

Lecture 8.2

Structure from Motion

Thomas Opsahl

More-than-two-view geometry

Correspondences (matching)

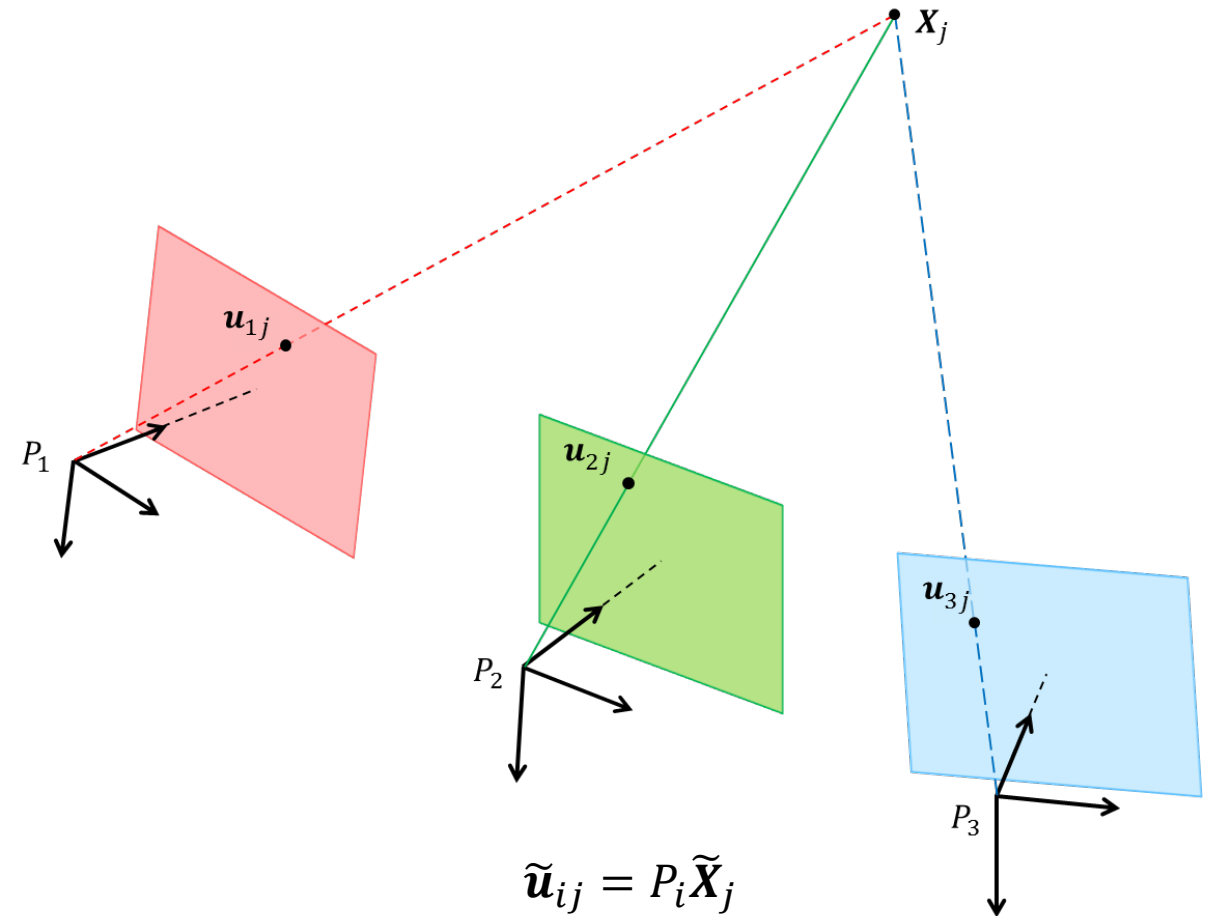
- More views enables us to reveal and remove more mismatches than we can do in the two-view case
- More views also enables us to predict correspondences that can be tested with or without the use of descriptors

Scene geometry (structure)

- Effect of more views on determining the 3D structure of the scene?

Camera geometry (motion)

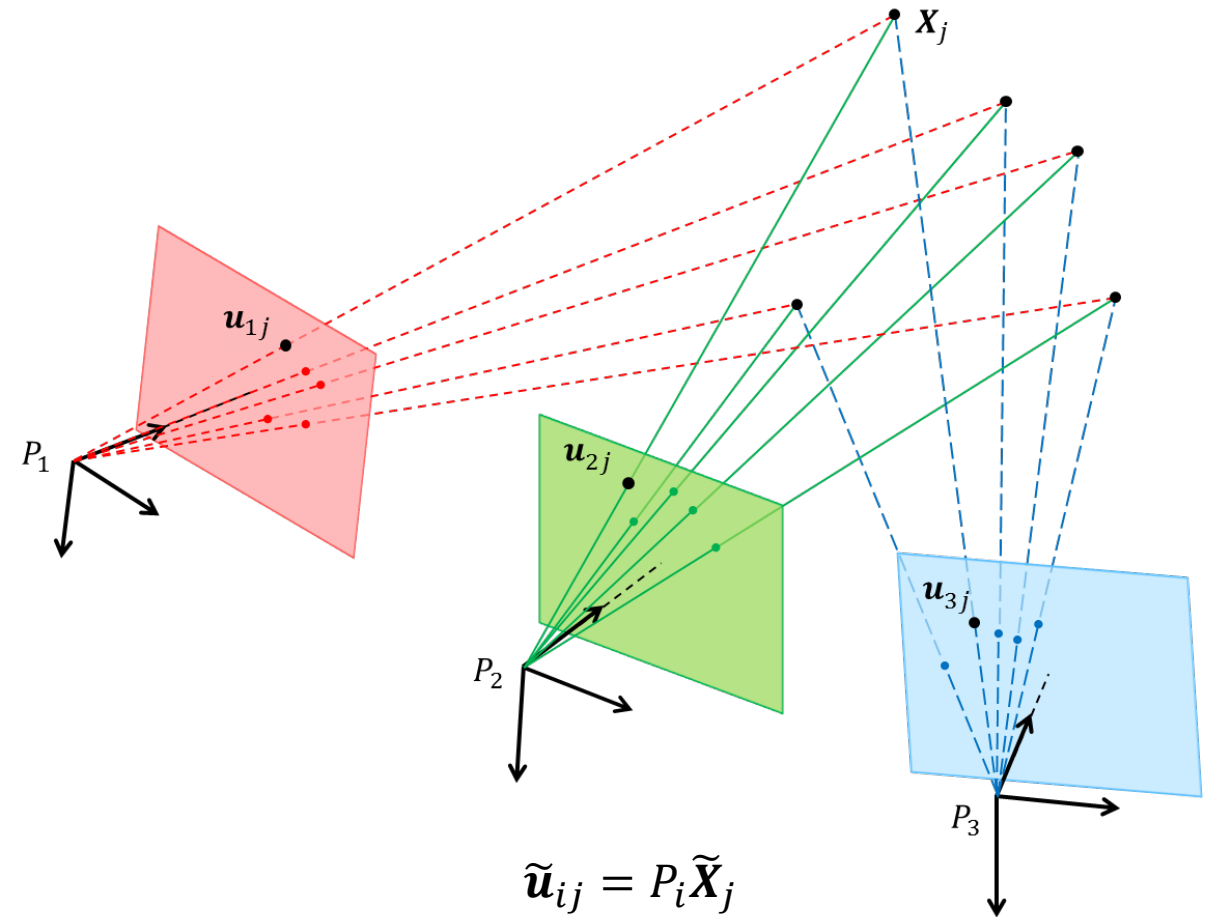
- Effect of more views on determining camera poses?



