UiO : Department of Technology Systems
University of Oslo

The perspective camera model revisited

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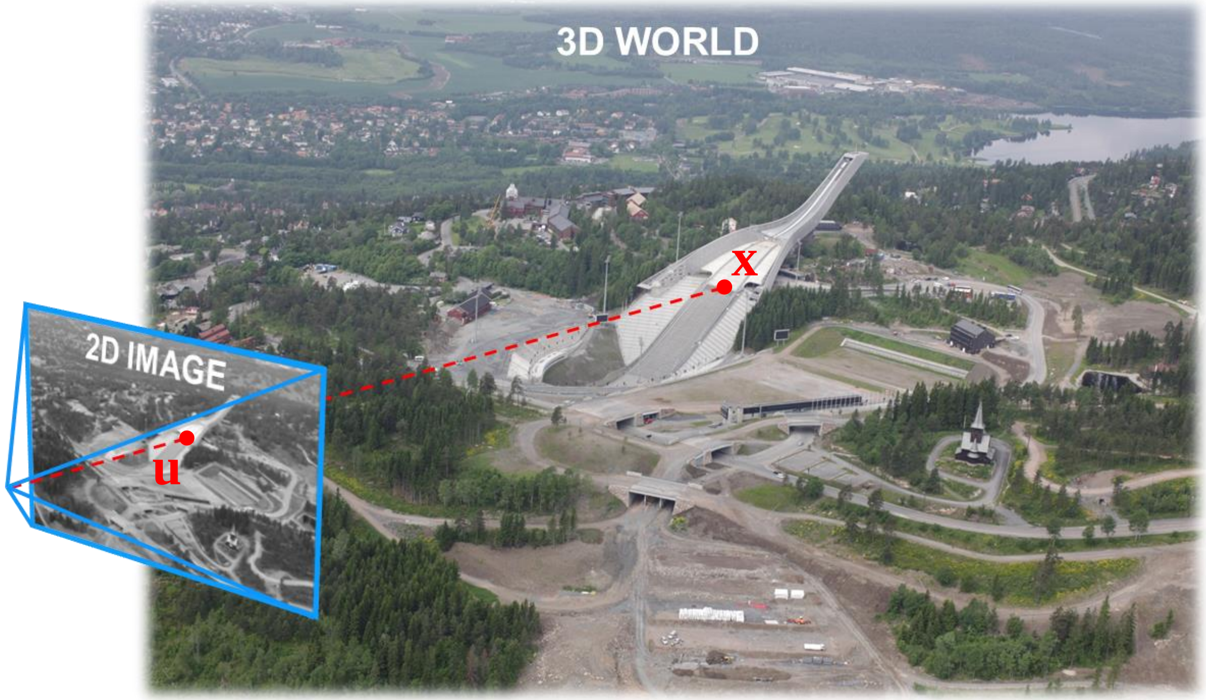
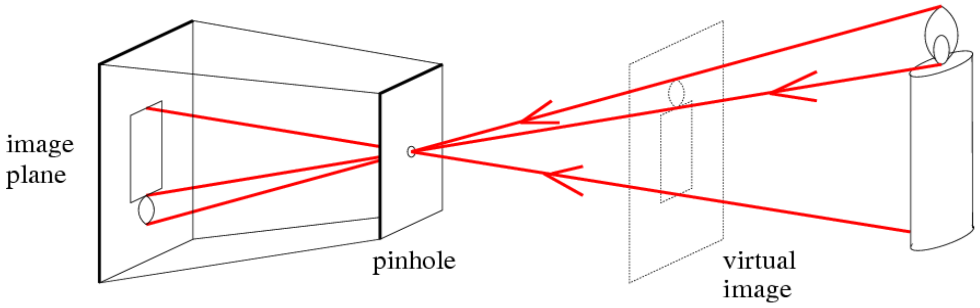
The perspective camera model

A mathematical model that describes the viewing geometry of pinhole cameras

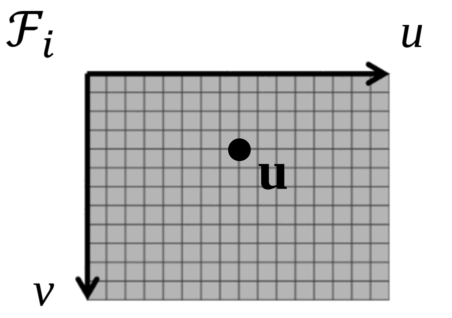
It describes how the perspective projection maps 3D points in the world to 2D points in the image

