UiO : Department of Technology Systems
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Estimating pose by minimizing reprojection error

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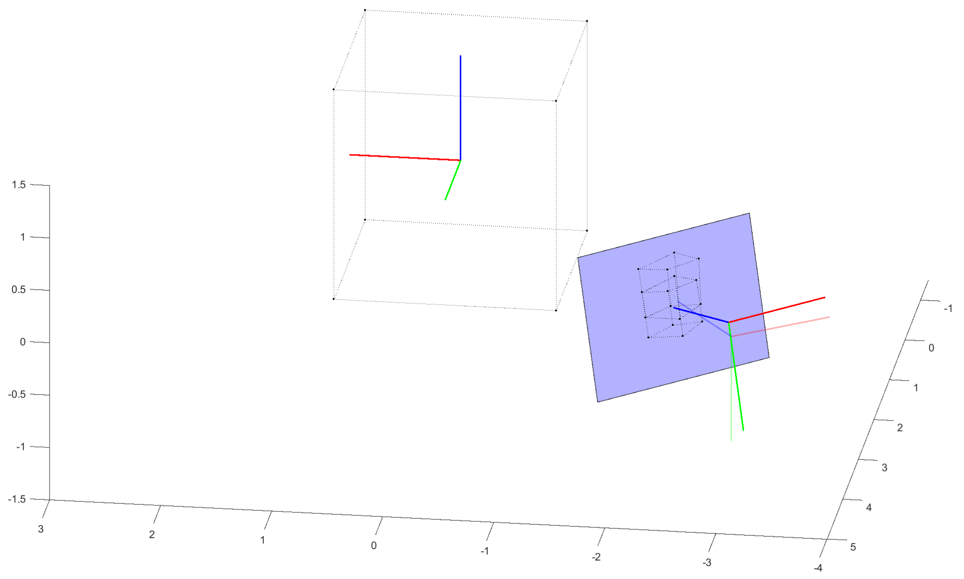


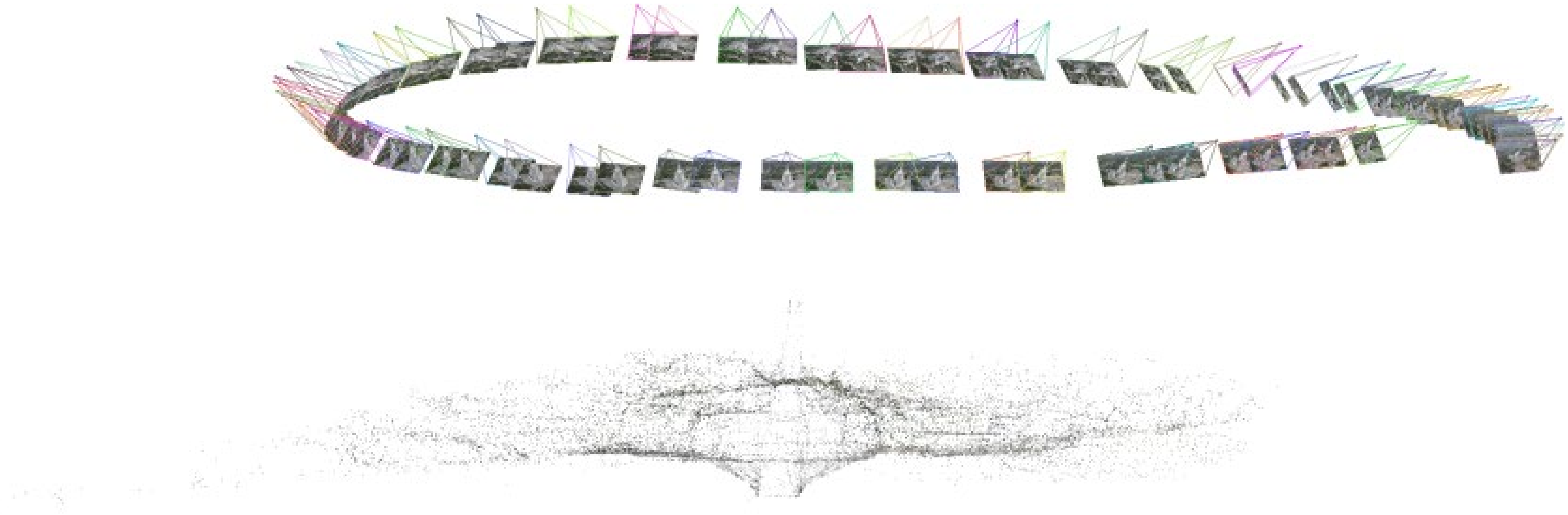
TEK5030

Bundle adjustment

Bundle Adjustment (BA)