Structure from Motion

Problem

Given m images of n fixed 3D points, estimate the m projection matrices P_j and the n points X_j from the $m \cdot n$ correspondences $u_{ij} \leftrightarrow u_{kj}$

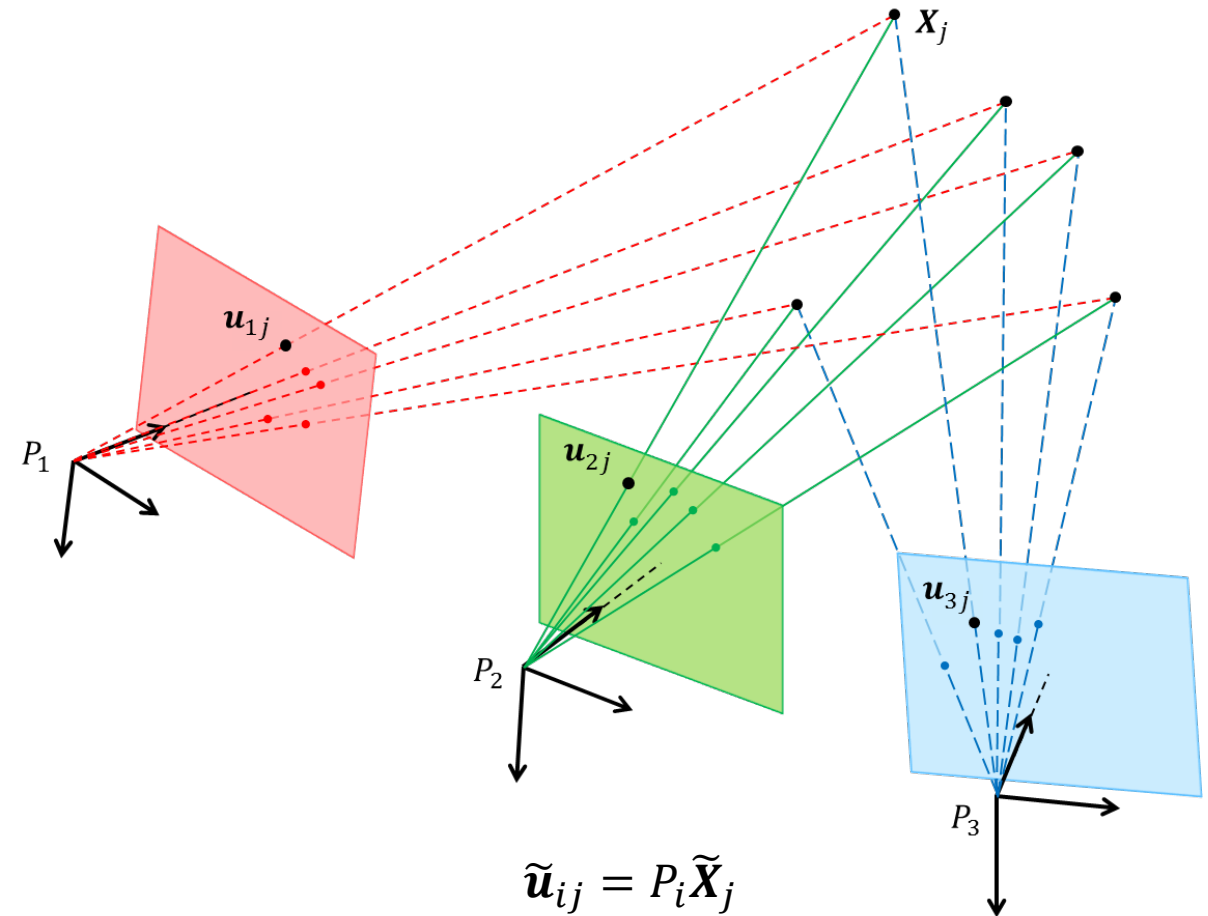


Structure from Motion

Problem

Given m images of n fixed 3D points, estimate the m projection matrices P_j and the n points X_j from the $m \cdot n$ correspondences $u_{ij} \leftrightarrow u_{kj}$

- We can solve for structure and motion when
$$2mn \geq 11m + 3n - 15$$
- In the general/uncalibrated case, cameras and points can only be recovered up to a projective ambiguity ($\tilde{u}_{ij} = P_i Q^{-1} Q \tilde{X}_j$)
- In the calibrated case, they can be recovered up to a similarity (scale)
 - Known as Euclidean/metric reconstruction

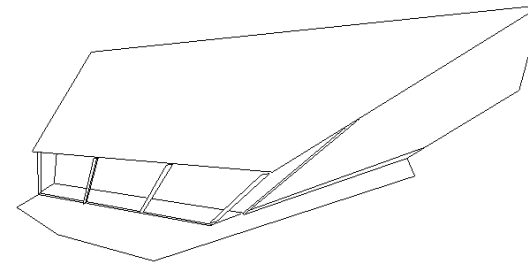
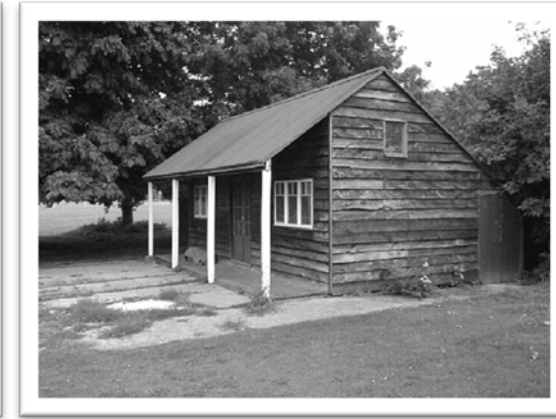
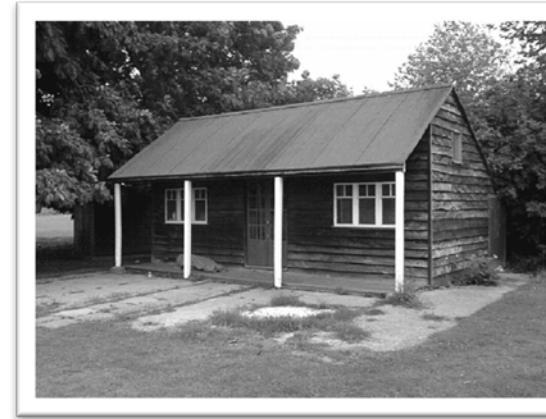


Structure from Motion

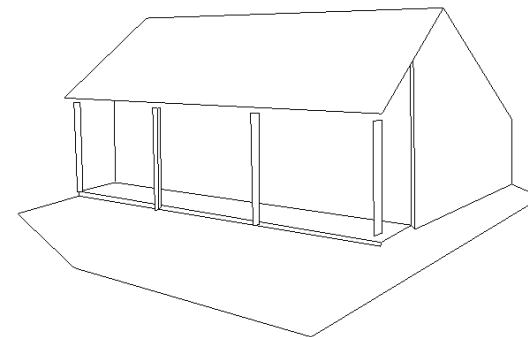
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Projective reconstruction



Metric reconstruction

Images courtesy of Hartley & Zisserman <http://www.robots.ox.ac.uk/~vgg/hzbook/>

Structure from Motion

Problem

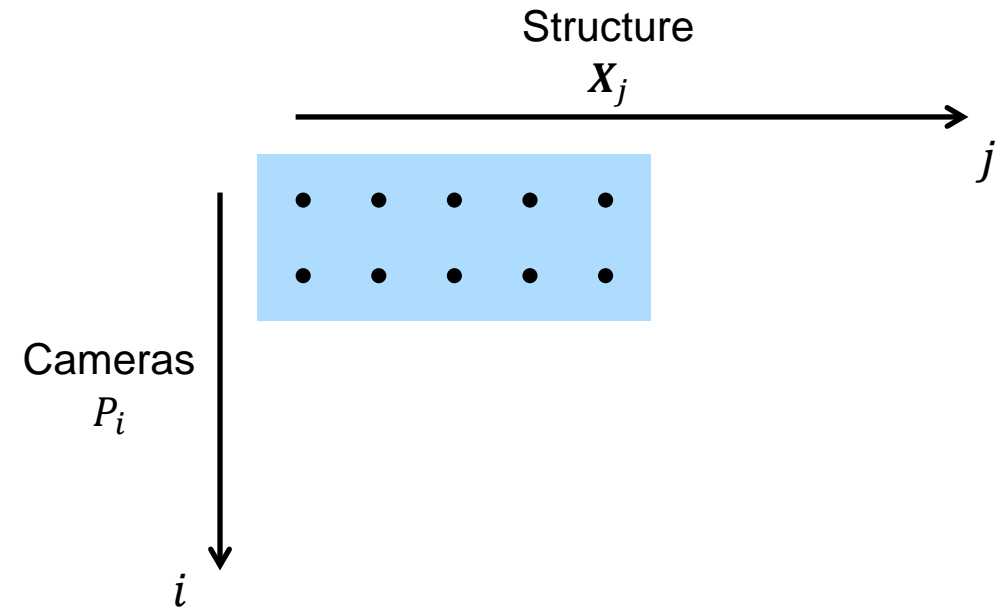
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- This problem has been studied extensively and several different approaches have been suggested
- We will take a look at a couple of these
 - Sequential structure from motion
 - Bundle adjustment

