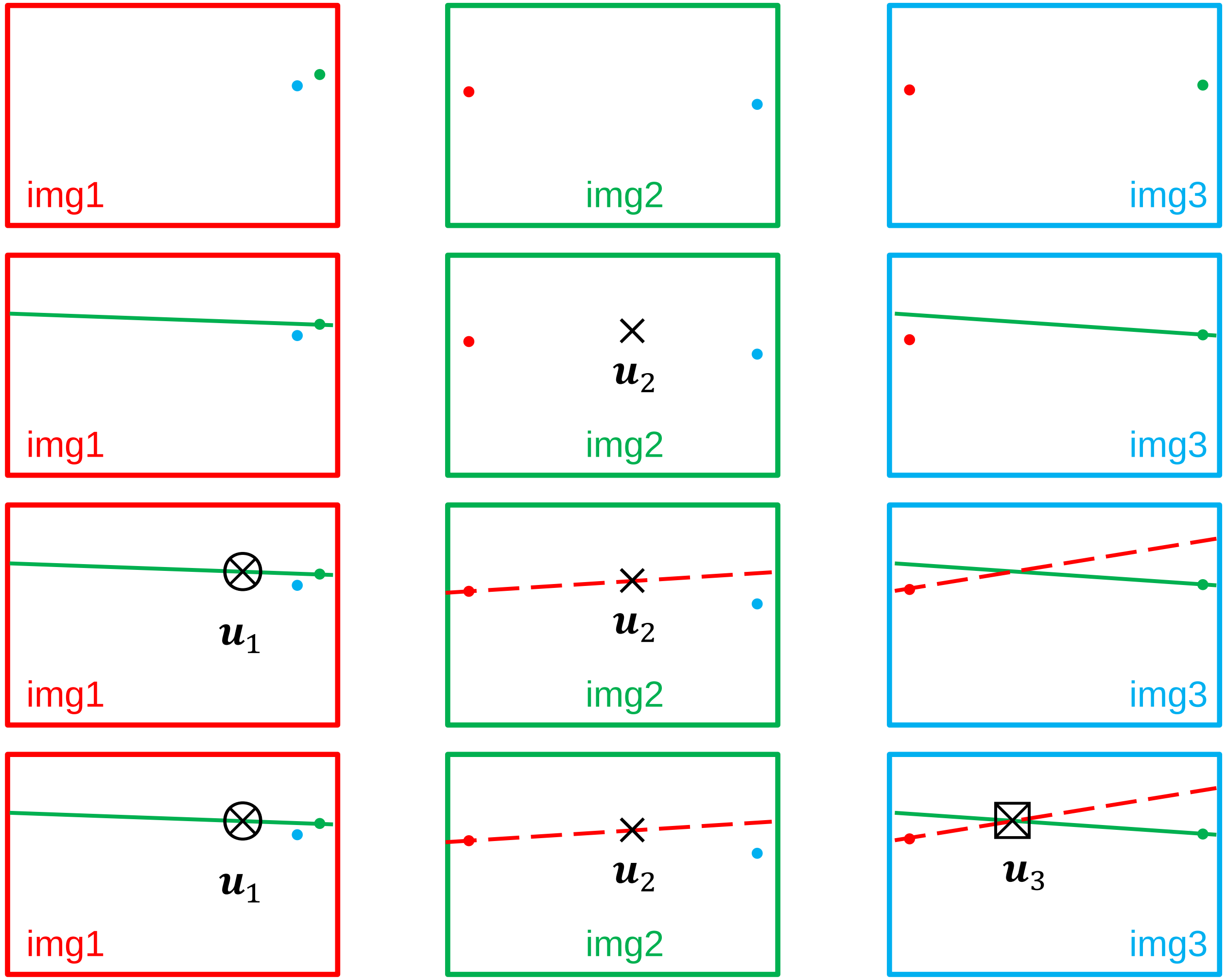
Three views

Epipoles

A point $\boldsymbol{u}_2$ in img2 correspond to lines in img1 and img3

A point $\boldsymbol{u}_1$ in img1 correspond to lines in img2 and img3

Points $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ define a point $\boldsymbol{u}_3$ in img3