Estimating the imaging geometry
based on minimizing reprojection error

- Motion-only BA
- Structure-only BA
- Full BA

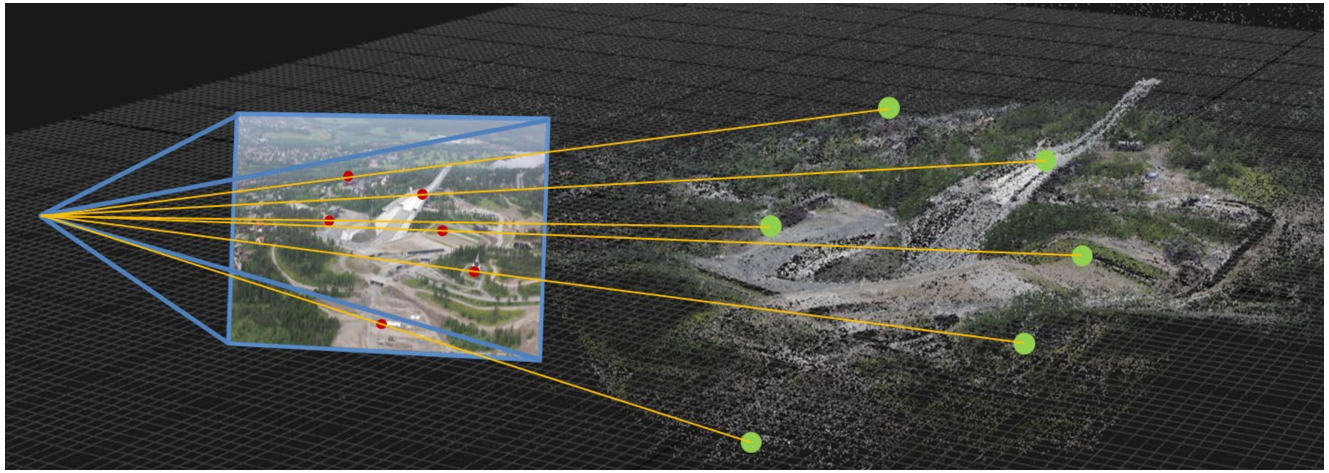


Pose estimation by minimizing reprojection error

Minimize **geometric error** over the **camera pose** given **known structure**

This is also sometimes called **Motion-Only Bundle Adjustment**

$$\mathbf{T}_{wc}^{*} = \underset{\mathbf{T}_{wc}}{\operatorname{argmin}} \sum_{j} \left\| \pi(\mathbf{T}_{wc}^{-1} \cdot \mathbf{x}_{j}^{w}) - \mathbf{u}_{j} \right\|^{2}$$



Pose estimation by minimizing reprojection error

Given:

- World points $\mathbf{x}_j^w$

Measurements:

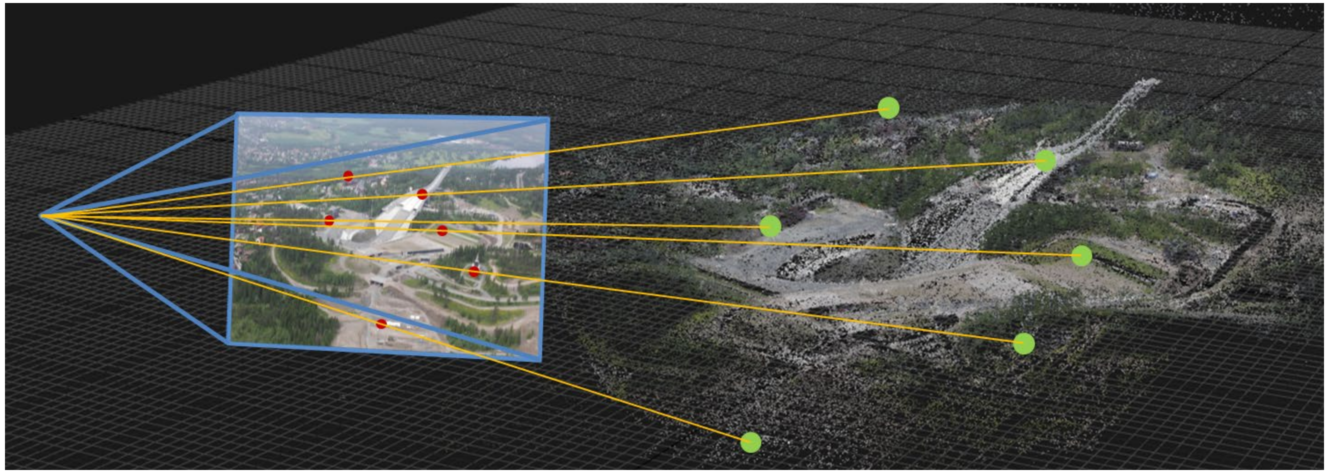
- Correspondences $\mathbf{u}_j \leftrightarrow \mathbf{x}_j^w$ with measurement noise $\mathbf{\Sigma}_j$

State we wish to estimate:

- Camera pose $\mathbf{T}_{wc}$

Initial estimate:

- Motion model
- Pose from homography
- PnP (P3P, EPnP, …)