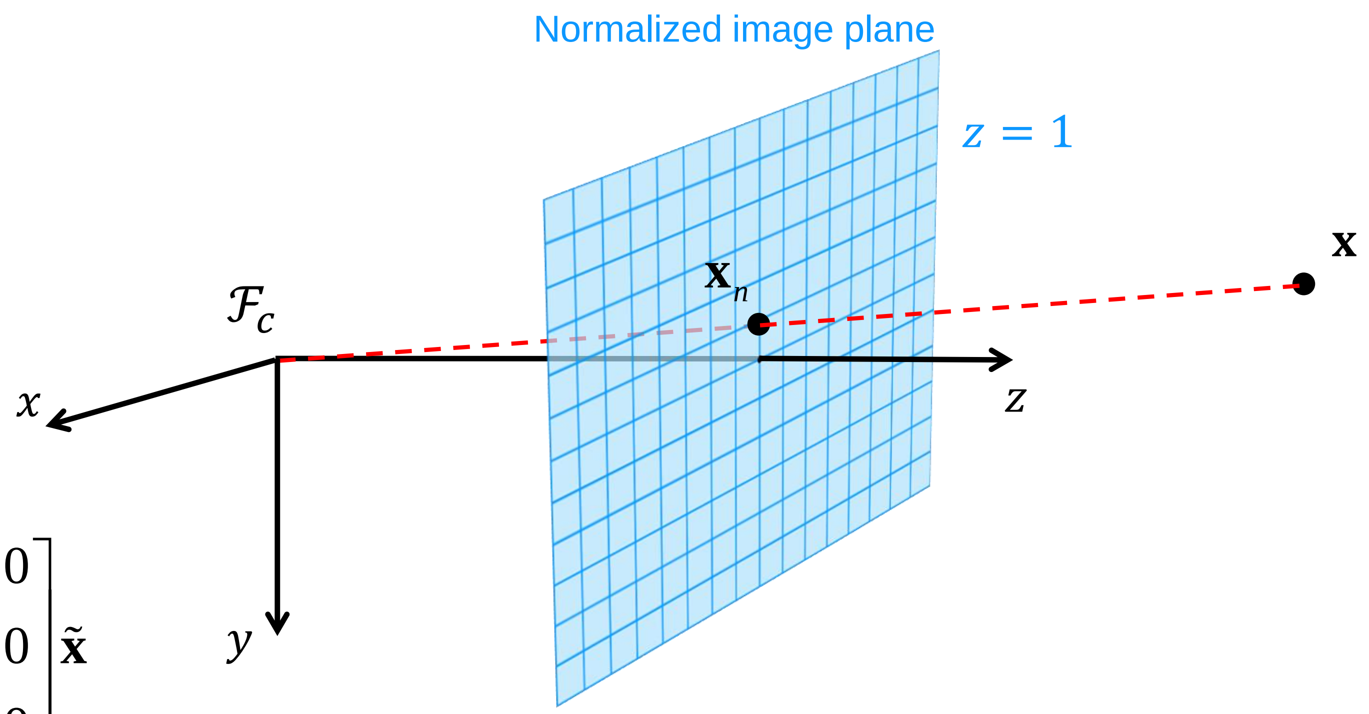
Combined with a distortion model, the perspective camera model can describe the viewing geometry of most cameras