Correspondences

Three views

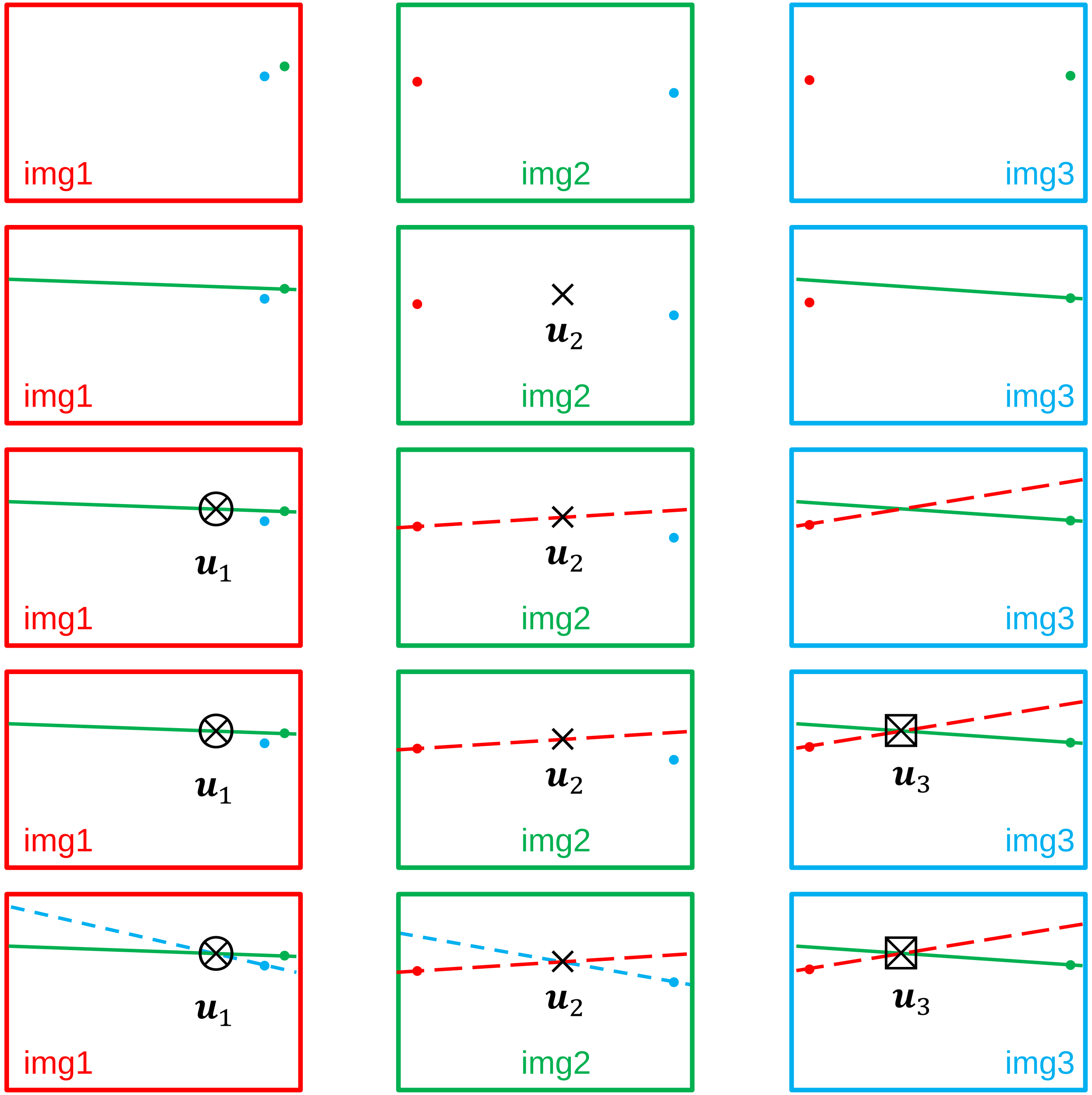
Epipoles

A point $\boldsymbol{u}_2$ in img2 correspond to lines in img1 and img3

A point $\boldsymbol{u}_1$ in img1 correspond to lines in img2 and img3

Points $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ define a point $\boldsymbol{u}_3$ in img3

Any two of the points $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ define the third