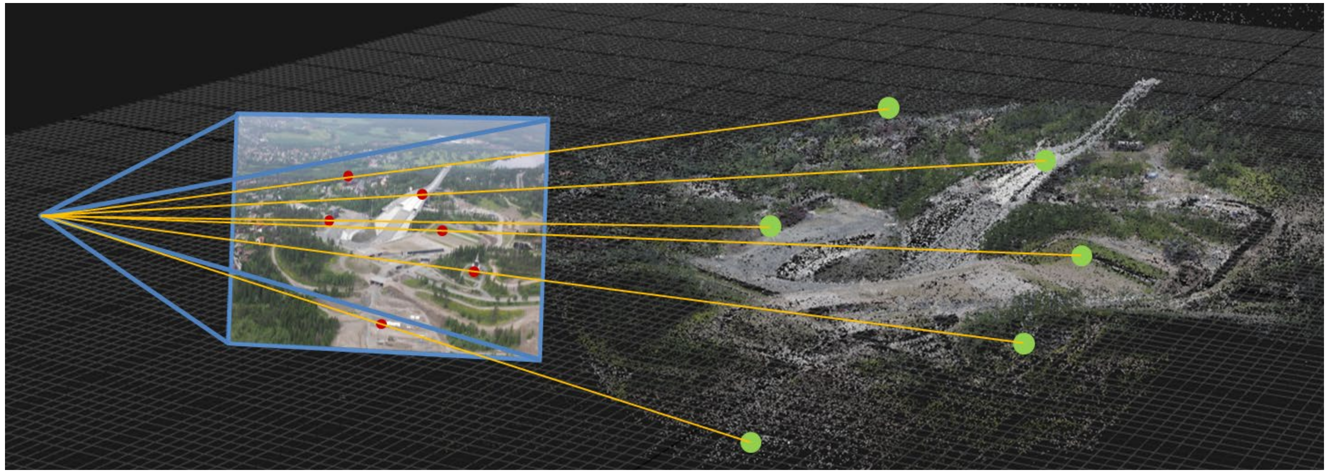


n-Point Pose Problem (P*n*P)

- Typically fast non-iterative methods
- Minimal in number of points, perfect for RANSAC!
- Accuracy can be comparable to iterative methods
- Good as initial estimates

Examples

- P3P, EPnP
- P4Pf
  - Estimate pose and focal length
- P6P
  - Estimates **P** with DLT
- R6P
  - Estimate pose with rolling shutter



Applying the MAP framework

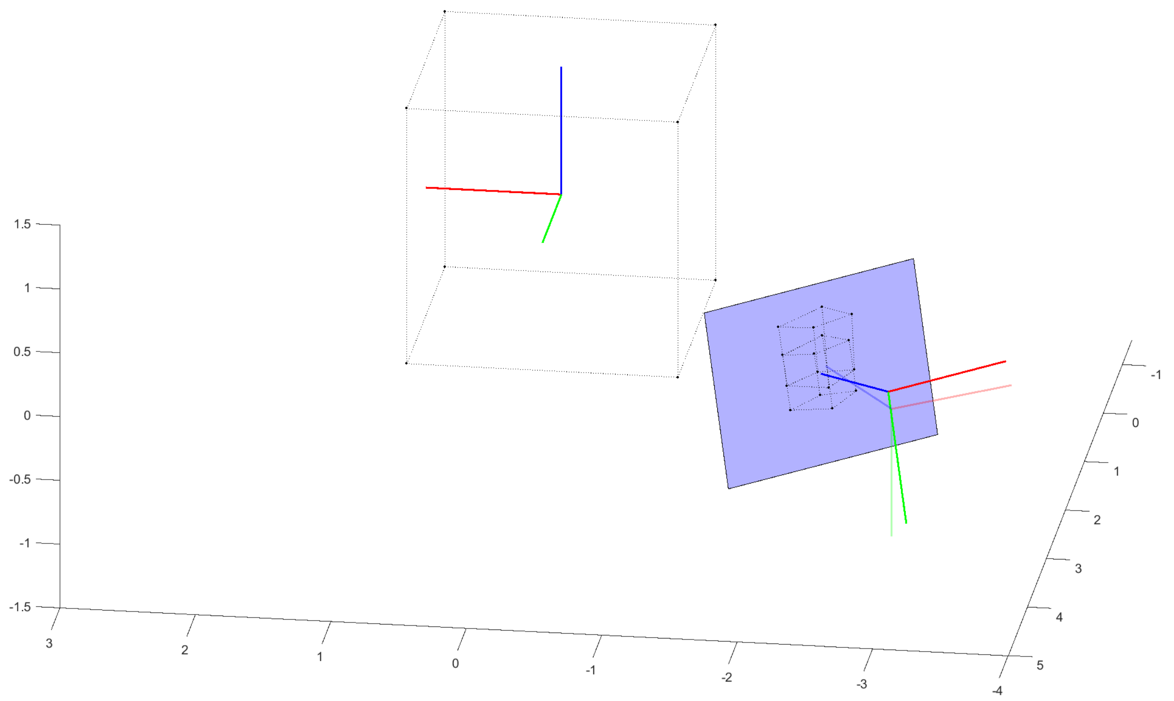
For simplicity,
we pre-calibrate to normalized image coordinates (and propagate the noise)