The perspective camera model



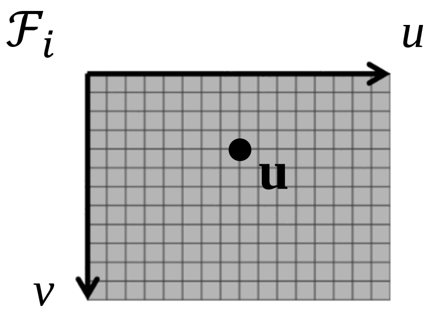


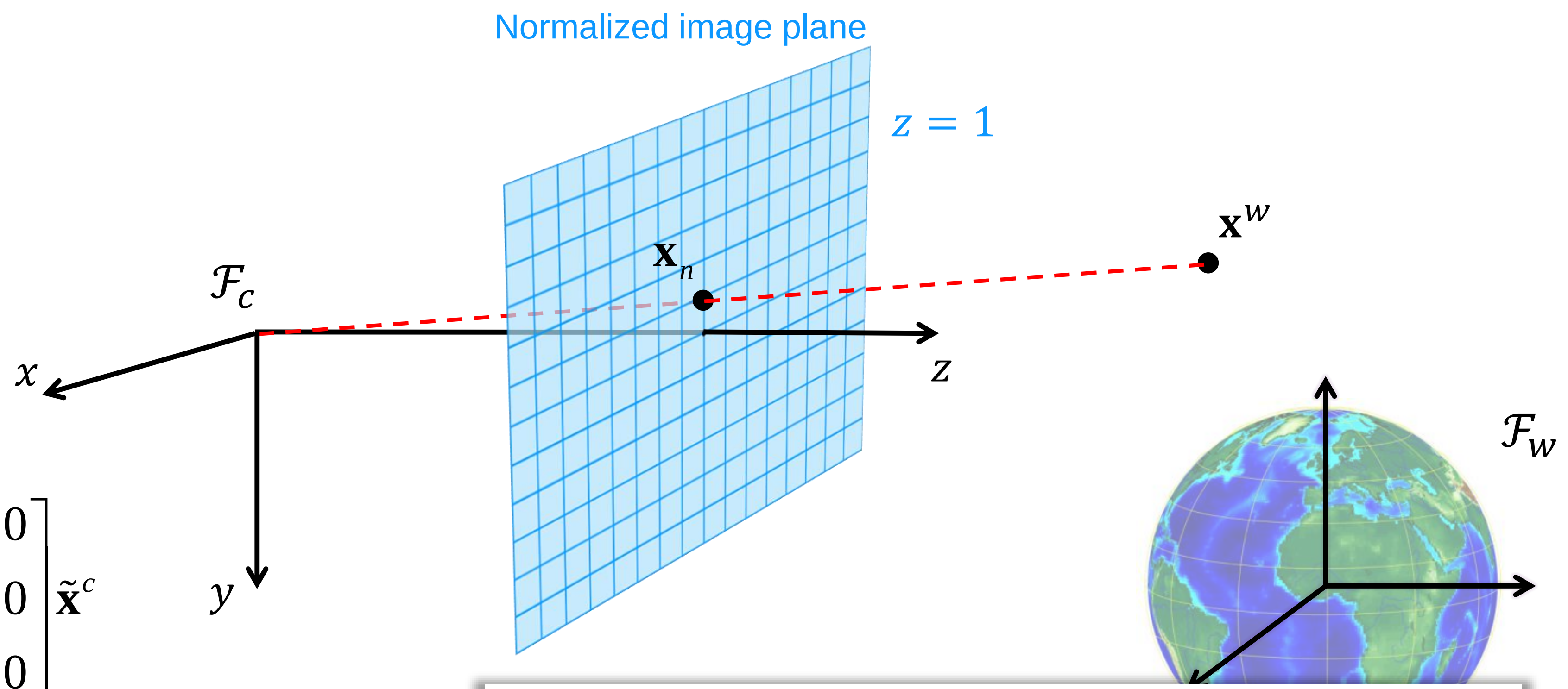
$$\tilde{\mathbf{u}} = \underbrace{\begin{bmatrix} f_u & 0 & c_u \\ 0 & f_v & c_v \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{\Pi}_0} \tilde{\mathbf{x}}$$

$\tilde{\mathbf{u}} \leftarrow \tilde{\mathbf{x}}_n$

$\tilde{\mathbf{x}}_n \leftarrow \tilde{\mathbf{x}}$

The perspective camera model





$$\tilde{\mathbf{u}} = \begin{bmatrix} f_u & 0 & c_u \\ 0 & f_v & c_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tilde{\mathbf{x}}^c$$

$\mathbf{K}$ $\quad$ $\mathbf{\Pi}_0$

$\tilde{\mathbf{u}} \leftarrow \tilde{\mathbf{x}}_n$ $\quad$ $\tilde{\mathbf{x}}_n \leftarrow \tilde{\mathbf{x}}^c$

In the current description, the world point is given relative to the camera frame $\mathcal{F}_c$

What changes if the world point is given relative to another coordinate frame $\mathcal{F}_w$?