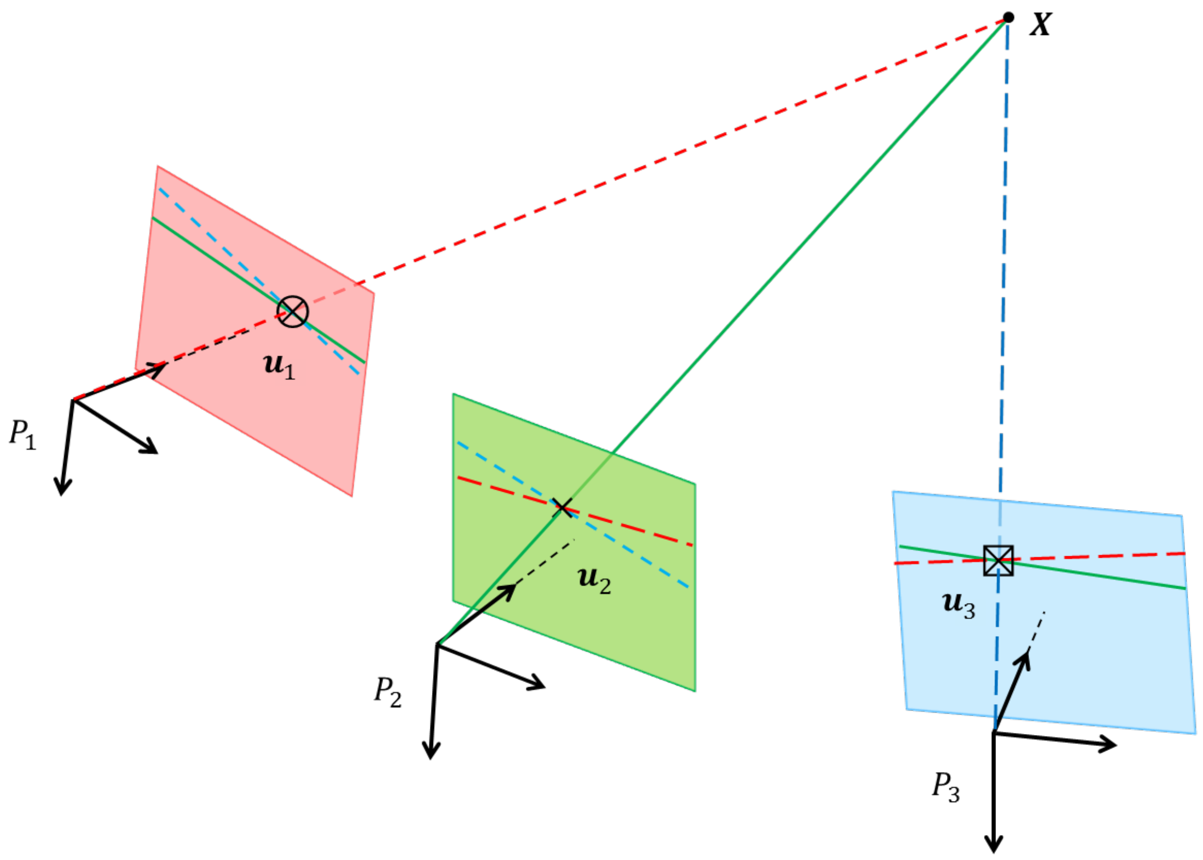


Correspondences

Three views

- This construction shows that the three points $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ and $\boldsymbol{u}_3$ are connected by some geometric constraint
  - Any one of them can be computed from the two others

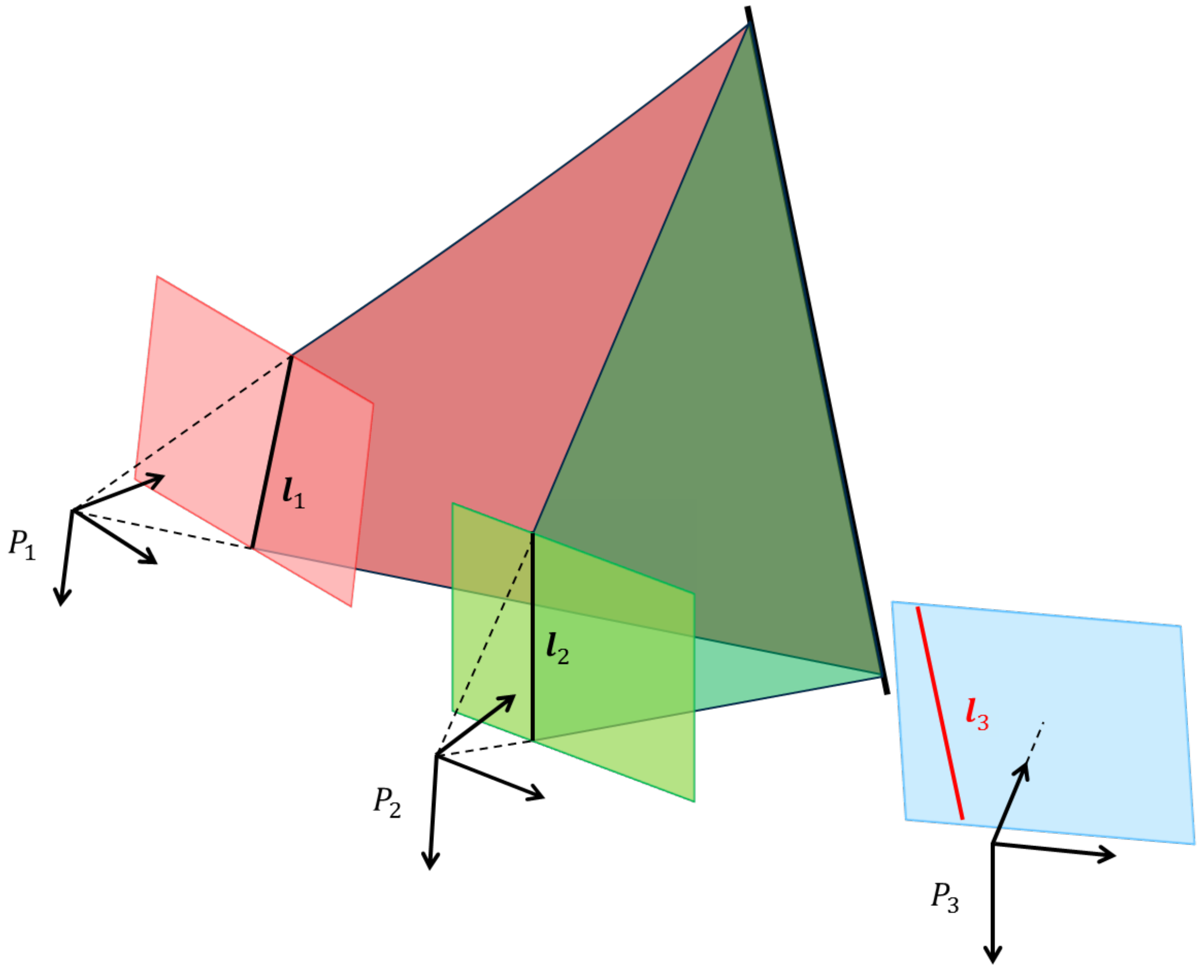
- But it is not clear if this three-view constraint governs more than the three epipolar constraints put together



Correspondences

Three views

- The difference between two-view geometry and three-view geometry becomes evident if we consider lines instead of points
- In two-view geometry no constraints are available for lines
- In three-view geometry, lines $\boldsymbol{l}_1$ and $\boldsymbol{l}_2$ in two views will in general generate a line $\boldsymbol{l}_3$ in a third view