This gives us the measurement prediction function

$$h_j(\mathbf{T}_{wc}) = \pi_n(\mathbf{T}_{wc}^{-1} \cdot \mathbf{x}_j^w)$$

and measurement error function

$$e_j(\mathbf{T}_{wc}) = \pi_n(\mathbf{T}_{wc}^{-1} \cdot \mathbf{x}_j^w) - \mathbf{x}_{n\,j}$$



Applying the MAP framework

$$\mathbf{x}_n = \pi_n(\mathbf{x}^c) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \frac{1}{z^c} \mathbf{x}^c = \begin{bmatrix} x^c / z^c \\ y^c / z^c \end{bmatrix}$$

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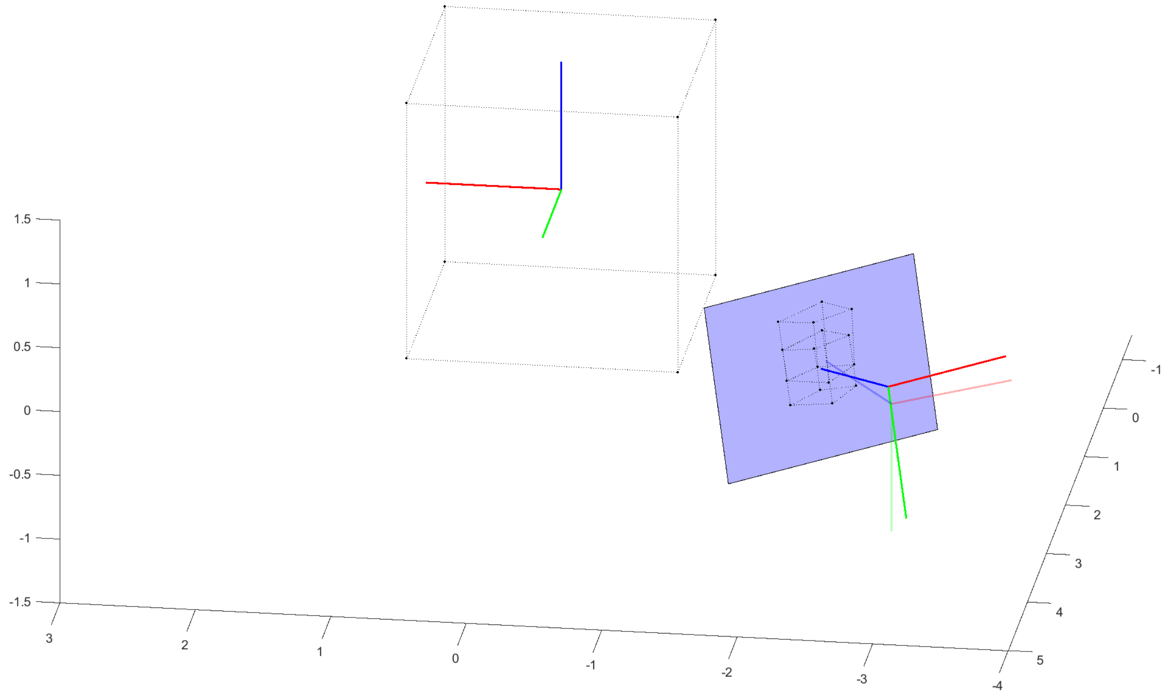

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The measurement Jacobian is given by

$$\begin{aligned}
\mathbf{J}^h_{\mathbf{T}_{wc}} &= \mathbf{J}^{\pi_n(\mathbf{T}_{wc}^{-1}\cdot\mathbf{x}^w)}_{\mathbf{T}_{wc}^{-1}\cdot\mathbf{x}^w} \mathbf{J}^{\mathbf{T}_{wc}^{-1}\cdot\mathbf{x}^w}_{\mathbf{T}_{wc}^{-1}} \mathbf{J}^{\mathbf{T}_{wc}^{-1}}_{\mathbf{T}_{wc}} \\
&= \mathbf{J}^{\pi_n(\mathbf{x}^c)}_{\mathbf{x}^c} \mathbf{J}^{\mathbf{T}_{wc}^{-1}\cdot\mathbf{x}^w}_{\mathbf{T}_{wc}^{-1}} \mathbf{J}^{\mathbf{T}_{wc}^{-1}}_{\mathbf{T}_{wc}} \\
&= \frac{1}{z^c}\begin{bmatrix} 1 & 0 & -x^c/z^c \\ 0 & 1 & -y^c/z^c \end{bmatrix} \begin{bmatrix} \mathbf{R}_{wc}^\top & -\mathbf{R}_{wc}^\top[\mathbf{x}^w]_\times \end{bmatrix} \cdot - \begin{bmatrix} \mathbf{R}_{wc} & [\mathbf{t}^w_{wc}]_\times \mathbf{R}_{wc} \\ \mathbf{0} & \mathbf{R}_{wc} \end{bmatrix} \\
&= d \begin{bmatrix} 1 & 0 & -x_n \\ 0 & 1 & -y_n \end{bmatrix} \begin{bmatrix} -\mathbf{I} & [\mathbf{x}^c]_\times \end{bmatrix} \\
&= \begin{bmatrix} -d & 0 & dx_n & x_n y_n & -1 - x_n^2 & y_n \\ 0 & -d & dy_n & 1 + y_n^2 & -x_n y_n & -x_n \end{bmatrix},
\end{aligned}$$