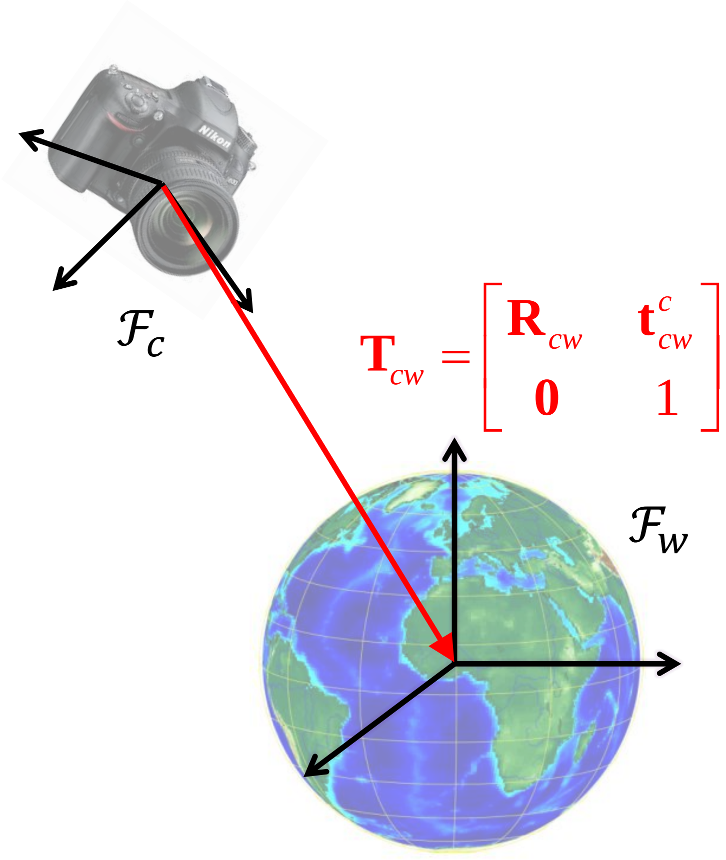
The perspective camera model

- The pose of the world frame relative to the camera frame, denoted by $\mathbf{T}_{cw}$, is also a point transformation from $\mathcal{F}_w$ to $\mathcal{F}_c$

- General perspective camera model

$$\tilde{\mathbf{u}} = \underbrace{\begin{bmatrix} f_u & s & c_u \\ 0 & f_v & c_v \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{\Pi}_0} \underbrace{\begin{bmatrix} \mathbf{R}_{cw} & \mathbf{t}_{cw}^c \\ \mathbf{0} & 1 \end{bmatrix}}_{\mathbf{T}_{cw}} \tilde{\mathbf{x}}^w$$

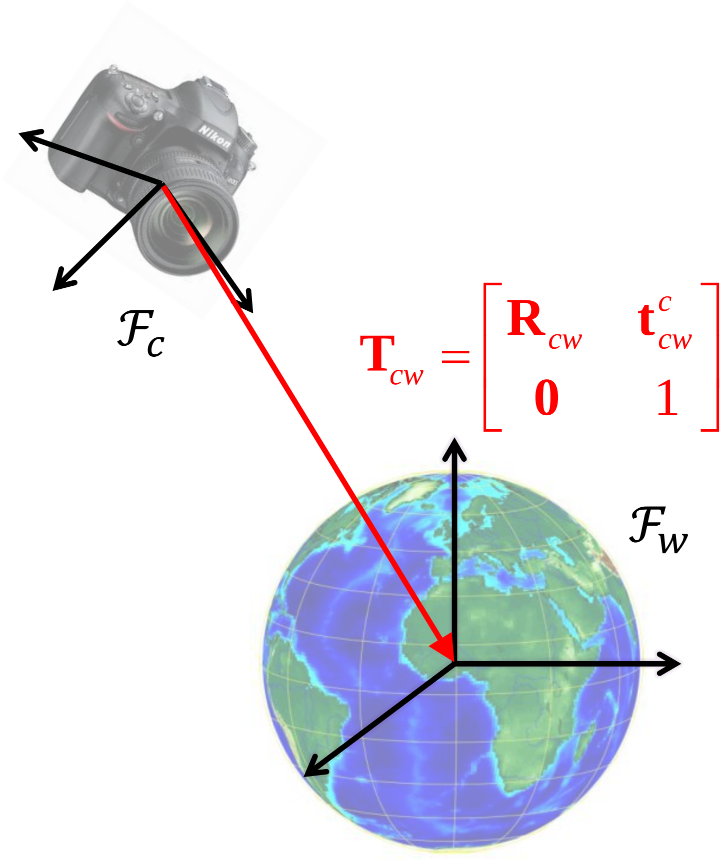
$\tilde{\mathbf{u}} \leftarrow \tilde{\mathbf{x}}_n$	$\tilde{\mathbf{x}}_n \leftarrow \tilde{\mathbf{x}}^c$	$\tilde{\mathbf{x}}^c \leftarrow \tilde{\mathbf{x}}^w$



The perspective camera model

- By multiplying $\mathbf{\Pi}_0$ with $\mathbf{T}_{cw}$ we get a very compact expression that is commonly used to represent the perspective camera model

$$\tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R}_{cw} & \mathbf{t}_{cw}^{c}\end{bmatrix}\tilde{\mathbf{x}}^{w}$$