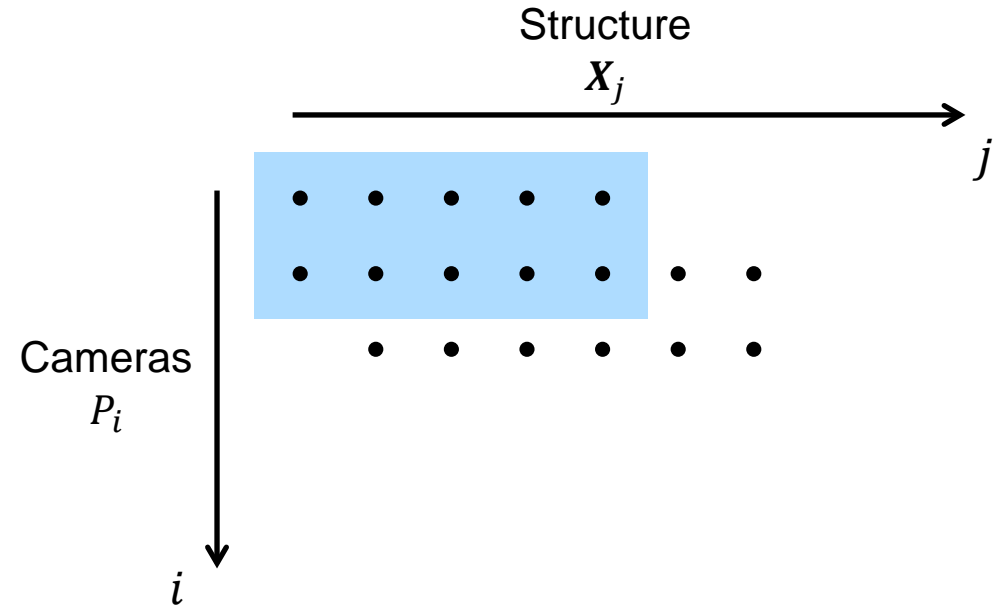
Sequential SfM

- Initialize motion from two images
 - $F \rightarrow (P_1, P_2)$
 - $E \rightarrow (P_1, P_2) = (K_1[I \quad \mathbf{0}], K_2[^1R_2 \quad ^1\mathbf{t}_2])$
- Initialize the 3D structure by triangulation



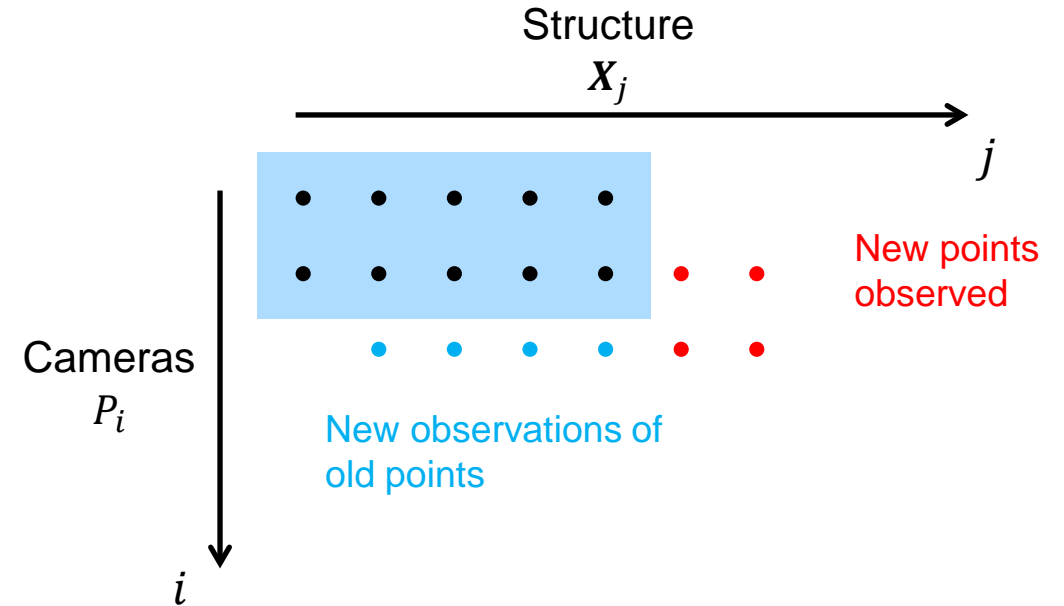
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- Initialize the 3D structure by triangulation
- For each additional view
 - Determine the projection matrix P_i , e.g. from 2D-3D correspondences $\mathbf{u}_{ij} \leftrightarrow \mathbf{X}_j$
 - Refine and extend the 3D structure by triangulation