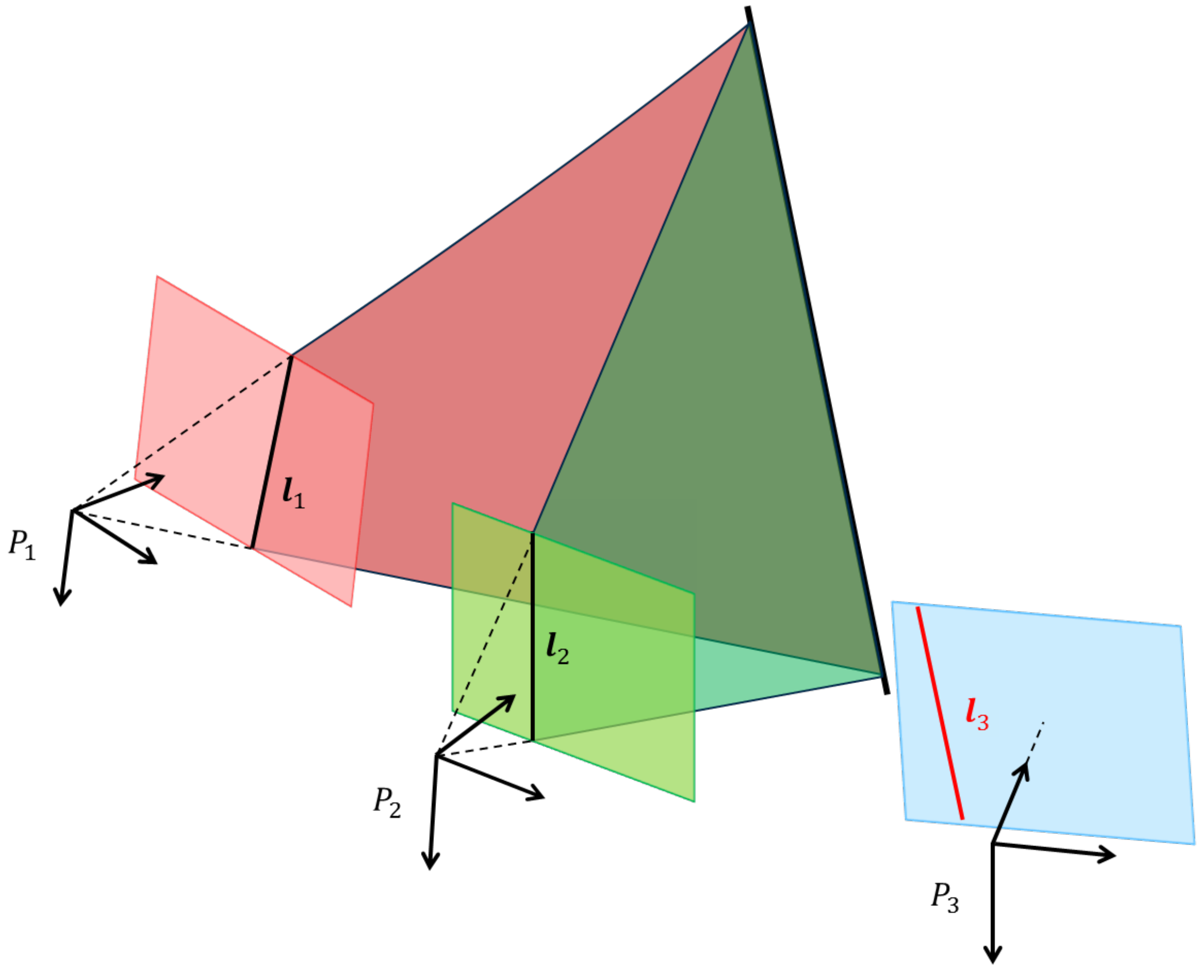
Correspondences

Three views

- The three view geometry has an algebraic representation known as the trifocal tensor T
  - A $3 \times 3 \times 3$ array with 18dof

- This tensor governs the relationship between points and lines in three views
  - Point-point-point
  - Point-point-line
  - Point-line-line
  - Point-line-point
  - Line-line-line

- It may be used to transfer a two-view point/line correspondence into a point/line in a third view