The perspective camera model

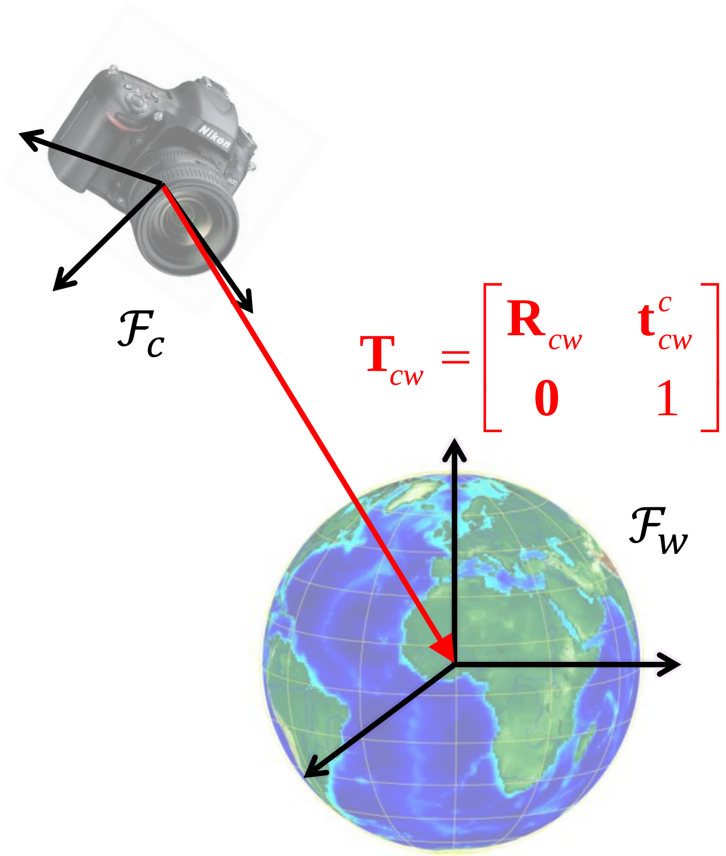
- By multiplying $\mathbf{\Pi}_0$ with $\mathbf{T}_{cw}$ we get a very compact expression that is commonly used to represent the perspective camera model

$$\tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R}_{cw} & \mathbf{t}_{cw}^{c}\end{bmatrix}\tilde{\mathbf{x}}^{w}$$

- We refer to $\mathbf{K}$ as the **intrinsic** part and $[\mathbf{R}_{\mathrm{cw}} \quad \mathbf{t}_{\mathrm{cw}}^{\mathrm{c}}]$ as the **extrinsic** part of the perspective camera model

- The matrix $\mathbf{K}[\mathbf{R}_{\mathrm{cw}} \quad \mathbf{t}_{\mathrm{cw}}^{\mathrm{c}}]$ is often denoted by $\mathbf{P}$ and referred to as the camera's projection matrix

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R}_{cw} & \mathbf{t}_{cw}^{c}\end{bmatrix}$$



The perspective camera model

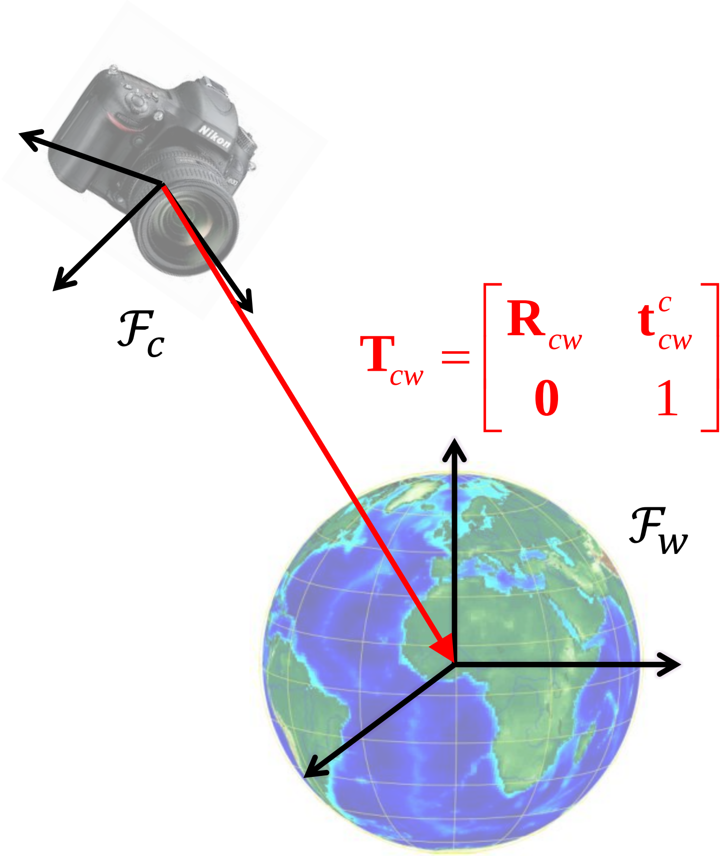
- Note that $\mathbf{R}_{cw}$ and $\mathbf{t}_{cw}^{c}$ are the orientation and position of the world relative to the camera