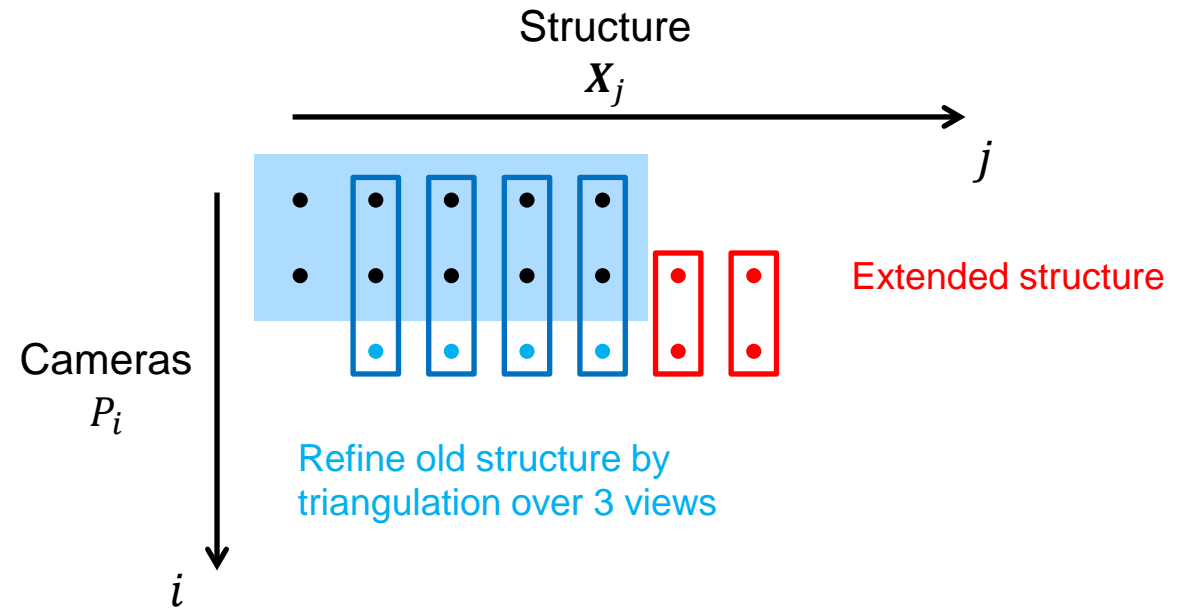
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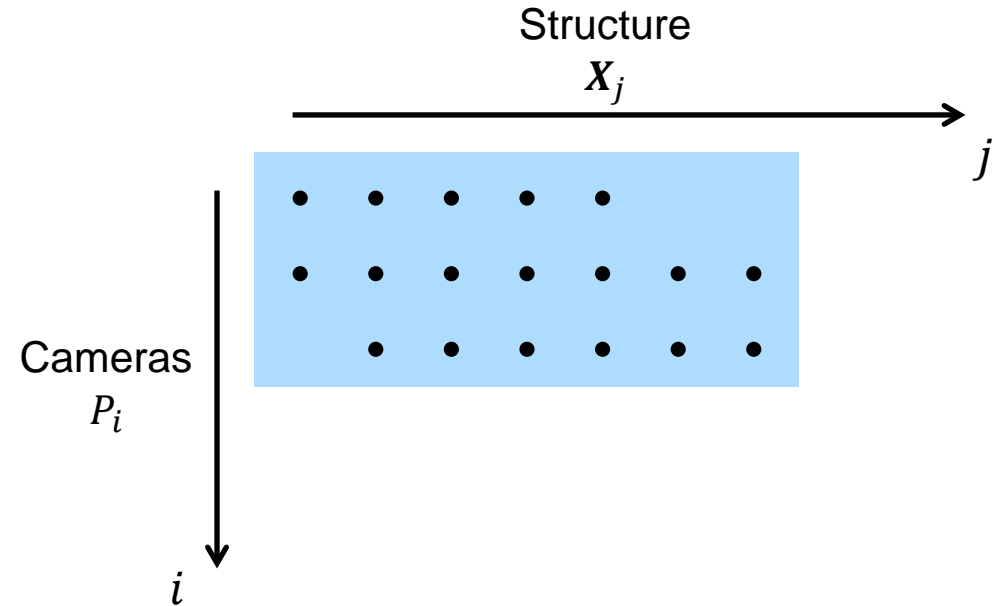
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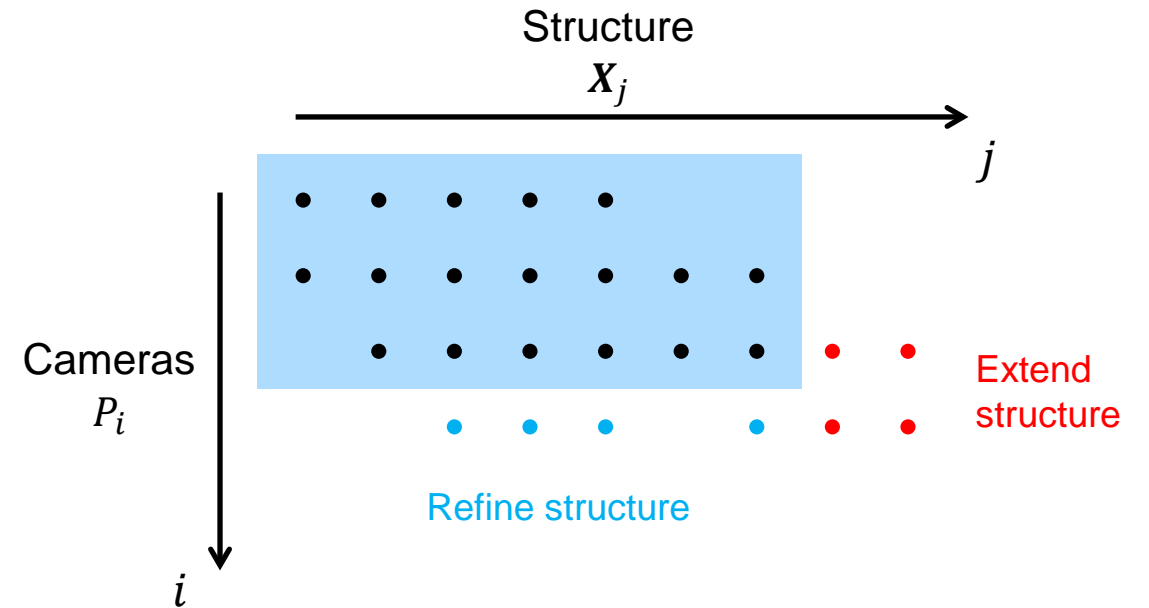
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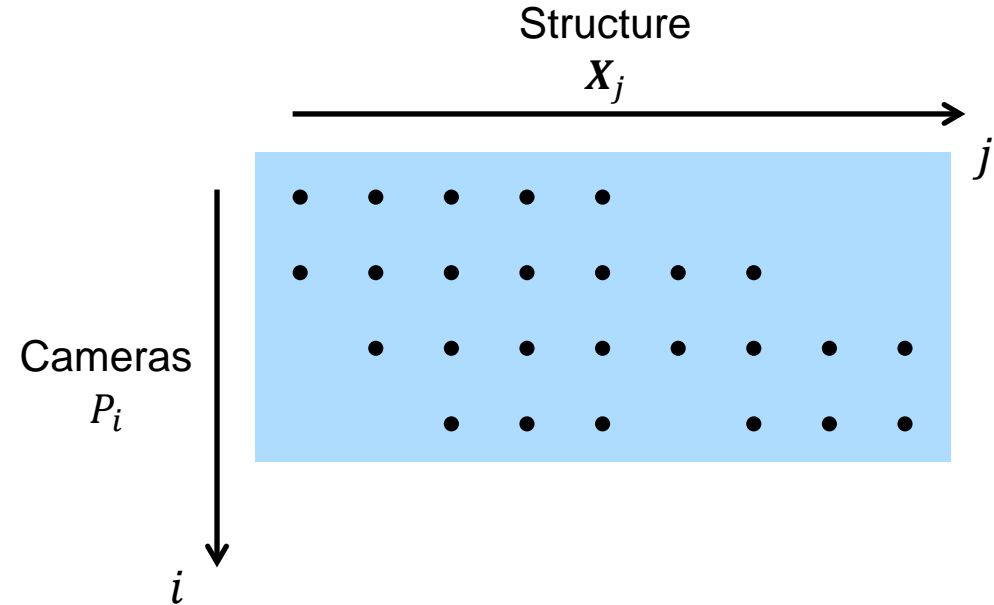
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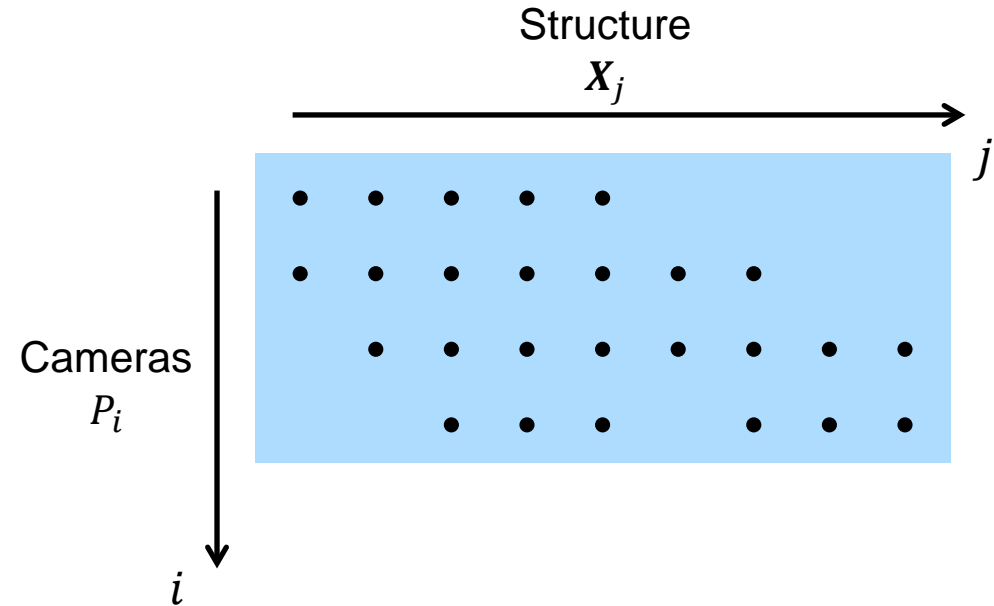
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 - Refine and extend the 3D structure by triangulation
- The resulting structure and motion can be refined in a process known as bundle adjustment