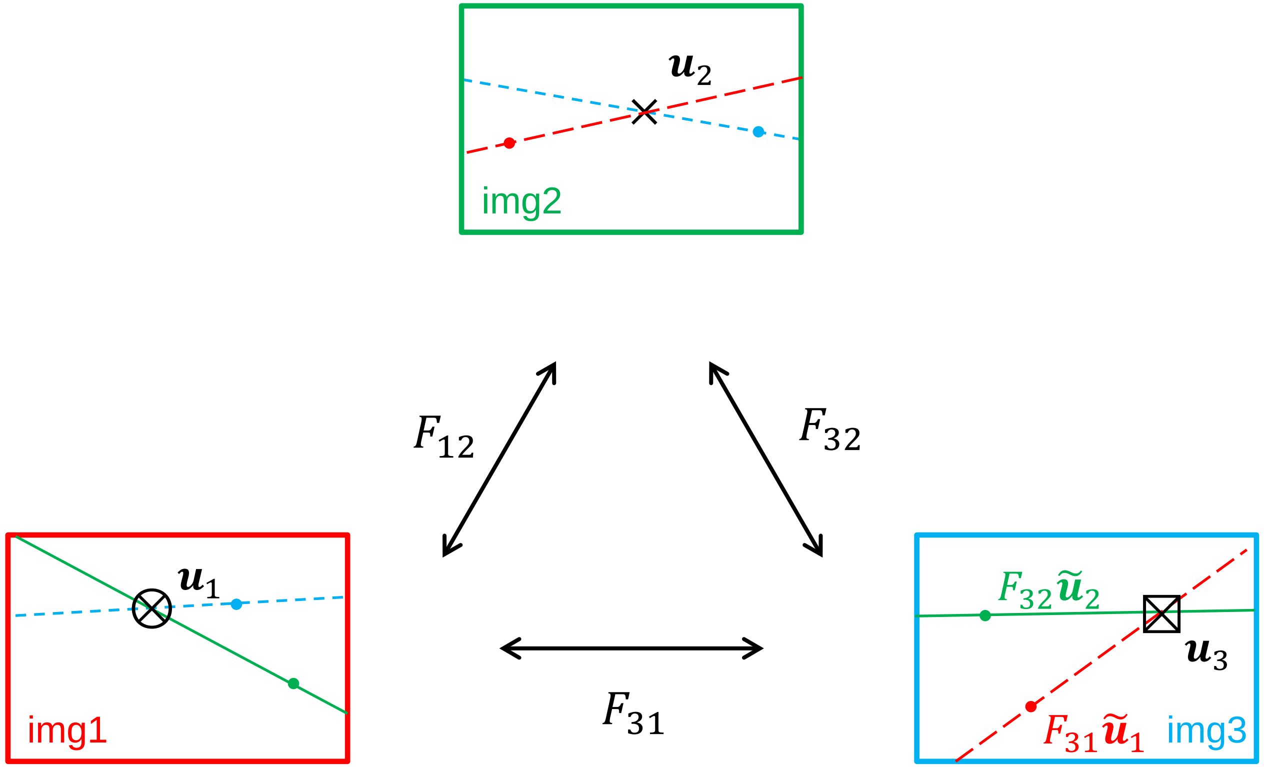
Correspondences

Three views

- As we just saw, point transfer can be done directly from the epipolar constraints
$$\tilde{\boldsymbol{u}}_3 = (F_{31}\tilde{\boldsymbol{u}}_1) \times (F_{32}\tilde{\boldsymbol{u}}_2)$$

- However, this fails for points in the plane defined by the three camera centers – the trifocal plane – since the epipolar lines then will coincide

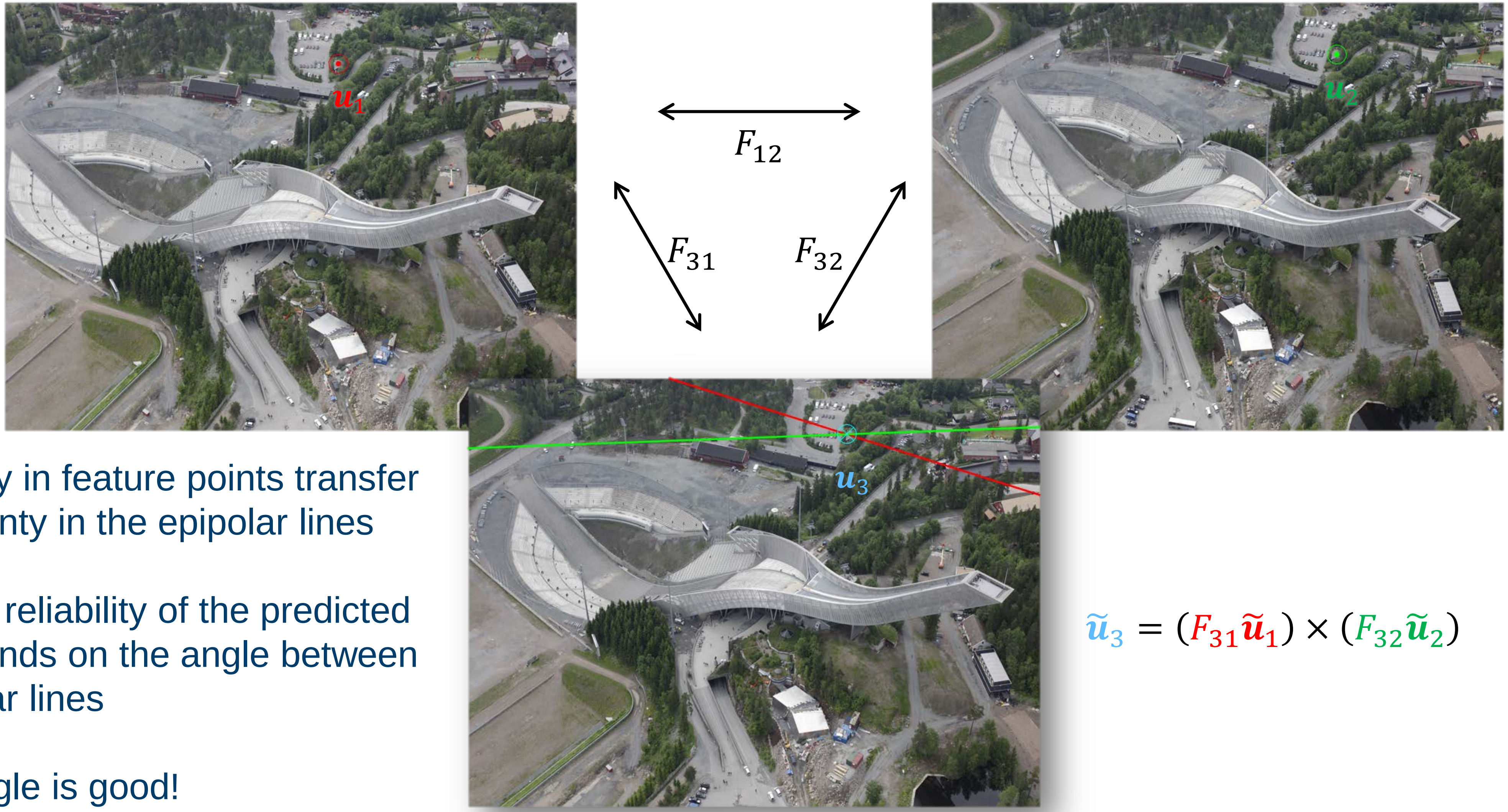
- The trifocal tensor allows point transfer also for points in the trifocal plane