Applying the MAP framework

This results in the linearized weighted least squares problem

$$\begin{aligned}\boldsymbol{\xi}^* &= \arg\min_{\boldsymbol{\xi}} \sum_{j=1}^{n} \left\| \mathbf{A}_j \boldsymbol{\xi} - \mathbf{b}_j \right\|^2 \\ &= \arg\min_{\boldsymbol{\xi}} \left\| \mathbf{A} \boldsymbol{\xi} - \mathbf{b} \right\|^2 ,\end{aligned}$$

where

$$\begin{aligned}\mathbf{A}_j &= \boldsymbol{\Sigma}_{n\,j}^{-1/2} \mathbf{J}_{\mathbf{T}_{wc}}^{h_j} \\ \mathbf{b}_j &= \boldsymbol{\Sigma}_{n\,j}^{-1/2} (\mathbf{x}_{n\,j} - h_j(\mathbf{T}_{wc})),\end{aligned} \qquad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_n \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_n \end{bmatrix} .$$

Applying the MAP framework

For an example with *three points*,
the measurement Jacobian $\mathbf{A}$ and the prediction error $\mathbf{b}$ are

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \end{bmatrix}$$

Applying the MAP framework

The solution can be found by solving the normal equations

$$\left(\mathbf{A}^T\mathbf{A}\right)\boldsymbol{\xi}^* = \mathbf{A}^T\mathbf{b}$$

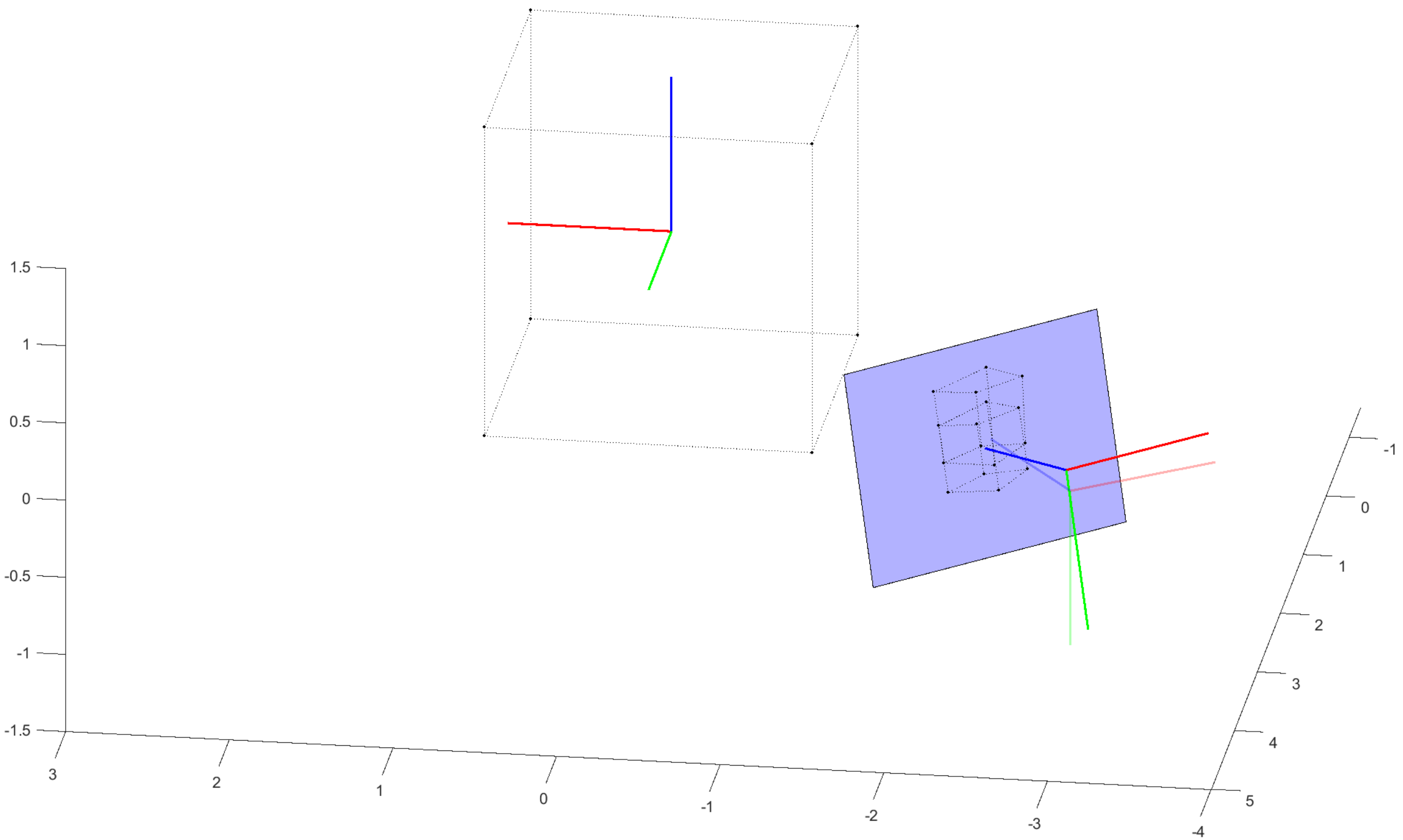
Choose a suitable inital estimate $\underline{\hat{\mathcal{X}}}^0$