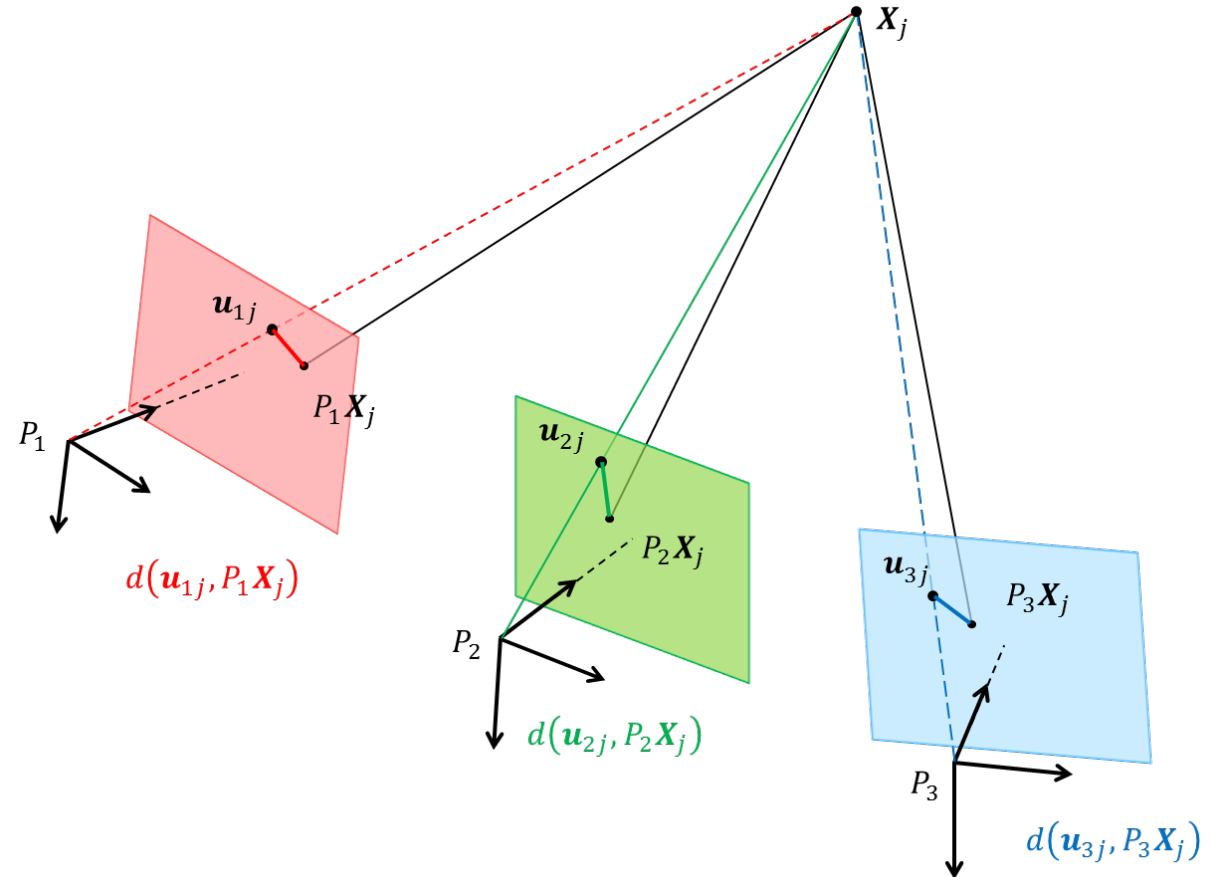


Bundle adjustment

- Non-linear method that refines structure and motion by minimizing the sum of squared reprojection errors

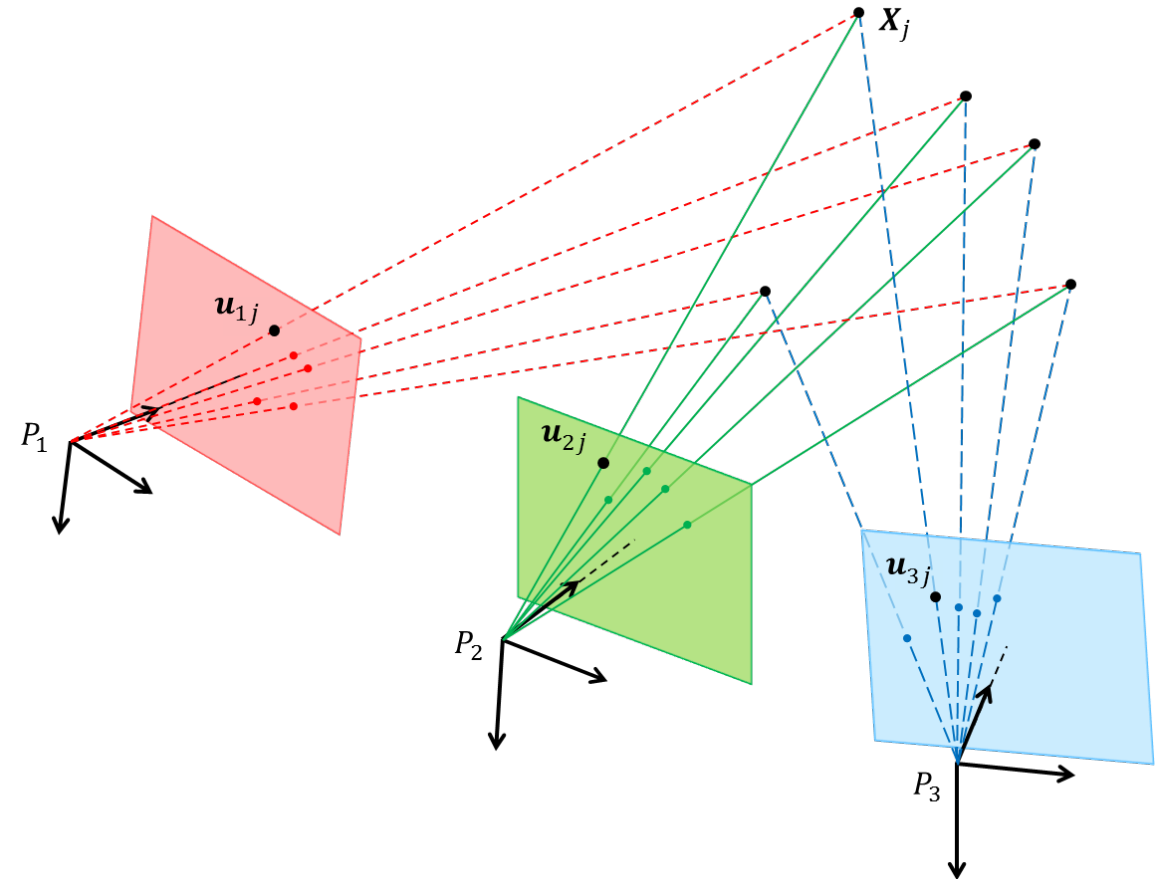
$$\epsilon = \sum_{i=1}^m \sum_{j=1}^n d(\tilde{\mathbf{u}}_{ij}, P_i \tilde{\mathbf{X}}_j)^2$$

- Camera calibration can be solved as part of bundle adjustment by including intrinsic parameters and skew parameters in the cost function
- Need initial estimates for all parameters!
 - 3 per 3D point
 - ~12 per camera depending on parameterization
 - Some intrinsic parameters, like the focal length, can be initialized from image EXIF data