$$\tilde{\boldsymbol{u}}_3 = (F_{31}\tilde{\boldsymbol{u}}_1) \times (F_{32}\tilde{\boldsymbol{u}}_2)$$

Example

Point transfer based on epipolar constraints



$$\tilde{u}_3 = (F_{31}\tilde{\boldsymbol{u}}_1) \times (F_{32}\tilde{\boldsymbol{u}}_2)$$

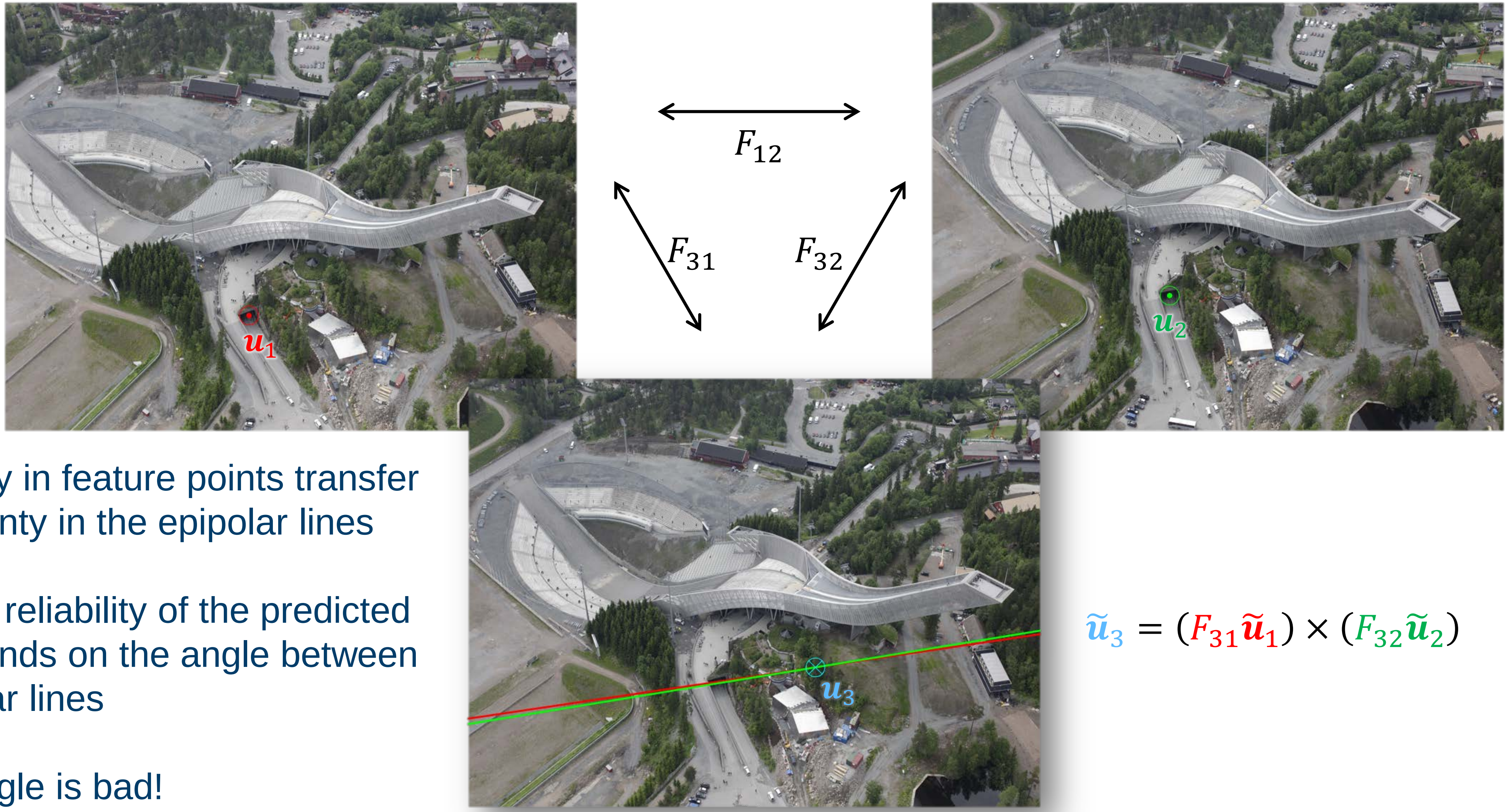
Uncertainty in feature points transfer to uncertainty in the epipolar lines

Hence the reliability of the predicted point depends on the angle between the epipolar lines

A large angle is good!