- Alternative formulation

$$\tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix} \mathbf{R}_{wc}^{T} & -\mathbf{R}_{wc}^{T}\mathbf{t}_{wc}^{w} \end{bmatrix}\tilde{\mathbf{x}}^{w}$$

where we have used that

$$\mathbf{T}_{cw} = \mathbf{T}_{wc}^{-1} = \begin{bmatrix} \mathbf{R}_{wc}^{T} & -\mathbf{R}_{wc}^{T}\mathbf{t}_{wc}^{w} \\ \mathbf{0} & 1 \end{bmatrix}$$



Example

- This image was captured from a platform with a good onboard navigation system
- So we know the camera's position and orientation when the image was taken
- Based on what we now know about the perspective camera model and pose, we can
  - Project points in the scene into the image

Example

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- So we know the camera's position and orientation when the image was taken
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North direction

A terrain model projected into the image

Example

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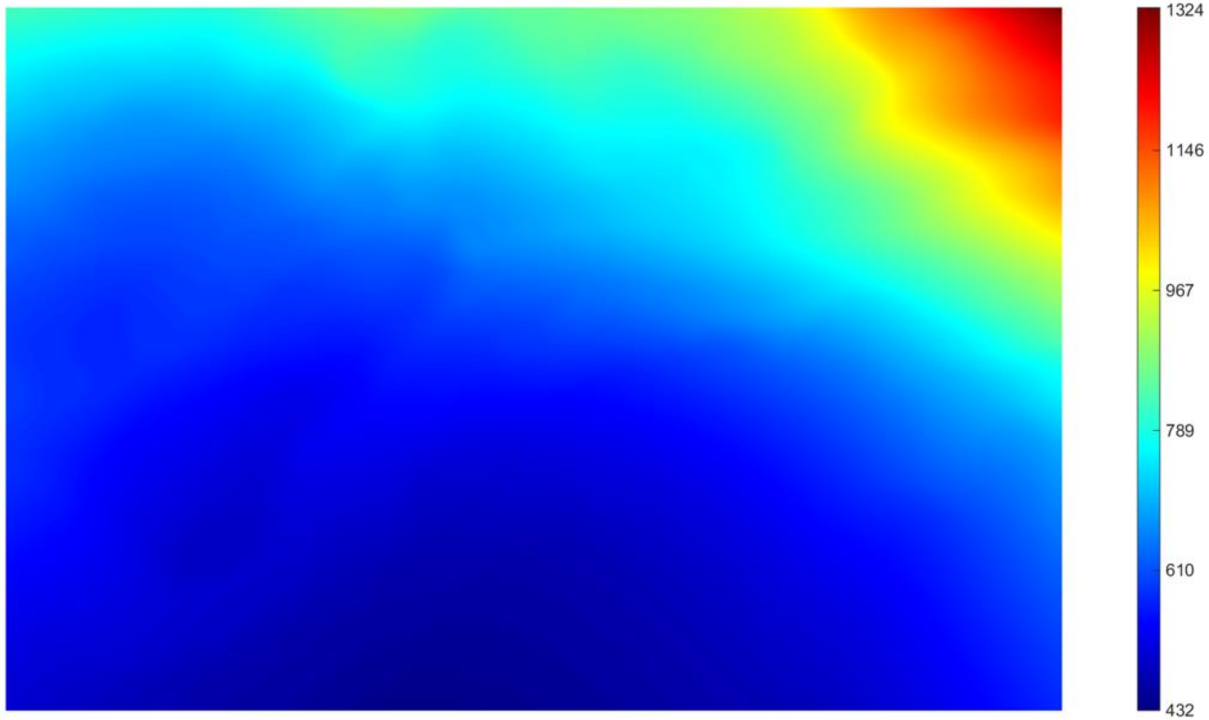
Roads, railroad and a stream from a vector map