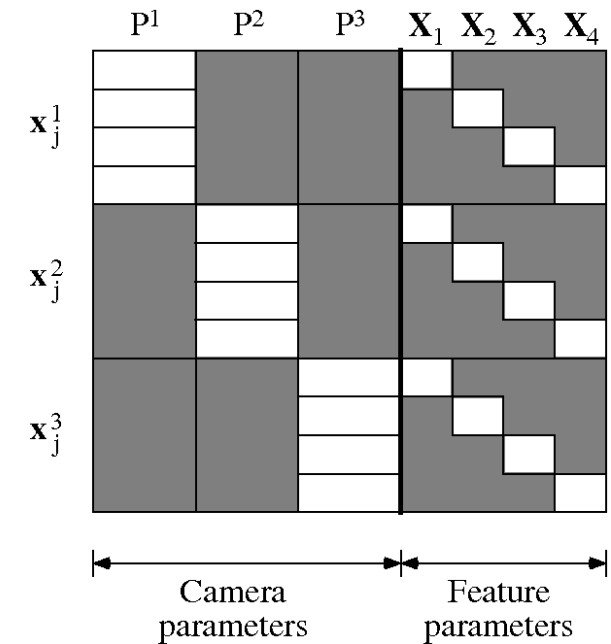
Bundle adjustment

- There are several strategies that deal with the potentially extreme number of parameters
- Reduce the number of parameters by not including all the views and/or all the points
 - Perform bundle adjustment only on a subset and compute missing views/points based on the result
 - Divide views/points into several subsets which are bundle adjusted independently and merge the results
- Interleaved bundle adjustment
 - Alternate minimizing the reprojection error by varying only the cameras or only the points
 - This is viable since each point is estimated independently given fixed cameras, and similarly each camera is estimated independently from fixed points



Bundle adjustment

- Sparse bundle adjustment
 - For each iteration, iterative minimization methods need to determine a vector Δ of changes to be made in the parameter vector
 - In Levenberg-Marquardt each such step is determined from the equation
$$(J^T J + \lambda I)\Delta = -J^T \epsilon$$
where J is the Jacobian matrix of the cost function and ϵ is the vector of errors
 - For the bundle adjustment problem the Jacobian matrix has a sparse structure that can be exploited in computations



The sparse structure of the Jacobian matrix for a bundle adjustment problem with 3 cameras and 4 3D points

Bundle adjustment

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- Combined with parallel processing the before mentioned strategies has made it possible to solve extremely large SfM problems

Bundle adjustment

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 - For the bundle adjustment problem the Jacobian matrix has a sparse structure that can be exploited in computations
- Combined with parallel processing the before mentioned strategies has made it possible to solve extremely large SfM problems
- S. Agarwal et al, *Building Rome in a Day*, 2011
 - Cluster of 62-computers
 - 150 000 unorganized images from Rome
 - ~37 000 image registered
 - Total processing time ~21 hours
 - SfM time ~7 hours
- J. Heinly et al, *Reconstructing the World in Six Days*, 2015
 - 1 dual processor PC with 5 GPU's (CUDA)
 - ~96 000 000 unordered images spanning the globe
 - ~1.5 000 000 images registered
 - Total processing time ~5 days
 - SfM time ~17 hours

Bundle adjustment

- SBA – Sparse Bundle Adjustment
 - A generic sparse bundle adjustment C/C++ package based on the Levenberg-Marquardt algorithm
 - Code (C and Matlab mex) available at <http://www.ics.forth.gr/~lourakis/sba/>
 - CVSBA is an OpenCV wrapper for SBA www.uco.es/investigacion/grupos/ava/node/39/
- Ceres
 - By Google (used in production since 2010)
 - A C++ library for modeling and solving large, complicated optimization problems like SfM
 - Homepage: www.ceres-solver.org
 - Code available on GitHub <https://github.com/ceres-solver/ceres-solver>
- GTSAM – Georgia Tech Smoothing and Mapping
 - A C++ library based on factor graphs that is well suited for SfM ++
 - Code (C++ library and Matlab toolbox) available at <https://borg.cc.gatech.edu/borg/download>
- g²o – General Graph Optimization
 - Open source C++ framework for optimizing graph-based nonlinear error functions
 - Homepage: <https://openslam.org/g2o.html>
 - Code available on GitHub <https://github.com/RainerKuemmerle/g2o>

Bundle adjustment

- Bundler
 - A structure from motion system for unordered image collections written in C and C++
 - SfM based on a modified version SBA (default) or Ceres
 - Homepage: <http://www.cs.cornell.edu/~snavely/bundler/>
 - Code available on GitHub https://github.com/snavely/bundler_sfm
- RealityCapture
 - A state-of-the-art photogrammetry software that automatically extracts accurate 3D models from images, laser-scans and other input
 - Homepage: <https://www.capturingreality.com/>
- VisualSfM
 - A GUI application for 3D reconstruction using structure from motion
 - Output works with other tools that performs dense 3D reconstruction
 - Homepage: <http://ccwu.me/vsfm/>

Example

Holmenkollen 2-view SfM

More than 10 000 points are registered in addition to the 2 cameras

1Ds

- Images: 2 images
 - _MG_8045.JPG f: 48.7mm
 - _MG_8046.JPG f: 48.7mm
- Control points: empty
- Component 0: 2/2 cams, 0 models
 - Camera poses: 2 registered
 - _MG_8046.JPG 10681 tie points
 - _MG_8045.JPG 10581 tie points

Selected input(0)

File name	< Different values >
Model	Canon Canon EOS 5D...
Width	5516
Height	3744
Features	40000
Features source	Use all image features
Enable alignment	True
Enable meshing	True
Enable texturing and co...	True
Weight in texturing	1.000000
Downscale for depth-map	1
Registered	True
Enable in component	True
Lock pose for continue	False
Prior pose	
Locked pose group	
Absolute pose	Unknown
Prior calibration	
Calibration group	-1
Prior	Approximate
Focal length (35mm)	48.678719
Principal point x	0.000000
Principal point y	0.000000
Skew	0.000000
Aspect ratio	1.000000
Prior lens distortion	
Lens group	-1
Prior	Approximate
Model	No lens distortion
Registration calibration	
Focal length (35mm)	< Different values >