Example

Point transfer based on epipolar constraints



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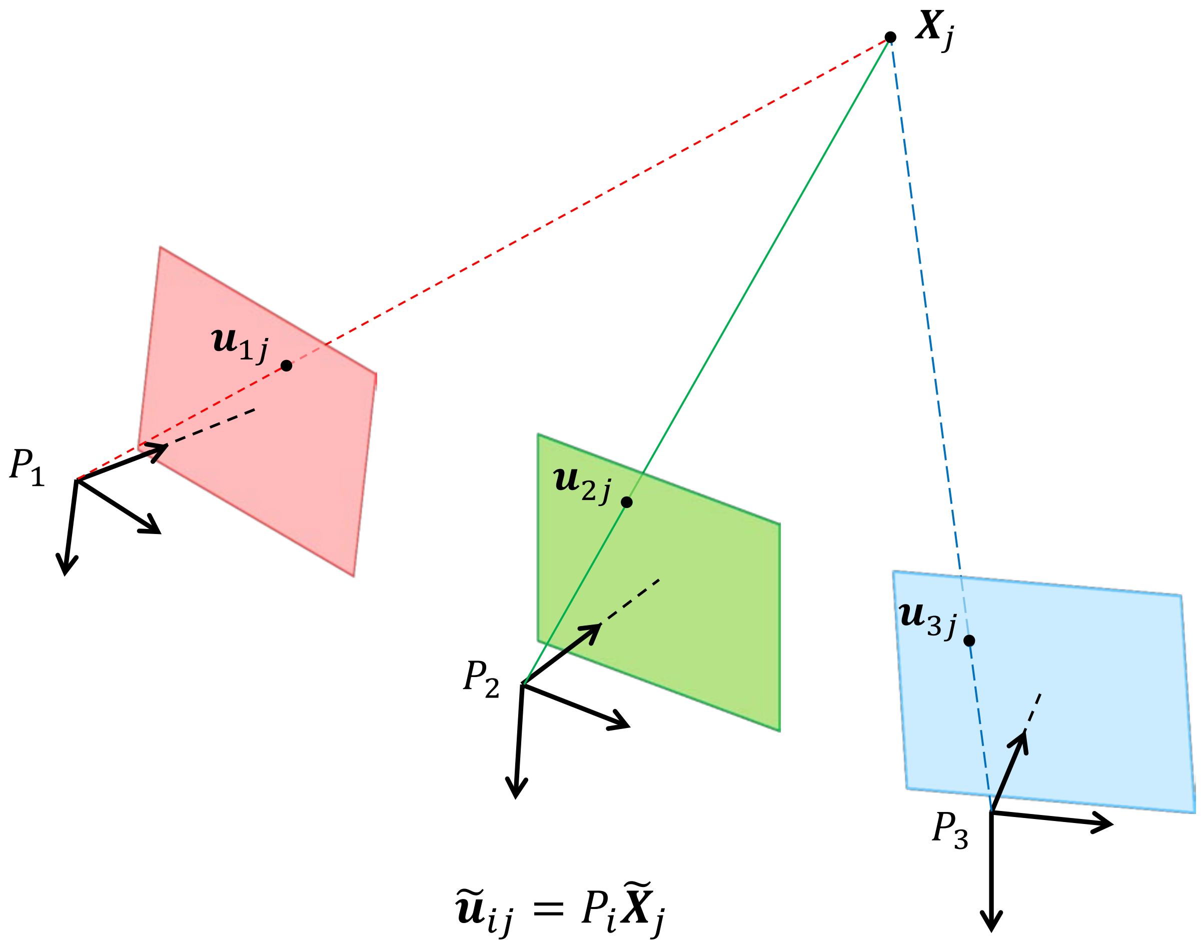
Hence the reliability of the predicted point depends on the angle between the epipolar lines

A small angle is bad!