$$\mathbf{A}, \mathbf{b} \leftarrow \text{Linearize at } \underline{\hat{\mathcal{X}}}^t$$

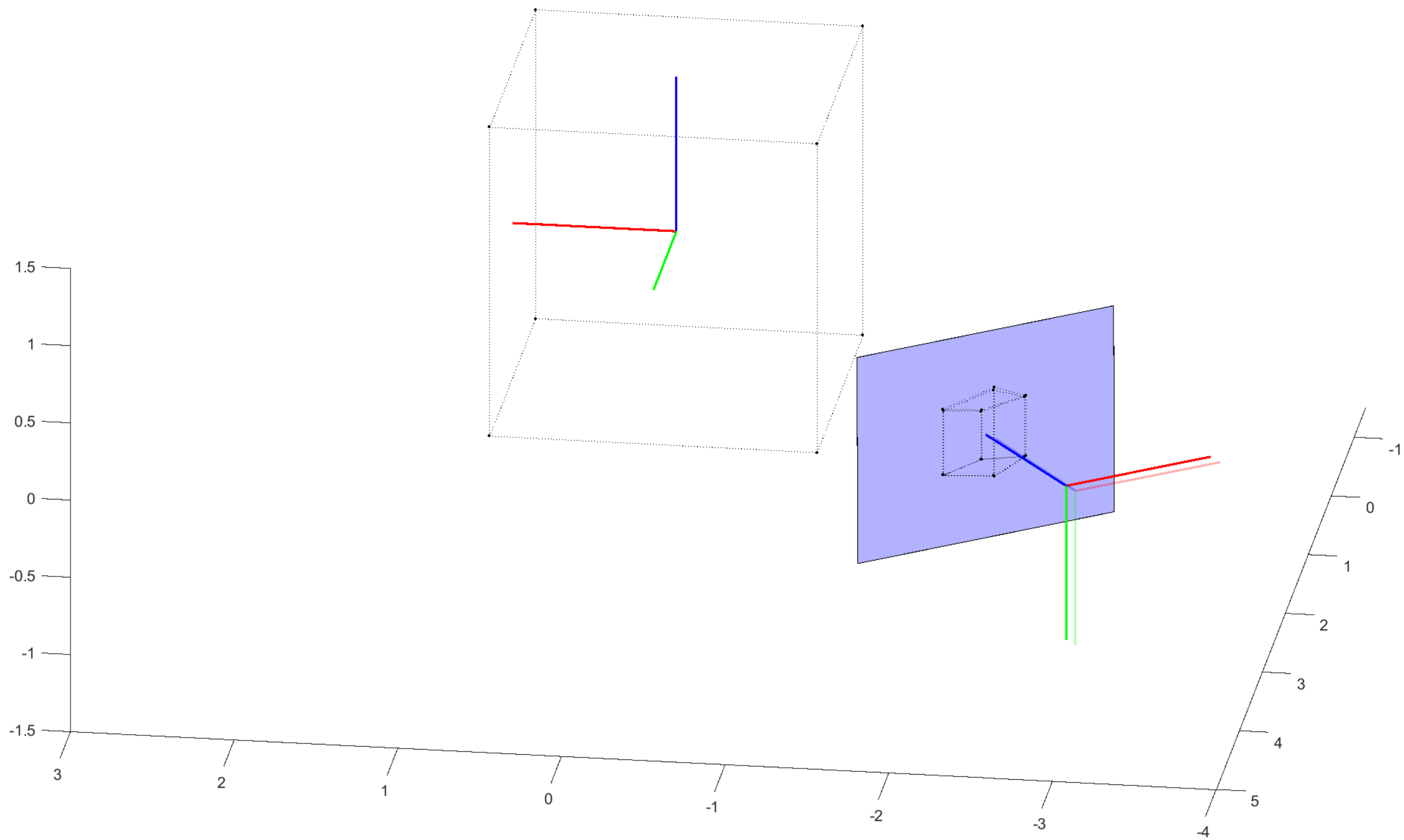
$$\underline{\boldsymbol{\tau}}^* \leftarrow \mathit{S}\text{olve } \underset{\underline{\boldsymbol{\tau}}}{\operatorname{argmin}} \left\| \mathbf{A}\underline{\boldsymbol{\tau}} - \mathbf{b} \right\|^2$$

$$\underline{\hat{\mathcal{X}}}^{t+1} \leftarrow \underline{\hat{\mathcal{X}}}^t \oplus \underline{\boldsymbol{\tau}}^*$$

