Problem 1

Do the integration by parts in Eq. (14) on page 12 in Ch. 2 in the textbook by Joannopoulos et al., for two fields that are Bloch waves, *i.e.*, where the magnetic field may be written

$$\boldsymbol{H}(r,t) = Re[\boldsymbol{u_k}(r,\omega)\exp(i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t)\,]$$

Here $\boldsymbol{k}$ is called the Bloch vector of the wave, and $\boldsymbol{u_k}$ is periodic in space. We need to make the assumption that the two Bloch waves that go into the volume integral have the same Bloch vector, and that the volume integral has to be done over a single unit cell of the photonic crystal. For the sake of the argument, we make the simplifying assumption that the unit cell is a cube.

Hint: The procedure is outlined in footnote 11 in Ch. 2 of the textbook. Remember that the divergence theorem of vector analysis states that the surface integral of the normal component of a vector, integrated over a closed surface, is equal to the volume integral of the divergence of a vector, integrated over the volume enclosed by the surface.

a) Show that surface integral contributions vanish if the Bloch vector is real.

b) Show that this does not hold for an evanescent Bloch "wave", where one component of the Bloch vector is imaginary or complex.

Problem 2

Try to go through the steps of mathematical derivation needed to go from Equation (26) to Equation (28) in Chapter 2 of the textbook by Joannopoulos et al.

Hint 1: Find a textbook in Linear Algebra or Quantum Mechanics that you are familiar with, and read about first-order perturbation theory.

Hint 2: Make the assumption that with $\varepsilon = \varepsilon_o$ you obtain $\omega = \omega_o$, and that with $\varepsilon = \varepsilon_o + \varepsilon_\Delta$ you obtain $\omega = \omega_o + \omega_\Delta$.

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