Example

- This image was captured from a platform with a good onboard navigation system

- So we know the camera's position and orientation when the image was taken

- Based on what we now know about the perspective camera model and pose, we can
  - Project points in the scene into the image
  - Backproject the image to the scene



Distance from the camera to the terrain model

Example

- This image was captured from a platform with a good onboard navigation system
- So we know the camera's position and orientation when the image was taken
- Based on what we now know about the perspective camera model and pose, we can
  - Project points in the scene into the image
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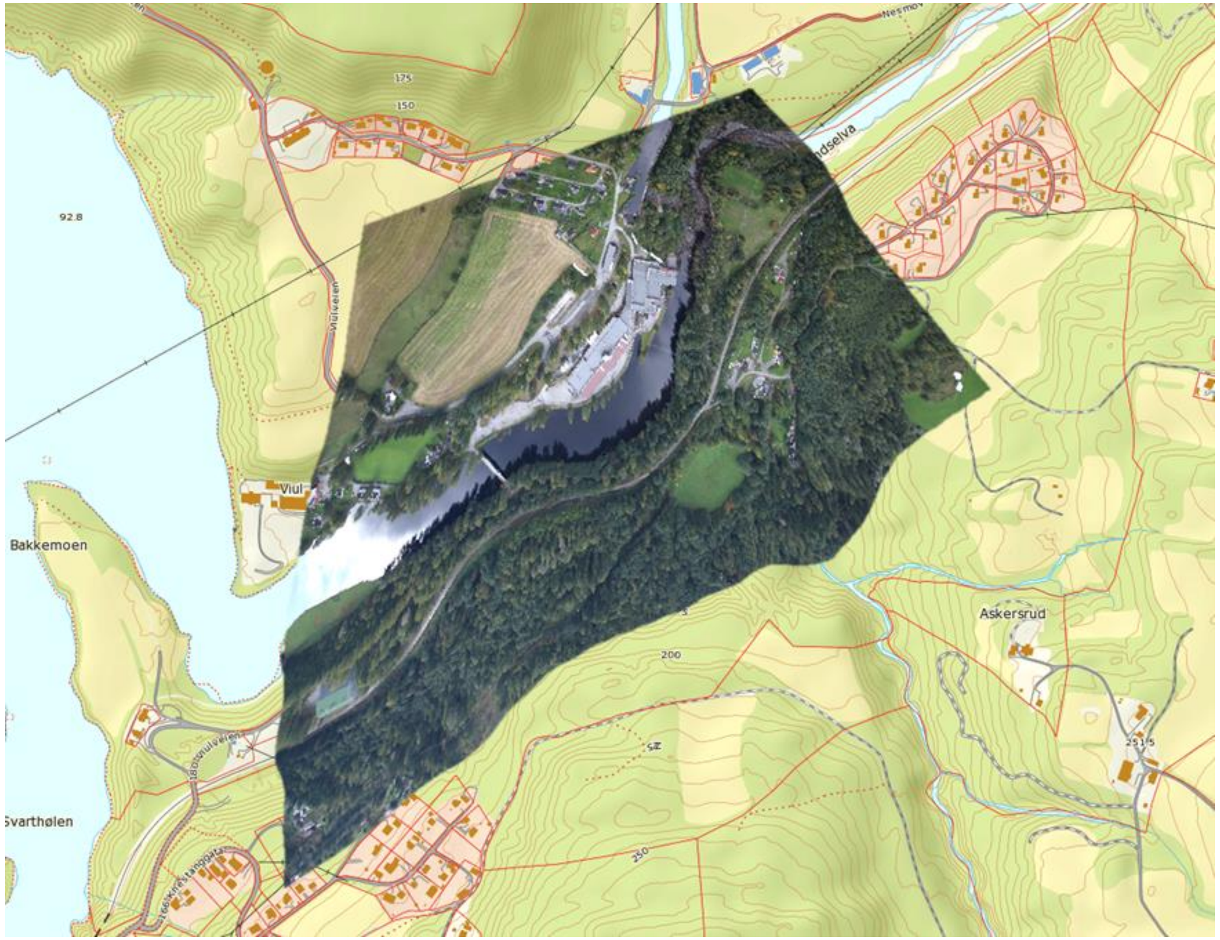
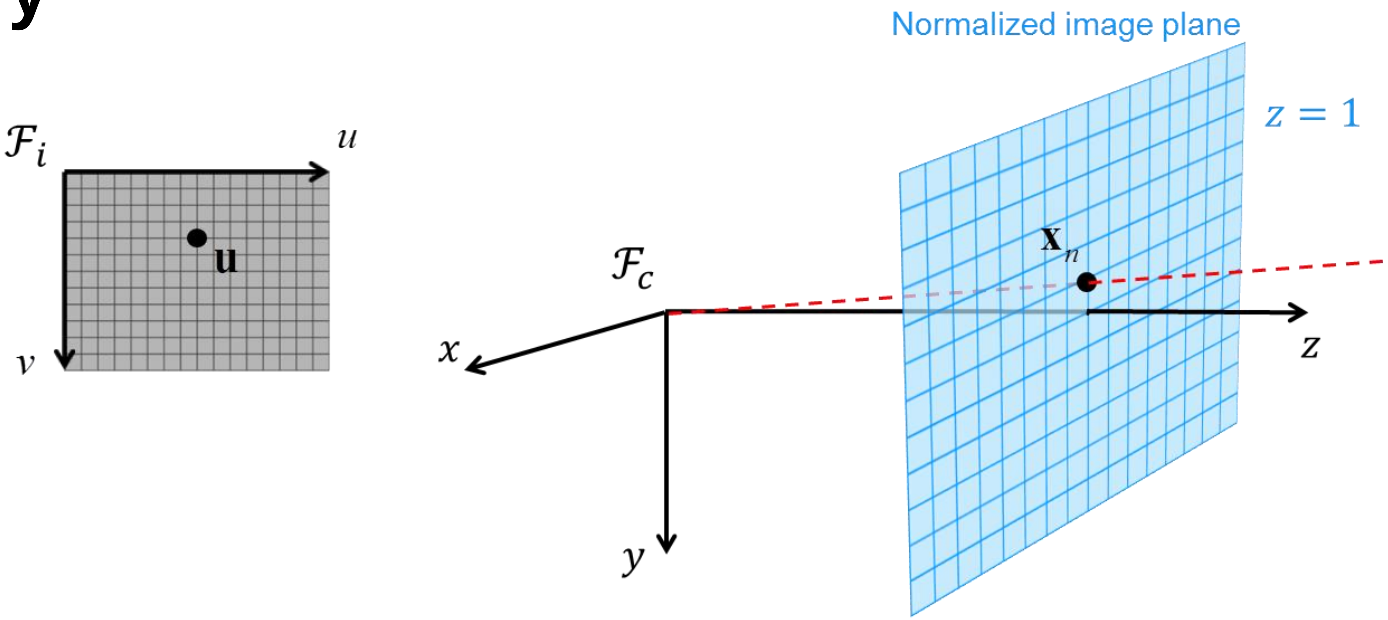
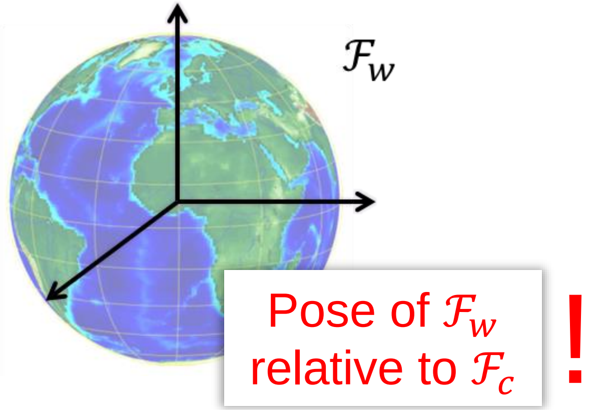


Image georeferenced by backprojection

Summary





The perspective camera model when we consider 3D points in a frame $\mathcal{F}_w$ instead of the camera frame $\mathcal{F}_c$

$$\tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R}_{cw} & \mathbf{t}_{cw}^{c}\end{bmatrix}\tilde{\mathbf{x}}^w$$

$$\tilde{\mathbf{u}} = \underbrace{\begin{bmatrix} f_u & s & c_u \\ 0 & f_v & c_v \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{\Pi}_0} \underbrace{\begin{bmatrix} \mathbf{R}_{cw} & \mathbf{t}_{cw}^{c} \\ \mathbf{0} & 1 \end{bmatrix}}_{\mathbf{T}_{cw}} \tilde{\mathbf{x}}^w$$

Supplementary material

Recommended

- *Richard Szeliski: Computer Vision: Algorithms and Applications 2nd ed*
  - Chapter 2 “Image formation”, in particular section 2.1.4 “3D to 2D projections” and section 2.1.5 “Lens distortions”

- *T. V. Haavardsholm: A Handbook In Visual SLAM*
  - Chapter 3 “Camera geometry”, in particular section 3.1 “Geometric camera models”