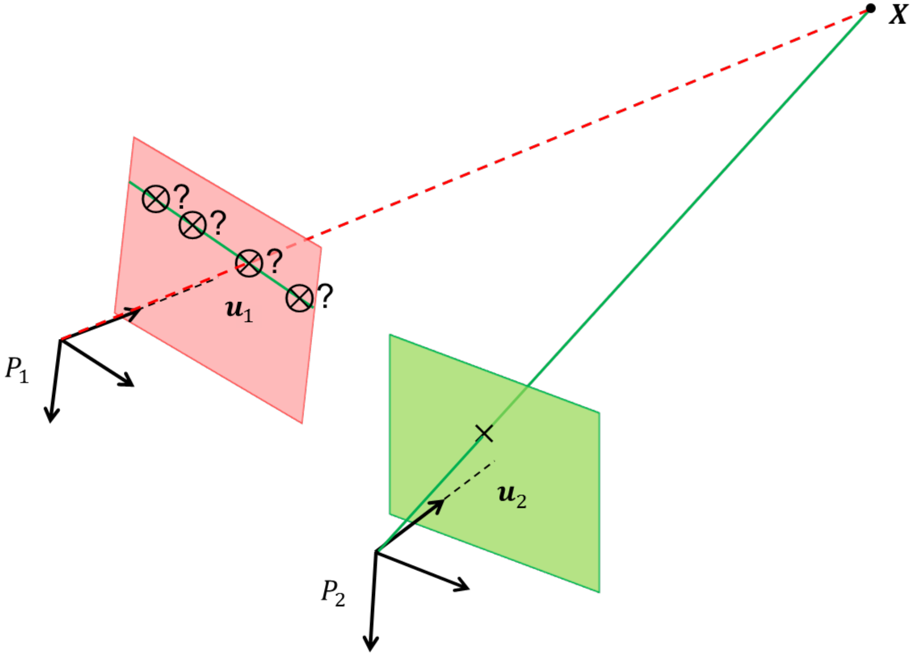
More-than-two-view geometry

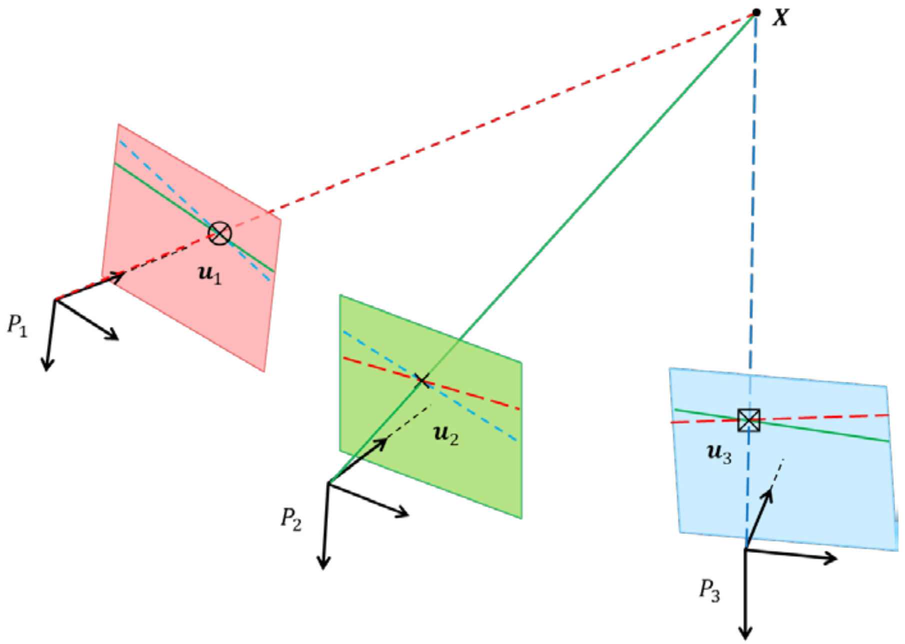
Correspondences (matching)

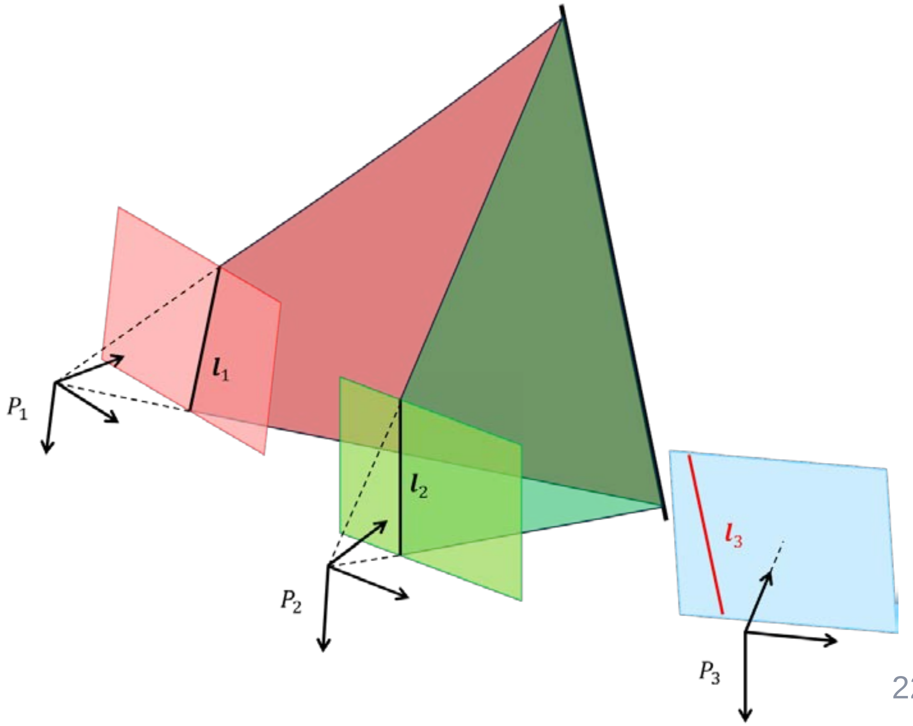
- More views enables us to reveal and remove more mismatches than we can do in the two-view case
- More views also enables us to predict correspondences that can be tested with or without the use of descriptors
- Uncertainties in these predictions will in general decrease with the number of views



Summary







- Multiple-view geometry
- Correspondences
  - Two-view vs Three-view
  - Fundamental matrix vs Trifocal tensor