2D

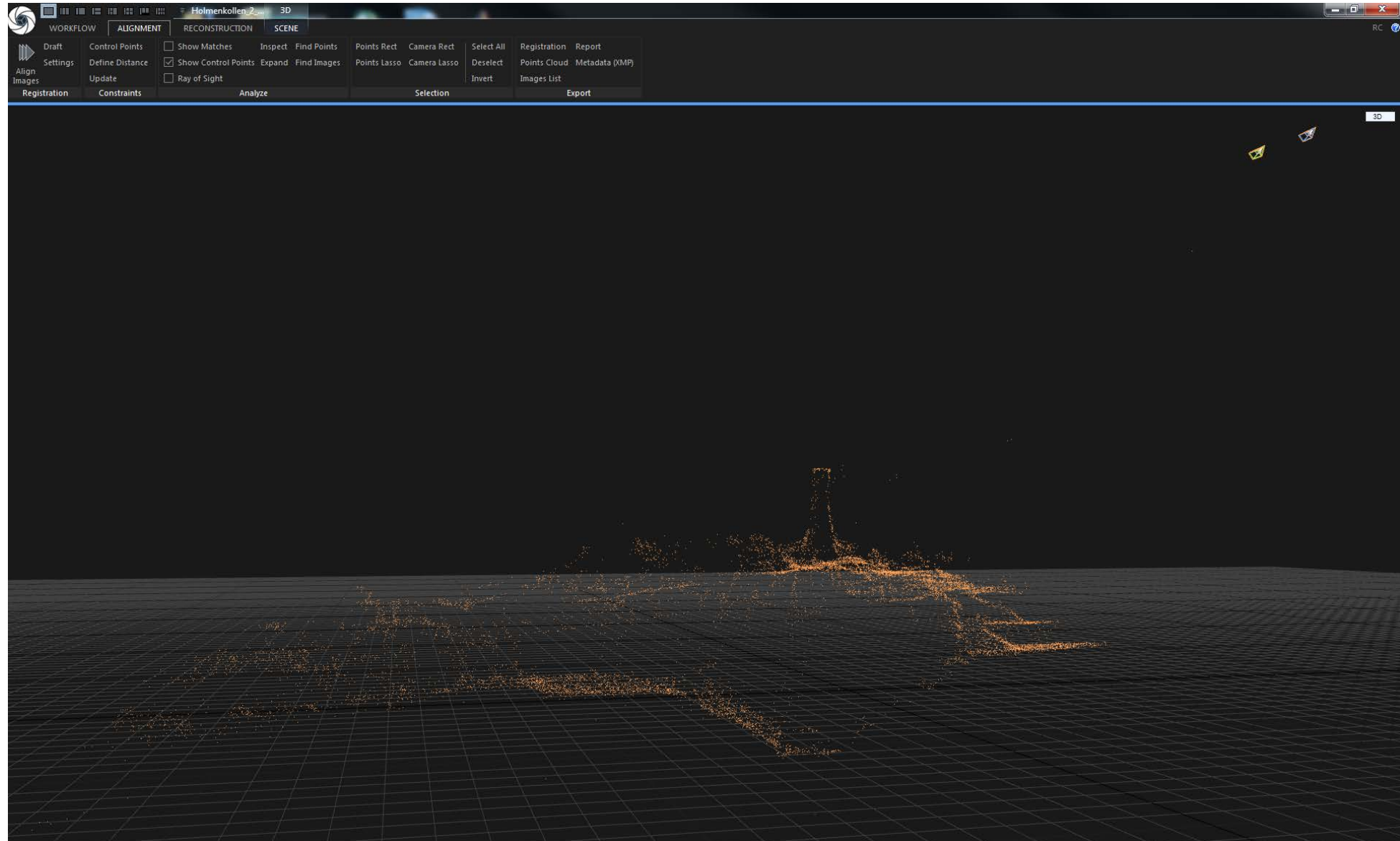
F:\3DModeler\Holmenkollen 2view\
_MG_8045.JPG [registered]

3D

F:\3DModeler\Holmenkollen 2view\
_MG_8046.JPG [registered]