Example



Example



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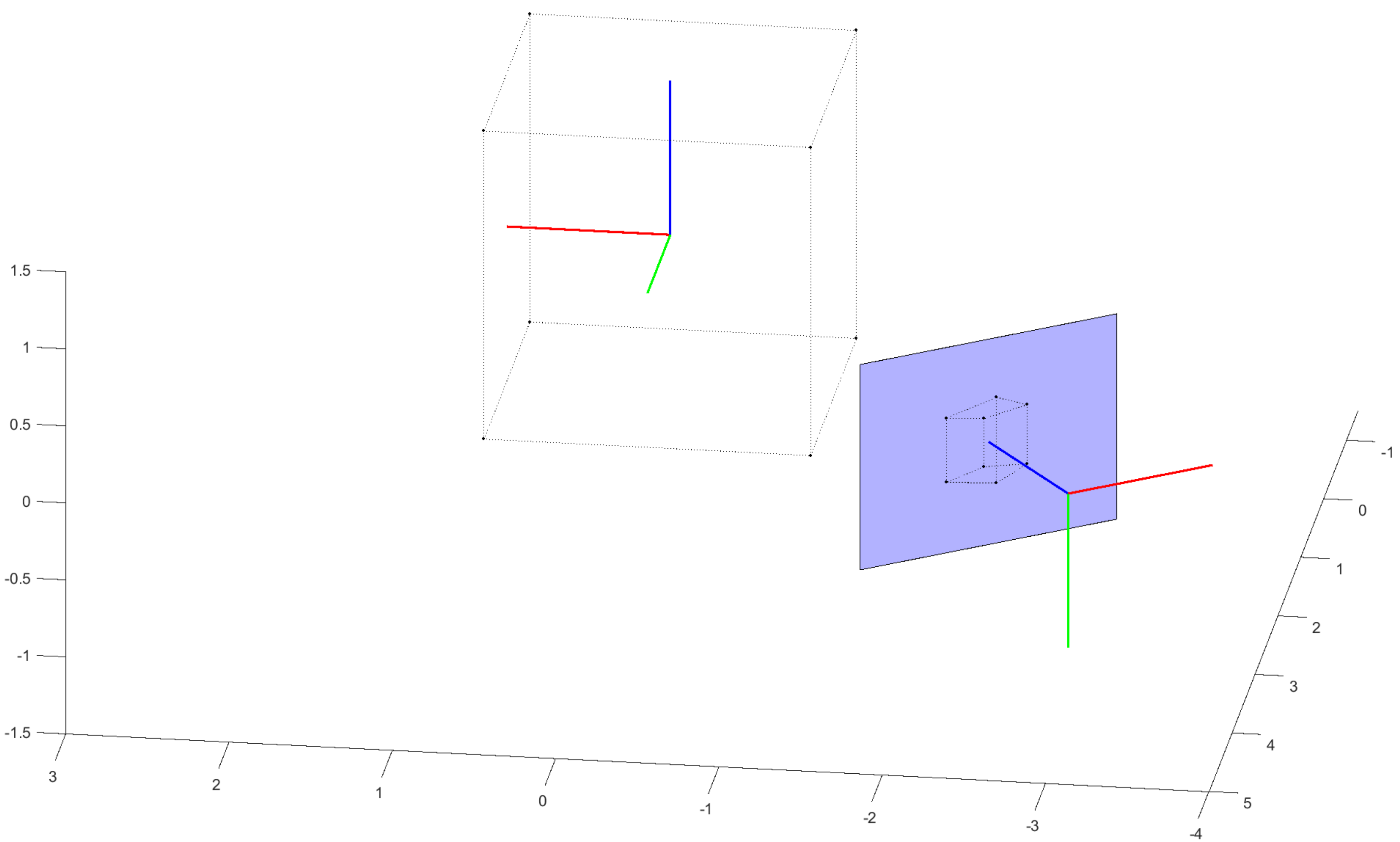