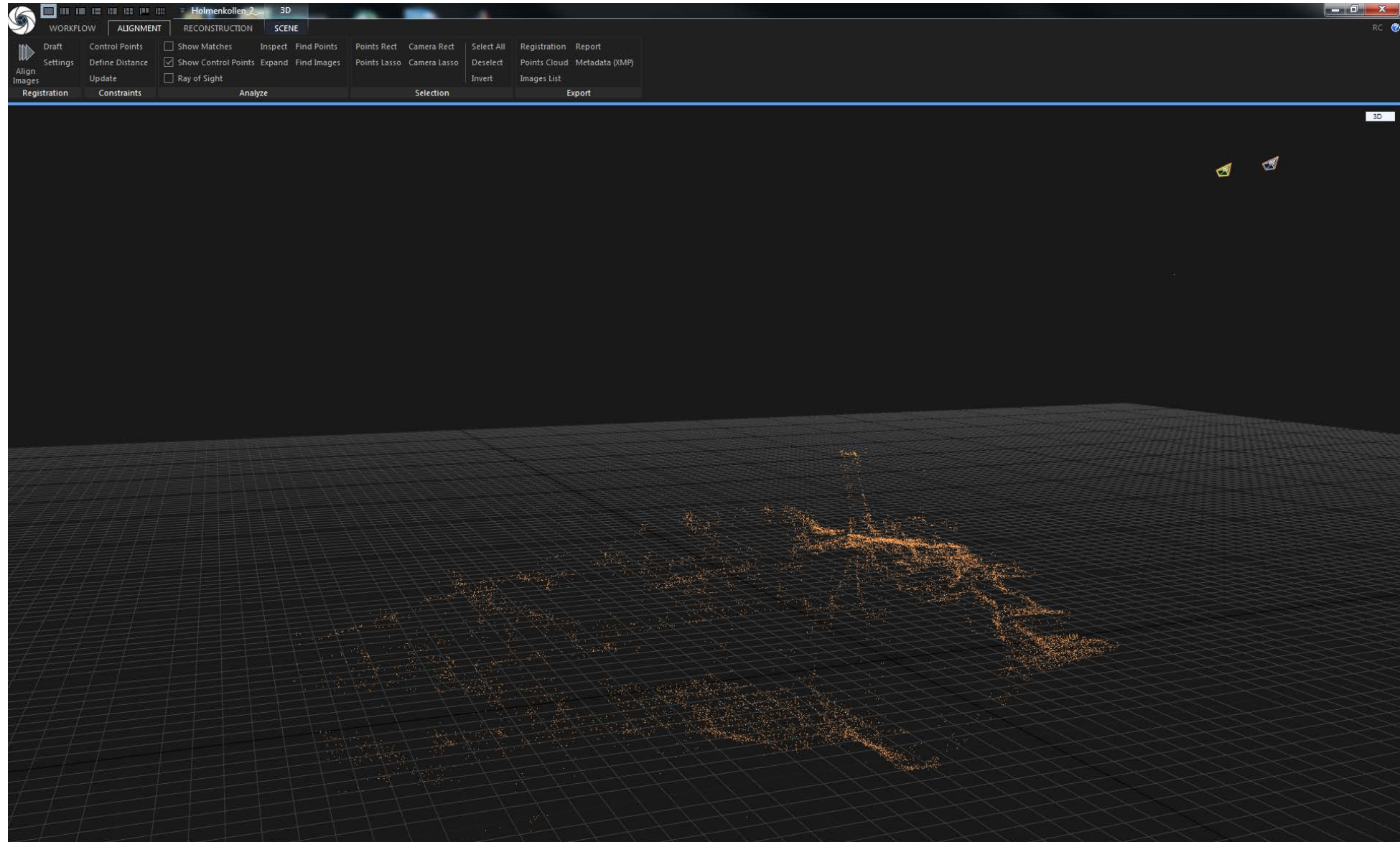
Example

Holmenkollen 2-view SfM



Example

Holmenkollen 2-view SfM



Example

Holmenkollen 3-view SfM

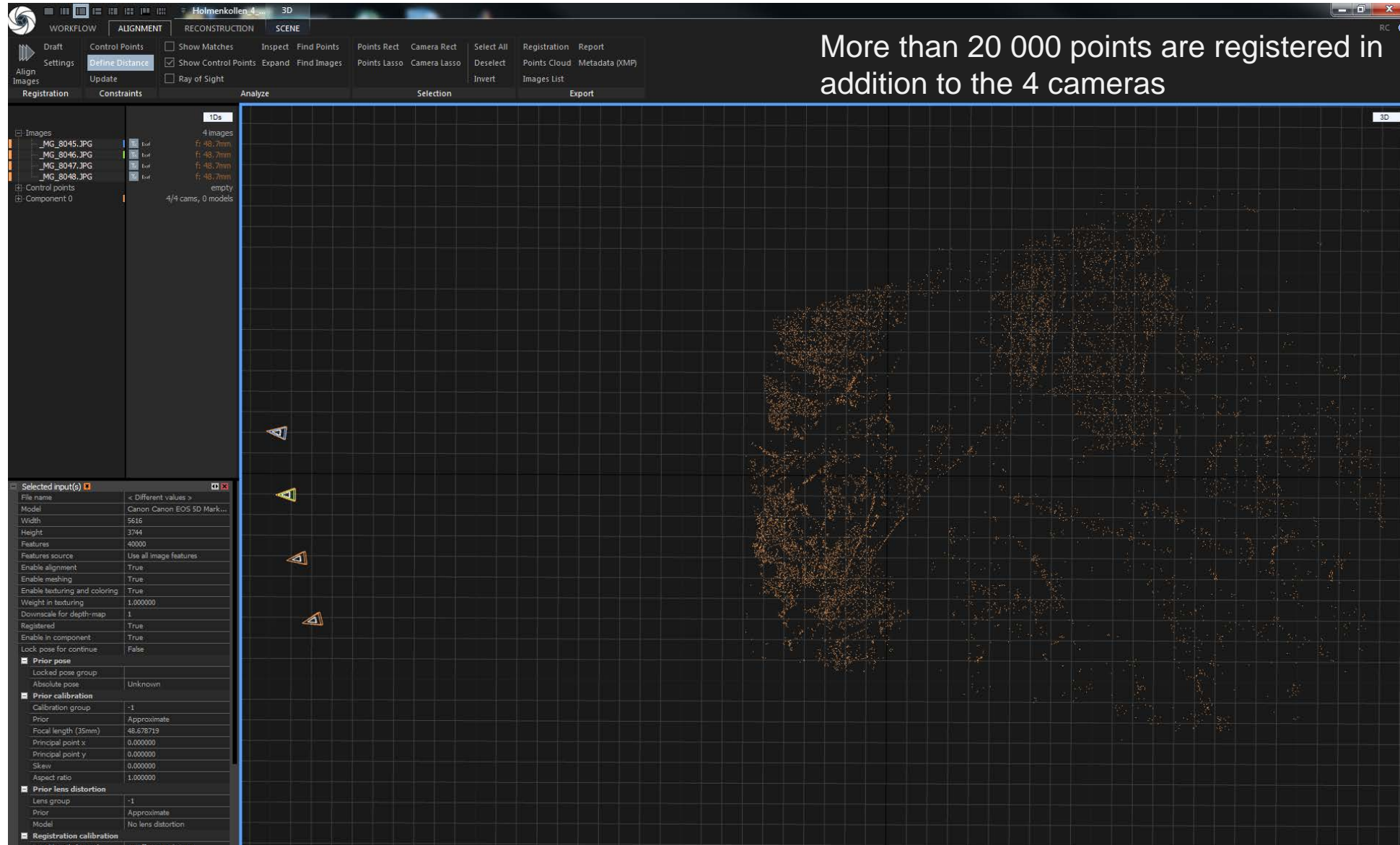
More than 16 000 points are registered in addition to the 3 cameras

The screenshot displays the Agisoft Metashape software interface for a 3-view SfM project. The top toolbar shows the workflow from Draft to SCENE. The left sidebar lists three images: _MG_8045.JPG, _MG_8046.JPG, and _MG_8047.JPG, each with a focal length of 48.7mm and 40000 points. Below the images, it shows 0 control points and 3 cameras. The bottom-left panel shows the 'Selected input' settings for a Canon Canon EOS 5D Mark II camera, including focal length (48.678719mm), principal points, and lens distortion. The main workspace is divided into three views: two 2D image views showing different perspectives of the Holmenkollen stadium and one 3D view showing a point cloud of the stadium structure.

File name	< Different values >
Model	Canon Canon EOS 5D Mark II
Width	5616
Height	3744
Features	40000
Features source	Use all image features
Enable alignment	True
Enable meshing	True
Enable texturing and coloring	True
Weight in texturing	1.000000
Downscale for depth-map	1
Registered	True
Enable in component	True
Lock pose for continue	False
Prior pose	
Locked pose group	
Absolute pose	Unknown
Prior calibration	
Calibration group	-1
Prior	Approximate
Focal length (35mm)	48.678719
Principal point x	0.000000
Principal point y	0.000000
Skew	0.000000
Aspect ratio	1.000000
Prior lens distortion	
Lens group	-1
Prior	Approximate
Model	No lens distortion
Registration calibration	
Focal length (35mm)	< Different values >

Example

Holmenkollen 4-view SfM



Example

Holmenkollen 110-view SfM

The screenshot displays the Agisoft Metashape software interface. The main window shows a 3D point cloud reconstruction of a helmet, rendered in orange and yellow. The interface includes a menu bar at the top with options like WORKFLOW, ALIGNMENT, RECONSTRUCTION, and SCENE. Below the menu bar is a toolbar with various tools. On the left side, there is a list of 110 images, each with a thumbnail and a focal length of 48.7mm. At the bottom left, a settings panel for the selected input is visible, showing various parameters such as File name, Model (Canon Canon EOS 5D Mark...), Width (5616), Height (3744), Features (40000), and various calibration and registration options.

~470 000 points are registered in addition to the 110 cameras

Example

Holmenkollen 110-view SfM

Maximal reprojection error ~3 pixels
Mean reprojection error ~0.7 pixels

File name	Model	Width	Height	Features	Features so...	Enable align...	Enable mes...	Enable text...	Weight in L...	Downscale ...	Registered	Enable in c...	Lock pose ...	Prior pose	Locked...	Absolut...	Prior calibration	Calibrat...	Prior	Focal le...	Princip...	Princip...	Skew	Aspect ...	Prior lens distortion	Lens gr...	Prior	Model	Registration calibrat...
< Different...	Canon Ca...	5616	3744	40000	Use all ima...	True	True	True	1.000000	1	True	True	False			Unknown		-1	Approximate	48.678719	0.000000	0.000000	0.000000	1.000000		Approximate	No lens dis...		

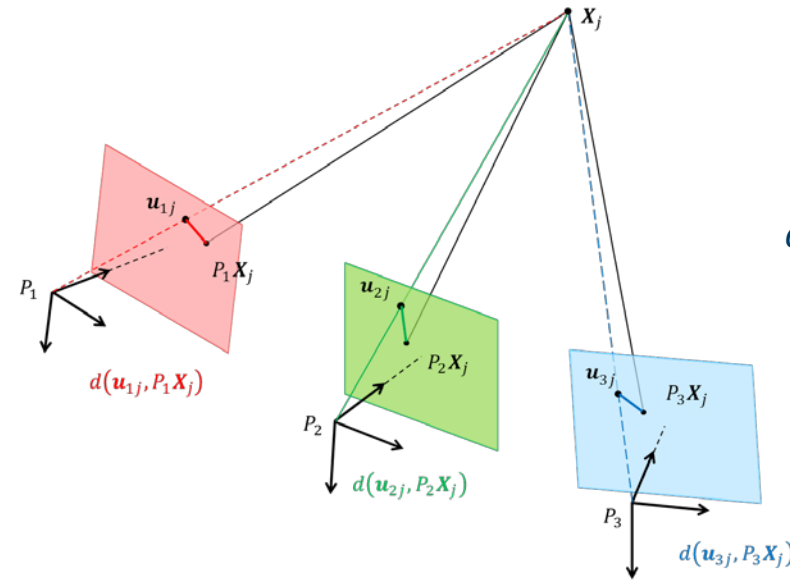
Name	Component...	Reconstruc...	Cameras c...	Points count	Control p...	Constraints
Component...	[C6C66918...	[889C29FE...	110	469618	0	0

Alignment report	
Total pr...	1740241
Averag...	3.706504
Maxima...	2.999642
Median ...	0.526006
Mean e...	0.706922
Georef...	false
Metric	false
Alignm...	1131

Alignment settings	
Alignm...	RealityCapt...
Alignm...	High
Max fea...	10000
Max fea...	40000
Detecto...	High
Presele...	10000
Image ...	1
Max fea...	3.000000
Use ca...	false
Lens dis...	Brown3
Final o...	true

Summary

- Structure from motion
 - Sequential SfM
 - Bundle adjustment
- Additional reading:
 - Szeliski: 7.3-7.5
- Optional reading:
 - Snavely N. Seitz S. M., Szeliski R., *Modeling the World from Internet Photo Collections*, 2007
 - S. Agarwal et al, *Building Rome in a Day*, 2011
 - J. Heinly et al, *Reconstructing the World in Six Days*, 2015



$$\epsilon = \sum_{i=1}^m \sum_{j=1}^n d(\tilde{\mathbf{u}}_{ij}, P_i \tilde{\mathbf{X}}_j)^2$$