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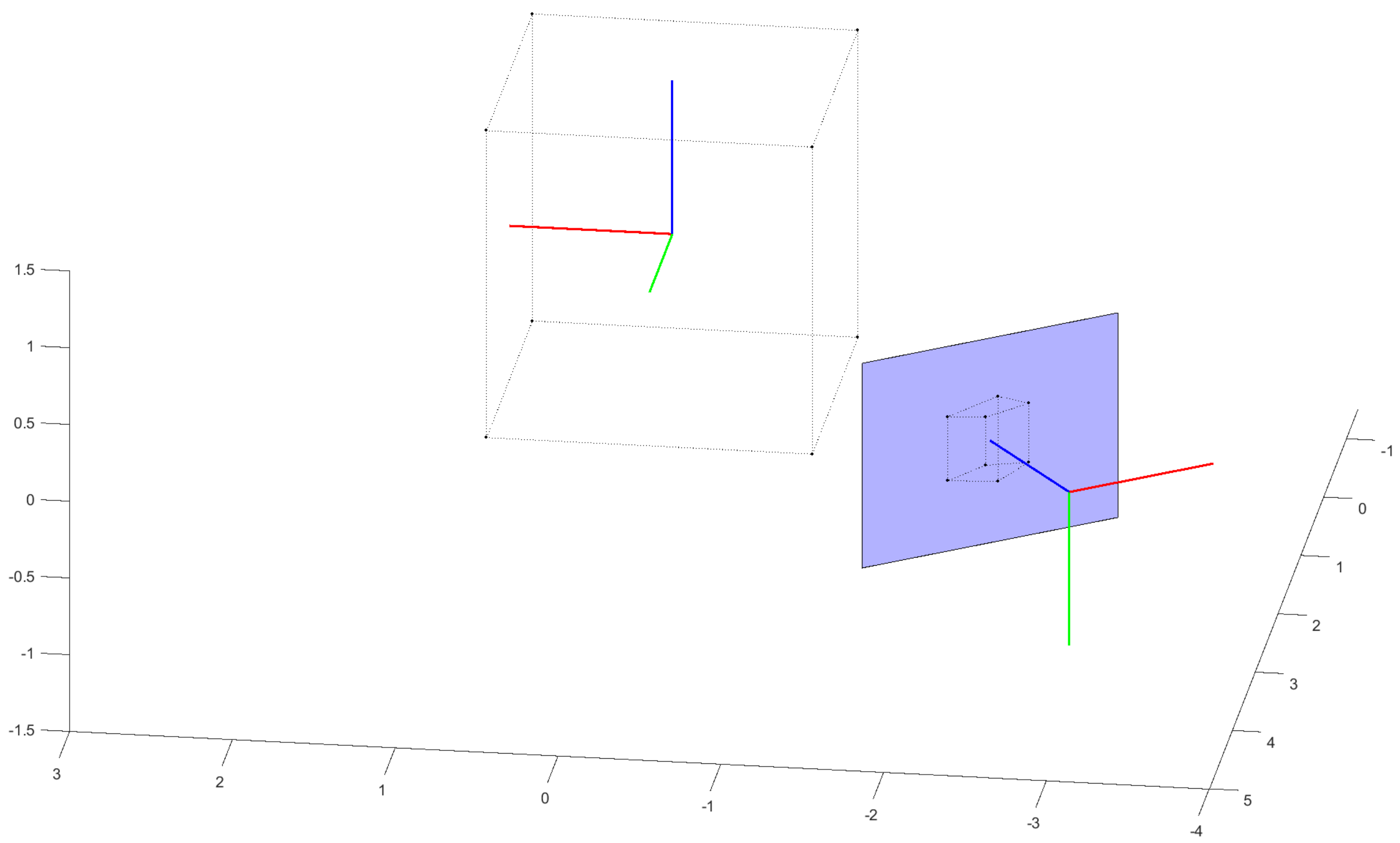
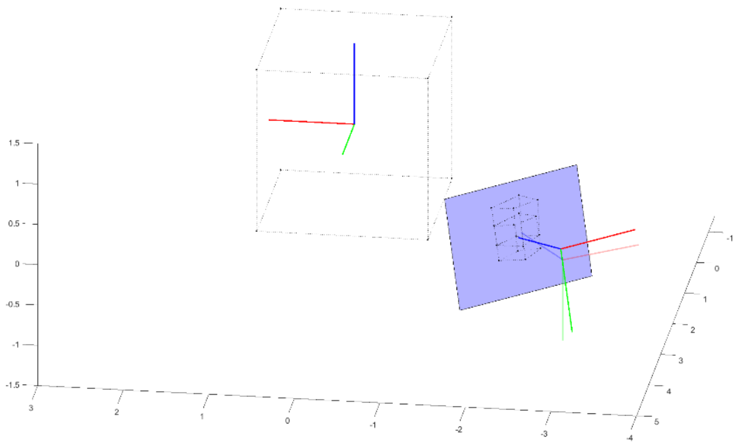


Summary

- Nonlinear MAP estimation of states such as poses

- Pose estimation relative to known 3D points
  - Pose from homography
  - PnP
  - Minimising reprojection error



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$$\underline{\boldsymbol{\tau}}^* \leftarrow \text{Solve } \underset{\underline{\boldsymbol{\tau}}}{\operatorname{argmin}} \left\| \mathbf{A}\underline{\boldsymbol{\tau}} - \mathbf{b} \right\|^2$$

$$\underline{\hat{\mathcal{X}}}^{t+1} \leftarrow \underline{\hat{\mathcal{X}}}^t \oplus \underline{\boldsymbol{\tau}}^*$$

Supplementary material

- The compendium!
  - Let me know if you would like to go through the derivations in greater detail!

- Python implementation of the bundle adjustment examples:
  - https://github.com/ttk21/lab_05